THE BISHOP'S SCHOOL - CAMP

SUBJECT: MATHEMATICS CLASS: 8

TOPIC: Chp 1: Rational Numbers

Introduction to Rational Numbers

N represents natural numbers i.e. 1, 2, 3, 4, ...
W represents whole numbers i.e. 0, 1, 2, 3, 4, ...
Z/I represents integers i.e. ...., −3, −2, −1, 0, 1, 2, 3, ...

When we add, subtract or multiply 2 integers the result is an integer.

eg. $-5 + (-3) = -8$ → which is an integer
$-5 - (-3) = -2$ → which is an integer
$-5 \times -3 = 15$ → which is an integer.

But when we divide 2 integers i.e. $-5 \div (-3)$
$= \frac{-5}{-3} = \frac{5}{3}$ → which is not an integer

$\frac{5}{3}$ is called a RATIONAL NUMBER.

Q represents rational numbers.

A rational no. is defined as a no. which can be written in the form $\boxed{p/q}$, where p and q are integers and $q \neq 0$.

* q cannot be 0 because division by 0 is not possible.

If we take the natural no. 7 then 7 can be written as $\frac{7}{1}$ i.e. in the form $\frac{p}{q}$, which implies that all natural nos. can be written as rational numbers.

Similarly $0 = \frac{0}{10}$ or $\frac{0}{111}$ or $\frac{0}{-7}$ ∴ 0 is also a rational no.

∴ All natural nos, whole nos. and integers are also rational nos.

POSITIVE and NEGATIVE RATIONAL NUMBERS :-

In $\frac{p}{q}$, if both p and q have the same sign, i.e. either both +ve or both -ve, then $\frac{p}{q}$ is a +ve rational no.

eg. $\frac{2}{3}$, $\frac{-11}{-12}$, $\frac{3}{4}$

If p and q have opposite signs then it is a -ve rational no.

eg. $\frac{-2}{3}$, $\frac{11}{-12}$, $\frac{-3}{4}$.

LOWEST FORM OF A RATIONAL NO:-

When the HCF of p and q in p/q is 1, then p/q is in the lowest form.

i.e. $\frac{12}{13}$ cannot be reduced further. ∴ It is in its lowest form.

But $\frac{25}{30} = \frac{5}{6}$. 25 and 30 have the HCF as 5.

∴ By dividing 25 and 30 both by 5, we get $\frac{5}{6}$.
So 5/6 is the lowest form of 25/30.

STANDARD FORM OF A RATIONAL NO. :-

A rational no. is in its lowest form and its denominator is +ve, is the standard form of a rational no.

eg. $\frac{24}{49}$, $\frac{-3}{11}$ are in the standard form

But $\frac{2}{-4}$, $\frac{3}{9}$, $\frac{-12}{36}$ are not in the standard form.

COMPARING RATIONAL NUMBERS:-

ex.3. Fill in the blanks with >, < or =.

1). $\frac{-11}{13}$ ___ $\frac{-2}{13}$.

⇒ When 2 rational nos. have the same denominators, we compare their numerators.

$-11 < -2$

$\therefore \frac{-11}{13} < \frac{-2}{13}$.

2). $\frac{-3}{7}$ ___ $\frac{-24}{21}$

⇒ First we make both denominators the same by taking their LCM.

LCM of 7, 21 = 21.

$\therefore \frac{-3}{7} = \frac{-3\times3}{7\times3} = \frac{-9}{21}$ (We have to multiply both num. and denominator by 3).

$-9 > -24$ (Comparing numerators)

$\therefore \frac{-9}{21} > \frac{-24}{21}$

i.e. $\frac{-3}{7} > \frac{-24}{21}$

Ex1.1. Q1b). $\frac{-9}{-13}$ ___ $\frac{36}{52}$. Fill in the blanks with >, < or =

⇒ First we write both fractions in standard form.

i.e. $\frac{-9}{-13} = \frac{9}{13}$

$\frac{36}{52} = \frac{9}{13}$ (as HCF of 36 and 52 = 4, so we write 36/52 in its lowest form).

$\therefore \frac{9}{13} = \frac{9}{13}$

Q1d). $\frac{7}{-13}$ ___ $-\frac{8}{15}$

$\Rightarrow$ $\frac{7}{-13} = -\frac{7}{13}$ (writing it in std. form).

Now LCM of 13 and ~~13~~ 15 = 195

$\therefore -\frac{7}{13} = \frac{-7\times15}{13\times15} = \frac{-105}{195}$ (Multiplying num. and den. by 15).

$-\frac{8}{15} = \frac{-8\times13}{15\times13} = \frac{-104}{195}$.

$-105 < -104$

$\therefore \frac{-105}{195} < \frac{-104}{195}$

i.e. $-\frac{7}{13} \underline{<} -\frac{8}{15}$

HW sums:- (Fill in the blanks with >, < or =).

Q1 a). $\frac{-11}{16}$ ___ $\frac{13}{-16}$ Q1c). $\frac{5}{8}$ ___ $\frac{-9}{14}$.

Q2b). Rewrite the following in ascending order:-

$\frac{9}{25}, \frac{2}{5}, \frac{14}{-75}, \frac{-19}{10}, \frac{8}{15}$

$\Rightarrow$ $\frac{14}{-75}$ and $-\frac{19}{10}$ are the only -ve rational nos.

So first we compare only these two as -ve nos. are always smaller than +ve nos.

i.e. $-\frac{14}{75}$ and $-\frac{19}{10}$ (writing them in std. form).

LCM of 75 and 10 = 150.

$$\begin{array}{r|l} 2 & 75, 10 \\ \hline 5 & 75, 5 \\ 5 & 15, 1 \\ 3 & 3, 1 \\ & 1, 1. \end{array}$$

$\therefore -\frac{14}{75} = \frac{-14\times2}{75\times2} = -\frac{28}{150}$

$-\frac{19}{10} = \frac{-19\times15}{10\times15} = -\frac{285}{150}$

$-285 < -28$

$\therefore \frac{-285}{150} < \frac{-28}{150}$

i.e. $\frac{-19}{10} < \frac{-14}{75}$ — ①.

Now we compare +ve rational nos.

$\frac{9}{25}, \frac{2}{5}, \frac{8}{15}$

$$\begin{array}{r|l} 3 & 25, 5, 15 \\ 5 & 25, 5, 5 \\ 5 & 5, 1, 1 \\ & 1, 1, 1 \end{array}$$

LCM of 25, 5, 15 = 75

$\frac{9}{25} = \frac{9 \times 3}{25 \times 3} = \frac{27}{75}$

$\frac{2}{5} = \frac{2 \times 15}{5 \times 15} = \frac{30}{75}$

$\frac{8}{15} = \frac{8 \times 5}{15 \times 5} = \frac{40}{75}$

$27 < 30 < 40$

$\therefore \frac{27}{75} < \frac{30}{75} < \frac{40}{75}$

i.e. $\frac{9}{25} < \frac{2}{5} < \frac{8}{15}$ — ②

Combining steps ① and ② we get

Ans). $\frac{-19}{10} < \frac{-14}{75} < \frac{9}{25} < \frac{2}{5} < \frac{8}{15}$. This is the required arrangement in ascending order.

HW. Q2a. Rewrite the following in ascending order.

$-\frac{3}{8}, \frac{5}{-12}, \frac{-7}{16}, \frac{-13}{18}, \frac{11}{-24}$.

Q3b). Rewrite the following in descending order.

$\frac{-5}{6}, -3, \frac{-13}{-4}, \frac{17}{-6}, \frac{-11}{12}, -\frac{13}{9}$.

i.e. $\frac{-5}{6}, \frac{-3}{1}, \frac{13}{4}, \frac{-17}{6}, \frac{-11}{12}, \frac{-13}{9}$ (writing in std. form).

$\frac{13}{4}$ is the only +ve no. which will the greatest out of all the nos.

so we compare the remaining nos. which are all -ve. i.e. $\frac{-5}{6}, \frac{-3}{1}, \frac{-17}{6}, \frac{-11}{12}, \frac{-13}{9}$.

LCM of 6, 1, 6, 12, 9 = 36.

2	6, 1, 6, 12, 9
2	3, 1, 3, 6, 9
3	3, 1, 3, 3, 9
3	1, 1, 1, 1, 3
	1, 1, 1, 1, 1

$\frac{-5}{6} = \frac{-5\times 6}{6\times 6} = \frac{-30}{36}$

$\frac{-3}{1} = \frac{-3\times \cancel{12}\,36}{1\times 36} = \frac{-108}{36}$

$\frac{-17}{6} = \frac{-17\times 6}{6\times 6} = \frac{-102}{36}$

$\frac{-11}{12} = \frac{-11\times 3}{12\times 3} = \frac{-33}{36}$

$\frac{-13}{9} = \frac{-13\times 4}{9\times 4} = \frac{-52}{36}$

Comparing numerators

$-30 > -33 > -52 > -102 > -108$

$\therefore \frac{-30}{36} > \frac{-33}{36} > \frac{-52}{36} > \frac{-102}{36} > \frac{-108}{36}$

i.e. $\frac{-5}{6} > \frac{-11}{12} > \frac{-13}{9} > \frac{-17}{6} > \frac{-3}{1}$

Ans). $\therefore \frac{13}{4} > \frac{-5}{6} > \frac{-11}{12} > \frac{-13}{9} > \frac{-17}{6} > -3$ is the required descending order.

HW sum:-

Q3a). Rewrite in descending order:-

$\frac{7}{8}, \frac{11}{12}, \frac{15}{-16}, \frac{1}{4}, \frac{14}{-15}$.

— Mrs. R. Oswal