Unit 1. DC CIRCUITS

Direct Current Circuits

Unit 1. DC CIRCUITS

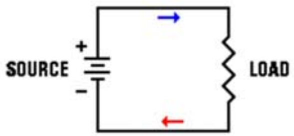
CONTENTS :

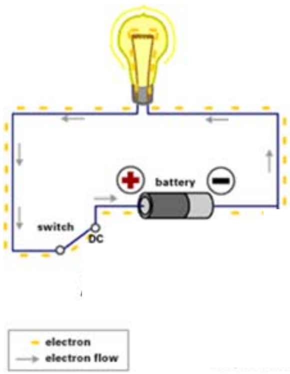
Unit 1. DC CIRCUITS

BASICS OF DC CIRCUITS

Direct Current

- Current flows in a constant direction
- Voltage has constant polarity.





http://www.pbs.org/wgbh/amex/edison/sfeature/acdc.html

Unit 1. DC CIRCUITS

BASICS OF DC CIRCUITS

Brief history of Direct Current

- In 1880, **Thomas Edison** developed the first commercial electric power transmission using direct current (DC).
- Simultaneously, **Nikola Tesla** developed electric power transmission using alternating current (AC).
- Edison promoted the use of DC for electric power distribution, while **George Westinghouse** promoted the use of AC. Westinghouse and Edison became adversaries.
- Unlike DC, AC could be stepped up to very high voltages with transformers, sent over thinner and cheaper wires, and stepped down again at the destination for distribution to users. These are the main reasons for the prevalent use of AC instead of DC transmission systems.
- In the mid 1950s, HVDC transmission was developed. The advantage of HVDC is the ability to transmit large amounts of power over long distances with lower capital costs and with lower losses than AC.
- HVDC is used in undersea cables.

DC APPLICATIONS

- **Low-voltage DC applications:**
  - Batteries
  - Electronic circuits
  - Solar photovoltaic cells
  - Automotive applications

- **Electrical traction (old units, new ones are fed in AC)**

- **Electrolytic processes industry**
  - Production of aluminum, Cl_2, etc.

- **High Voltage DC (HVDC) applications:**
  - Overhead power lines (high power, long dist.)
  - Undersea cables (run under the sea)
  - Offshore wind-driven generators

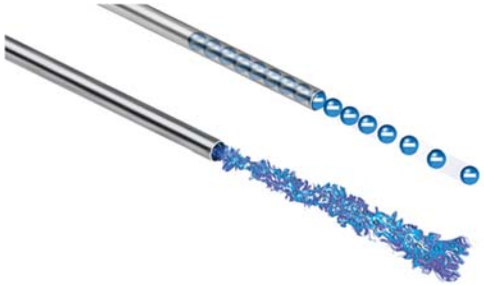
ELECTRICAL CURRENT

Electrical current

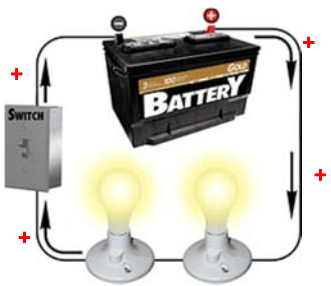
- Electrical current consists on a flow of electric charge.
- The flowing electric charge is carried by:
  - Moving electrons in a conductor such as wire
  - Moving ions in an electrolyte
  - Moving electrons and moving ions in plasma
- The SI unit for measuring the rate of flow of electric charge is the ampere (A).
- Electric current is measured using an ammeter.

- A solid conductive metal contains mobile or free electrons (conduction electrons).
- These electrons are bound to the metal lattice, but no longer to any individual atom.
- Even with no external electric field applied, these electrons move about randomly due to thermal energy. However, on average, there is zero net current within the metal.

ELECTRICAL CURRENT



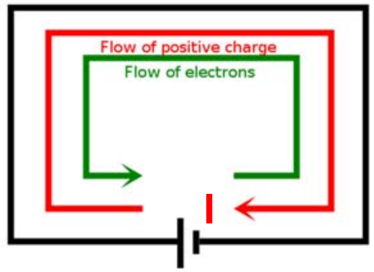
The electrical current (flow of electricity) is similar in many ways to water flowing through a pipe.



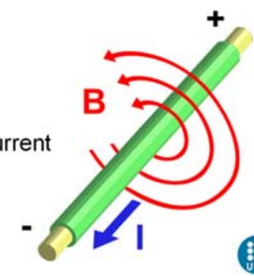
The electrical current direction is chosen as the motion direction of the **positive charges**.

ELECTRICAL CURRENT

When a metal wire is connected across the two terminals of a DC voltage source, the free electrons of the conductor are forced to drift toward the positive terminal. The free electrons are therefore the current carrier in a typical solid conductor.



According to Ampère's law, an electric current produces a magnetic field.

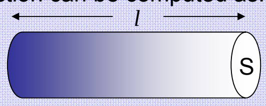


ELECTRICAL RESISTANCE

Electrical resistance

- The electrical resistance of an object is a measure of its opposition to the passage of a steady electric current.
- It was discovered by George Ohm in 1827
- The SI unit of electrical resistance is the ohm (Ω).
- The resistance R of a conductor of uniform cross section can be computed as:

$$R = \rho \cdot \frac{l}{S}$$



l is the length of the conductor, measured in meters [m]

S is the cross-sectional area of the current flow, measured in square meters [m²]

ρ (Greek: rho) is the electrical resistivity of the material, measured in ohm-metres (Ω m). Resistivity is a measure of the material's ability to oppose electric current.

ELECTRICAL RESISTANCE: temperature dependence

Electrical resistance: temperature dependence

- The electric resistance of a typical metal increases linearly with rising temperature as:

$$R = R_0 \cdot [1 + \alpha \cdot (T - T_0)]$$

Where

- T: metal's temperature
- T_0: the reference temperature (usually room temperature)
- R_0: resistance at T_0
- α: **Temperature coefficient**. Percentage change in resistivity per unit temperature. It depends only on the material being considered

Material	Resistivity (Ω·m) at 20 °C	Temperature coefficient K^{-1}
Silver	1.59×10^{-8}	0.0038
Copper	1.68×10^{-8}	0.0039
Gold	2.44×10^{-8}	0.0034
Aluminium	2.82×10^{-8}	0.0039
Tungsten	5.60×10^{-8}	0.0045
Zinc	5.90×10^{-8}	0.0037
Nickel	6.99×10^{-8}	0.006
Iron	1.0×10^{-7}	0.005
Platinum	1.06×10^{-7}	0.00392

SERIES/PARALLEL CONNECTED LOADS

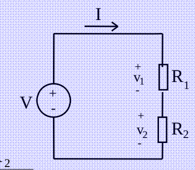
	SERIES	PARALLEL
Resistors	$R_{equivalent} = \sum_{i=1}^{n} R_i$	$\frac{1}{R_{equivalent}} = \sum_{i=1}^{n} \frac{1}{R_i}$ $R_1//R_2$: $R_{equivalent} = \frac{R_1 . R_2}{R_1 + R_2}$
Inductors	$L_{equivalent} = \sum_{i=1}^{n} L_i$	$\frac{1}{L_{equivalent}} = \sum_{i=1}^{n} \frac{1}{L_i}$
Capacitors	$\frac{1}{C_{equivalent}} = \sum_{i=1}^{n} \frac{1}{C_i}$	$C_{equivalent} = \sum_{i=1}^{n} C_i$

VOLTAGE AND CURRENT DIVIDERS

The voltage divider

- It is a simple linear circuit that produces an output voltage that is a fraction of its input voltage.
- Voltage is partitioned among the components of the divider.

General case: $V_{Rj} = V \cdot \frac{R_j}{\sum_i R_i}$

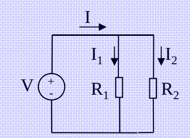


In the example shown: $V_{R_2} = V \cdot \frac{R_2}{R_1 + R_2}$

The current divider

- A current divider is a simple linear circuit that produces an output current that is a fraction of its input current.
- Current is split between the branches of the divider.

General case: $I_i = I \cdot \frac{R_1 \cdot R_2 \cdots R_{i-1} \cdot R_{i+1} \cdot R_n}{\sum_i R_i}$



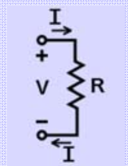
In the example shown: $I_1 = I \cdot \frac{R_2}{R_1 + R_2}$

OHM'S LAW

Ohm's law

The current through a conductor between two points is directly proportional to the potential difference - or voltage across the two points - and inversely proportional to the resistance between them.

I = V/R or **V = I·R**



- The electrical resistance of an object is a measure of its opposition to the passage of a steady electric current.
- The electrical resistance was discovered by George Ohm in 1827
- The SI unit of electrical resistance is the ohm (Ω).

The resistance R of a conductor of uniform cross section can be computed as:

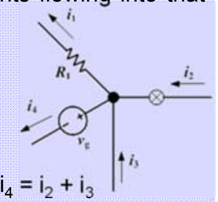
$$R = \rho \cdot \frac{l}{S}$$

KIRCHHOFF'S CIRCUIT LAWS

Kirchhoff's current law (KCL)

At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

$$\sum_i I_i = 0 \quad \text{or} \quad \sum I_{input} = \sum I_{output}$$

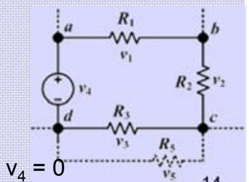


The current entering any junction is equal to the current leaving that junction: $i_1 + i_4 = i_2 + i_3$

Kirchhoff's voltage law (KVL)

The direct sum of the electrical potential differences (voltage) around any closed circuit must be zero.

$$\sum_i V_i = 0$$

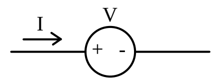


The sum of all the voltages around the loop is equal to zero: $v_1 + v_2 + v_3 + v_4 = 0$

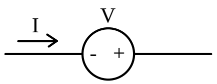
ELECTRIC POWER

Electric power (W)

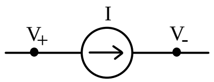
- **This is the rate at which electrical energy is transferred by an electric circuit or consumed by an electric load.**
- The SI unit of power is the **watt**.
- The power consumed by a load is considered positive
- The power delivered by a source is considered negative



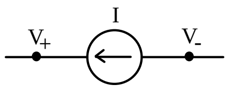
P = + V·I Absorbed power



P = - V·I Delivered power



P = + V·I Absorbed power

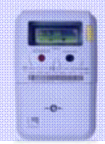


P = - V·I Delivered power

ENERGY

Energy (J, W·h)

- Energy results form the integral: $W = \int_0^t P(t) \cdot dt$
- If power is constant, it results: $W = P \cdot t = V \cdot I \cdot t$
- Energy is usually measured in kWh (kilowatt hours). Energy in watt hours is the multiplication of power in watts and time in hours.
- 1 kWh = 3.6 MJoules
- Electric power meters:



	joule	watt hour	electronvolt	calorie
1 J = 1 kg m^2 s^{-2} =	1	2.778×10^{-4}	6.241×10^{18}	0.239
1 W·h =	3600	1	2.247×10^{22}	859.8
1 eV =	1.602×10^{-19}	4.45×10^{-23}	1	3.827×10^{-20}
1 cal =	4.1868	1.163×10^{-3}	2.613×10^{19}	1

Don't confuse power and energy !

POWER AND ENERGY

In domestic and industrial supplies the kilowatt hour (kWh) is usually used. One kWh is the energy used when a power of 1kW is supplied for one hour (3600s). Notice that power = energy/time, therefore energy = power × time and. Since 1kWh is a power multiplied by a time, it is in fact a unit of energy (not power).

Example. How much energy is supplied to a 100Ω resistor that is connected to a 150V supply for 1 hour?

Power: $P = U^2/R = 150^2/100 = 225$ W

Energy: $W = P\cdot t = 225W\cdot 1h = 225$ Wh = 0.225 kWh
$W = P\cdot t = 225W\cdot 1\cdot 60\cdot 60s = 810000$ J = 810 kJ

The number of joules or kWh supplied is a measure of the amount of electrical energy supplied.

Example. A DC motor takes 15 A from a 200 V supply. It is used for 40 mins. What will it cost to run if the tariff is 0.13 €/kWh?

Power: $P = U\cdot I = 200.15 = 3000$ W = 3 kW

Energy in kWh: $W = P\cdot t = 3\cdot(40/60) = 2$ kWh

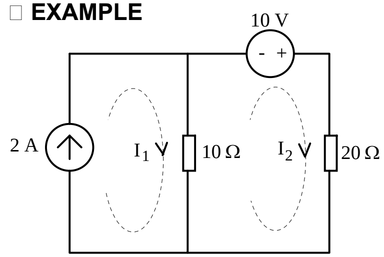
Cost in €: $C = 2\cdot 0.13 = 0.26$ €

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

The mesh current method

- Also known as the loop current method.
- It uses simultaneous equations, Kirchhoff's voltage law, and Ohm's law to determine unknown currents in a network.
- It is a method used to solve planar circuits for the voltage and currents at any place in the circuit.

- **EXAMPLE**



Mesh current equations:

loop 1: $I_1 = 2$ A
loop 2: $10 = -10\cdot I_1 + 30\cdot I_2$

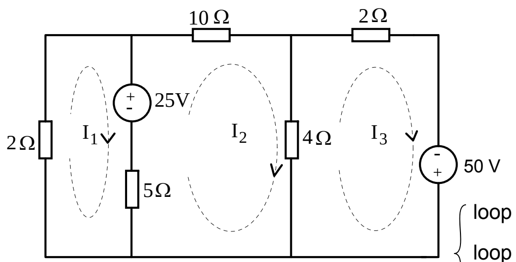
It results: **$I_1 = 2$ A, $I_2 = 1$ A**

$P_{10V} = -1\cdot 10 = -10$ W (delivers power)
$P_{2A} = -2\cdot 10 = -20$ W (delivers power)
$P_{10\Omega} = +1^2\cdot 10 = +10$ W (absorbs power)
$P_{20\Omega} = +1^2\cdot 20 = +20$ W (absorbs power)

$$\sum_i P_i = 0$$

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

- **EXAMPLE**



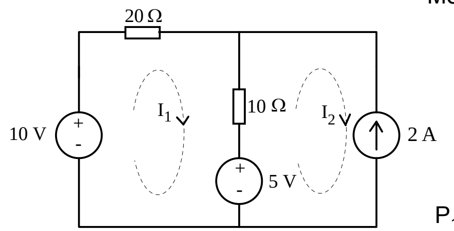
Mesh current equations:

loop 1: $-25 = +I_1\cdot(2+5) - I_2\cdot(5) - I_3\cdot(0)$
loop 2: $+25 = -I_1\cdot(5) + I_2\cdot(5+10+4) - I_3\cdot(4)$
loop 3: $+50 = -I_1\cdot(0) - I_2\cdot(4) + I_3\cdot(4+2)$

$$I_1 = \frac{\begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = -1'306 A \qquad I_2 = \frac{\begin{vmatrix} 7 & -25 & 0 \\ -5 & 25 & -4 \\ 0 & 50 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = 3'172 A \qquad I_3 = \frac{\begin{vmatrix} 7 & -5 & -25 \\ -5 & 19 & 25 \\ 0 & -4 & 50 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = 10'448 A$$

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

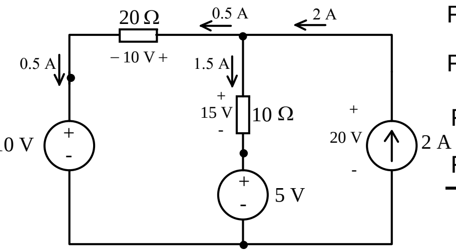
- **EXAMPLE**



Mesh current equations:

loop 1: $10 - 5 = 30\cdot I_1 - 10\cdot I_2$
loop 2: $I_2 = -2$ A

It results: $I_1 = -0.5$ A, $I_2 = -2$ A.

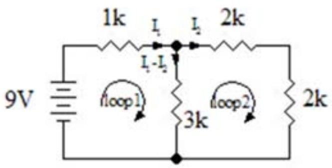


$P_{10V} = +0'5\cdot 10 = +5$ W (absorbs power)
$P_{5V} = +1'5\cdot 5 = +7'5$ W (absorbs power)
$P_{2A} = -2\cdot 20 = -40$ W (delivers power)
$P_{10\Omega} = +10\cdot 1'52 = +22'5$ W (dissipates power)
$P_{20\Omega} = +20\cdot 12 = +5$ W (dissipates power)

$$\sum_i P_i = 0$$

Unit 1. DC CIRCUITS
CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

EXAMPLE



Mesh current equations:

loop 1: $9 = 4000 \cdot I_1 - 3000 \cdot I_2$

loop 2: $0 = -3000 \cdot I_1 + 7000 \cdot I_2$

It results: $I_1 = 3.32$ mA, $I_2 = 1.42$ mA.

$P_{9V} = -3.32 \cdot 9$ mW = -29.88 mW (delivers power)

$P_{1000\Omega} = +(3.32)^2 \cdot 1$ mW = 11.02 mW (dissipates power)

$P_{3000\Omega} = +(3.32-1.42)^2 \cdot 3$ mW = +10.83 m W (dissipates power)

$P_{2000\Omega} = +(1.42)^2 \cdot 2$ mW = 4.03 mW (dissipates power)

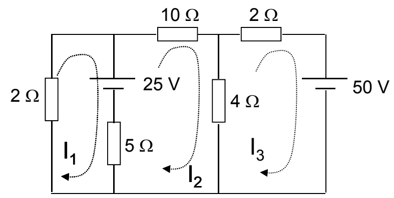
$P_{2000\Omega} = +(1.42)^2 \cdot 2$ mW = 4.03 mW (dissipates power)

$$\sum_i P_i = 0$$

Unit 1. DC CIRCUITS
TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

EXAMPLE

Determine currents in each loop and perform a power balance.

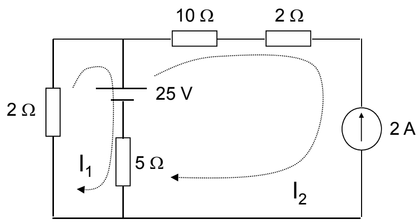


Answer: a) I_1 = -5'037 A, I_2 = -2'052 A, I_3 = -9'701 A b) $\Sigma P_{source} = \Sigma P_{resist.}$ = 559'65 W

Unit 1. DC CIRCUITS
CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

EXAMPLE

Determine currents in each loop and perform a power balance.

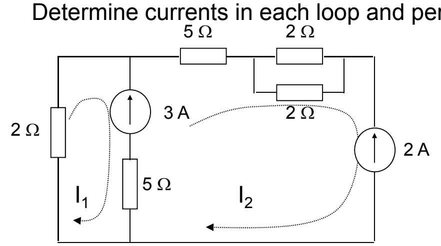


Answer a) I_1 = -5 A, I_2 = -2 A b) $\Sigma P_{source} = \Sigma P_{resist.}$ = 143 W

Unit 1. DC CIRCUITS
CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

EXAMPLE

Determine currents in each loop and perform a power balance.

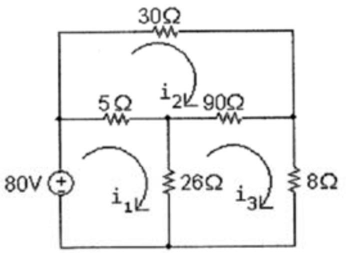


Answer a) I_1 = -5 A, I_2 = -2 A b) $\Sigma P_{source} = \Sigma P_{resist.}$ = 119 W

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

- **EXAMPLE**

Determine currents in each loop and perform a power balance.



The mesh current equations are:

$$31i_1 - 5i_2 - 26i_3 = 80$$
$$-5i_1 + 125i_2 - 90i_3 = 0$$
$$-26i_1 - 90i_2 + 124i_3 = 0$$

$$\Delta = \begin{vmatrix} 31 & -5 & -26 \\ -5 & 125 & -90 \\ -26 & -90 & 124 \end{vmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 80 & -5 & -26 \\ 0 & 125 & -90 \\ 0 & -90 & 124 \end{vmatrix}}{\Delta} = 5A$$

$$i_2 = \frac{\begin{vmatrix} 31 & 80 & -26 \\ -5 & 0 & -90 \\ -26 & 0 & 124 \end{vmatrix}}{\Delta} = 2A$$

(a) $p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

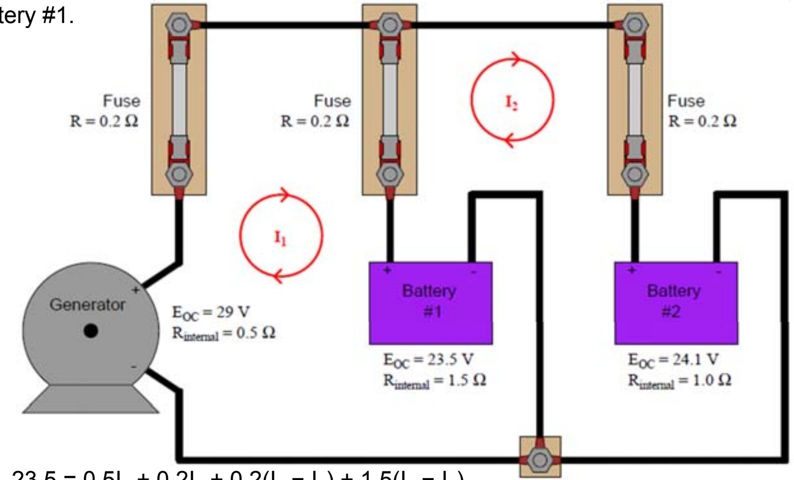
Therefore the 80 V source is delivering 400 W to the circuit.

(b) $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the 8 Ω resistor dissipates 50 W.

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

- **EXAMPLE**

Given the following mesh current directions, solve for the charging current through battery #1.



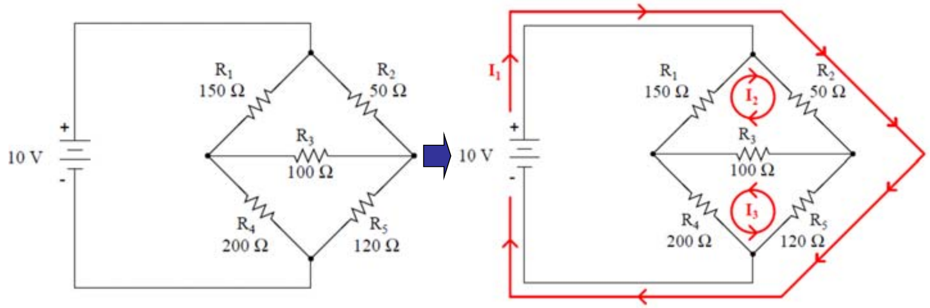
$$\begin{cases} 29 - 23.5 = 0.5I_1 + 0.2I_1 + 0.2(I_1 - I_2) + 1.5(I_1 - I_2) \\ 23.5 - 24.1 = 1.5(I_2 - I_1) + 0.2(I_2 - I_1) + 0.2I_2 + I_2 + 0 \end{cases}$$

The result is I_{bat1} = 1.7248 A

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

- **EXAMPLE**

Given the following mesh current directions, solve for the mesh currents.



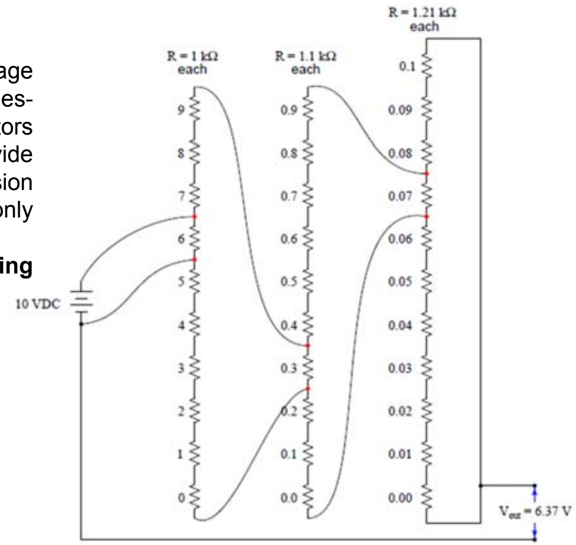
$$\begin{cases} \text{Loop 1:} \quad 10 = 170I_1 + 50I_2 - 120I_3 \\ \text{Loop 2:} \quad 0 = 50I_1 + 300I_2 + 100I_3 \\ \text{Loop 3:} \quad 0 = -120I_1 + 100I_2 + 420I_3 \end{cases}$$

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method

- **EXAMPLE**

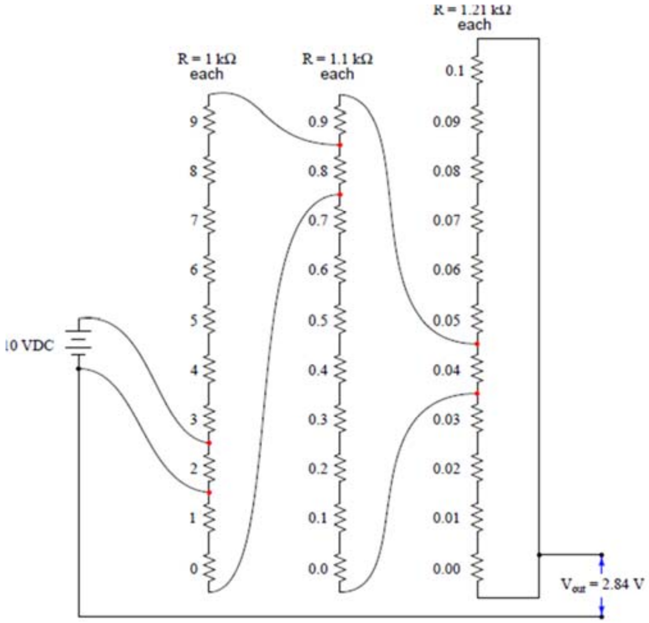
A very interesting style of voltage divider which uses three series-connected strings of resistors and connection clips to provide 1000 steps of voltage division with only 31 resistors, andf only 3 different resistance values.

Can you solve the following two circuits?



http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

CIRCUIT ANALYSIS TECHNIQUES: the mesh current method



CIRCUIT ANALYSIS TECHNIQUES: the node voltage method

The node-voltage method

- Also known as the nodal analysis or branch current method.
- It uses simultaneous equations, Kirchhoff's voltage law, and Ohm's law to determine unknown currents in a network.
- It is a method used to solve planar circuits for the voltage and currents at any place in the circuit.

THÉVENIN'S THEOREM

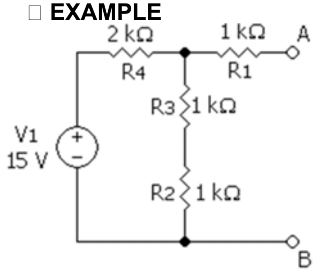
Thévenin's theorem

- Any two-terminal circuit composed of a combination of voltage sources, current sources and resistors is electrically equivalent to a single voltage source V in series with a single resistor R.

Steps for calculating the Thévenin's equivalent circuit:

- Calculate the output voltage, V_{AB}, in **open circuit** condition. This is V_{Th}.
- Replace the voltage sources with short circuits and the current sources with open circuits. Measure the total resistance, R_{AB}. This is R_{Th}

- **EXAMPLE**



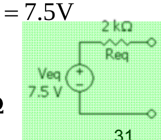
V_{TH}:

$$15 = I \cdot (2000 + 1000 + 1000) \rightarrow I = 0.00375A$$

$$V_{AB} = I \cdot (1000 + 1000) \rightarrow V_{AB} = V_{Th} = 7.5V$$

R_{TH}:

$$R_{Th} = (2000//2000) + 1000 = 2000\Omega$$



NORTON'S THEOREM

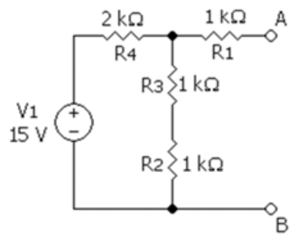
Norton's theorem

- Any two-terminals circuit composed of a combination of voltage sources, current sources and resistors is electrically equivalent to a single current source in parallel with a single resistor R.

Steps for calculating the Norton's equivalent circuit:

- Calculate the output current, I_{AB}, with a short circuit as the load. This is I_{No}.
- Replace the voltage sources with short circuits and the current sources with open circuits. Measure the total resistance, R_{AB}. This is R_{No}

- **EXAMPLE**



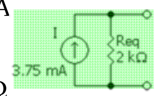
I_{No}:

$$I_{tot} = \frac{15}{2000 + 2000//1000} = 0.005625A$$

Current divider: $I_{No} = I_{tot} \cdot \frac{2000}{3000} = 0.00375A$

R_{No}:

$$R_{No} = R_{Th} = (2000//2000) + 1000 = 2000\Omega$$

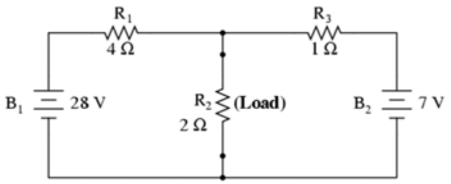


THÉVENIN-NORTON RELATIONSHIP

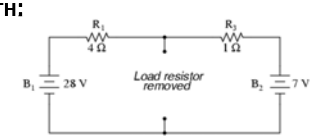
Equivalence between Thévenin and Norton

- $R_{Th} = R_{No}$
- $V_{Th} = R_{No} \cdot I_{No}$
- $I_{No} = V_{Th}/R_{Th}$

EXAMPLE



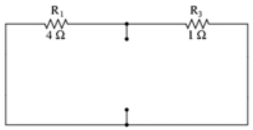
1) VTH:



$$28 - 7 = I\cdot(4+1) \rightarrow I = 4.2A$$

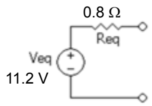
$$V_{AB} = 28 - I\cdot 4 \rightarrow V_{AB} = V_{Th} = 11.2V$$

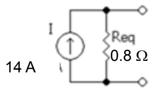
2) RTH, RNo



$$R_{Th} = R_{No} = 4//1 = 0.8\Omega$$

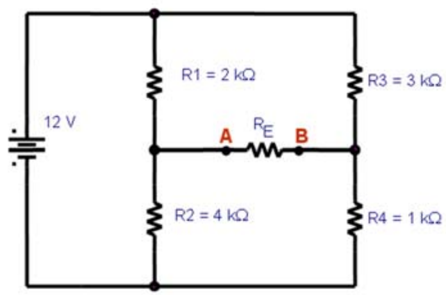
3) INo: $I_{No} = V_{Th}/R_{No} = 11.2/0.8 = 14A$



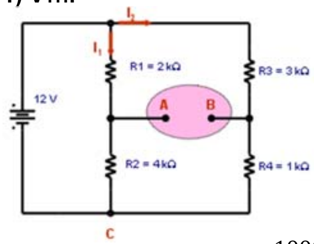


THÉVENIN-NORTON RELATIONSHIP

EXAMPLE



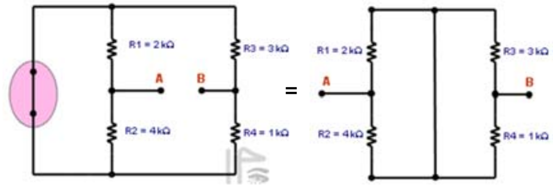
1) VTH:



$$V_{AC} = 12\cdot\frac{4000}{6000} = 8V \quad V_{BC} = 12\cdot\frac{1000}{4000} = 3V$$

$$V_{Th} = V_{AB} = V_{AC} - V_{BC} = 8 - 3 = 5V$$

2) RTH, RNo



$$R_{Th} = R_{No} =$$

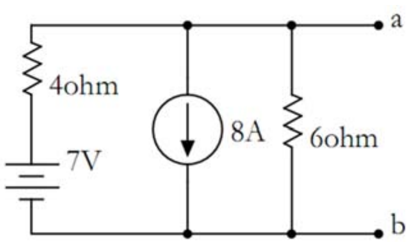
$$= 2//4 + 3//1k\Omega = 2.08\hat{3}k\Omega$$

3) INo:

$$I_{No} = V_{Th}/R_{No} = 2.4mA$$

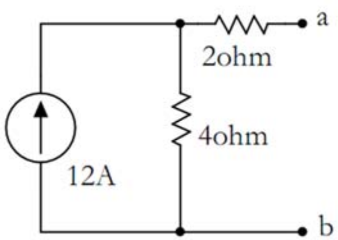
THÉVENIN-NORTON RELATIONSHIP

EXAMPLE



$I_N = 6.25A.,\ R_{Th} = 2.4\ \text{Ohm}$

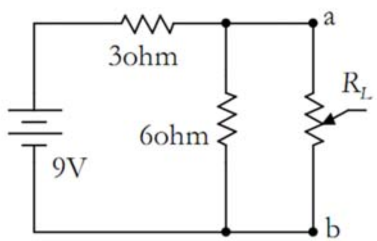
EXAMPLE



$R_{Th} = 6\Omega,\ V_{Th} = 48V$

THÉVENIN-NORTON RELATIONSHIP

EXAMPLE



$R_{Th} = 2\Omega,\ V_{Th} = 6V$

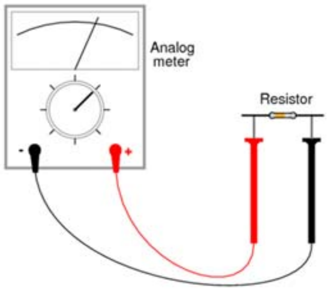
Online resources:

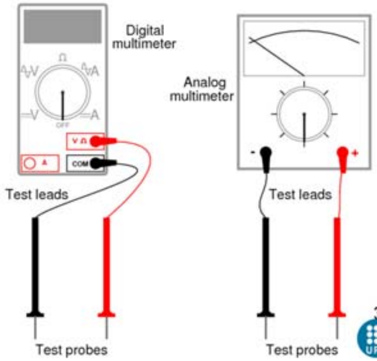
http://utwired.engr.utexas.edu/rgd1/lesson07.cfm

Voltmeters

- Instruments used for measuring the electrical potential difference between two points in an electric circuit.
- They have an internal impedance of several MΩ (infinite).
- They are connected in parallel.

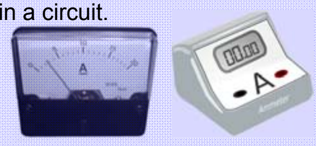
Voltmeter usage



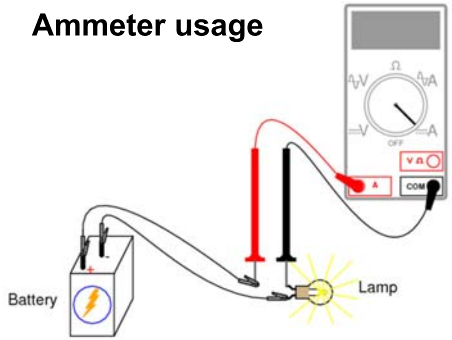


Ammeters

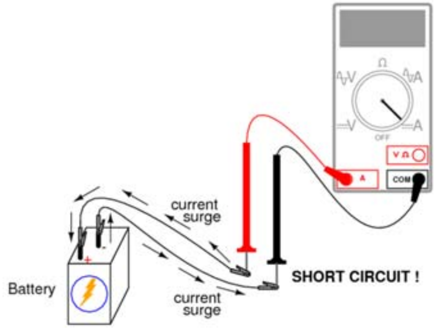
- Instruments used for measuring the electric current in a circuit.
- They have an internal impedance close to zero.
- They are connected in series.



Ammeter usage



WELL DONE



DANGER!!!!

Multimeters

- Electronic measuring instruments that combines several measurement functions in one unit: voltage, current and resistance.

Clamp multimeters

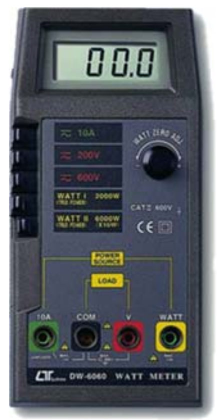
- **Current clamp**: electrical device that has two jaws which open to clamp around an electrical conductor. Thus, the electric current in the conductor can be measured without having to make physical contact with it or disconnect it to insert it through the probe.
- Can also be used to measure voltage, current and resistance.

clamp

Wattmeters

- Instruments for measuring the electric power in watts of any given circuit.
- They have 4 terminals (2 for voltage and 2 for current measurement)

Analog



Digital

ELECTRICAL MEASUREMENTS

Wattmeter connection