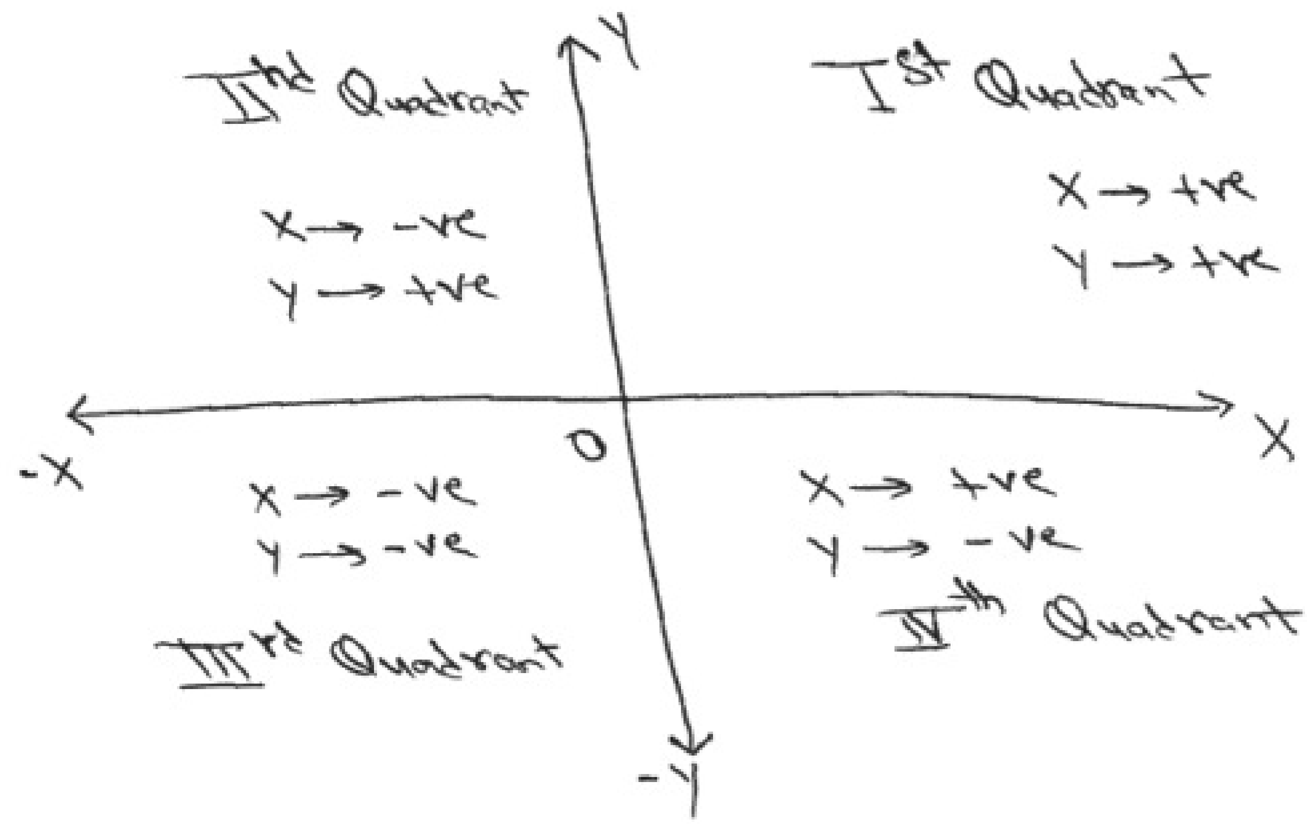
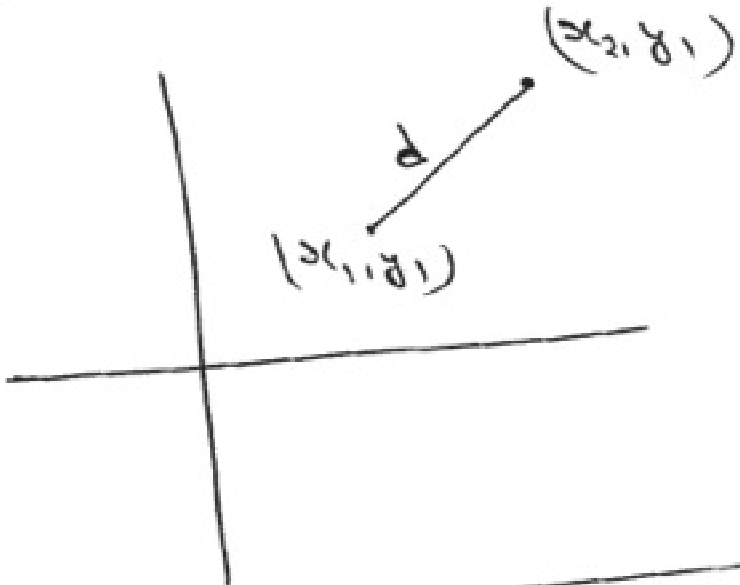
Coordinate Geometry

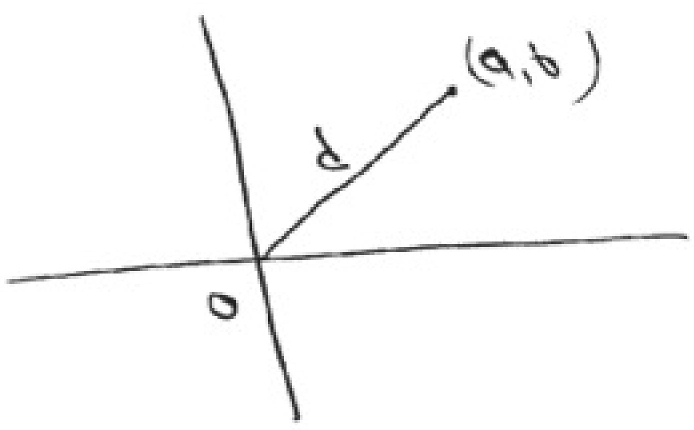


Distance between two points



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point from origin

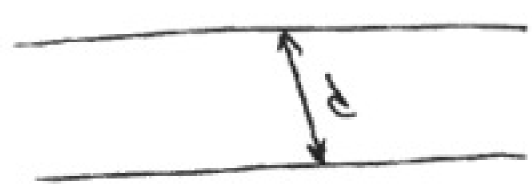


$$d = \sqrt{a^2 + b^2}$$

Distance between Parallel lines

$ax + by + c_1 = 0$

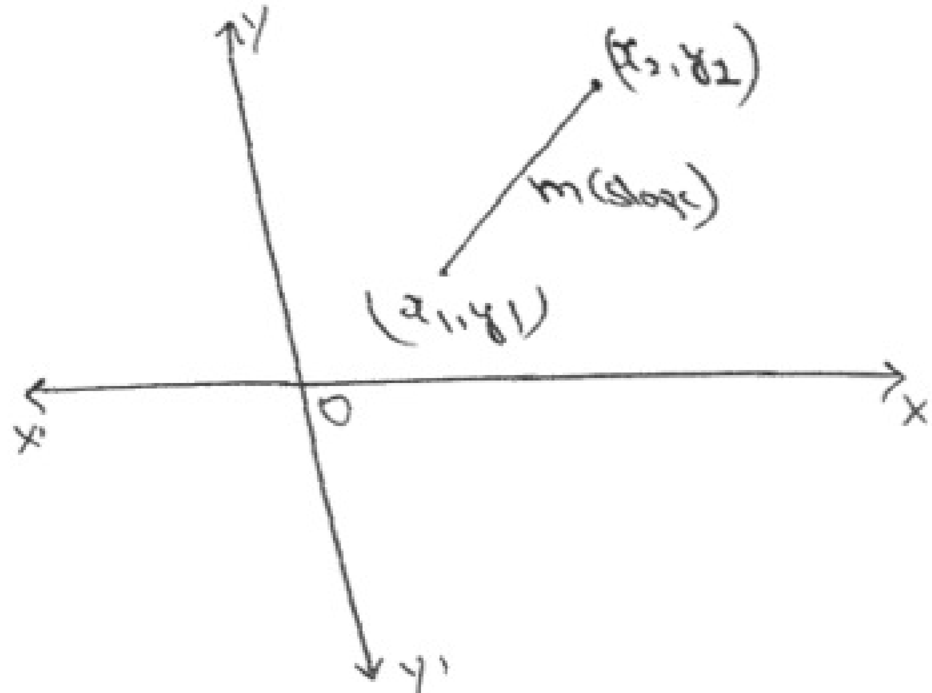
$ax + by + c_2 = 0$



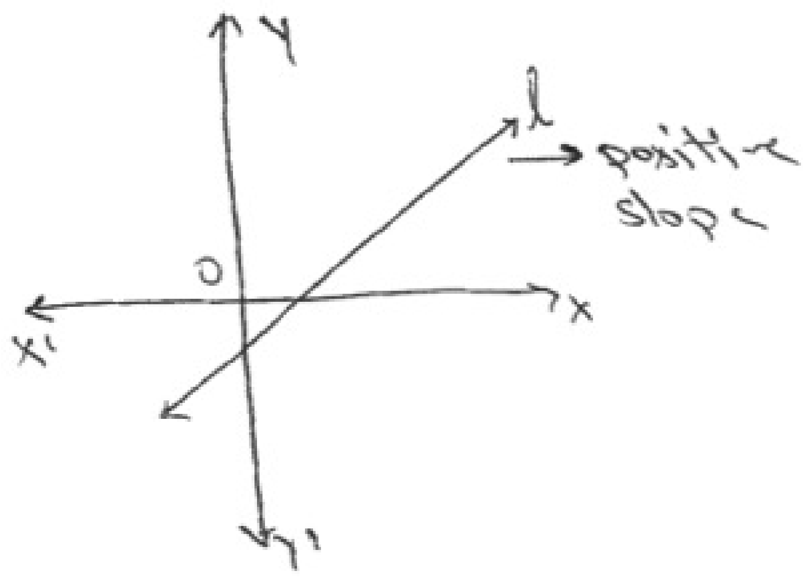
Distance between above parallel lines

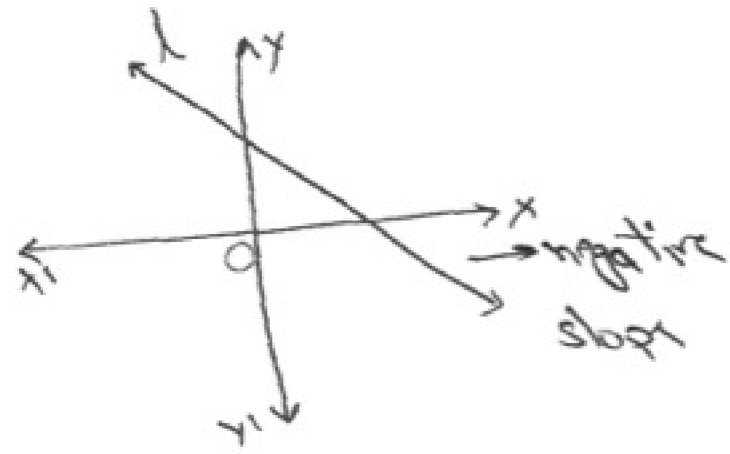
$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

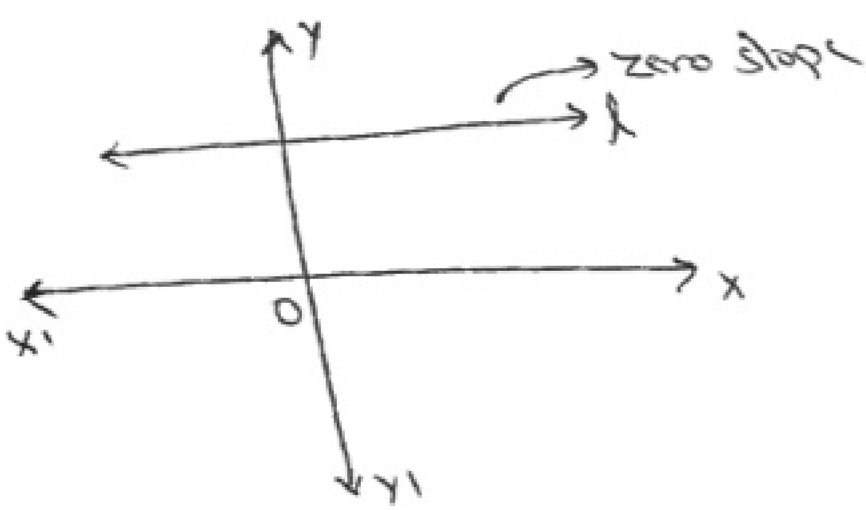
Slope of a line

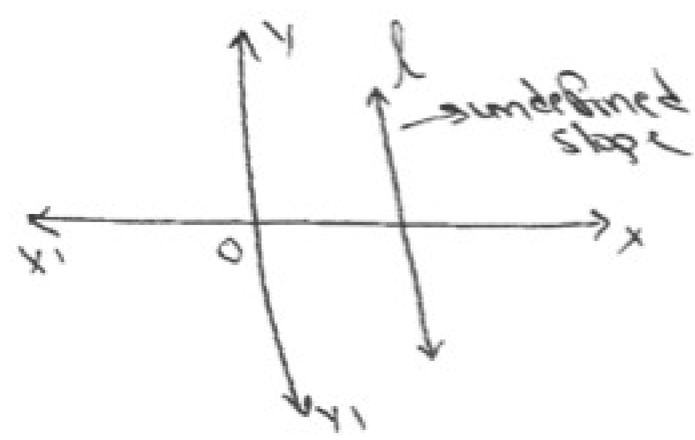


Slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$









Parallel lines

$y = m_1 x + c_1$

$y = m_2 x + c_2$

→ If two lines are parallel then their slope is same

$m_1 = m_2$

Perpendicular lines

$y = m_1 x + c_1$

$y = m_2 x + c_2$

→ If two lines are perpendicular then product of their slopes is equal to -1

$m_1 \times m_2 = -1$

Point of Intersection of two lines

→ just solve the two equation to get values of (x, y), which is the point of intersection.

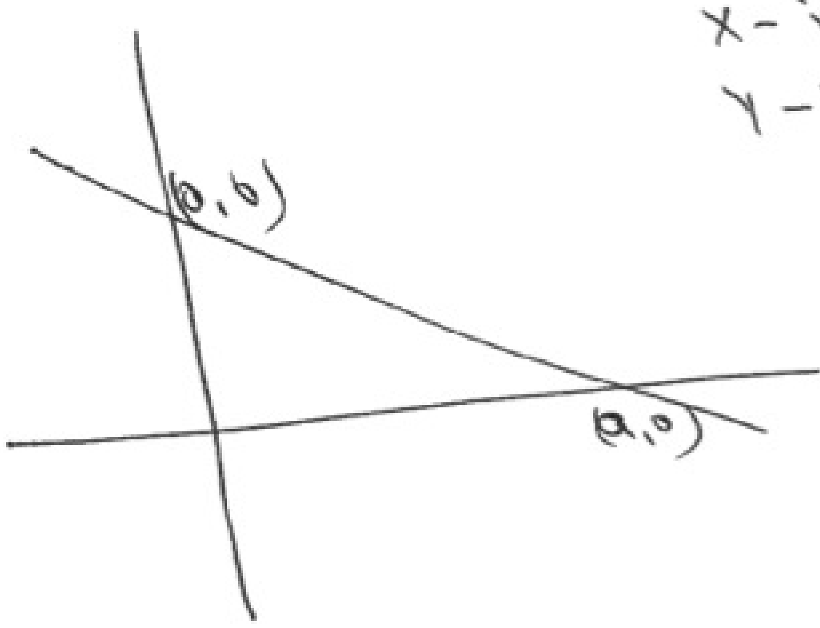
Lines

Equation of a line passing through two points

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1}$$

Equation of a line (intercept formula)



x - intercept $= a$
y - intercept $= b$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\text{slope} = \frac{-b}{a}$$

General equation of a line.

$y = mx + c$

where, m = slope of the line.

Equation of a line passing through origin.

$y = mx$ where m is the slope

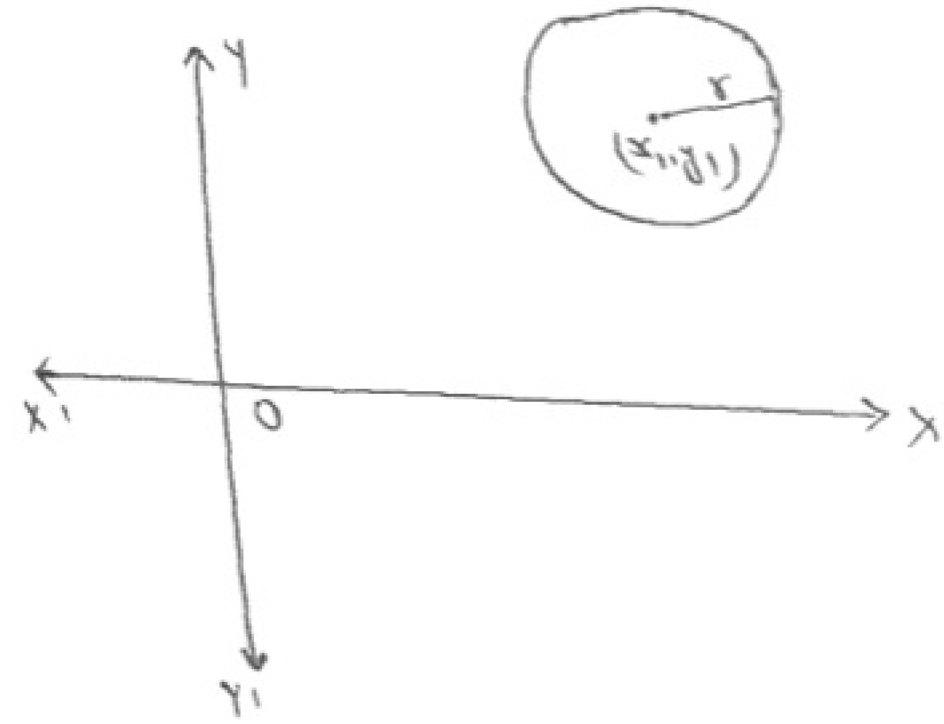
Equation of a line ~~passing thro~~ parallel to x-axis

$y = k$ where k is a constant

Equation of a line parallel to y-axis

$x = k$ where k is a constant.

Equation of other figures



Equation of a circle with center at (x_1,y_1) & radius r

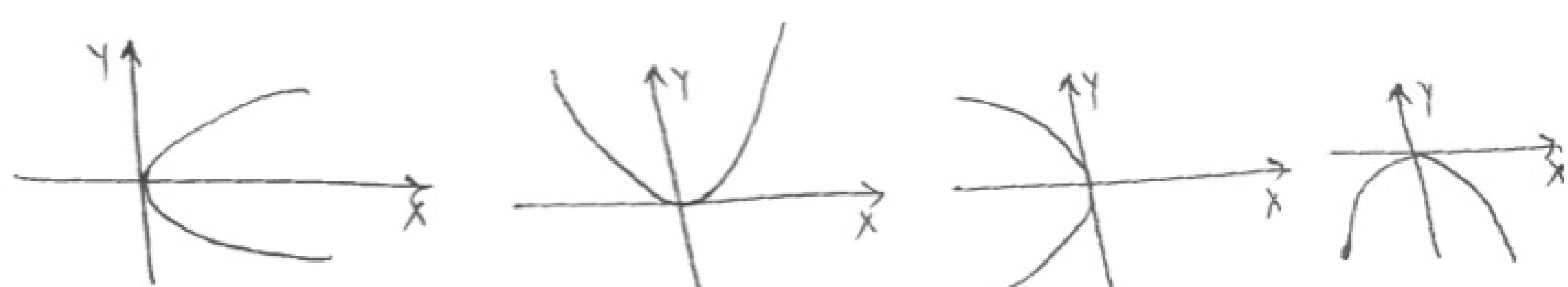
$$(x-x_1)^2+(y-y_1)^2=r^2$$

Equation of a circle with center at (0,0) & radius r

$$x^2+y^2=r^2$$

⇓ Not required in GMAT but good to know

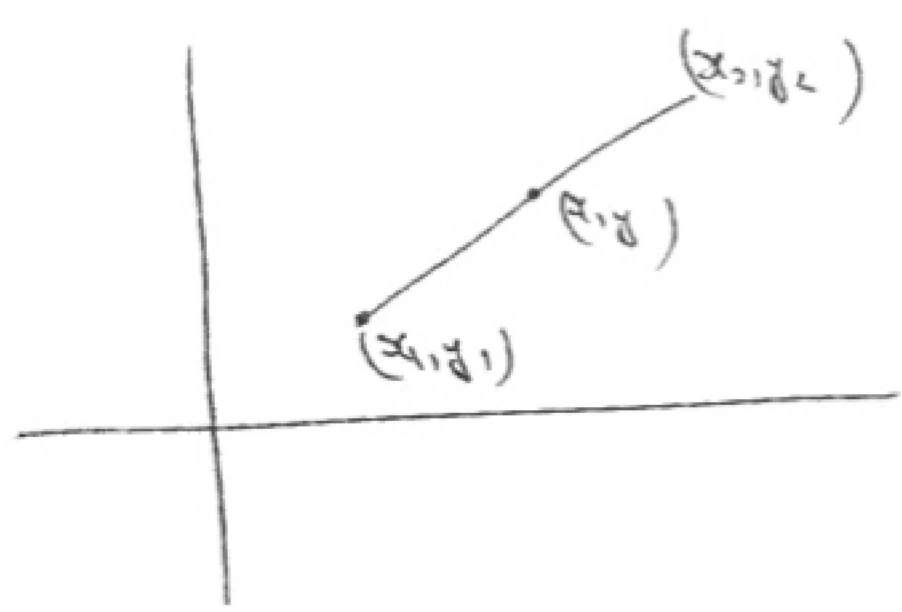
Parabola



$y^2=4ax$ $\quad$ $x^2=4ay$ $\quad$ $y^2=-4ax$ $\quad$ $x^2=-4ay$

⇑

Co-ordinates of mid-point

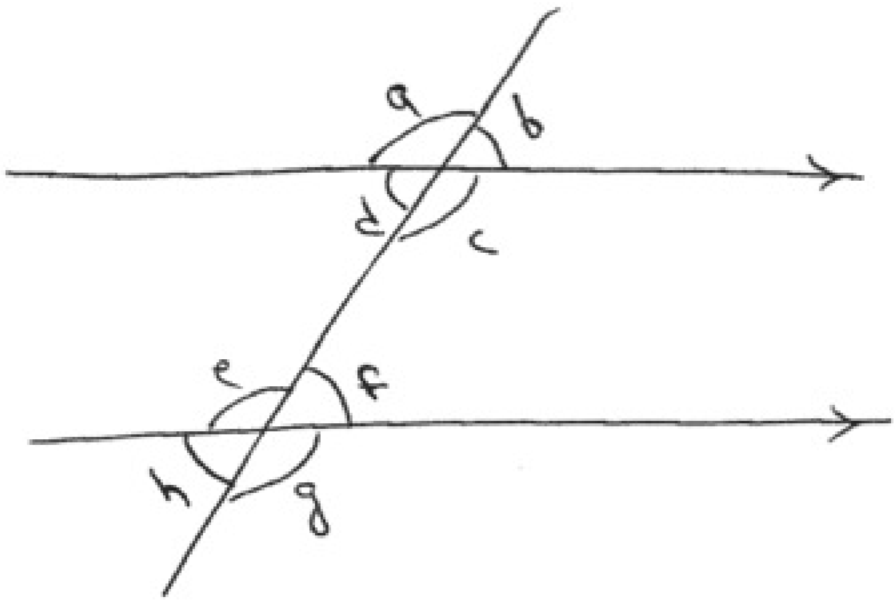


Coordinates of midpoint (x,y) of (x_1,y_1) & (x_2,y_2) =

$$x=\frac{x_1+x_2}{2}$$

$$y=\frac{y_1+y_2}{2}$$

Angles



Parallel lines

$\angle a = \angle e$ $\quad \angle c = \angle g$
$\angle b = \angle f$ $\quad \angle d = \angle h$ } Corresponding Angles

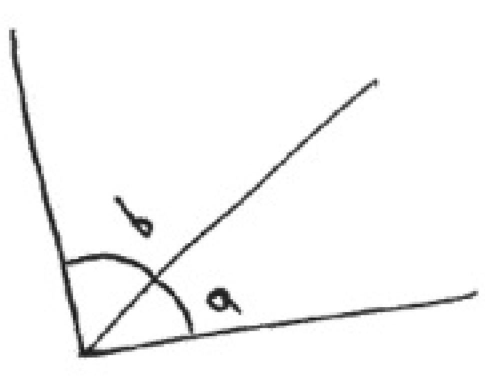
$\angle d = \angle f$ $\quad \angle c = \angle e$ → Alternate interior angles.

$\angle a = \angle g$ $\quad \angle b = \angle h$ → Alternate exterior angles

$\angle a = \angle c$, $\angle b = \angle d$
$\angle e = \angle g$, $\angle f = \angle h$ } → Veritical angles or opposite angles

$\angle c + \angle f = 180^\circ$
$\angle d + \angle e = 180^\circ$ } → Consecutive Interior angles

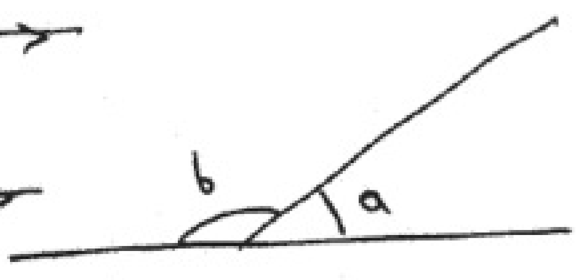
Complementary Angles



Two angles are complementary if their sum is 90°

$a + b = 90^\circ$

Supplementary Angles

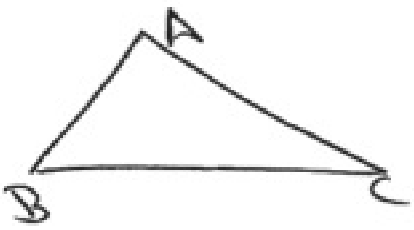


Two angles are supplementary if their sum is 180°.

$a + b = 180^\circ$
$c + d = 180^\circ$

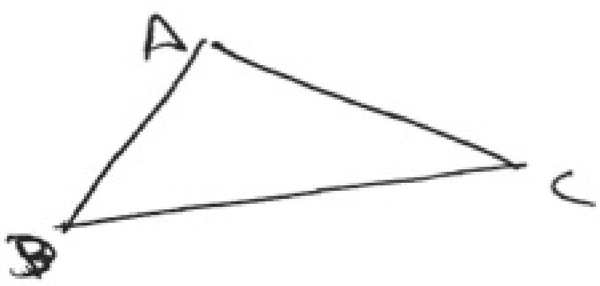
Triangles

- Sum of all the angles of a triangle is 180°



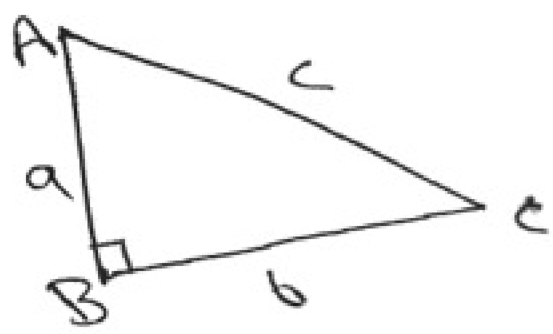
$\angle A + \angle B + \angle C = 180°$

- Sum of length of two sides of a triangle is ALWAYS greater than the third side



$AB + AC > BC$
$AB + BC > AC$
$BC + AC > AB$

- Pythagorean Theorm

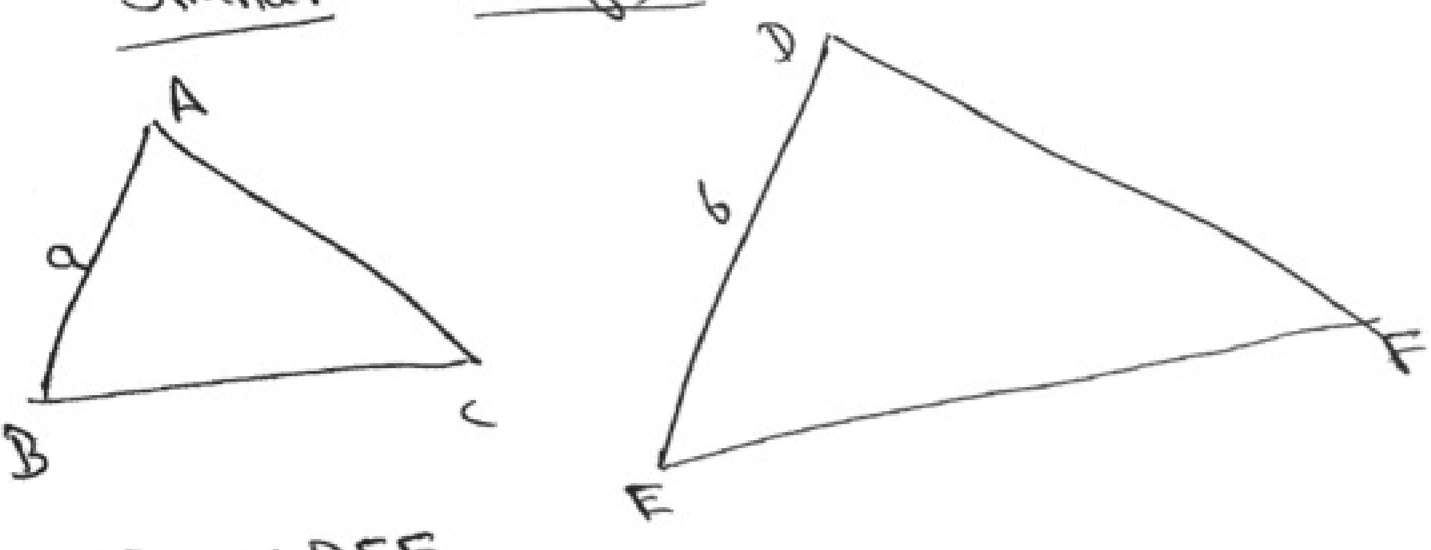


$a^2 + b^2 = c^2$

Common triplets (pythagorean)

3 - 4 - 5
5 - 12 - 13
6 - 8 - 10
8 - 15 - 17
7 - 24 - 25

Similar Triangles



$ABC \sim DEF$

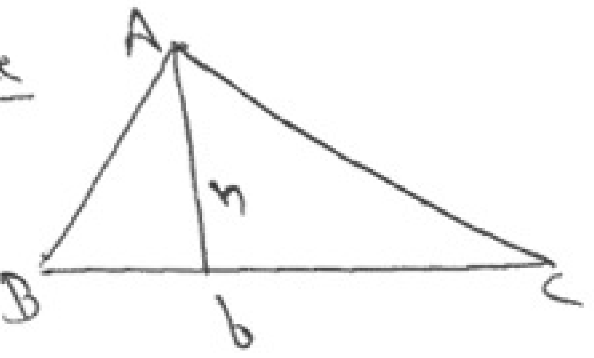
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{a}{b}$$

$$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF} = \frac{a^2}{b^2}$$

$$\frac{\text{Perimeter } ABC}{\text{Perimeter } DEF} = \frac{a}{b}$$

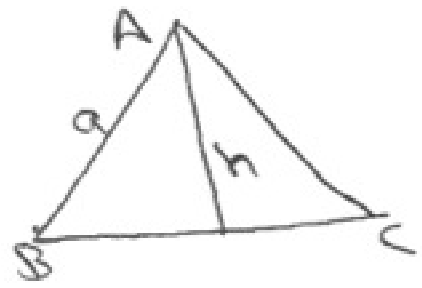
Area of a △gle

Scalene △gle



Area ABC $= \frac{1}{2} \times b \times h$

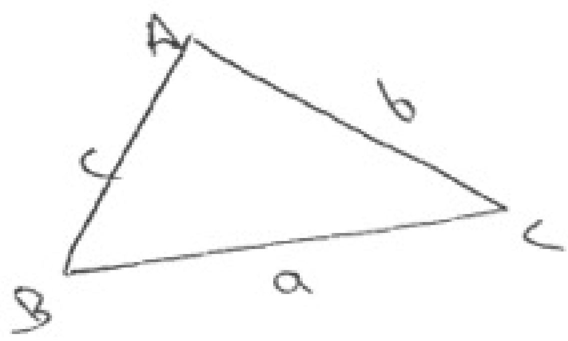
Equilateral △gle



$h = \frac{\sqrt{3}}{2} a$

Area of ABC $= \frac{\sqrt{3}}{4} a^2$

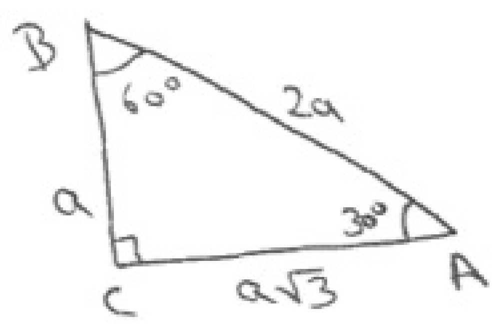
Heron's Formula



$S = \frac{a+b+c}{2}$

Area. A $= \sqrt{S(S-a)(S-b)(S-c)}$

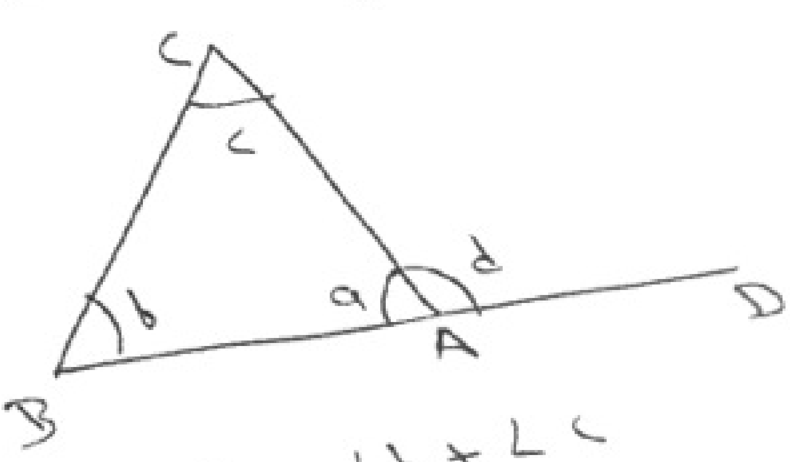
30-60-90 △gle



Sides are in the ratio

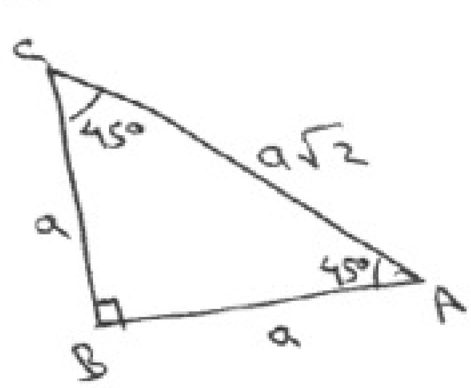
$1 : 2 : \sqrt{3}$

External angle is equal to sum of interior opposite angles



$\angle d = \angle b + \angle c$

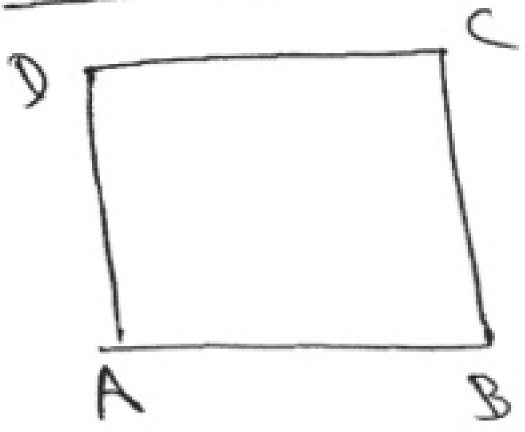
45-45-90 △gle



Sides are in the ratio $1 : 1 : \sqrt{2}$

Properties of various Figures

Square

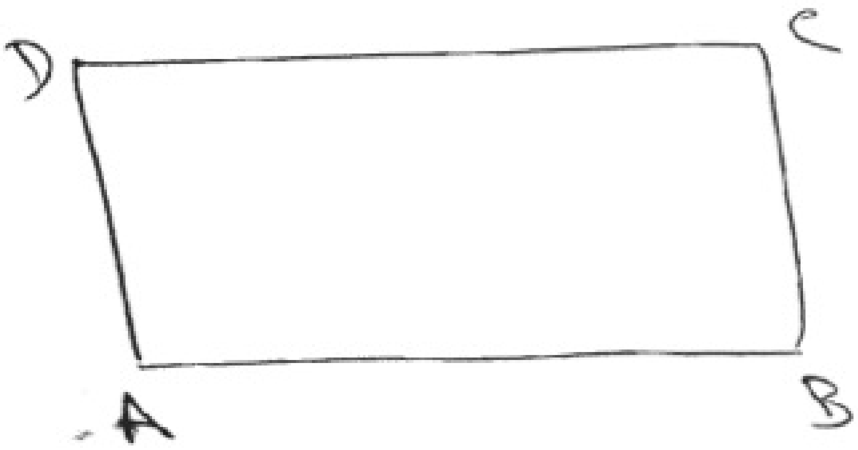


- all four sides are equal
- opposite sides are parallel
- all angles are 90°
- diagonals are equal & are perpendicular bisector of each other.

What is sufficient to prove a figure as square

- any figure which has all sides equal
- and all angles equal to 90°

Rectangle

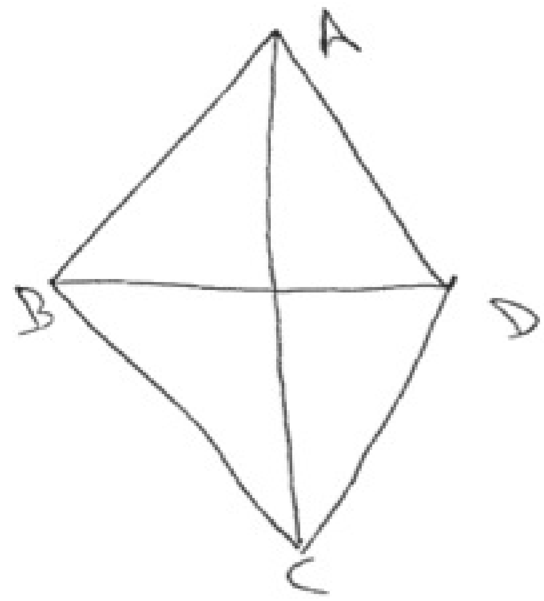


- all angles are 90°
- opposite sides are equal & parallel
- diagonals are equal & bisect each other

Sufficiency

- any quadilateral with four right angles.

Rhombus

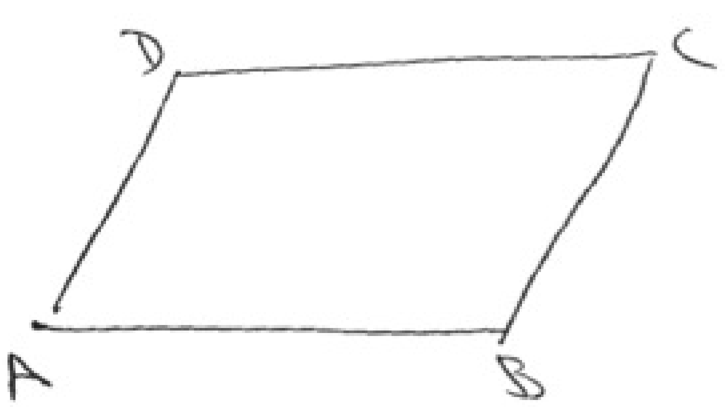


- → all sides are equal
- → diagonals bisect pair of opposite angles
- → diagonal are perpendicular bisector of each other

Sufficiency

→ any quadilateral with four equal sides.

Parallelogram



- → opposite sides are parallel & equal
- → opposite angles are equal
- → consecutive angles are supplementary
- → diagonals bisect each other.

Sufficiency

→ opposite sides should be equal or parallel

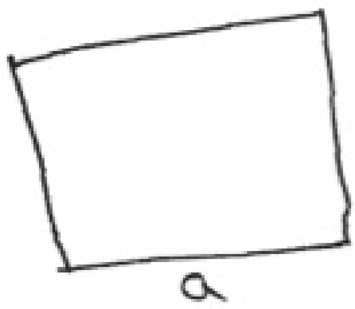
→ or two pairs of opposite angles should be equal

→ or if the diagonals bisect each other

→ or one pair of opposite sides are equal & parallel.

Area (A) & Perimeter (P)

Square



$A = a^2$

$P = 4a$

Rectangle

$A = a \times b$

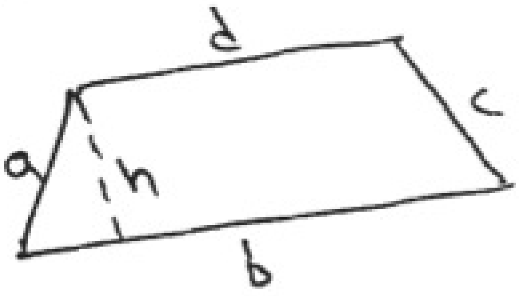
$P = 2(a+b)$

Parallelogram

$A = h \times b$

$P = 2(a+b)$

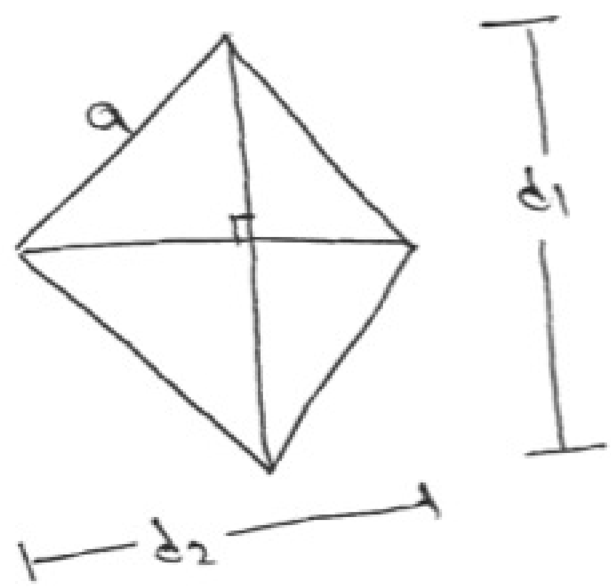
Trapezium



$A = \frac{1}{2} \times h \times (b+d)$

$P = a+b+c+d$

Rhombus

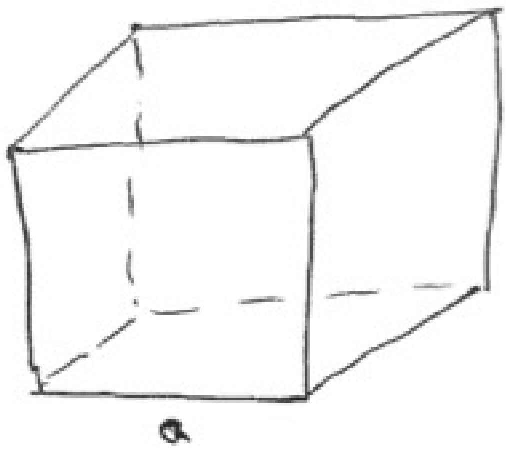


$A = \frac{1}{2} \times d_1 \times d_2$

$P = 4a$

Surface Area & Volume

Cube

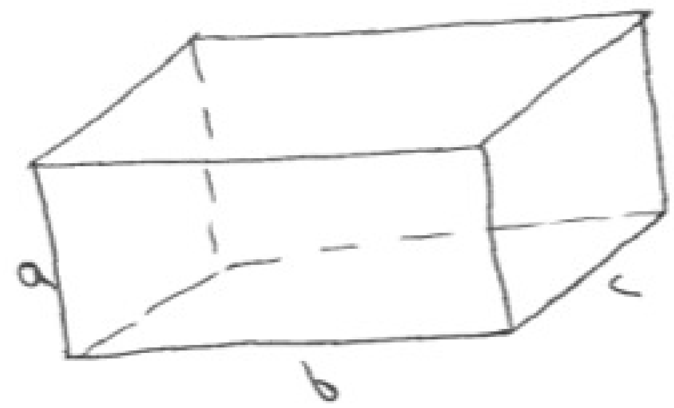


Volume $= a^3$

Surface Area $= 6a^2$

longest diagonal $= a\sqrt{3}$

Cuboid

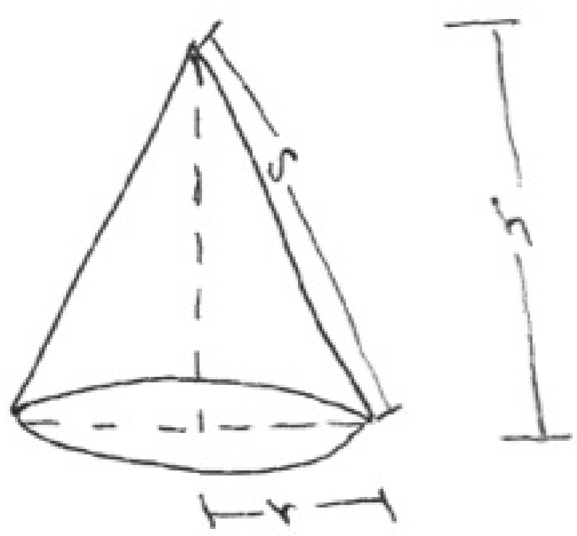


Volume $= abc$

Surface Area $= 2(ab+bc+ac)$

Longest Diagonal $= \sqrt{a^2+b^2+c^2}$

Cone

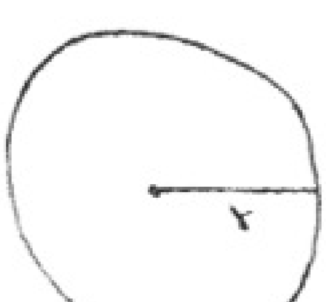


Volume $= \frac{1}{3}\pi r^2 h$

Surface Area $= \pi r s + \pi r^2$

$s = \sqrt{r^2 + h^2}$

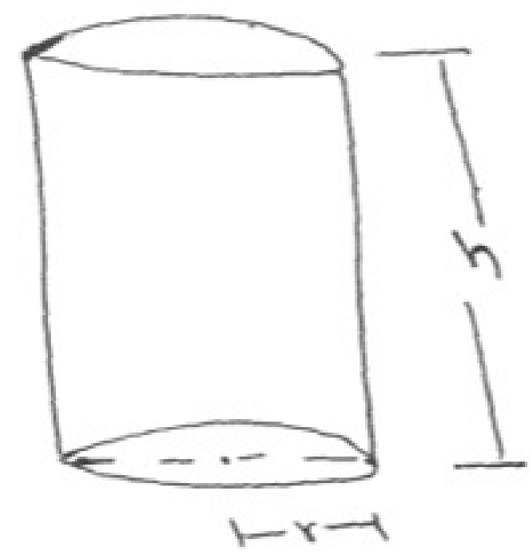
Sphere



Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

Cylinder



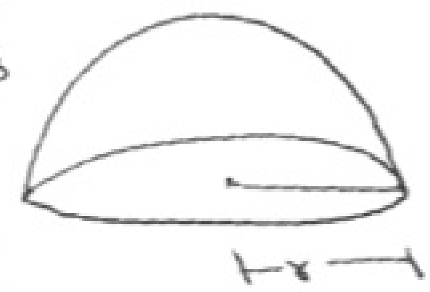
Volume $= \pi r^2 h$

Surface Area of Solid Cylinder $= 2\pi r^2 + 2\pi r h$

Surface Area of hollow cylinder $= 2\pi r h$

Hemisphere

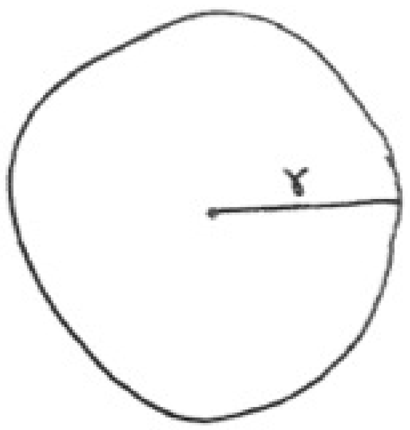
Volume $= \frac{2}{3}\pi r^3$



Surface Area (Solid Hemisphere) $= 3\pi r^2$

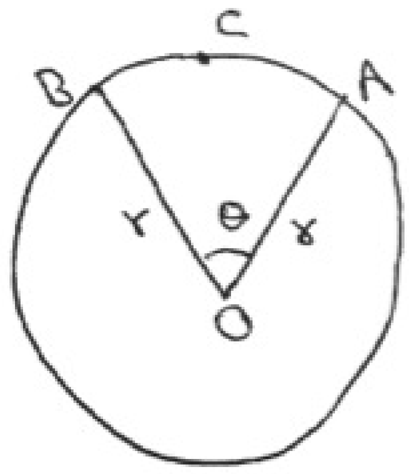
Surface Area (Hollow Hemisphere) $= 2\pi r^2$

Circle



Area $= \pi r^2$

Circumference $= 2\pi r$

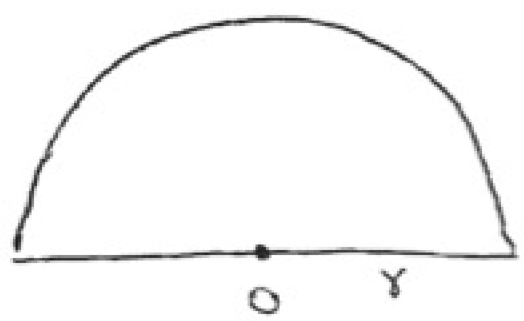


Area of sector OACB $= \frac{\theta}{360} \times \pi r^2$

Length of arc ACB $= \frac{\theta}{360} \times 2\pi r$

Length of sector OACB

$= 2r + \left(\frac{\theta}{360} \times 2\pi r\right)$



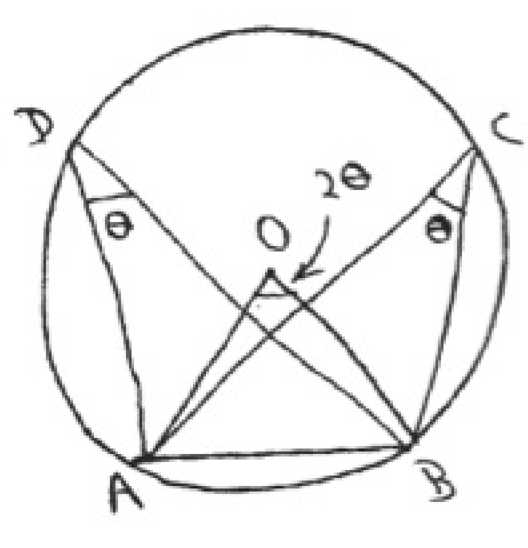
← Semicircle.

Area $= \frac{\pi r^2}{2}$

Perimeter (Circumference) $= \pi r + 2r$

Properties

(1)



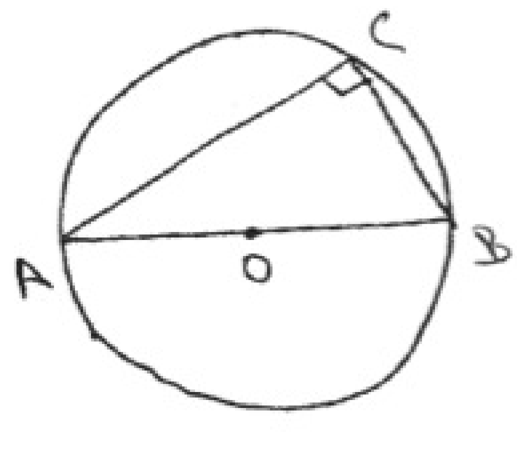
An arc substends same angle at any point on the circle.

$\angle ACB = \angle ADB = \theta$

Angle substended by an arc at the center is twice the angle substended by the arc at any point on the circle.

$\angle AOB = 2\angle ACB = 2\angle ADB = 2\theta$

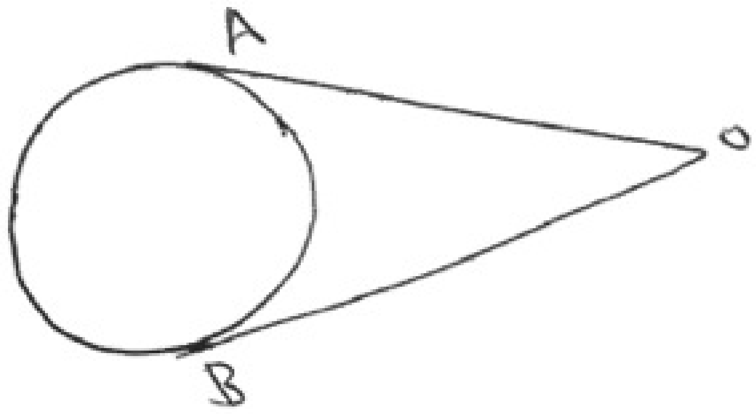
(2)



Diameter substends a right angle on any point on the circle.

$\angle ACB = 90°$ (AB is diameter)

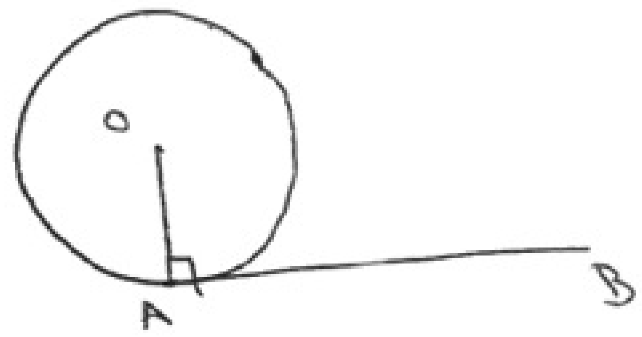
(3)



Length of all the tangents (only two are possible) draw to a circle from the same point is equal

OA = OB

(4)

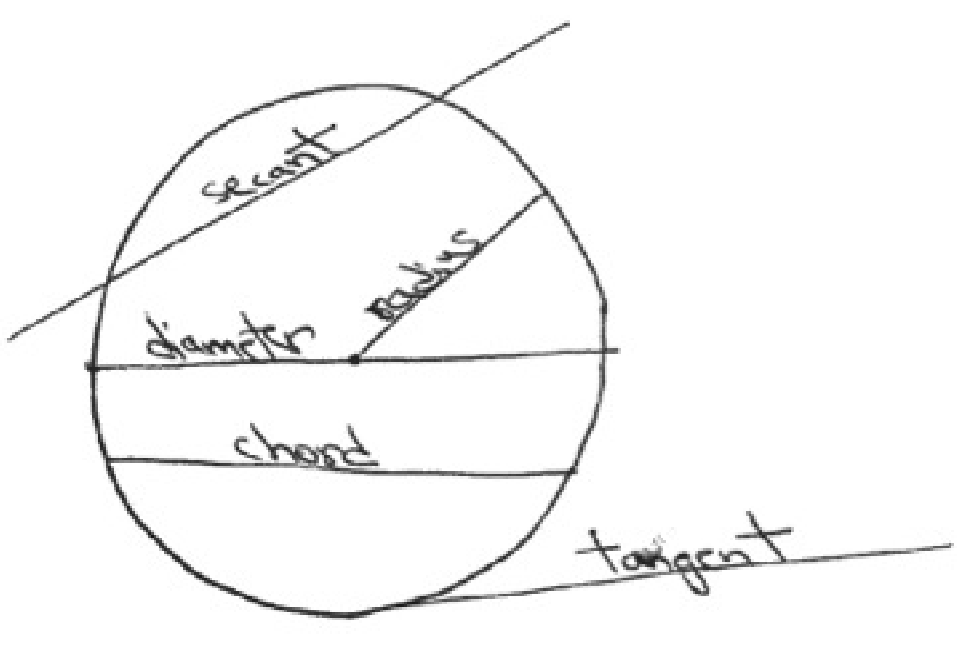


AB is a tangent.

Tangent always makes a 90° angle with the line joining the point of intersection of the tangent with the circle (A) and center of the circle (O)

OA ⊥ AB

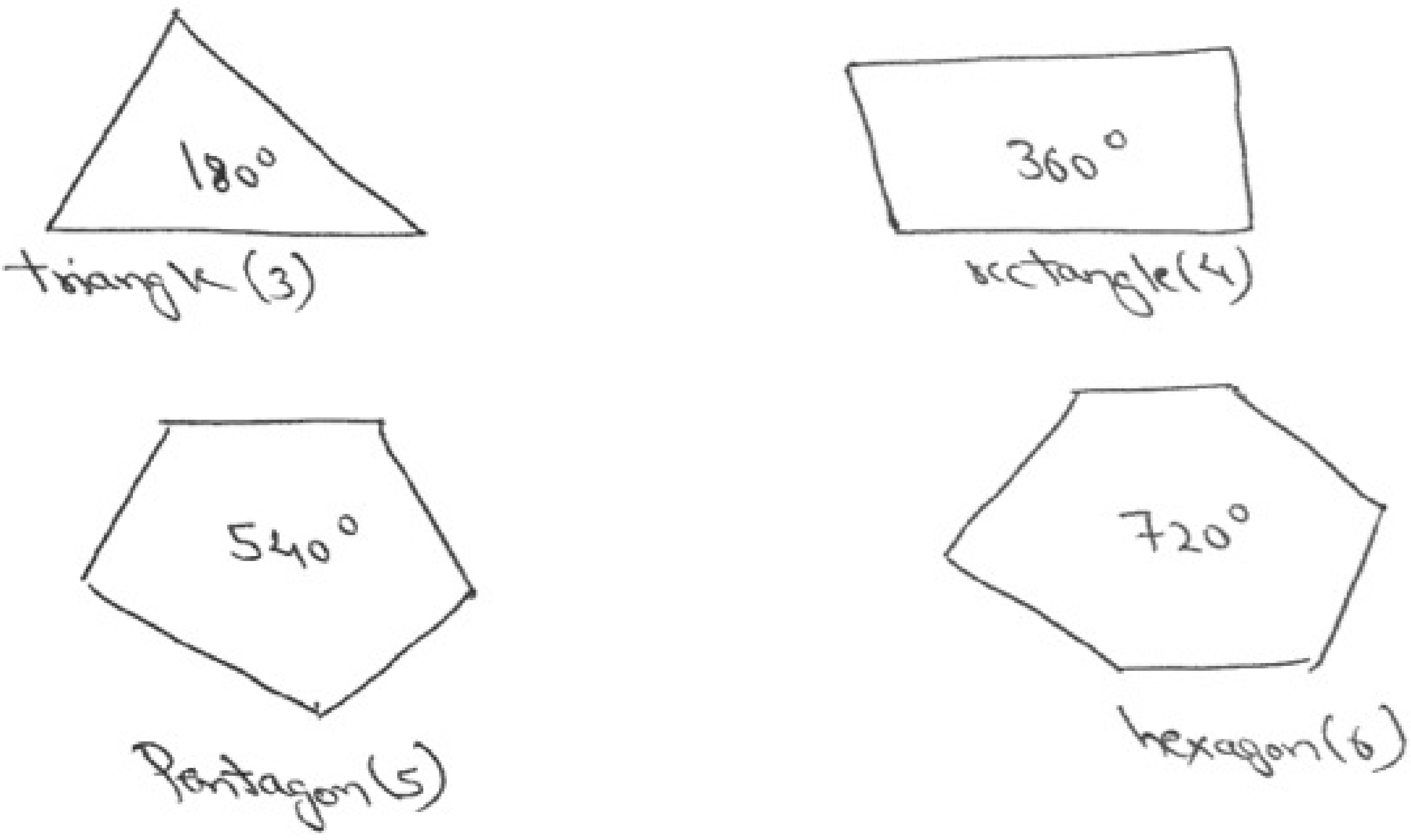
(5)



Polygons

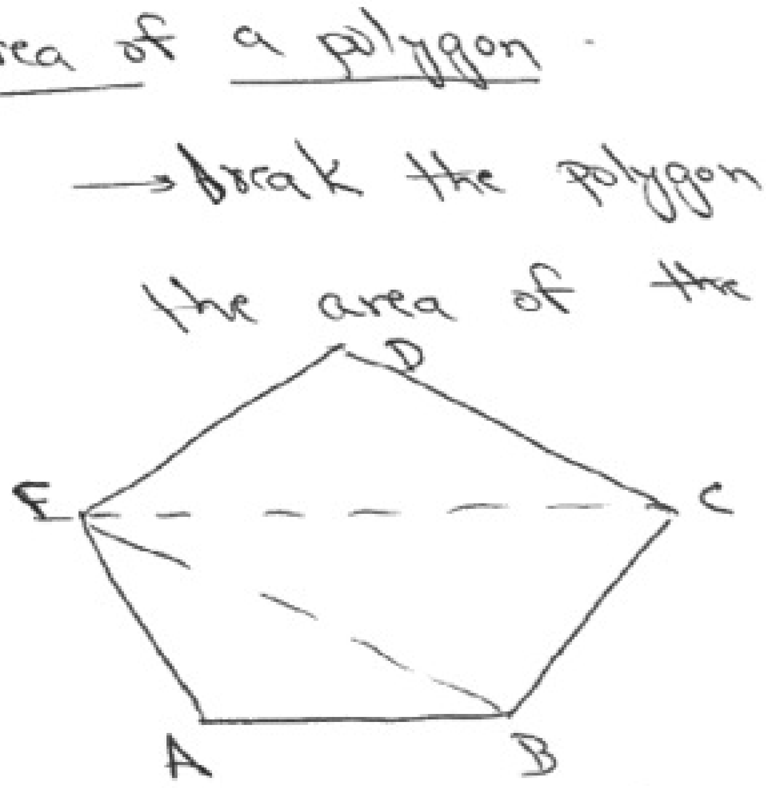
Sum of interior angles of a polygon

$$= (n-2) \times 180$$



Area of a polygon

→ break the polygon into triangles & find the area of the triangles & add them up



Area of ABCDE = Area Δ ABE + Area Δ BCE + Area Δ CDE

Perimeter of a polygon

→ it is the sum of lengths of all the sides of the polygon

Perimeter of ABCDE (above) = AB + BC + CD + DE + EA