P K Nag Exercise problems - Solved

Thermodynamics

Contents

Solved by

Er. S K Mondal

IES Officer (Railway), GATE topper, NTPC ET-2003 batch, 12 years teaching experienced, Author of Hydro Power Familiarization (NTPC Ltd)

Benefits of solving Exercise (unsolved) problems of P K Nag

- *The best ways is to study thermodynamics is through problems, you must know how to apply theoretical concepts through problems and to do so you must solve these problems*
- *It contains Expected Questions of IES, IAS, IFS and GATE examinations*
- *It will enable the candidates to understand thermodynamics properly*
- *It will clear all your doubts*
- *There will be no fear of thermodynamics after solving these problems*
- *Candidate will be in a comfortable position to appear for various competitive examinations*
- *Thermodynamics- "the Backbone of Mechanical Engineering" therefore Mastering Thermodynamics is most important many of the subjects which come in later like Heat and Mass Transfer, Refrigeration and Air Conditioning, Internal Combustion Engine will require fundamental knowledge of Thermodynamics*

Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). I would be thankful to the readers if they are brought to my attention at the following e-mail address: swapan_mondal_01@yahoo.co.in

S K Mondal

1. Introduction

Some Important Notes

- Microscopic thermodynamics or statistical thermodynamics
- Macroscopic thermodynamics or classical thermodynamics
- A quasi-static process is also called a reversible process

Intensive and Extensive Properties

Intensive property*:* Whose value is independent of the size or extent i.e. mass of the system. e.g., pressure *p* and temperature *T*.

Extensive property: Whose value depends on the size or extent i.e. mass of the system (upper case letters as the symbols). e.g., Volume, Mass (V, M). If mass is increased, the value of extensive property also increases. e.g., volume *V,* internal energy *U,* enthalpy *H,* entropy S, etc.

Specific property: It is a special case of an intensive property. It is the value of an extensive property per unit mass of system. (Lower case letters as symbols) e.g: specific volume, density (v, ρ).

Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure, velocity etc.) and fluid properties vary continuously from one point to another. Mathematical descriptions of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established.
For example density at a point is normally defined as

$$\rho = \lim_{\triangle\forall \to 0}\left(\frac{m}{\triangle\forall}\right)$$

Here $\triangle\forall$ is the volume of the fluid element and m is the mass

If $\triangle\forall$ is very large ρ is affected by the in-homogeneities in the fluid medium. Considering another extreme if $\triangle\forall$ is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of $\triangle\forall$, before statistical fluctuations become significant. This is called continuum limit and is denoted by $\triangle\forall_C$.

$$\rho = \lim_{\triangle\forall \to \triangle\forall_C}\left(\frac{m}{\triangle\forall}\right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is characterized by mean free path (λ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analyzed by the molecular theory.

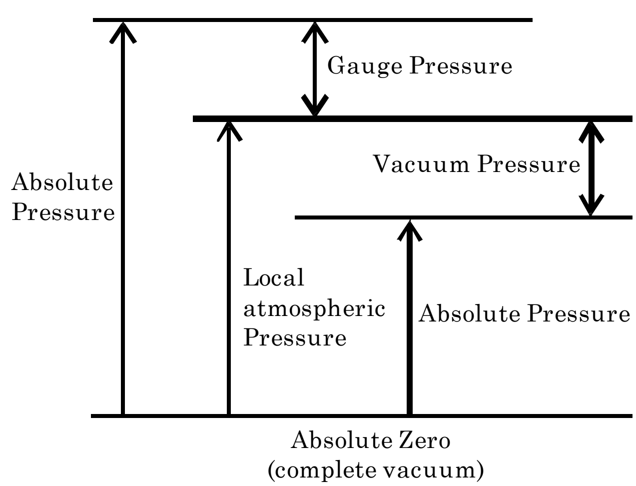
A dimensionless parameter known as Knudsen number, $K_n = \lambda / L$, where λ is the mean free path and L is the characteristic length. It describes the degree of departure from continuum. Usually when $K_n > 0.01$, the concept of continuum does not hold good.

In this, K_n is always less than 0.01 and it is usual to say that the fluid is a continuum.

Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good.

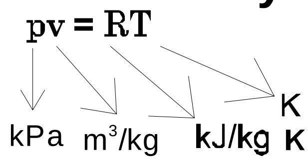
In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

The Scale of Pressure

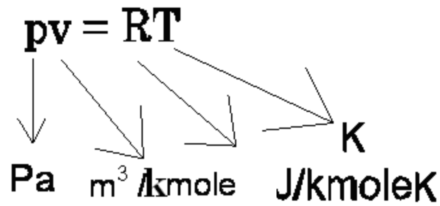


At sea-level, the international standard atmosphere has been chosen as P_{atm} = 101.325 kN/m^2

Some special units for Thermodynamics



Note: Physicists use below units



Universal gas constant, R_u= 8.314 kJ/kmole – K

Characteristic gas constant, $R_c = \dfrac{R_u}{M}$

$$\text{For Air R} = \frac{8.314}{29} = \frac{\text{kJ/kmole-K}}{\text{kg/kmole}}$$

$$= 0.287 \text{ kJ/kg- K}$$

$$\text{For water R} = \frac{8.314}{18}\ \frac{\text{kJ/kmole-K}}{\text{kg/kmole}}$$

$$= 0.461 \text{ kJ/kg -K}$$

Units of heat and work is kJ

Units of pressure is kPa

1 atm = 101.325 kPa

1 bar = 100 kPa

1 MPa =1000 kPa.

Questions with Solution P. K. Nag

Q1.1 **A pump discharges a liquid into a drum at the rate of 0.032 m³/s. The drum, 1.50 m in diameter and 4.20 m in length, can hold 3000 kg of the liquid. Find the density of the liquid and the mass flow rate of the liquid handled by the pump.**

(**Ans.** 12.934 kg/s)

Solution:

$$\text{Volume of drum} = \frac{\pi d^2}{4} \times h$$

$$= \frac{\pi \times 1.50^2}{4} \times 4.2\ m^3$$

$$= 7.422\ m^3$$

$$\text{density} = \frac{\text{mass}}{\text{Volume}} = \frac{3000}{7.422}\ kg/m^3 = 404.203\ kg/m^3$$

$$\text{mass flow rate} = \text{Vloume flow rate} \times \text{density}$$

$$= 0.032 \times 404.203\ kg/s$$

$$= 12.9345\ kg/s$$

Q1.2 **The acceleration of gravity is given as a function of elevation above sea level by**

$$g = 980.6 - 3.086 \times 10^{-6} H$$

Where *g* is in cm/s² and *H* is in cm. If an aeroplane weighs 90,000 N at sea level, what is the gravity force upon it at 10,000 m elevation? What is the percentage difference from the sea-level weight?

(**Ans.** 89,716.4 N, 0.315%)

Solution:

$$g' = 980.6 - 3.086 \times 10^{-6} \times 10,000 \times 100$$

$$= 977.514\ cm/s^2 = 9.77514\ m/s^2$$

$$W_{sea} = 90,000\ N = \frac{90,000}{9.806}\ kgf$$

$$= 9178.054\ kgf$$

$$W_{ete} = 9178.054 \times 9.77514\ N = 89716.765\ N$$

$$\%\ \text{less} = \frac{90,000 - 89716.765}{90,000} \times 100\%$$

$$= 0.3147\%\ (\text{less})$$

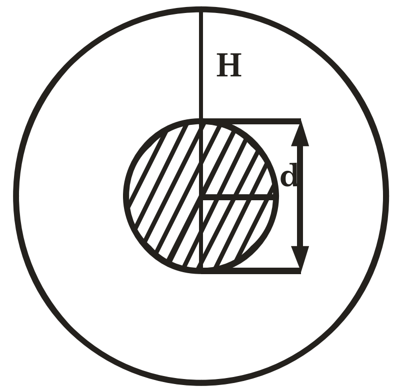
Q1.3 **Prove that the weight of a body at an elevation *H* above sea-level is given by**

$$W = \frac{mg}{g_0}\left(\frac{d}{d + 2H}\right)^2$$

Where *d* is the diameter of the earth.

Solution: According to Newton's law of gravity it we place a man of m at an height of H then

Force of attraction = $\dfrac{GMm}{\left(\frac{d}{2}+H\right)^2}$ ... (i)



1.3

If we place it in a surface of earth then

$$\text{Force of attraction} = \frac{GMm}{\left(\frac{d}{2}\right)^2} = mg_o$$

or $$g_o = \frac{GM}{\left(\frac{d}{2}\right)^2}$$

$$\therefore \quad \text{Weight } (W) = \frac{GMm}{\left(\frac{d}{2}+H\right)^2}$$

$$= \frac{mg_o\left(\frac{d}{2}\right)^2}{\left(\frac{d}{2}+H\right)^2} \quad \text{from equation...(i)}$$

$$= mg_o\left(\frac{d}{d+2H}\right)^2 \quad \text{Proved.}$$

Q1.4 **The first artificial earth satellite is reported to have encircled the earth at a speed of 28,840 km/h and its maximum height above the earth's surface was stated to be 916 km. Taking the mean diameter of the earth to be 12,680 km, and assuming the orbit to be circular, evaluate the value of the gravitational acceleration at this height.**

The mass of the satellite is reported to have been 86 kg at sea-level. Estimate the gravitational force acting on the satellite at the operational altitude.

(**Ans.** 8.9 m/s²; 765 N)

Solution: Their force of attraction = centrifugal force

$$\text{Centirfugal force} = \frac{mv^2}{r}$$

$$= \frac{86\times\left(\dfrac{28840\times1000}{60\times60}\right)^2}{\left(\dfrac{12680\times10^3}{2}+916\times10^3\right)}N$$

$$= 760.65 \text{ N (Weight)}$$

Q1.5 **Convert the following readings of pressure to kPa, assuming that the barometer reads 760 mmHg:**

(a) 90 cmHg gauge (b) 40 cmHg vacuum

(c) 1.2 m H_2O gauge (d) 3.1 bar

Solution: 760 mm Hg = 0.760 × 13600 × 9.81 Pa

= 10139.16 Pa

≃ 101.4 kPa

(a) 90 cm Hg gauge

$= 0.90 \times 13600 \times 9.81 \times 10^{-3} + 101.4$ kPa

$= 221.4744$ kPa

(b) 40 cm Hg vacuum

$= (76 - 40)$ cm (absolute)

$= 0.36 \times 43.600 \times 9.81$ kPa

$= 48.03$ kPa

(c) 1.2 m H_2O gauge

$= 1.2 \times 1000 \times 9.81 \times 10^{-3} + 101.4$ kPa

$= 113.172$ kPa

(d) 3.1 bar = 3.1×100 kPa = 310 kPa

Q1.6 **A 30 m high vertical column of a fluid of density 1878 kg/m³ exists in a place where *g* = 9.65 m/s². What is the pressure at the base of the column.**

(**Ans.** 544 kPa)

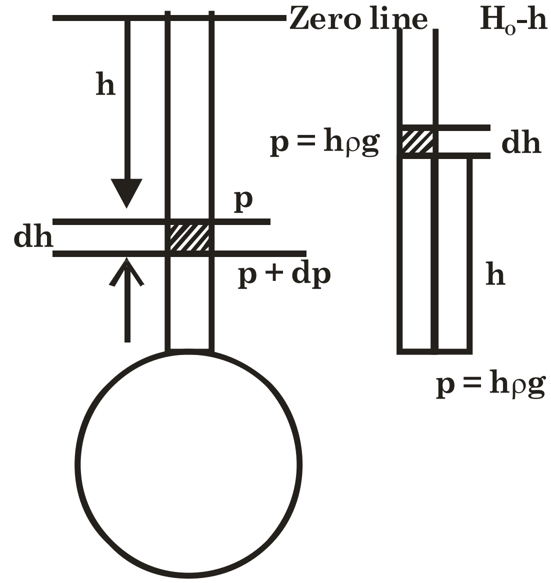
Solution: p = z ρg

$= 30 \times 1878 \times 9.65$ Pa

$= 543.681$ kPa

Q1.7 **Assume that the pressure *p* and the specific volume *v* of the atmosphere are related according to the equation $pv^{1.4} = 2.3 \times 10^5$, where *p* is in N/m² abs and *v* is in m³/kg. The acceleration due to gravity is constant at 9.81 m/s². What is the depth of atmosphere necessary to produce a pressure of l.0132 bar at the earth's surface? Consider the atmosphere as a fluid column.**

(**Ans.** 64.8 km)



Solution: $dp = dh\,\rho g$

or $dp = dh \times \frac{1}{v} \times g$

or $v = \frac{g\,dh}{dp}$

$pv^{1.4} = 2.3 \times 10^3 = 2300$

or $v = \left(\frac{2300}{p}\right)^{\frac{1}{1.4}} = \left(\frac{2300}{p}\right)^n$ where $n = \frac{1}{1.4}$

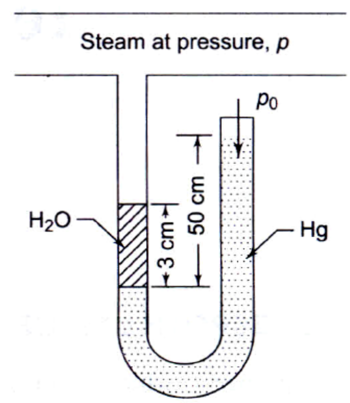
or $\frac{g\,dh}{dp} = \left(\frac{2300}{p}\right)^n$

or $g\,dh = \left(\frac{2300}{p}\right)^n dp$

or $\int_0^H dh = \frac{2300^n}{g}\int_0^{101320}\frac{dp}{p^n}$

or $h = \frac{2300^n}{g(1-n)}\left[(101320)^{(1-n)} - 0\right] = 2420\,\text{m} = 2.42\ \text{km}$

Q1.8 **The pressure of steam flowing in a pipeline is measured with a mercury manometer, shown in Figure. Some steam condenses into water. Estimate the steam pressure in kPa. Take the density of mercury as $13.6 \times 10^3\,kg/m^3$, density of water as 10^3 kg/m³, the barometer reading as 76.1 cmHg, and *g* as 9.806 m/s².**



Solution: $p_o + 0.50 \times \rho_{Hg} \times g = 0.03 \times \rho_{H_2O} \times g + p$

or $p = 0.761 \times 13.6 \times 10^3 \times 9.806 + 0.5 \times 13.6 \times 10^3 \times 9.806 - 0.03 \times 1000 \times 9.806$ Pa.

$= 167.875$ kPa

Q1.9 **A vacuum gauge mounted on a condenser reads 0.66 mHg. What is the absolute pressure in the condenser in kPa when the atmospheric pressure is 101.3 kPa?**

(**Ans.** 13.3 kPa)

Solution: Absolute = atm. – vacuum

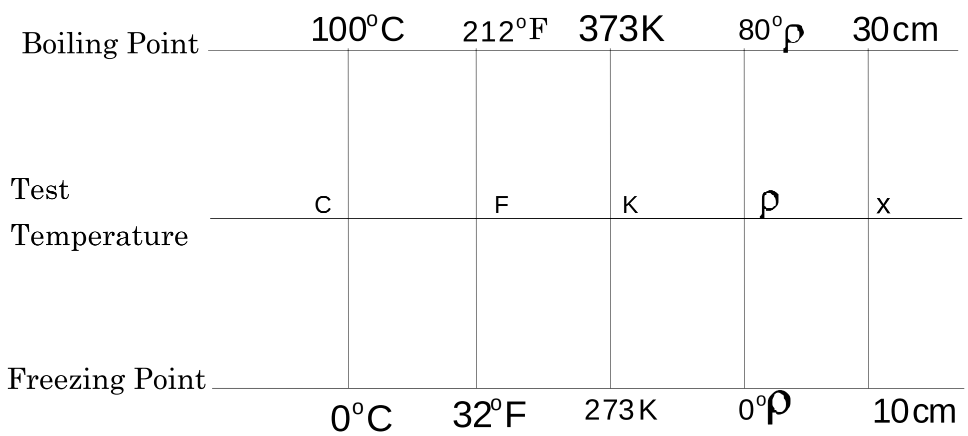
$= 101.3 - 0.66 \times 13.6 \times 10^3 \times 9.81 \times 10^{-3}$ kPa

= 13.24 kPa

2. Temperature

Some Important Notes

Comparison of Temperature scale



Relation: $\dfrac{C-0}{100-0}=\dfrac{F-32}{212-32}=\dfrac{K-273}{373-273}=\dfrac{\rho-0}{80-0}=\dfrac{x-10}{30-10}$

Questions with Solution P. K. Nag

Q2.1 **The limiting value of the ratio of the pressure of gas at the steam point and at the triple point of water when the gas is kept at constant volume is found to be 1.36605. What is the ideal gas temperature of the steam point?**

(**Ans.** 100°C)

Solution:

$$\frac{p}{p_t} = 1.36605$$

$$\therefore \quad \theta_{(v)} = 273.16 \times \frac{p}{p_t}$$

$$= 273.16 \times 1.36605$$

$$= 373.15°C$$

Q2.2 **In a constant volume gas thermometer the following pairs of pressure readings were taken at the boiling point of water and the boiling point of sulphur, respectively:**

Water b.p.	**50.0**	**100**	**200**	**300**
Sulphur b.p.	**96.4**	**193**	**387**	**582**

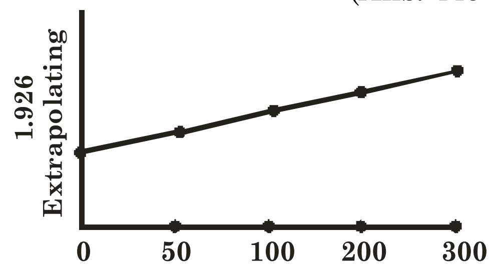
The numbers are the gas pressures, mm Hg, each pair being taken with the same amount of gas in the thermometer, but the successive pairs being taken with different amounts of gas in the thermometer. Plot the ratio of $S_{b.p.}$:$H_2O_{b.p.}$ against the reading at the water boiling point, and extrapolate the plot to zero pressure at the water boiling point. This

gives the ratio of $S_{b.p.}$: $H_2O_{b.p.}$ On a gas thermometer operating at zero gas pressure, i.e., an ideal gas thermometer. What is the boiling point of sulphur on the gas scale, from your plot?

(**Ans.** 445°C)

Solution :

Water b.p.	50.0	100	200	300
Sulphur b.p.	96.4	193	387	582
Ratio $\frac{S_{b.p}}{W_{b.p}}$ =	1.928	1.93	1.935	1.940



$\therefore \quad T_1 = 100°C = 373K$

$T_2 = ?$

$\frac{p_1}{p_2} = 1.926$

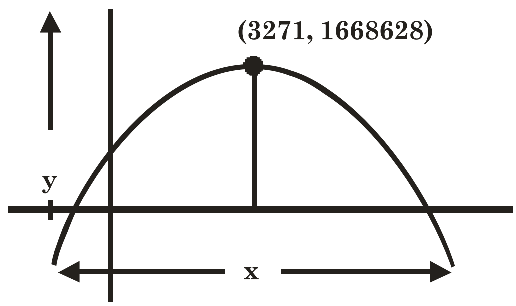
$\therefore \quad T_2 = 373 \times 1.926 = 718K = 445°C$

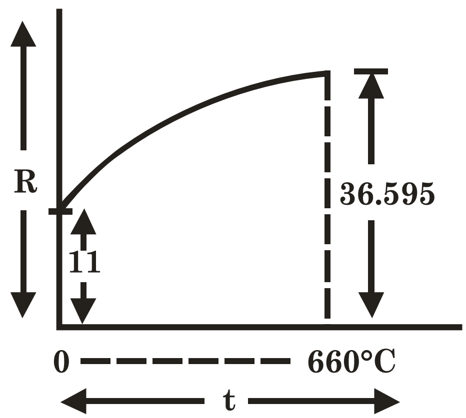
Q2.3 **The resistance of a platinum wire is found to be 11,000 ohms at the ice point, 15.247 ohms at the steam point, and 28.887 ohms at the sulphur point. Find the constants A and B in the equation**

$$\boldsymbol{R = R_0(1 + At + Bt^2)}$$

And plot R against t in the range 0 to 660°C.

Solution:





$$R_0 = 11.000\,\Omega$$

$$R_{100} = R_0\left\{1 + A \times 100 + B \times 100^2\right\}$$

or $\quad 15.247 = 11.000 + 1100A + 11 \times 10^4 B$

or $\quad 3.861 \times 10^{-3} = A + 100B \quad \ldots(i)$

$$28.887 = 11.00 + 445 \times 11A + 445^2 \times 11B$$

$3.6541 \times 10^{-3} = A + 445B \quad \ldots(ii)$

equation (ii) – (i) gives.

$$B = -6 \times 10^{-7}$$

$$A = 3.921 \times 10^{-3}$$

$$R = 11\left\{1 + 3.921 \times 10^{-3} t - 6 \times 10^{-7} t^2\right\}$$

or $\quad Y = 11\left(1 + 3.921 \times 10^{-3} t - 6 \times 10^{-7} t^2\right)$

or $(t - 3271)^2 = -4 \times 37922 \times (Y - 1668628)$

$$R_{660} = 36.595$$

Q2.4 **when the reference junction of a thermocouple is kept at the ice point and the test junction is at the Celsius temperature *t*, and e.m.f. *e* of the thermocouple is given by the equation**

$$\varepsilon = at + bt^2$$

Where *a* = 0.20 mV/deg, and *b* = - 5.0 × 10^{-4} mV/deg^2

(a) Compute the e.m.f. when *t* = - l00°C, 200°C, 400°C, and 500°C, and draw graph of ε against *t* in this range.

(b) Suppose the e.m.f. ε is taken as a thermometric property and that a temperature scale *t is defined by the linear equation.**

$$t^* = a'\varepsilon + b'$$

And that *t = 0 at the ice point and *t** = 100 at the steam point. Find the numerical values of *a'* and *b'* and draw a graph of ε against *t**.**

(c) Find the values of *t when *t* = -100°C, 200°C, 400°C, and 500°C, and draw a graph of *t** against *t*.**

(d) Compare the Celsius scale with the *t scale.**

Solution: Try please

Q2.5 **The temperature *t* on a thermometric scale is defined in terms of a property *K* by the relation**

$$t = a \ln K + b$$

Where *a* and *b* are constants.
The values of *K* are found to be 1.83 and 6.78 at the ice point and the steam point, the temperatures of which are assigned the numbers 0 and 100 respectively. Determine the temperature corresponding to a reading of *K* equal to 2.42 on the thermometer.

(**Ans.** 21.346°C)

Solution:

$$t = a \ln x + b$$

$$0 = a \text{ x } \ln 1.83 + b \qquad \ldots \text{(i)}$$

$$100 = a \text{ x } \ln 6.78 + b \qquad \ldots \text{(ii)}$$

Equation {(ii) – (i)} gives

$$a \cdot \ln \cdot \left(\frac{6.78}{1.83}\right) = 100$$

or $\quad a = 76.35$

$\therefore \quad b = -a \times \ln 1.83$

$\qquad = -46.143$

$\therefore \quad t = 76.35 \ln k - 46.143$

$\therefore \quad t^* = 76.35 \times \ln 2.42 - 46.143$

$\qquad = 21.33°C$

Q2.6 **The resistance of the windings in a certain motor is found to be 80 ohms at room temperature (25°C). When operating at full load under steady state conditions, the motor is switched off and the resistance of the windings, immediately measured again, is found to be 93 ohms. The windings are made of copper whose resistance at temperature t°C is given by**

$$R_t = R_0\,[1 + 0.00393\,t]$$

Where R_0 is the resistance at 0°C. Find the temperature attained by the coil during full load.

(**Ans.** 70.41°C)

Solution: $R_{25} = R_0\,[1 + 0.00393 \times 25]$

$\therefore \quad R_0 = \dfrac{80}{[1 + 0.00393 \times 25]} = 72.84\ \Omega$

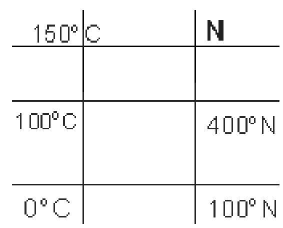
$\therefore \quad 93 = 72.84\{1 + 0.00393 \times t\}$

or $\quad t = 70.425°C$

Q2.7 **A new scale *N* of temperature is divided in such a way that the freezing point of ice is 100°N and the boiling point is 400°N. What is the temperature reading on this new scale when the temperature is 150°C? At what temperature both the Celsius and the new temperature scale reading would be the same?**

(**Ans.** 550°N, – 50°C.)

Solution:



$$\frac{150-0}{100-0} = \frac{N-100}{400-100}$$

or $N = 550^o N$

let N= C for x^o

then $\dfrac{C-0}{100-0} = \dfrac{N-100}{400-100}$

or $\dfrac{x}{100} = \dfrac{x-100}{300}$

or $\quad x = \dfrac{x-100}{3}$

or $\quad 3x = x - 100$

or $\quad 2x = -100$

or $\quad x = -50^o\,C$

Q2.8 **A platinum wire is used as a resistance thermometer. The wire resistance was found to be 10 ohm and 16 ohm at ice point and steam point respectively, and 30 ohm at sulphur boiling point of 444.6°C. Find the resistance of the wire at 500°C, if the resistance varies with temperature by the relation.**

$$\boldsymbol{R = R_0(1 + \alpha t + \beta t^2)}$$

(**Ans.** 31.3 ohm)

Solution:

$10 = R_0(1 + 0 \times \alpha + \beta \times 0^2)$

$16 = R_0(1 + 100 \times \alpha + \beta \times 100^2)$

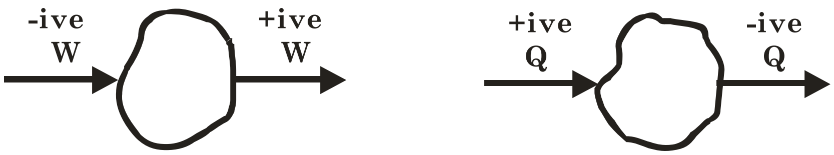
$30 = R_0(1 + \alpha \times 444.6 + \beta \times 444.6^2)$

Solve R_0 , α & β then

$R = R_0(1 + 500 \times \alpha + \beta \times 500^2)$

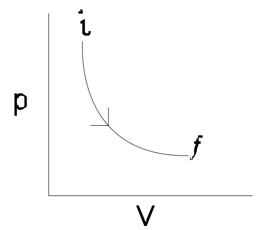
3. Work and Heat Transfer

Some Important Notes



Our aim is to give heat to the system and gain work output from it.
So heat input $\rightarrow$ +ive (positive)
Work output $\rightarrow$ +ive (positive)

$$W_{i-f} = \int_i^f pdV = \int_{vi}^{vf} pdv$$



$$\bar{d}Q = du + \bar{d}W$$

$$\int_i^f \bar{d}Q = u_f - u_i + \int_i^f \bar{d}W$$

$$Q_{i-f} = u_f - u_i + \int_{vi}^{vf} pdV$$

Questions with Solution P. K. Nag

Q3.1 **(a)A pump forces 1 m³/min of water horizontally from an open well to a closed tank where the pressure is 0.9 MPa. Compute the work the pump must do upon the water in an hour just to force the water into the tank against the pressure. Sketch the system upon which the work is done before and after the process.**

(**Ans.** 5400 kJ/h)

(b)If the work done as above upon the water had been used solely to raise the same amount of water vertically against gravity without change of pressure, how many meters would the water have been elevated?

(**Ans.** 91.74 m)

(c)If the work done in (a) upon the water had been used solely to accelerate the water from zero velocity without change of pressure or elevation, what velocity would the water have reached? If the work had been used to accelerate the water from an initial velocity of 10 m/s, what would the final velocity have been?

(**Ans.** 42.4 m/s; 43.6 m/s)

Solution: (a) Flow rate $1 m^3/hr$.
Pressure of inlet water = 1 atm = 0.101325 MPa
Pressure of outlet water = 0.9 MPa

$\therefore$ Power $= \Delta p\dot{v}$

$$= (0.9 - 0.101325) \times 10^3 \text{kPa} \times \frac{1}{60} \text{ m}^3/\text{s}$$

$$= 13.31 \text{ kJ/s}$$

(b) So that pressure will be 0.9 MPa

$\therefore$ $h\rho g = 0.9$ MPa

or $h = \dfrac{0.9 \times 10^6}{1000 \times 9.81}$ m = 91.743 m

(c) $\dfrac{1}{2}\dot{m}\left(V_2^2 - V_1^2\right) = \Delta p\dot{v}$ where $\dot{m} = \dot{v}\rho$

or $\dfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \Delta p$

or $V_2^2 - V_1^2 = 2\dfrac{\Delta p}{\rho}$

or $V_2^2 = V_1^2 + 2\dfrac{\Delta p}{\rho}$

$$= 10^2 + \frac{2 \times (0.9 - 0.101325) \times 10^6}{1000}$$

$V_2 = 41.2$ m/s.

Q3.2 The piston of an oil engine, of area 0.0045 m², moves downwards 75 mm, drawing in 0.00028 m³ of fresh air from the atmosphere. The pressure in the cylinder is uniform during the process at 80 kPa, while the atmospheric pressure is 101.325 kPa, the difference being due to the flow resistance in the induction pipe and the inlet valve. Estimate the displacement work done by the air finally in the cylinder.

(Ans. 27 J)

Solution : Volume of piston stroke

$= 0.0045 \times 0.075 \text{m}^3$

$= 0.0003375 \text{m}^3$

$\therefore$ $\Delta V = 0.0003375$ m³

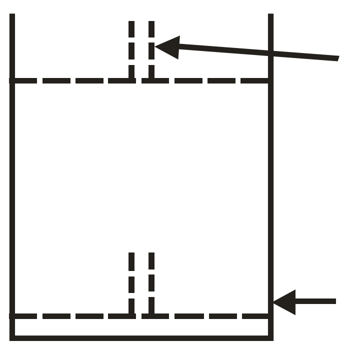
as pressure is constant

= 80 kPa

So work done $= p\Delta V$

$= 80 \times 0.0003375$ kJ

$= 0.027$ kJ = 27 J



Q3.3 An engine cylinder has a piston of area 0.12 m³ and contains gas at a pressure of 1.5 MPa. The gas expands according to a process which is represented by a straight line on a pressure-volume diagram. The final pressure is 0.15 MPa. Calculate the work done by the gas on the piston if the stroke is 0.30 m.

(Ans. 29.7 kJ)

Solution: Initial pressure $(p_1) = 1.5$ MPa

Final volume $(V_1) = 0.12\text{m}^2 \times 0.3\text{m}$

$= 0.036 \text{ m}^3$

Final pressure $(p_2) = 0.15$ MPa

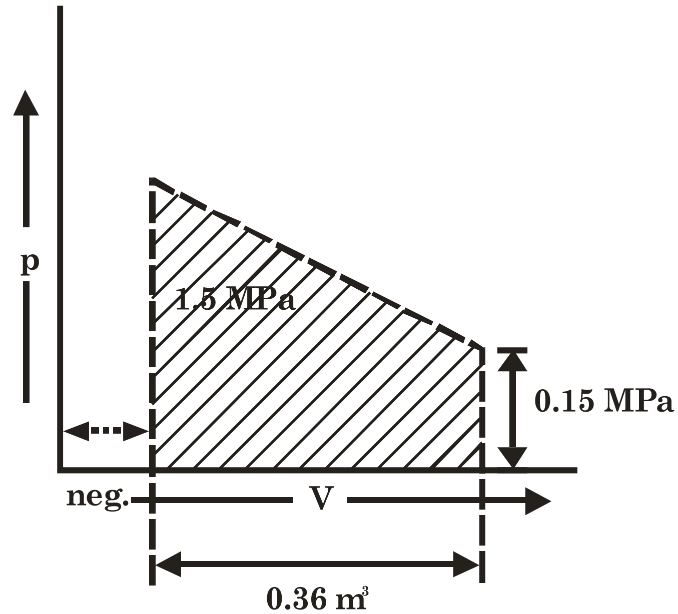
As initial pressure too high so the volume is neglected.

Work done = Area of pV diagram

$$= \frac{1}{2}(p_1 + p_2) \times V$$

$$= \frac{1}{2}(1.5 + 0.15) \times 0.036 \times 10^3 \text{ kJ}$$

$$= 29.7 \text{ kJ}$$



Q3.4 **A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which *pv* = constant. The initial density of air is 1.16 kg/m³. Find the work done by the piston to compress the air.**

(**Ans.** 251.62 kJ)

Solution: **For quasi-static process**

Work done $= \int p dV$ [given $pV = C$

$$= p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} \qquad \therefore \quad p_1 V_1 = pV = p_2 V_2 = C$$

$$= p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \qquad \therefore \quad p = \frac{p_1 V_1}{V}$$

$$= p_1 V_1 \ln\left(\frac{p_1}{p_2}\right) \qquad \therefore \quad \frac{p_1}{p_2} = \frac{V_2}{V_1}$$

$$= 0.1 \times 1.2931 \times \ln\left|\frac{0.1}{0.7}\right| \text{MJ} \qquad \text{given } p_1 = 0.1 \text{ MPa}$$

$$= 251.63 \text{ kJ} \qquad V_1 = \frac{m_1}{\rho_1} = \frac{1.5}{1.16} \text{ m}^3$$

$$p_2 = 0.7 \text{ MPa}$$

Q3.5 **A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m³ to 0.4 MPa, 0.03 m³. Assuming that the pressure and volume are related by *pvⁿ* = constant, find the work done by the gas system.**

(**Ans.** –11.83 kJ)

Solution: Given initial pressure $(p_1) = 80\text{kPa}$

Initial volume $(V_1) = 0.1 \text{ m}^3$

Final pressure $(p_2) = 0.4$ MPa = 400 kPa

Final volume$(V_2) = 0.03$ m^3

As p-V relation $pV^n = C$

$\therefore \quad p_1 V_1^n = p_2 V_2^n$

taking $\log_e$ both side

$\ln p_1 + n \ln V_1 = \ln p_2 + n \ln V_2$

or $\quad n[\ln V_1 - \ln V_2] = \ln p_2 - \ln p_1$

or $\quad n \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$

$$\text{or} \quad n = \frac{\ln\left(p_2/p_1\right)}{\ln\left(V_1/V_2\right)} = \frac{\ln\left(\frac{400}{80}\right)}{\ln\left(\frac{0.1}{0.03}\right)} = \frac{1.60944}{1.20397} \approx 1.3367 \approx 1.34$$

$$\therefore \quad \text{Work done}(W) = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{80 \times 0.1 - 400 \times 0.03}{1.34 - 1} = -11.764 \text{ kJ}$$

Q3.6 **A single-cylinder, double-acting, reciprocating water pump has an indicator diagram which is a rectangle 0.075 m long and 0.05 m high. The indicator spring constant is 147 MPa per m. The pump runs at 50 rpm. The pump cylinder diameter is 0.15 m and the piston stroke is 0.20 m. Find the rate in kW at which the piston does work on the water.**

(**Ans.** 43.3 kW)

Solution: Area of indicated diagram $(a_d) = 0.075 \times 0.05$ m^2 = 3.75×10^{-3} m^2

Spring constant (k) = 147 MPa/m

$$p_m = \frac{a_d}{l_d} \times k$$

$$= \frac{0.075 \times 0.05}{0.075} \times 147 \text{ MPa}$$

$$= 7.35 \text{ MPa} = 7.35 \times 10^3 \text{ kPa}$$

$L = 0.20$ m

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (0.15)^2}{4} \text{m}^2 = 0.01767 \text{ m}^2$$

$N = 50$ rpm

No. of stroke per minute = 2N

$$\text{Power} = \frac{p_m LA(2N)}{60} = \frac{7.35 \times 10^3 \times 0.20 \times 0.01767 \times 2 \times 50}{60} \text{kW}$$

$$= 43.29 \text{ kW}$$

Q3.7 **A single-cylinder, single-acting, 4 stroke engine of 0.15 m bore develops an indicated power of 4 kW when running at 216 rpm. Calculate the area of the indicator diagram that would be obtained with an indicator having a spring constant of 25 × 10⁶ N/m³. The length of the indicator diagram is 0.1 times the length of the stroke of the engine.**

(**Ans.** 505 mm^2)

Solution: Given Diameter of piston (D) = 0.15 m

I.P = 4 kW = 4 × 1000 W

Speed (N) = 216 rpm

Spring constant (k) = 25×10^6 N/m

Length of indicator diagram (l_d) = 0.1 × Stoke (L)

Let Area of indicator diagram = (a_d)

$$\therefore \quad \text{Mean effective pressure } (p_m) = \frac{a_d}{l_d} \times k$$

$$\text{and} \quad \text{I.P.} = \frac{p_m \, LAN}{120} \quad [\text{as 4 stroke engine}]$$

$$\therefore \quad \text{or} \quad \text{I.P.} = \frac{a_d \times k}{l_d} \times \frac{L \times A \times N}{120}$$

$$\text{or} \quad a_d = \frac{\text{I.P} \times l_d \times 120}{k \times L \times A \times N}$$

$$= \frac{\text{I.P} \times 0.1 \cancel{L} \times 120 \times 4}{k \times \cancel{L} \times \pi \times D^2 \times N} \qquad \begin{bmatrix} \text{area } A = \dfrac{\pi D^2}{4} \\ \text{and } l_d = 0.1L \end{bmatrix}$$

$$= \frac{4 \times 0.1 \times 120 \times 4 \times 1000}{25 \times 10^6 \times \pi \times 0.15^2 \times 216} \text{ m}^2$$

$$= 5.03 \times 10^{-4} \text{ m}^2$$

$$= 503 \text{ mm}^2$$

Q3.8 **A six-cylinder, 4-stroke gasoline engine is run at a speed of 2520 RPM. The area of the indicator card of one cylinder is 2.45 × 10³ mm² and its length is 58.5 mm. The spring constant is 20 × 10⁶ N/m³. The bore of the cylinders is 140 mm and the piston stroke is 150 mm. Determine the indicated power, assuming that each cylinder contributes an equal power.**

(**Ans.** 243.57 kW)

Solution:

$$p_m = \frac{a_d}{l_d} \times k$$

$$= \frac{2.45 \times 10^3}{58.5} \times 20 \times 10^3 \text{ Pa} \qquad \therefore \frac{mm^2}{mm} \times \frac{N}{m^3} \Rightarrow \frac{mm \times N}{m \times m^2} = \left(\frac{1}{1000}\right) N/m^2$$

$$= 837.607 \text{ kPa}$$

$$L = 0.150 \text{ m}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.14^2}{4}$$

$$N = 2520$$

$$n = 6$$

$$\therefore \quad \text{I.P.} = \frac{p_m \text{ LAN}}{120} \times n \qquad [\text{as four stroke}]$$

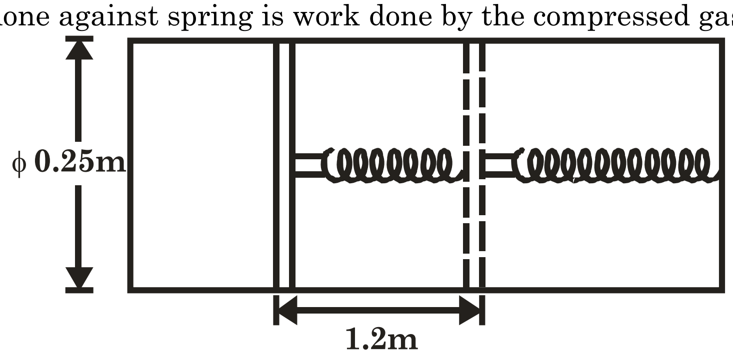
$$= 837.607 \times 0.15 \times \frac{\pi \times 0.14^2}{4} \times \frac{2520 \times 6}{120} \text{ kW}$$

$$= 243.696 \text{ kW}$$

Q3.9 **A closed cylinder of 0.25 m diameter is fitted with a light frictionless piston. The piston is retained in position by a catch in the cylinder wall and the volume on one side of the piston contains air at a pressure of 750 kN/m². The volume on the other side of the piston is evacuated. A helical spring is mounted coaxially with the cylinder in this evacuated space to give a force of 120 N on the piston in this position. The catch is released and the piston travels along the cylinder until it comes to rest after a stroke of 1.2 m. The piston is then held in its position of maximum travel by a ratchet mechanism. The spring force increases linearly with the piston displacement to a final value of 5 kN. Calculate the work done by the compressed air on the piston.**

(**Ans.** 3.07 kJ)

Solution: Work done against spring is work done by the compressed gas



$$\text{Mean force} = \frac{120 + 5000}{2}$$

$$= 2560 \text{ N}$$

$$\text{Travel} = 1.2 \text{ m}$$

$$\therefore \quad \text{Work Done} = 2560 \times 1.2 \text{ N.m}$$

$$= 3.072 \text{ kJ}$$

By Integration

At a travel (x) force (F_x) = 120 + kx

At 1.2 m then 5000 = 120 + k × 1.2

$\therefore$ $F_x = 120 + 4067\,x$

$$\therefore \qquad W = \int_0^{1.2} F_x dx$$
$$= \int_0^{1.2} [120 + 4067x] dx$$
$$= \left[120x + 4067 \times \frac{x^2}{2}\right]_0^{1.2}$$
$$= 120 \times 1.2 + 4067 \times \frac{1.2^2}{2} \text{ J}$$
$$= 144 + 2928.24 \text{ J}$$
$$= 3072.24\text{J} = 3.072 \text{ kJ}$$

Q 3.10 **A steam turbine drives a ship's propeller through an 8: 1 reduction gear. The average resisting torque imposed by the water on the propeller is 750 × 10^3 mN and the shaft power delivered by the turbine to the reduction gear is 15 MW. The turbine speed is 1450 rpm. Determine (a) the torque developed by the turbine, (b) the power delivered to the propeller shaft, and (c) the net rate of working of the reduction gear.**

(**Ans.** (a) T = 98.84 km N, (b) 14.235 MW, (c) 0.765 MW)

Solution: Power of the propeller = Power on turbine shaft

$$\text{Torque developed by turbine} = \frac{P}{W}$$
$$= \frac{15 \times 1000 \times 60}{2\pi \times 1450} \text{ mN}$$
$$= 98.786 \text{ k mN}$$
$$= 98786 \text{ mN}$$

Power developered by propeller shaft

$$= T \times \omega$$
$$= 750 \times 10^3 \times \frac{2\pi}{60}\left(\frac{1450}{8}\right)$$
$$= 14.235 \text{ MW}$$

The net rate of working of the reduction gear

$$= (15 - 14.235) \text{ MW}$$
$$= 0.7647 \text{ MW}$$

Q 3.11 **A fluid, contained in a horizontal cylinder fitted with a frictionless leak proof piston, is continuously agitated by means of a stirrer passing through the cylinder cover. The cylinder diameter is 0.40 m. During the stirring process lasting 10 minutes, the piston slowly moves out a distance of 0.485 m against the atmosphere. The net work done by the fluid during the process is 2 kJ. The speed of the electric motor driving the stirrer is 840 rpm. Determine the torque in the shaft and the power output of the motor.**

(**Ans.** 0.08 mN, 6.92 W)

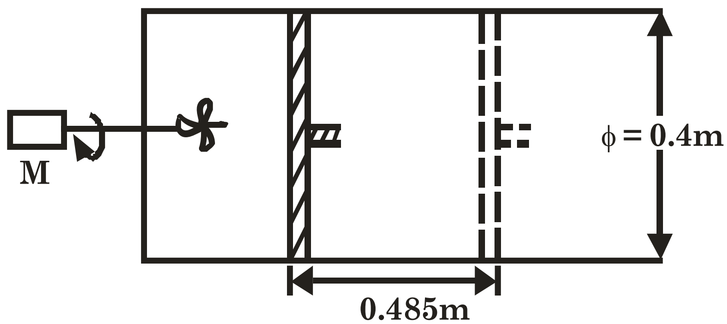
Solution: Change of volume = A L

$$= \frac{\pi d^2}{4} \times L$$

$$= \frac{\pi \times 0.4^2}{4} \times 0.485 \text{ m}^3$$

$$= 0.061 \text{ m}^3$$

As piston moves against constant atmospheric pressure then work done $= p\Delta V$



$$= 101.325 \times 0.061 \text{ kJ}$$

$$= 6.1754 \text{ kJ}$$

Net work done by the fluid = 2 kJ

$\therefore$ Net work done by the Motor = 4.1754 kJ

There for power of the motor

$$= \frac{4.1754 \times 10^3}{10 \times 60} \text{ W}$$

$$= 6.96 \text{ W}$$

$$\text{Torque on the shaft} = \frac{P}{W}$$

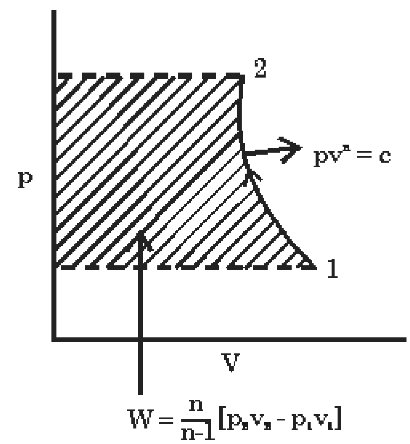
$$= \frac{6.96 \times 60}{2\pi \times 840}$$

$$= 0.0791 \text{ mN}$$

Q3.12 **At the beginning of the compression stroke of a two-cylinder internal combustion engine the air is at a pressure of 101.325 kPa. Compression reduces the volume to 1/5 of its original volume, and the law of compression is given by $pv^{1.2}$ = constant. If the bore and stroke of each cylinder is 0.15 m and 0.25 m, respectively, determine the power absorbed in kW by compression strokes when the engine speed is such that each cylinder undergoes 500 compression strokes per minute.**

(**Ans.** 17.95 kW)

Solution:



Initial volume $(V_1) = \frac{\pi d^2}{4} \times L$

$$= \frac{\pi \times (0.15)^2}{4} \times 0.25 \text{ m}^3$$

$$= 0.00442 \text{ m}^3$$

Initial pressure $(p_1) = 101.325$ kPa.

Final volume $(V_2) = \frac{V_1}{5} = 0.000884 \text{ m}^3$

$$p_1V_1^{1.2} = p_2V_2^{1.2}$$

Or $\quad p_2 = \frac{p_1V_1^{1.2}}{V_2^{1.2}} = 699.41 \approx 700$ kPa

Work done/unit stroke – unit cylinder (W)

$$= \left(\frac{1.2}{1.2-1}\right) \times [p_1V_1 - p_2V_2]$$

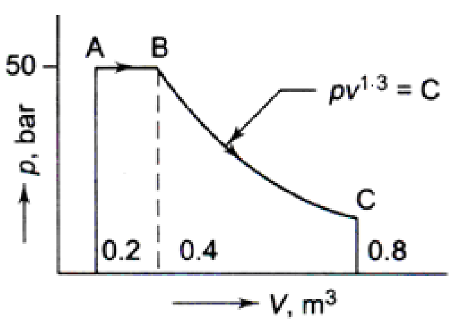
$$= \left(\frac{101.325 \times 0.00442 - 700 \times 0.000884}{1.2-1}\right) \times 1.2$$

(-ive work, as work done on the system)

$$\text{Power} = \frac{W \times 500 \times 2 \times 1.2}{60} \text{ kW}$$

$$= 17.95 \text{ kW}$$

Q3.13 **Determine the total work done by a gas system following an expansion process as shown in Figure.**

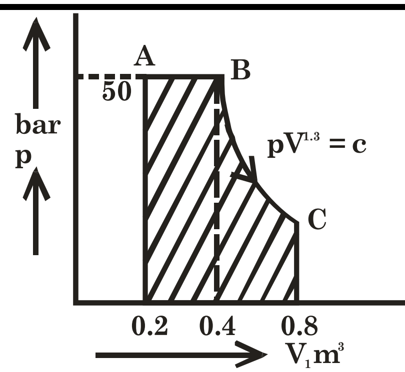


(**Ans.** 2.253 MJ)

Solution: Area under AB

$$= (0.4 - 0.2) \times 50 \times 10^5 \text{ J}$$

$$= 10^6 \text{ W} = 1 \text{ MJ}$$



Area under BC

$$= \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{50\times10^5\times0.4 - 20.31\times10^5\times0.8}{1.3-1}\,W$$

$$= 1.251\,MJ$$

Here $p_B = p_B = 50 \text{ bar} = 50\times10^5\,Pa$

$V_B = 0.4m^3$

$V_C = 0.8m^3$

$$p_C = \frac{p_B V_B^{1.3}}{V_C^{1.3}} = \frac{50\times10^5\times0.4^{1.3}}{0.8^{1.3}}$$

$$= 20.31\times10^5\,Pa$$

Total work = 2.251MJ

Q3.14 **A system of volume *V* contains a mass *m* of gas at pressure *p* and temperature *T*. The macroscopic properties of the system obey the following relationship:**

$$\left(p + \frac{a}{V^2}\right)(V - b) = mRT$$

Where *a, b,* and *R* are constants.
Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume V_1 to volume V_2. Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m³ to 10 m³ at a temperature of 293 K. Use the values $a = 15.7\times10\ Nm^4$, $b = 1.07\times10^{-2}\ m^3$, and R = 0.278 kJ/kg-K.

(**Ans.** 1742 kJ)

Solution: As it is constant temp-expansion then

$$\left(p + \frac{a}{V^2}\right)(V - b) = \text{constant } (mRT)\ (k) \text{as T = constant}$$

$$\therefore \quad \left(p_1 + \frac{a}{V_1^2}\right)(V_1 - b) = \left(p_2 + \frac{a}{V_2^2}\right)(V_2 - b) = (k)$$

$$W = \int_1^2 p\,dV \qquad \therefore \quad \left(p + \frac{a}{V}\right) = \frac{\text{constant}(k)}{V-b}$$

$$= \int_1^2 \left(\frac{k}{V-b} - \frac{a}{V^2}\right) dV \qquad \text{or} \quad p = \frac{k}{V-b} - \frac{a}{V^2}$$

$$= \left[k \ln(V-b) + \frac{a}{V}\right]_1^2 \qquad -\int \frac{1}{V^2}\,dv = \frac{1}{V} + c$$

$$= k \ln\left(\frac{V_2 - b}{V_1 - b}\right) + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$= \left[\left(p_1 + \frac{a}{V_1^2}\right)(V_1 - b)\ln\left|\frac{V_2 - b}{V_1 - b}\right| + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)\right]$$

$$\left(p + \frac{a}{V^2}\right)(V - b) = \text{constant } (mRT)\ (k) \text{as } T = \text{constant}$$

Given m = 10 kg; T = 293 K; R = 0.278 kJ/kg. K

$\therefore$ Constant k = 10 × 293 × 0.278 kJ = 814.54 kJ

a = 15.7 × 10 Nm^4; b = 1.07 × $10^{-2}m^3$

$\Rightarrow V_2 = 10m^3$, $V_1 = 1m^3$

$$\therefore \quad W = 814.54 \ln\left(\frac{10 - 1.07\times10^{-2}}{1 - 1.07\times10^{-2}}\right) + a\left(\frac{1}{10} - \frac{1}{1}\right)$$

$$= (1883.44 - a\times0.9)\text{ kJ}$$

$$= (1883.44 - 157\times0.9)\text{ kJ}$$

$$= 1742.14\text{ kJ}$$

Q3.15 **If a gas of volume 6000 cm³ and at pressure of 100 kPa is compressed quasistatically according to *pV*² = constant until the volume becomes 2000 cm³, determine the final pressure and the work transfer.**

(**Ans.** 900 kPa, – 1.2 kJ)

Solution: Initial volume (V_1) = 6000 cm³

= 0.006 m³

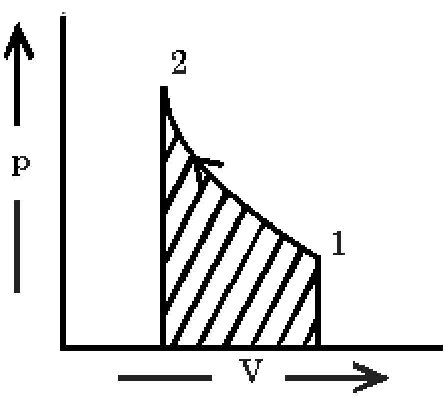
Initial pressure (p_1) = 100 kPa

Final volume (V_2) = 2000 cm³

= 0.002 m³

If final pressure (p_2)

$$\therefore \quad p_2 = \frac{p_1 V_1^2}{V_2^2} = \frac{100\times(0.006)^2}{(0.002)^2} = 900 \text{ kPa}$$



work done on the system $= \frac{1}{n-1}\left[p_2V_2 - p_1V_1\right]$

$= \frac{1}{2-1}\left[900 \times 0.002 - 100 \times 0.006\right]$ kJ

$= 1.2$ kJ

Q3.16 **The flow energy of 0.124 m³/min of a fluid crossing a boundary to a system is 18 kW. Find the pressure at this point.**

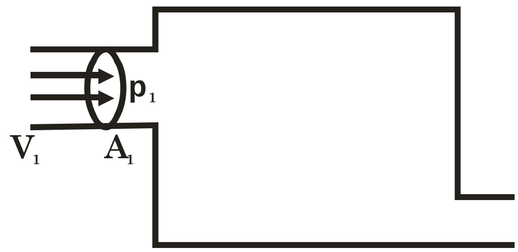
(**Ans.** 8709 kPa)

Solution: If pressure is p_1
Area is A_1
Velocity is V_1
Volume flow rate (Q) = A_1V_1

∴ Power = force × velocity

$= p_1A_1 \times V_1$

$= p_1 \times (Q)$



∴ $18 = p_1 \times \frac{0.124}{60}$

or $p_1 = \frac{18 \times 60}{0.124}$ kPa

$= 8.71$ MPa

Q3.17 **A milk chilling unit can remove heat from the milk at the rate of 41.87 MJ/h. Heat leaks into the milk from the surroundings at an average rate of 4.187 MJ/h. Find the time required for cooling a batch of 500 kg of milk from 45°C to 5°C. Take the c_p of milk to be 4.187 kJ/kg K.**

(**Ans.** 2h 13 min)

Solution: Heat to be removed (H) = mst

= 500 × 4.187 × (45-5) kJ

= 83.740 MJ

Net rate of heat removal

$= \dot{H}_{rej} - H_{leak}$

$= (41.87 - 4.187)$ MJ / h

$= 37.683$ MJ / h

∴ Time required $= \frac{83.740}{37.683}$ hr

= 2 hr. 13 min. 20 sec.

Q3.18 **680 kg of fish at 5°C are to be frozen and stored at – 12°C. The specific heat of fish above freezing point is 3.182, and below freezing point is 1.717 kJ/kg K. The freezing point is – 2°C, and the latent heat of fusion is 234.5 kJ/kg. How much heat must be removed to cool the fish, and what per cent of this is latent heat?**

(**Ans.** 186.28 MJ, 85.6%)

Solution: Heat to be removed above freezing point

= 680 × 3.182 × {5 – (-2)} kJ

= 15.146 MJ

Heat to be removed latent heat

$= 680 \times 234.5$ kJ

$= 159.460$ MJ

Heat to be removed below freezing point

$= 680 \times 1.717 \times \{-2 - (-12)\}$ kJ

$= 11.676$ MJ

$\therefore$ Total Heat = 186.2816 MJ

% of Latent heat = $\frac{159.460}{186.2816} \times 100 = 85.6$ %

4. First Law of Thermodynamics

Some Important Notes

- $đQ$ is an inexact differential, and we write

$$\int_1^2 đQ = Q_{1-2} \quad or \quad {}_1Q_2 \neq Q_2 - Q_1$$

- $đW$ is an inexact differential, and we write

$$W_{1-2} = \int_1^2 đW = \int_1^2 pdV \neq W_2 - W_1$$

- $(\Sigma Q)_{cycle} = (\Sigma W)_{cycle}$ or $\oint \delta Q = \oint \delta W$

  The summations being over the entire cycle.

- **$\delta Q - \delta W = dE$**

  E consists of ➡ E=U+KE+PE

  U -internal energy

  KE - the kinetic energy

  PE - the potential energy

  For the whole process A ➡ $Q-W=E_2-E_1$

  Similarly for the process B ➡ $Q-W=E_1-E_2$

- An isolated system which does not interact with the surroundings Q = 0 and W = 0. Therefore, E remains constant for such a system.
- The Zeroth Law deals with thermal equilibrium and provides a means for measuring temperatures.
- The First Law deals with the conservation of energy and introduces the concept of internal energy.
- The Second Law of thermodynamics provides with the guidelines on the conversion heat energy of matter into work. It also introduces the concept of entropy.
- The Third Law of thermodynamics defines the absolute zero of entropy. The entropy of a pure crystalline substance at absolute zero temperature is zero.

Summation of 3 Laws

- Firstly, there isn't a meaningful temperature of the source from which we can get the full conversion of heat to work. Only at infinite temperature one can dream of getting the full 1 kW work output.
- Secondly, more interestingly, there isn't enough work available to produce 0K. In other words, 0 K is unattainable. This is precisely the Third law.

- Because, we don't know what 0 K looks like, we haven't got a starting point for the temperature scale!! That is why all temperature scales are at best empirical.

You can't get something for nothing:

To get work output you must give some thermal energy.

You can't get something for very little:

To get some work output there is a minimum amount of thermal energy that needs to be given.

You can't get every thing:

However much work you are willing to give 0 K can't be reached.

Violation of all 3 laws:

Try to get everything for nothing.

Questions with Solution P. K. Nag

Q4.1 **An engine is tested by means of a water brake at 1000 rpm. The measured torque of the engine is 10000 mN and the water consumption of the brake is 0.5 m³/s, its inlet temperature being 20°C. Calculate the water temperature at exit, assuming that the whole of the engine power is ultimately transformed into heat which is absorbed by the cooling water.**

(**Ans.** 20.5°C)

Solution: Power = T.ω

$$= 10000 \times \left(\frac{2\pi \times 1000}{60}\right)$$

$$= 1.0472 \times 10^6 \text{ W}$$

$$= 1.0472 \text{MW}$$

Let final temperature = t°C

$\therefore$ Heat absorb by cooling water / unit $= \dot{m}\ s\ \Delta t$

$$= \dot{v}\rho s\ \Delta t$$

$$= 0.5 \times 1000 \times 4.2 \times (t - 20)$$

$\therefore \quad 0.5 \times 1000 \times 4.2 \times (t - 20) = 1.0472 \times 10^6$

$\therefore \quad t - 20 = 0.4986 \approx 0.5$

$\therefore \quad t = 20.5°\text{C}$

Q4.2 **In a cyclic process, heat transfers are + 14.7 kJ, – 25.2 kJ, – 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process?**

(**Ans.** 17.34 kJ)

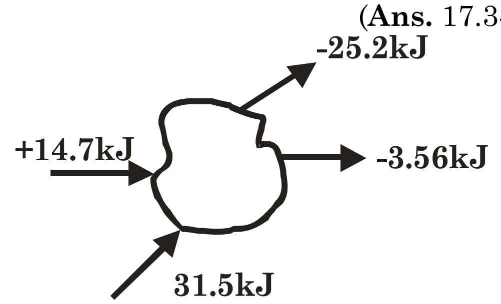
Solution : $\sum Q = (14.7 + 31.5 - 25.2 - 3.56)$ kJ

$= 17.44$ kJ

From first law of thermodynamics (for a cyclic process)

$\sum Q = \sum W$

$\therefore \sum W = 17.44$ kJ



Q4.3 **A slow chemical reaction takes place in a fluid at the constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is then removed and 105 kJ of heat flow to the surroundings as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3.**

State	**v (m³)**	**t (°C)**
1	**0.003**	**20**
2	**0.3**	**370**
3	**0.06**	**20**

For the fluid system, calculate E_2 and E_3, if $E_1 = 0$

(**Ans.** $E_2 = -29.7$ kJ, $E_3 = -110.7$ kJ)

Solution: From first law of thermodynamics

$$đQ = \Delta E + pdV$$

$$\therefore \quad Q = \Delta E + \int pdV$$

$$\therefore \quad Q_{1-2} = (E_2 - E_1) + \int_1^2 pdV$$

or $= (E_2 - E_1) + 0.1 \times 10^3(0.3 - 0.003)$ $\quad [\text{as insulated } Q_{2-3} = 0]$

or $E_2 = -29.7$ kJ

$$Q_{2-3} = (E_3 - E_2) + \int_2^3 pdV$$

or $-105 = (E_3 - E_2) + 0.1 \times 10^3 (0.06 - 0.3)$

or $-105 = E_3 + 29.7 + 0.1 \times 10^3 (0.06 - 0.3)$

or $-105 = E_3 + 29.7 - 24$

or $E_3 = -105 - 29.7 + 24$

$= -110.7$ kJ

Q4.4 **During one cycle the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid, and three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer.**

(**Ans.** – 6 kJ)

Solution: From first law of thermodynamics

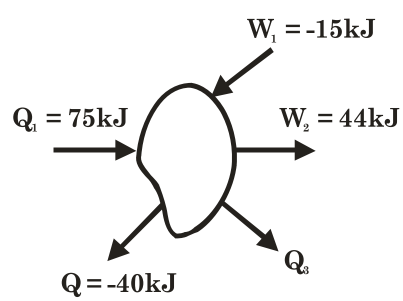
$$\sum đQ = \sum đW$$

$\therefore \quad Q_1 + Q_2 + Q_3 = W_1 + W_2$

or $75 - 40 + Q_3 = -15 + 44$

$Q_3 = -6$kJ

i.e. 6kJ from the system



Q4.5 **A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transfer for the system.**

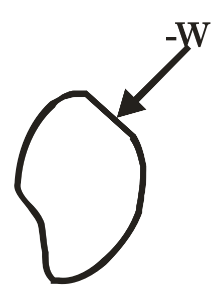
(**Ans.** – 8.6 MJ)

Solution: $Q = \Delta E + W$

$$Q_{2-1} = (E_2 - E_1) + W_{2-1}$$

$$= -5000\text{kJ} + \frac{-1000 \times 3600}{1000}\text{kJ}$$

$$= -8.6\text{MJ}$$



Q4.6 **1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg K is stirred in a well-insulated chamber causing the temperature to rise by 15°C. Find ΔE and W for the process.**

(**Ans.** Δ E = 56.25 kJ, W = – 56.25 kJ)

Solution: Heat added to the system = 1.5 × 2.5 × 15kJ

= 56.25 kJ

∴ ΔE rise = 56.25kJ

As it is insulated then $\cancel{d}Q = 0$

$$\therefore \quad \Delta Q = \Delta E + W$$

$$\text{or} \quad 0 = 56.25 + W$$

$$\text{or} \quad W = -56.25 \text{ kJ}$$

Q4.7 **The same liquid as in Problem 4.6 is stirred in a conducting chamber. During the process 1.7 kJ of heat are transferred from the liquid to the surroundings, while the temperature of the liquid is rising to 15°C. Find ΔE and W for the process.**

(**Ans.** Δ E = 54.55 kJ, W = 56.25 kJ)

Solution: As temperature rise is same so internal energy is same

ΔE = 56.25 kJ

As heat is transferred form the system so we have to give more work = 1.7 kJ to the system

So W = – 56.25 – 1.7 kJ

= –57.95 kJ

Q4.8 **The properties of a certain fluid are related as follows:**

$$u = 196 + 0.718t$$

$$pv = 0.287\ (t + 273)$$

Where u is the specific internal energy (kJ/kg), t is in °C, p is pressure (kN/m²), and v is specific volume (m³/kg). For this fluid, find c_v and c_p.

(**Ans.** 0.718, 1.005 kJ/kg K)

Solution:

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

$$= \left[\frac{\partial(u + pV)}{\partial T}\right]_p$$

$$= \left[\frac{\partial\{196 + 0.718t + 0.287(t + 273)\}}{\partial T}\right]_p$$

$$= \left[\frac{0 + 0.718\ \partial t + 0.287\ \partial t + 0}{\partial T}\right]_p$$

$$= \left[1.005\frac{\partial t}{\partial T}\right]_p \qquad \begin{bmatrix} T = t + 273 \\ \therefore \partial T = \partial t \end{bmatrix}$$

$$= 1.005 \text{ kJ / kg} - \text{K}$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$
$$= \left[\frac{\partial(196 + 0.718t)}{\partial T}\right]_v$$
$$= \left[0 + 0.718\frac{\partial t}{\partial T}\right]_v$$
$$= 0.718 \text{ kJ/kg} - \text{K}$$

Q4.9 **A system composed of 2 kg of the above fluid expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. If there is no heat transfer, find the net work for the process.**

(**Ans.** 100.52 kJ)

Solution: Heat transfer is not there so

$$Q = \Delta E + W$$
$$W = -\Delta E$$
$$= -\Delta U$$
$$= -\int_1^2 C_v dT$$
$$= -0.718(T_2 - T_1)$$
$$= -0.718(100 - 30)$$
$$= -50.26 \text{ kJ/kg}$$

∴ Total work (W) = 2 × (-50.26) = -100.52 kJ

Q 4.10 **If all the work in the expansion of Problem 4.9 is done on the moving piston, show that the equation representing the path of the expansion in the *pv*-plane is given by $pv^{1.4}$ = constant.**

Solution: Let the process is pV^n = constant.

Then

$$\text{Work done} = \frac{p_1V_1 - p_2V_2}{n-1} \qquad [\because pV = mRT]$$

$$= \frac{mRT_1 - mRT_2}{n-1} \qquad \left[\begin{aligned} R &= (c_p - c_v) \\ &= 1.005 - 0.718 \\ &= 0.287 \text{ kJ/kg} - \text{K} \end{aligned}\right]$$

$$= \frac{mR}{n-1}(T_1 - T_2)$$

or $$= \frac{2 \times 0.287 \times (100 - 30)}{n-1} = 100.52$$

or $n - 1 = 0.39972$

or $n = 1.39972 \approx 1.4$

Q4.11 **A stationary system consisting of 2 kg of the fluid of Problem 4.8 expands in an adiabatic process according to $pv^{1.2}$ = constant. The initial**

conditions are 1 MPa and 200°C, and the final pressure is 0.1 MPa. Find W and ΔE for the process. Why is the work transfer not equal to $\int pdV$?

(**Ans.** W= 217.35, ΔE = – 217.35 kJ, $\int pdV$ = 434.4 kJ)

Solution: $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{0.1}{1}\right)^{\frac{1.2-1}{1.2}}$

$$\therefore \quad T_2 = T_1 \times (0.10)^{\frac{0.2}{1.2}}$$
$$= 322.251$$
$$= 49.25°C$$

From first law of thermodynamics

$$\cancel{d}Q = \Delta E + \cancel{d}W$$
$$\therefore \quad 0 = \int C_v dT + \cancel{d}W$$
$$\therefore \quad \cancel{d}W = -\int C_v dT$$
$$= -0.718 \times \int_1^2 dT = -0.718 \times (200 - 49.25) \text{kJ/kg}$$
$$W = -2 \times W$$
$$= -2 \times 108.2356 \text{kJ}$$
$$= -216.5 \text{kJ}$$
$$\therefore \quad \Delta E = 216.5 \text{kJ}$$
$$\int pdV = \frac{p_1V_1 - p_2V_2}{n-1}$$
$$= \frac{mRT_1 - mRT_2}{n-1}$$
$$= \frac{mR(T_1 - T_2)}{n-1}$$
$$= \frac{2 \times 0.287(200 - 49.25)}{(1.2-1)}$$
$$= 432.65 \text{kJ}$$

As this is not quasi-static process so work is not $\int pdV$.

Q4.12 **A mixture of gases expands at constant pressure from 1 MPa, 0.03 m³ to 0.06 m³ with 84 kJ positive heat transfer. There is no work other than that done on a piston. Find DE for the gaseous mixture.**

(**Ans.** 54 kJ)

The same mixture expands through the same state path while a stirring device does 21 kJ of work on the system. Find ΔE, W, and Q for the process.

(**Ans.** 54 kJ, – 21 kJ, 33 kJ)

Solution: Work done by the gas $(W) = \int pdV$

$$= p(V_2 - V_1)$$

$$= 1 \times 10^3 (0.06 - 0.03) kJ$$

$$= 30 kJ$$

Heat added = 89kJ

$\therefore \quad Q = \Delta E + W$

or $\quad \Delta E = Q - W = 89 - 30 = 54 kJ$

Q4.13 **A mass of 8 kg gas expands within a flexible container so that the *p–v* relationship is of the from $pv^{1.2}$ = constant. The initial pressure is 1000 kPa and the initial volume is 1 m³. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.**

(**Ans.** + 2615 kJ)

Solution: $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\dfrac{V_1}{V_2}\right)^{n-1}$

$\therefore \quad \dfrac{p_2}{p_1} = \left(\dfrac{V_1}{V_2}\right)^{n}$

or $\quad \dfrac{V_2}{V_1} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{n}}$

or $\quad V_2 = V_1 \times \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{n}}$

$$= 1 \times \left(\frac{1000}{5}\right)^{\frac{1}{1.2}} = 82.7\,m^3$$

$\therefore \quad W = \dfrac{p_1V_1 - p_2V_2}{n-1}$

$$= \frac{1000 \times 1 - 5 \times 82.7}{1.2 - 1} = 2932.5 kJ$$

$$\Delta E = -8 \times 40 = -320\ kJ$$

$\therefore \quad Q = \Delta E + W = -320 + 2932.5 = 2612.5 kJ$

Q4.14 **A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship $p = a + bV$, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m³ and 1.20 m³. The specific internal energy of the gas is given by the relation**

$$u = 1.5\,pv - 85\ kJ/kg$$

Where p is the kPa and v is in m³/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

(**Ans.** 660 kJ, 503.3 kJ)

Solution:

$$1000 = a + b \times 0.2 \quad ....(i)$$
$$\underline{200 = a + b \times 1.2} \quad ...(ii)$$
$$(ii) - (i) \text{ gives}$$
$$-800 = b$$
$$\therefore \quad a = 1000 + 2 \times 800 = 1160$$
$$\therefore \quad p = 1160 - 800V$$
$$\therefore \quad W = \int_{v_1}^{v_2} pdV$$
$$= \int_{0.2}^{1.2} (1160 - 800V)dV$$
$$= \left[1160V - 400V^2\right]_{0.2}^{1.2}$$
$$= 1160 \times (1.2 - 0.2) - 400(1.2^2 - .2^2)\text{kJ}$$
$$= 1160 - 560\text{kJ} = 600\text{kJ}$$
$$u_1 = 1.5 \times 1000 \times \frac{0.2}{1.5} - 85 = 215\text{kJ/kg}$$
$$u_2 = 1.5 \times 200 \times \frac{1.2}{1.5} - 85 = 155\text{kJ/kg}$$
$$\therefore \quad \Delta u = u_2 - u_1 = (275 - 215) = 40\text{kJ/kg}$$
$$\therefore \Delta U = m\Delta u = 40 \times 1.5 = 60\text{kJ}$$
$$\therefore \quad Q = \Delta U + W = 60 + 600 = 660\text{kJ}$$
$$\Rightarrow \quad u = 1.5pv - 85\text{kJ/kg}$$
$$= 1.5\left(\frac{1160 - 800v}{1.5}\right)v - 85\text{kJ/kg}$$
$$= 1160v - 800v^2 - 85\text{kJ/kg}$$
$$\frac{\partial u}{\partial v} = 1160 - 1600v$$
$$\text{for maximum u,} \quad \frac{\partial u}{\partial v} = 0 \therefore \quad v = \frac{1160}{1600} = 0.725$$
$$\therefore \quad u_{max.} = 1160 \times 0.725 - 800 \times (0.725)^2 - 85\text{kJ/kg}$$
$$= 335.5\text{kJ/kg}$$
$$U_{max} = 1.5u_{max} = 503.25\text{kJ}$$

Q4.15 **The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as**

$$C_p = 2.093 + \frac{41.87}{t + 100} \text{ J/°C}$$

Where t is the temperature of the system in °C. The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2000 cm³ to 2400 cm³ and its temperature increases from 0°C to 100°C.

(a) Find the magnitude of the heat interaction.

(b) How much does the internal energy of the system increase?

(**Ans.** (a) 238.32 J (b) 197.79 J)

Solution:
$$Q = \int_{273}^{373} C_p dT \qquad t = T - 273$$

$$\therefore t + 100 = T - 173$$

$$= \int_{273}^{373} \left(2.093 + \frac{41.87}{T-173}\right) dT$$

$$= \left[2.093T + 41.87 \ln|T-173|\right]_{273}^{373}$$

$$= 2.093(373-273) + 41.87 \ln\left(\frac{200}{100}\right)$$

$$= 209.3 + 41.87 \ln 2$$

$$= 238.32J$$

$$Q = \Delta E + \int pdV$$

$$\Delta E = Q - \int pdV$$

$$= Q - p(V_2 - V_1)$$

$$= 238.32 - 101.325(0.0024 - 0.0020) \times \times 1000J$$

$$= (238.32 - 40.53)J$$

$$= 197.79J$$

Q4.16 **An imaginary engine receives heat and does work on a slowly moving piston at such rates that the cycle of operation of 1 kg of working fluid can be represented as a circle 10 cm in diameter on a *p–v* diagram on which 1 cm = 300 kPa and 1 cm = 0.1 m³/kg.**

(a) How much work is done by each kg of working fluid for each cycle of operation?

(b) The thermal efficiency of an engine is defined as the ratio of work done and heat input in a cycle. If the heat rejected by the engine in a cycle is 1000 kJ per kg of working fluid, what would be its thermal efficiency?

(**Ans.** (a) 2356.19 kJ/kg, (b) 0.702)

Solution: Given Diameter = 10 cm

$$\therefore \text{ Area} = \frac{\pi \times 10^2}{4} cm^2 = 78.54\ cm^2$$

$$1\ cm^2 \equiv 300kPa \times 0.1m^3/kg$$

$$= 30kJ$$

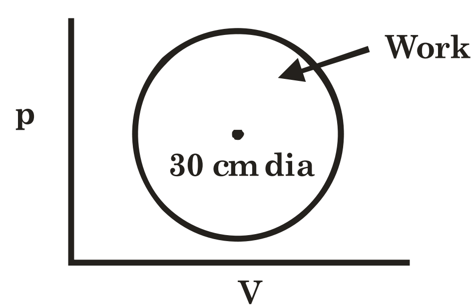
$$\therefore \text{ Total work done} = 78.54 \times 30kJ/kg$$

$$= 2356.2\,kJ/kg$$

$$\text{Heat rejected} = 1000kJ$$

$$\text{Therefore, } \eta = \frac{2356.2}{2356.2 + 1000} \times 100\%$$

$$= 70.204\%$$



Q4.17 **A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where p_1 = 1 bar, V_1 = 1.5 m³ and U_1 = 512 kJ. The processes are as follows:**

(i) Process 1–2: Compression with pV = constant to p_2 = 2 bar, U_2 = 690 kJ
(ii) Process 2–3: W_{23} = 0, Q_{23} = –150 kJ, and
(iii) Process 3–1: W_{31} = +50 kJ. Neglecting KE and PE changes, determine the heat interactions Q_{12} and Q_{31}.

(**Ans.** 74 kJ, 22 kJ)

Solution:

$$Q_{1-2} = \Delta E + \int pdV$$

$$Q_{1-2} = (u_2 - u_1) + p_1V_1\int_{v_1}^{v_2}\frac{dV}{V}$$

$$= (690 - 512) + 100 \times 1.5 \times \ln\left(\frac{p_1}{p_2}\right)$$

$$= 178 - 103.972$$

$$= 74.03 \text{kJ}$$

As $W_{2\text{-}3}$ is ZERO so it is constant volume process. As W_{31} is +ive (positive) so expansion is done.

$$\therefore\ u_3 = u_2 - 150 = 540\text{kJ}$$

$$\therefore Q_{31} = u_1 - u_3 + W$$

$$= \Delta E + W = -(540 - 512) + 50$$

$$= -28 + 50 = 22\text{kJ}$$

Q4.18 **A gas undergoes a thermodynamic cycle consisting of the following processes:**

(i) Process 1–2: Constant pressure p = 1.4 bar, V_1 = 0.028 m³, W_{12} = 10.5 kJ
(ii) Process 2–3: Compression with pV = constant, $U_3 = U_2$
(iii) Process 3–1: Constant volume, $U_1 - U_3$ = – 26.4 kJ. There are no significant changes in KE and PE.

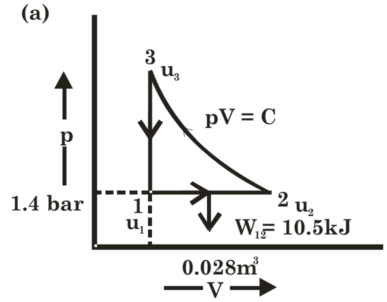
(a) Sketch the cycle on a p–V diagram
(b) Calculate the net work for the cycle in kJ
(c) Calculate the heat transfer for process 1–2
(d) Show that $\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$.

(**Ans.** (b) – 8.28 kJ, (c) 36.9 kJ)

Solution: (b) $W_{12} = 10.5$ kJ

$$W_{23} = \int_2^3 pdV$$
$$= p_2V_2\int_2^3 \frac{dV}{V}$$
$$= p_2V_2 \ln\left(\frac{V_3}{V_2}\right)$$
$$= p_2V_2 \ln\left(\frac{V_1}{V_2}\right)$$
$$= 1.4\times100\times0.103\times\ln\left(\frac{0.028}{0.103}\right)$$
$$= -18.783\text{kJ} \qquad \begin{bmatrix} \text{as} & W_{12} = p(V_2 - V_1) \\ & 10.5 = 1.4\times100(V_2 - 0.028) \\ \therefore & V_2 = 0.103 \text{ m}^3 \end{bmatrix}$$

(a)



$u_2 = u_3$
$u_1 - u_3 = -26.4$kJ
$\therefore\ u_1 = u_3 - 26.4$kJ
$= u_2 - 26.4$kJ
$u_2 = u_1 + 26.4$kJ

$W_{31} = 0$ as constant volume

$\therefore$ Net work output $= -8.283$ kJ ans.(b)

(c) $Q_{12} = U_2 - U_1 + W_{12}$
$= 26.4 + 10.5\text{kJ} = 36.9\text{kJ}$

(d) $Q_{23} = U_3 - U_2 + W_{23}$
$= 0 - 18.783\text{kJ} = -18.783\text{kJ}$

$Q_{31} = U_2 - U_3 + 0 = -26.4\text{kJ}$

$\therefore \sum Q = Q_{12} + Q_{23} + Q_{31} = 36.9\text{kJ} - 18.783 - 26.4$
$= -8.283\text{kJ}$

$\therefore \sum W = \sum Q$ Proved.

5. First Law Applied to Flow Process

Some Important Notes

- **S.F.E.E. per unit mass basis**

$$h_1 + \frac{V_1^2}{2} + gZ_1 + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2} + gZ_2 + \frac{đW}{dm}$$

[h, W, Q should be in **J/kg** and C in m/s and g in m/s²]

$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{đW}{dm}$$

[h, W, Q should be in **kJ/kg** and C in m/s and g in m/s²]

- **S.F.E.E. per unit time basis**

$$\mathrm{w}_1\left(h_1 + \frac{V_1^2}{2} + Z_1 g\right) + \frac{đQ}{d\tau}$$

$$= \mathrm{w}_2\left(h_2 + \frac{V_2^2}{2} + Z_2 g\right) + \frac{đW_x}{d\tau}$$

Where, w = mass flow rate (kg/s)

- **Steady Flow Process Involving Two Fluid Streams at the Inlet and Exit of the Control Volume**

Mass balance

$$w_1 + w_2 = w_3 + w_4$$

$$\frac{A_1 V_1}{v_1} + \frac{A_2 V_2}{v_2} = \frac{A_3 V_3}{v_3} + \frac{A_4 V_4}{v_4}$$

Where, v = specific volume (m³/kg)

Energy balance

$$w_1\left(h_1+\frac{V_1^2}{2}+Z_1 g\right)+w_2\left(h_2+\frac{V_2^2}{2}+Z_2 g\right)+\frac{\cancel{d}Q}{d\tau}$$

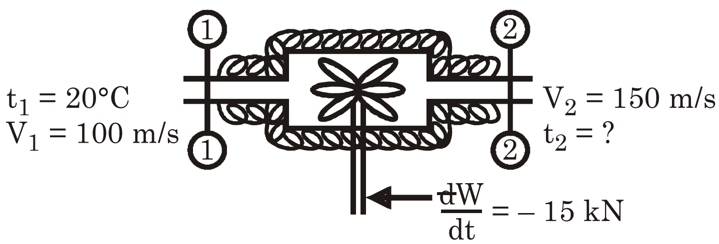
$$=w_3\left(h_3+\frac{V_3^2}{2}+Z_3 g\right)+w_4\left(h_4+\frac{V_4^2}{2}+Z_4 g\right)+\frac{\cancel{d}W_x}{d\tau}$$

Questions with Solution P. K. Nag

Q5.1 **A blower handles 1 kg/s of air at 20°C and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. Take c_p of air is 1.005 kJ/kg-K.**

(**Ans.** 28.38°C)

Solution:



From S.F.E.E.

$$w_1\left(h_1+\frac{V_1^2}{2000}+\frac{gZ_1}{1000}\right)+\frac{\cancel{d}Q}{dt}=w_2\left(h_2+\frac{V_2^2}{2000}+\frac{gZ_2}{1000}\right)+\frac{\cancel{d}W}{dt}$$

Here $w_1 = w_2 = 1\ \text{kg/s}$; $Z_1 = Z_2$; $\frac{\cancel{d}Q}{dt}=0.$

$$\therefore \quad h_1+\frac{100^2}{2000}+0=h_2+\frac{150^2}{2000}-15$$

$$\therefore \quad h_2-h_1=\left(15+\frac{100^2}{2000}-\frac{150^2}{2000}\right)$$

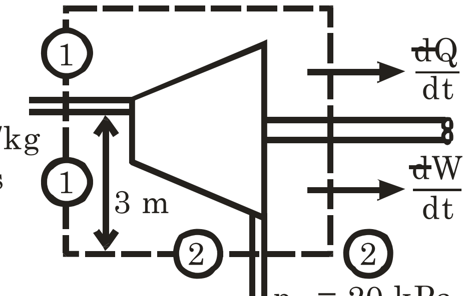
or $\quad C_p\left(t_2-t_1\right)=8.75$

or $\quad t_2=20+\frac{8.75}{1.005}=28.7°\text{C}$

Q5.2 **A turbine operates under steady flow conditions, receiving steam at the following state: Pressure 1.2 MPa, temperature 188°C, enthalpy 2785 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 20 kPa, enthalpy 2512 kJ/kg, velocity 100 m/s, and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/s. If the rate of steam flow through the turbine is 0.42 kg/s, what is the power output of the turbine in kW?**

(**Ans.** 112.51 kW)

Solution: $w_1 = w_2 = 0.42\,\text{kg/s}$



By S.F.E.E.

$$w_1\left(h_1 + \frac{V_1^2}{2000} + \frac{g Z_1}{1000}\right) + \frac{đQ}{dt} = w_2\left(h_2 + \frac{V_2^2}{2000} + \frac{g Z_2}{1000}\right) + \frac{đW}{dt}$$

or $$0.42\left\{2785 + \frac{33.3^2}{2000} + \frac{9.81 \times 3}{1000}\right\} - 0.29 = 0.42\left\{2512 + \frac{100^2}{2000} + 0\right\} + \frac{đW}{dt}$$

or $$1169.655 = 1057.14 + \frac{đW}{dt}$$

or $$\frac{đW}{dt} = 112.515 \text{ kW}$$

Q5.3 **A nozzle is a device for increasing the velocity of a steadily flowing stream. At the inlet to a certain nozzle, the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2762 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.**

(a) Find the velocity at exists from the nozzle.

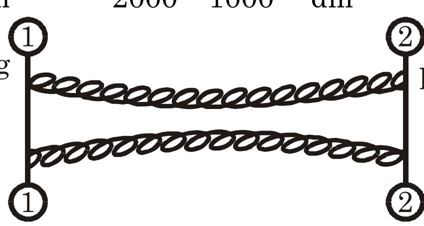
(b) If the inlet area is 0.1 m² and the specific volume at inlet is 0.187 m³/kg, find the mass flow rate.

(c) If the specific volume at the nozzle exit is 0.498 m³/kg, find the exit area of the nozzle.

(**Ans.** (a) 692.5 m/s, (b) 32.08 kg/s (c) 0.023 m²)

Solution: (a) Find V_2 i.e. Velocity at exit from S.F.E.E.

$$h_1 + \frac{V_1^2}{2000} + \frac{g Z_1}{1000} + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{g Z_2}{1000} + \frac{đW}{dm}$$



Here $Z_1 = Z_2$ and $\frac{\cancel{d}Q}{dm} = 0$ and $\frac{\cancel{d}W}{dm} = 0$

$$\therefore \quad h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$\text{or} \quad \frac{V_2^2 - V_1^2}{2000} = (h_1 - h_2)$$

$$\text{or} \quad V_2^2 = V_1^2 + 2000(h_1 - h_2)$$

$$\text{or} \quad V_2 = \sqrt{V_1^2 + 2000(h_1 - h_2)}$$

$$= \sqrt{60^2 + 2000(3000 - 2762)}\,\text{m/s}$$

$$= 692.532 \text{ m/s}$$

(b) Mass flow rate $(w) = \frac{A_1 V_1}{v_1}$

$$= \frac{0.1 \times 60}{0.187}\,\text{kg/s} = 32.1\text{kg/s}$$

(c) Mass flow rate is same so

$$32.0855613 = \frac{A_2 \times 692.532}{0.498}$$

$$\text{or} \quad A_2 = 8.023073 \text{ m}^2$$

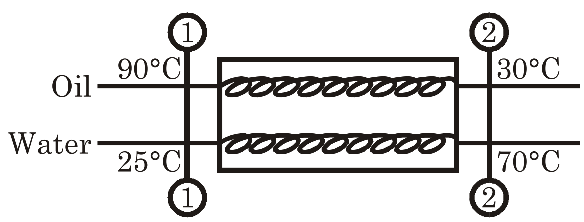
Q5.4 **In oil cooler, oil flows steadily through a bundle of metal tubes submerged in a steady stream of cooling water. Under steady flow conditions, the oil enters at 90°C and leaves at 30°C, while the water enters at 25°C and leaves at 70°C. The enthalpy of oil at t°C is given by**

$$\mathbf{h = 1.68\, t + 10.5 \times 10^{-4}\, t^2 \text{ kJ/kg}}$$

What is the cooling water flow required for cooling 2.78 kg/s of oil?

(**Ans.** 1.47 kg/s)

Solution: $w_o\,(h_{oi} + 0 + 0) + w_{H_2O}\,(h_{H_2O_i} + 0 + 0) + 0\ w_o\,(h_{o,o} + 0 + 0) + w_{H_2O}(h_{H_2O_o} + 0 + 0) + 0$



$$\therefore \quad w_o(h_{oi} - h_{o,o}) = w_{H_20}(h_{H_2O_o} - h_{H_2O_i})$$

$h_{oi} = 1.68 \times 90 + 10.5 \times 10^{-4} \times 90^2$ kJ/kg $= 159.705$ kJ/kg

$h_{o,o} = 1.68 \times 30 + 10.5 \times 10^{-4} \times 36^2$ kJ/kg $= 51.395$ kJ/kg

$$\therefore \quad W_{H_2o} = \frac{2.78 \times 108.36}{4.187\,(70 - 25)}\,\text{kg/s}$$

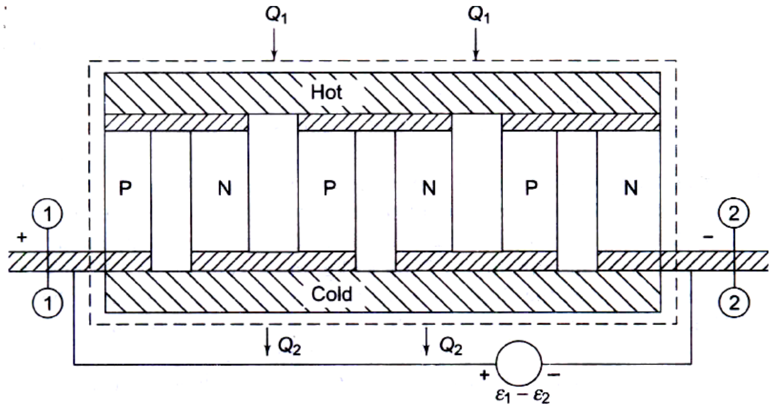
$$= 1.598815 \text{ kg/s} \simeq 1.6 \text{ kg/s}$$

Q5.5 **A thermoelectric generator consists of a series of semiconductor elements (Figure) heated on one side and cooled on the other. Electric current flow is produced as a result of energy transfer as heat. In a**

particular experiment the current was measured to be 0.5 amp and the electrostatic potential at

(1) Was 0.8 volt above that at

(2) Energy transfer as heat to the hot side of the generator was taking place at a rate of 5.5 watts. Determine the rate of energy transfer as heat from the cold side and the energy conversion efficiency.



(**Ans.** Q_2 = 5.1 watts, η = 0.073)

Solution: $\dot{Q}_1 = \dot{E} + \dot{Q}_2$

or $5.5 = 0.5 \times 0.8 + \dot{Q}_2$

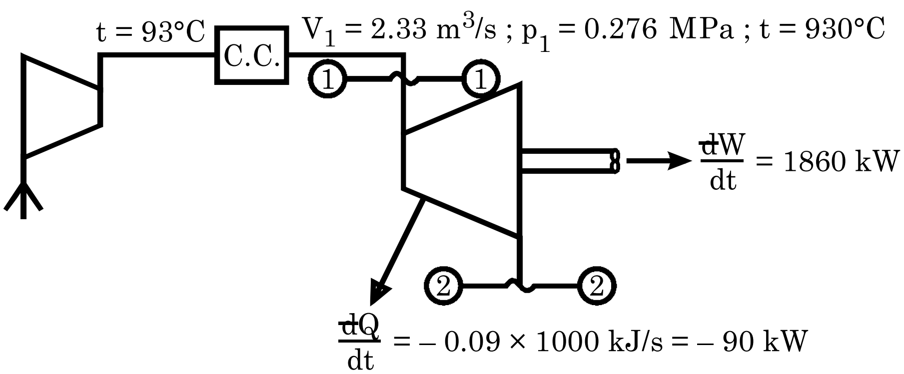
or $\dot{Q}_2 = 5.1$ watt

$$\eta = \frac{5.5 - 5.1}{5.5} \times 100\% = 7.273\%$$

Q5.6 **A turbo compressor delivers 2.33 m³/s at 0.276 MPa, 43°C which is heated at this pressure to 430°C and finally expanded in a turbine which delivers 1860 kW. During the expansion, there is a heat transfer of 0.09 MJ/s to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic and potential energy are negligible.**

(**Ans.** 157°C)

Solution:



$$w_1 h_1 + \frac{dQ}{dt} = w_2 h_2 + \frac{dW}{dt}$$

$$\therefore w_1 (h_1 - h_2) = \frac{dW}{dt} - \frac{dQ}{dt}$$

or $= 1860 - (-90) = 1950$ kW

$$p_1 V_1 = m_1 R T_1$$

$$\therefore \quad \dot{m}_1 = \frac{p_1 V_1}{RT_1} = \frac{276\ \text{kPa} \times 2.33\ \text{m}^3/\text{s}}{0.287\ \text{kJ/kg} \times 316K} = 7.091\ \text{kg/s}$$

Or $\quad h_1 - h_2 = 275$

$\therefore \quad C_p (t_1 - t_2) = 275$

or $\quad t_1 - t_2 = \dfrac{275}{1.005}\ 273.60$

$\therefore \quad t_2 = 430 - 273.60$

$\quad = 156.36^\circ$ C

Q5.7 **A reciprocating air compressor takes in 2 m³/min at 0.11 MPa, 20°C which it delivers at 1.5 MPa, 111°C to an aftercooler where the air is cooled at constant pressure to 25°C. The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in**
(a) The compressor
(b) The cooler

State your assumptions.

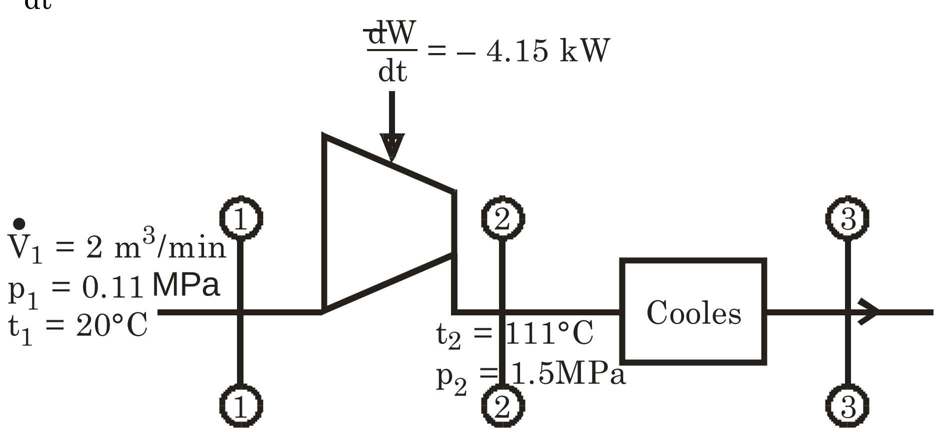
(**Ans.** – 0.17 kJ/s, – 3.76 kJ/s)

Solution:

(a) $\quad \therefore \quad w_1(h_1 + 0 + 0) + \dfrac{đQ}{dt} = w_1 h_2 + \dfrac{đW}{dt}$

$\therefore \quad 0.0436\ (111.555 - 20.1) - 4.15 = \left(\dfrac{đQ}{dt}\right)$

$\dfrac{đQ}{dt} = -0.1622$ kW $\quad$ i.e. 1622 kW loss by compressor



$$\text{Compressor work} = \frac{n}{n-1}(p_2V_2 - p_1V_1) = \frac{n}{n-1}(mRT_2 - mRT_1)$$

$$= \frac{1.4}{0.4} \times 0.0436 \times 0.287(111 - 20)\ \text{kW}$$

$$= 3.9854\ \text{kW}$$

$$\therefore \frac{đQ}{dt} = 3.9854 - 4.15 = -0.165\ \text{kW}$$

(b) $\quad \dfrac{đQ}{dt}$ For cooler

$= \dot{m}\, c_p(t_2 - t_1)$
$= 0.0436 \times 1.005 \times (111 - 25)$ kJ/s
$= 3.768348$ kW to surroundings

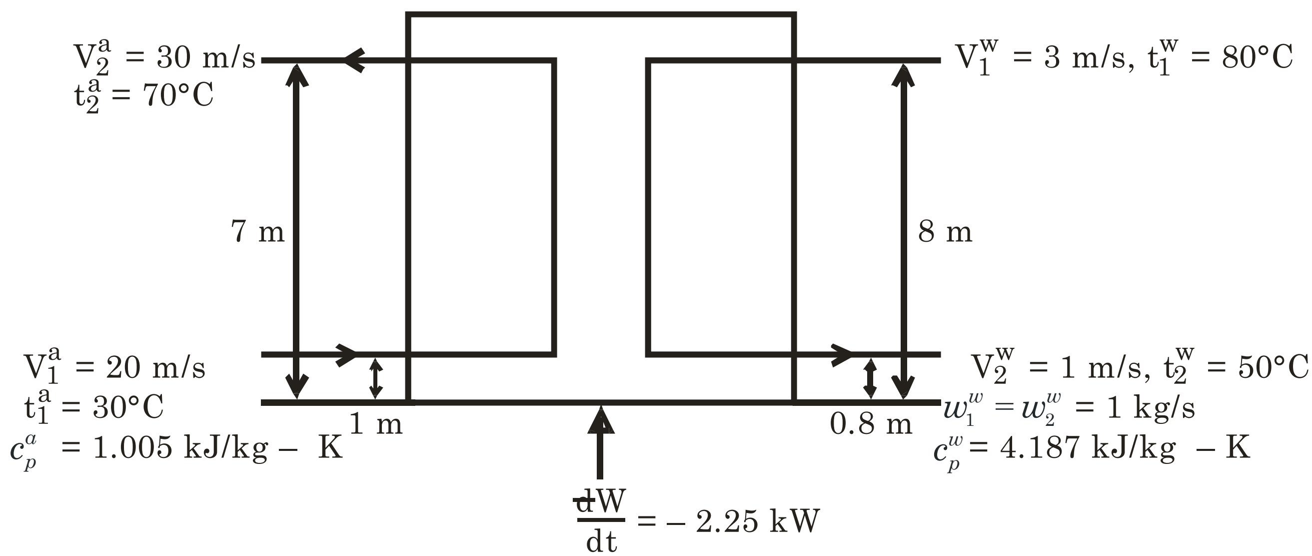
Q5.8 **In water cooling tower air enters at a height of 1 m above the ground level and leaves at a height of 7 m. The inlet and outlet velocities are 20 m/s and 30 m/s respectively. Water enters at a height of 8 m and leaves at a height of 0.8 m. The velocity of water at entry and exit are 3 m/s and 1 m/s respectively. Water temperatures are 80°C and 50°C at the entry and exit respectively. Air temperatures are 30°C and 70°C at the entry and exit respectively. The cooling tower is well insulated and a fan of 2.25 kW drives the air through the cooler. Find the amount of air per second required for 1 kg/s of water flow. The values of c_p of air and water are 1.005 and 4.187 kJ/kg K respectively.**

(**Ans.** 3.16 kg/s)

Solution: Let air required is w_1^a kg/s

$$\therefore\ w_1^a\left(h_1^a + \frac{V_1^{a2}}{2000} + \frac{gZ_1^a}{1000}\right) + w_1^w\left(h_1^w + \frac{V_1^{w^2}}{2000} + \frac{gZ_1^w}{1000}\right) + \frac{đQ}{dt}$$
$$= w_2^a\left(h_2^a + \frac{V_2^{a2}}{2000} + \frac{gZ_2^a}{1000}\right) + w_2^w\left(h_2^w + \frac{V_2^{w^2}}{2000} + \frac{gZ_2^w}{1000}\right) + \frac{đW}{dt}$$

$$\therefore\quad w_1^a = w_2^a = \text{w (say) and } \frac{đQ}{dt} = 0\ \ w_1^w = w_2^w = 1 \text{ kg/s}$$



$$\therefore\quad \left\{(h_1^a - h_2^a) + \frac{V_1^{a^2} - V_2^{a^2}}{2000} + \frac{g}{1000}(Z_1^a - Z_2^a)\right\}$$
$$= \left\{(h_2^w - h_1^w) + \frac{V_2^{w^2} - V_1^{w^2}}{2000} + \frac{g}{1000}(Z_1^w - Z_2^w)\right\} + \frac{đW}{dt}$$

Or $\quad w\left\{1.005 \times (30 - 70) + \dfrac{20^2 - 30^2}{2000} + \dfrac{9.81}{1000}(1 - 7)\right\}$

$$= 4.187\,(50-80) + \frac{1^2 - 3^2}{2000} + \frac{9.81}{1000} \times (0.8 - 8) - 2.25$$

or $\quad -\text{w} \times 40.509 = -127.9346$

$\therefore \qquad \text{w} = \dfrac{127.9346}{40.509} = 3.1582 \text{ kg/s} \approx 3.16 \text{ kg/s}$

Q5.9 **Air at 101.325 kPa, 20°C is taken into a gas turbine power plant at a velocity of 140 m/s through an opening of 0.15 m² cross-sectional area. The air is compressed heated, expanded through a turbine, and exhausted at 0.18 MPa, 150°C through an opening of 0.10 m² cross-sectional area. The power output is 375 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law**

pv = 0.287 (t + 273)

Where p is the pressure in kPa, v is the specific volume in m³/kg, and t is the temperature in °C. Take c_p = 1.005 kJ/kg K.

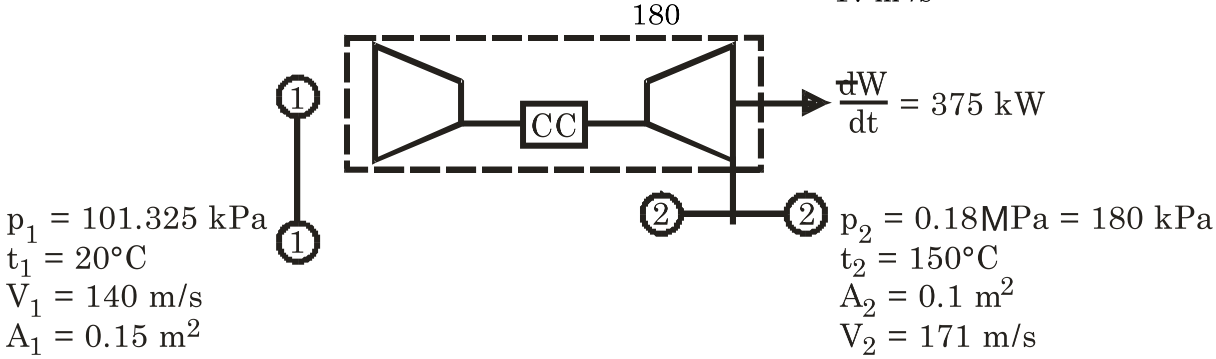
(**Ans.** 150.23 kJ/kg)

Solution: Volume flow rate at inlet $(\dot{V})_1 = V_1 A_1$ m³/s = 21 m³/s

Inlet mass flow rate $(\text{w}_1) = \dfrac{p_1 \dot{V}_1}{R\,T_1} = \dfrac{101.325 \times 21}{0.287 \times 293} = 25.304$ kg/s

Volume flow rate at outlet $= (\dot{V}_2) = \dfrac{w_2 RT_2}{p_2}$

$$= \frac{25.304 \times 0.287 \times 423}{180} = 17 \text{ m}^3\text{/s}$$



Velocity at outlet $= \dfrac{\dot{V}_2}{A_2} = \dfrac{17}{0.1} = 170.66$ m/s

$\therefore$ Using S.F.E.E.

$$w_1\left(h_1 + \frac{V_1^2}{2000} + 0\right) + \frac{đQ}{dt} = w_2\left(h_2 + \frac{V_2^2}{2000} + 0\right) + \frac{đW}{dt}$$

$$\text{w}_1 = \text{w}_2 = \text{w} = 25.304 \text{ kg/s}$$

$$\therefore \qquad \frac{đQ}{dt} = w\left\{(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000}\right\} + \frac{đW}{dt}$$

$$= w\left\{C_p(t_2 - t_1) + \frac{V_2^2 - V_1^2}{2000}\right\} + \frac{đW}{dt}$$

$$= 25.304\left\{1.005\,(150-20) + \frac{171^2 - 140^2}{2000}\right\} + 375\text{ kW}$$

$$= 3802.76\text{ kW}$$

$$\frac{\text{đ}Q}{\mathrm{d}m} = \frac{\text{đ}Q}{\mathrm{d}t}\Big/w = \frac{3802.76}{25.304} = 150.28\text{ kJ/kg}$$

Q5.10 **A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa, and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.**

(a) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible.

(b) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s.

(**Ans.** 143.3 kJ/kg, 152.9 kJ/kg)

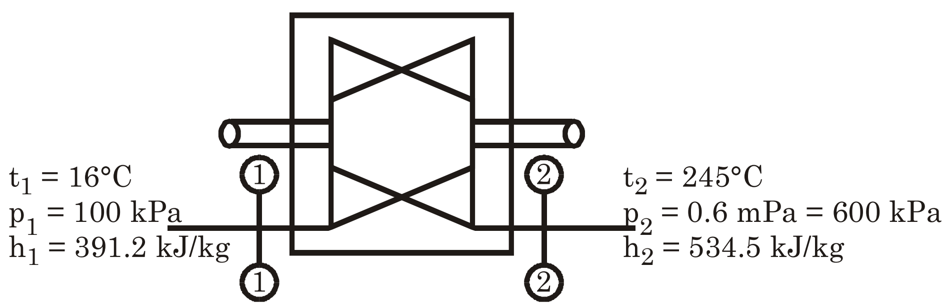
Solution: (a)

$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{\text{đ}Q}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{\text{đ}W}{dm}$$

For V_1 and V_2 negligible and $Z_1 = Z_2$ so

$$\frac{\text{đ}W}{dm} = h_1 - h_2 = (391.2 - 5345)\text{ kJ/kg}$$

$= -143.3$ kJ/kg i.e. work have to give



$$\therefore \frac{\text{đ}Q}{dt} = 0$$

(b) $V_1 = 80$ m/s; $V_2 = 160$ m/s

So $$\frac{\text{đ}W}{dm} = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2000}$$

$$= -143.3 + \frac{80^2 - 160^2}{2000}\text{ kJ/kg} = (-143.3 - 9.6)\text{ kJ/kg}$$

$= -152.9$ kJ/kg i.e. work have to give

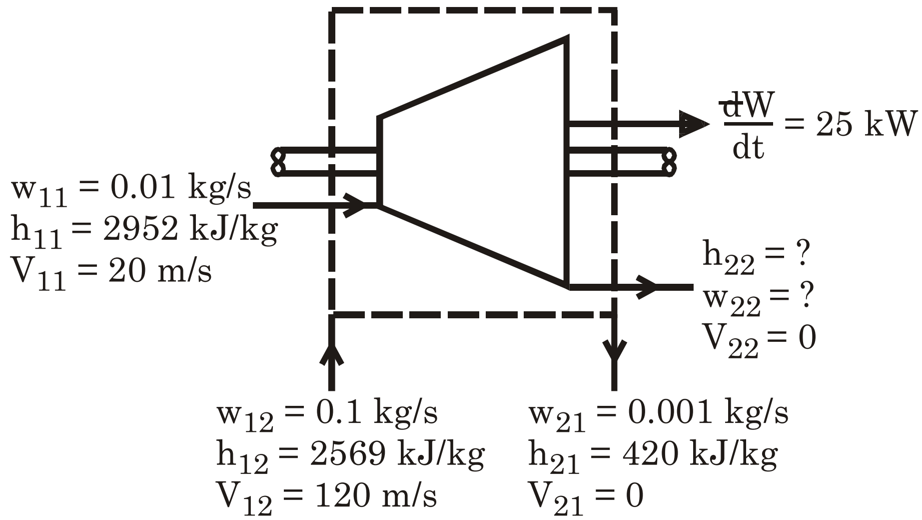
Q5.11 **The steam supply to an engine comprises two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2952 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2569 kJ/kg and a velocity of 120 m/s. At the exit from the engine the fluid leaves as two**

streams, one of water at the rate of 0.001 kg/s with an enthalpy of 420 kJ/kg and the other of steam; the fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

(**Ans.** 2402 kJ/kg)

Solution: $\therefore \frac{dQ}{dt} = 0$

By mass balance



$W_{11} + W_{12} = W_{21} + W_{22}$

$\therefore \quad W_{22} = 0.01 + 0.1 - 0.001 \text{ kg/s} = 0.109 \text{ kg/s}$

$$\therefore \quad W_{11}\left(h_{11} + \frac{V_{11}^2}{2000}\right) + W_{12}\left(h_{12} + \frac{V_{12}^2}{2000}\right) + \frac{dQ}{dt}$$

$$= W_{21}(h_{21}) + W_{22} \times h_{22} + \frac{dW}{dt}$$

$$\therefore \quad 0.01\left(2952 + \frac{20^2}{2000}\right) + 0.1\left(2569 + \frac{120^2}{2000}\right) + 0$$

$$= 0.001 \times 420 + 0.109 \times h_{22} + 25$$

or $\quad 29.522 + 257.62 = 0.42 + 0.109 \times h_{22} + 25$

or $\quad 286.722 = 0.109 \times h_{22} + 25$

or $\quad h_{22} = 2401.2$ kJ/kg

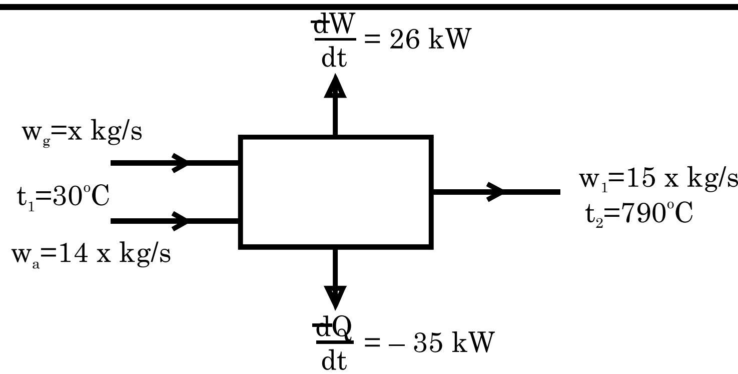
Q5.12 **The stream of air and gasoline vapour, in the ratio of 14: 1 by mass, enters a gasoline engine at a temperature of 30°C and leaves as combustion products at a temperature of 790°C. The engine has a specific fuel consumption of 0.3 kg/kWh. The net heat transfer rate from the fuel-air stream to the jacket cooling water and to the surroundings is 35 kW. The shaft power delivered by the engine is 26 kW. Compute the increase in the specific enthalpy of the fuel air stream, assuming the changes in kinetic energy and in elevation to be negligible.**

(**Ans.** – 1877 kJ/kg mixture)

Solution: In 1 hr. this m/c will produce 26 kWh for that we need fuel

$= 0.3 \times 26 = 7.8$ kg fuel/hr.

$\therefore$ Mass flow rate of fuel vapor and air mixture



$$w_1 = \frac{15 \times 7.8}{3600} \text{ kg/s} = 0.0325 \text{ kg/s}$$

Applying S.F.E.E.

$$w_1 h_1 + \frac{dQ}{dt} = w_1 h_2 + \frac{dW}{dt}$$

or $$w_1 (h_2 - h_1) = \frac{dQ}{dt} - \frac{dW}{dt}$$

$$\therefore \quad h_2 - h_1 = \frac{\frac{dQ}{dt} - \frac{dW}{dt}}{w_1}$$

$$= \frac{-35 - 26}{0.0325} = -1877 \text{ kJ/kg of mixture.}$$

Q5.13 **An air turbine forms part of an aircraft refrigerating plant. Air at a pressure of 295 kPa and a temperature of 58°C flows steadily into the turbine with a velocity of 45 m/s. The air leaves the turbine at a pressure of 115 kPa, a temperature of 2°C, and a velocity of 150 m/s. The shaft work delivered by the turbine is 54 kJ/kg of air. Neglecting changes in elevation, determine the magnitude and sign of the heat transfer per unit mass of air flowing. For air, take c_p = 1.005 kJ/kg K and the enthalpy h = c_p t.**

(**Ans.** + 7.96 kJ/kg)

Solution:

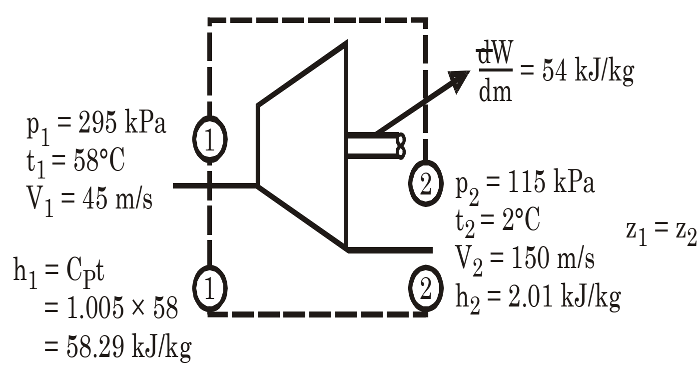
$$h_1 + \frac{V_1^2}{2000} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{dW}{dm}$$

or $$\frac{dQ}{dm} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000} + \frac{dW}{dm}$$

$$= (2.01 - 58.29) + \frac{150^2 - 45^2}{2000} + 54 \text{ kJ/kg}$$

$$= -56.28 + 10.2375 + 54 \text{ kJ/kg}$$

= 7.9575 kJ/kg (have to give to the system)



Q5.14 **In a turbo machine handling an incompressible fluid with a density of 1000 kg/m³ the conditions of the fluid at the rotor entry and exit are as given below:**

	Inlet	**Exit**
Pressure	**1.15 MPa**	**0.05 MPa**
Velocity	**30 m/s**	**15.5 m/s**

Height above datum 10 m 2 m

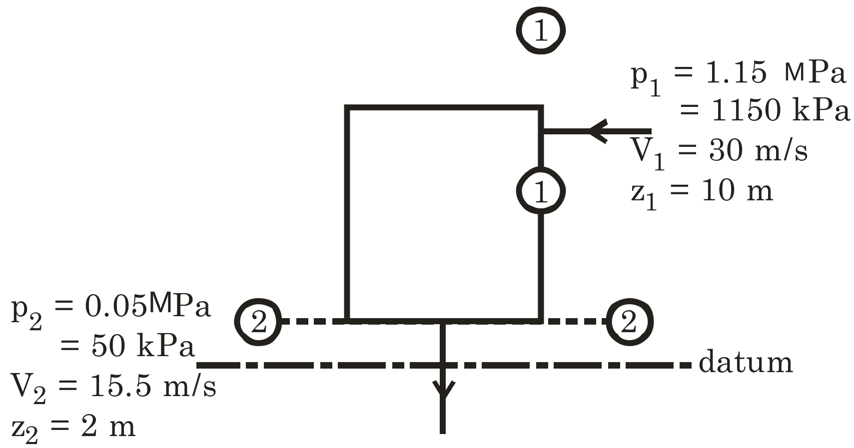
If the volume flow rate of the fluid is 40 m³/s, estimate the net energy transfer from the fluid as work.

(**Ans.** 60.3 MW)

Solution:

By S.F.E.E.

$$w\left(\frac{p_1}{\rho}+\frac{V_1^2}{2}+gZ_1\right)+\frac{dQ}{dt}=w\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}+gZ_2\right)+\frac{dW}{dt}$$



Flow rate = 40 m³/s ≡ 40 × 1000 kg/s = w (say)

$$\therefore \quad 40000\left(\frac{1150}{1000}+\frac{30^2}{2000}+\frac{9.81\times 10}{1000}\right)+0$$

$$\text{Or}\quad \frac{dW}{dt}=40000\left\{\frac{p_1-p_2}{\rho}+\frac{V_1^2-V_2^2}{2}+g(Z_1-Z_2)\right\}$$

$$=40000\left\{\frac{1150-50}{1000}+\frac{30^2-15.5^2}{2000}+\frac{9.81\times(10-2)}{1000}\right\}\text{kW}$$

$$=60.3342\text{ MW}$$

Q5.15 **A room for four persons has two fans, each consuming 0.18 kW power, and three 100 W lamps. Ventilation air at the rate of 80 kg/h enters with an enthalpy of 84 kJ/kg and leaves with an enthalpy of 59 kJ/kg. If each person puts out heat at the rate of 630 kJ/h determine the rate at which heat is to be removed by a room cooler, so that a steady state is maintained in the room.**

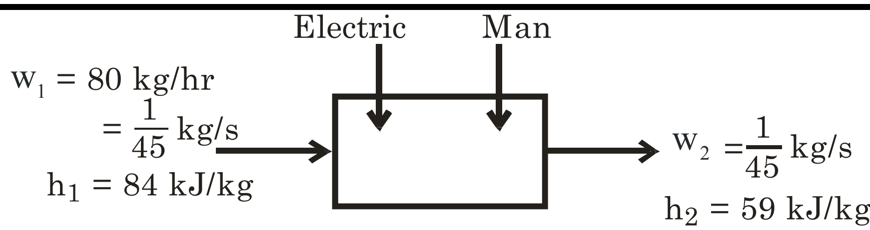
(**Ans.** 1.92 kW)

Solution:

$$\frac{dQ_{person}}{dt}=+\frac{4\times 630}{3600}\text{kJ/s}=0.7\text{ kW}$$

$$\frac{dQ_{electic}}{dt}=+0.18\times 2+\frac{3\times 100}{1000}\text{kW}=0.66\text{ kW}$$

$$\therefore \quad \frac{dQ}{dt}=1.36\text{ kW}$$



For steady state

$$w_1 h_1 + \frac{\cancel{d}Q}{dt} = w_2 h_2 + \frac{\cancel{d}W}{dt}$$

$$\therefore \quad \frac{\cancel{d}W}{dt} = w_1 h_1 - w_2 h_2 + \frac{\cancel{d}Q}{dt} = \frac{1}{45} \times (84 - 59) + 1.36 \text{ kW}$$

$$= 1.9156 \text{ kW}$$

Q5.16 **Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m³/kg, and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m³/kg. The internal energy of the air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 59 W. Calculate the power required to drive the compressor and the inlet and outlet cross-sectional areas.**

(**Ans.** 45.4 kW, 0.057 m², 0.0142 m²)

Solution: **By S.F.E.E.**

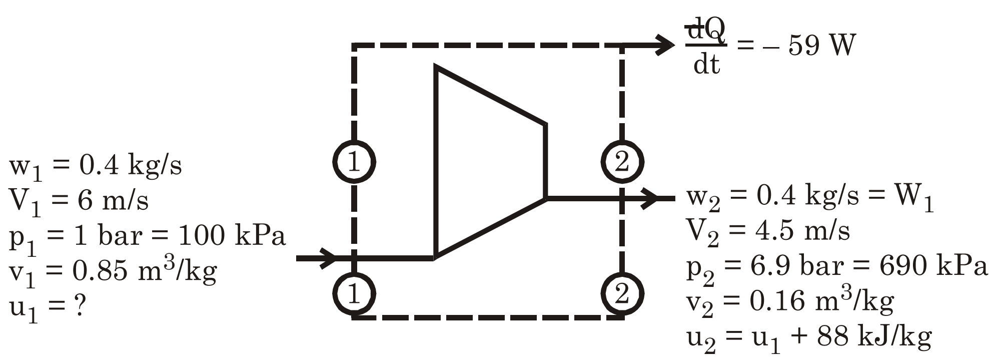
$$w_1\left(u_1 + p_1 v_1 + \frac{V_1^2}{2000} + 0\right) + \frac{\cancel{d}Q}{dt} = w_2\left(u_2 + p_2 v_2 + \frac{V_2^2}{2000} + 0\right) + \frac{\cancel{d}W}{dt}$$

$$\text{Or} \quad \frac{\cancel{d}W}{dt} = \left[(u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \frac{V_1^2 - V_2^2}{2000}\right] + \frac{\cancel{d}Q}{dt}$$

$$= 0.4\,[-88 + 85 - 110.4 + 0.0076] - 0.059$$

$$= -45.357 - 0.059$$

$$= -45.416 \text{ kW [have to give to compressor]}$$



$$w_1 = \frac{A_1 V_1}{v_1} \qquad \therefore A_1 = \frac{w_1 v_1}{V_1} = \frac{0.4 \times 0.85}{6} = 0.0567 \text{ m}^2$$

$$w_2 = \frac{A_2 V_2}{v_2} \qquad \therefore A_2 = \frac{w_2 v_2}{V_2} = \frac{0.4 \times 0.16}{4.5} = 0.01422 \text{ m}^2$$

6. Second Law of Thermodynamics

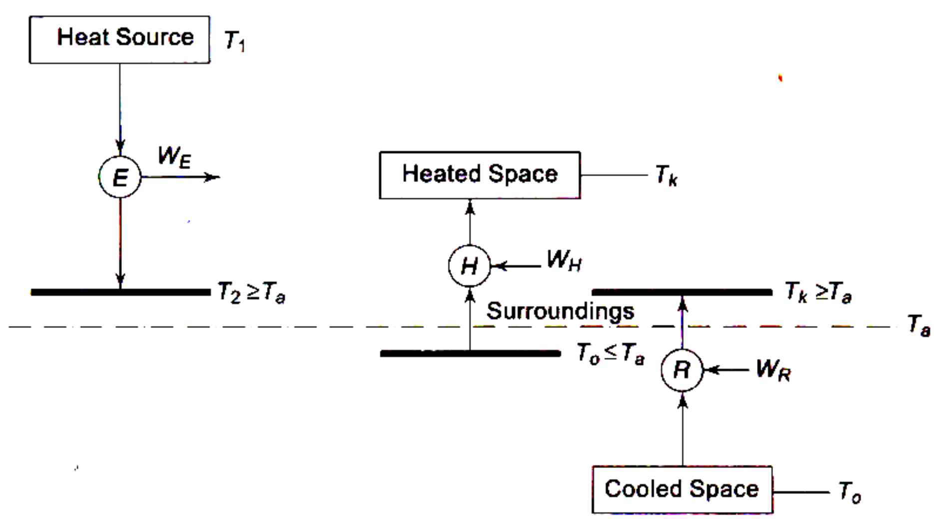
Some Important Notes

Regarding Heat Transfer and Work Transfer

- Heat transfer and work transfer are the energy interactions.
- Both heat transfer and work transfer are boundary phenomena.
- It is wrong to say 'total heat' or 'heat content' of a closed system, because heat or work is not a property of the system.
- Both heat and work are path functions and inexact differentials.
- Work is said to be a high grade energy and heat is low grade energy.
- HEAT and WORK are NOT properties because they depend on the path and end states.
- HEAT and WORK are not properties because their net change in a cycle is not zero.
- **Clausius' Theorem:** The cyclic integral of $đQ/T$ for a reversible cycle is equal to zero.

$$or \qquad \oint_R \frac{đQ}{T} = 0$$

- The more effective way to increase the cycle efficiency is **to decrease T_2.**
- **Comparison of heat engine, heat pump and refrigerating machine**



$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$hence,\ \eta_{Carnot,HE} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

$$COP_{Carnot,HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$COP_{Carnot,R} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

Questions with Solution P. K. Nag

Q6.1 **An inventor claims to have developed an engine that takes in 105 MJ at a temperature of 400 K, rejects 42 MJ at a temperature of 200 K, and delivers 15 kWh of mechanical work. Would you advise investing money to put this engine in the market?**

(**Ans.** No)

Solution: Maximum thermal efficiency of his engine possible

$$\eta_{max} = 1 - \frac{200}{400} = 50\%$$

$\therefore$ That engine and deliver output = $\eta \times$ input

$$= 0.5 \times 105 \text{ MJ}$$
$$= 52.5 \text{ MJ} = 14.58 \text{ kWh}$$

As he claims that his engine can deliver more work than ideally possible so I would not advise to investing money.

Q6.2 **If a refrigerator is used for heating purposes in winter so that the atmosphere becomes the cold body and the room to be heated becomes the hot body, how much heat would be available for heating for each kW input to the driving motor? The COP of the refrigerator is 5, and the electromechanical efficiency of the motor is 90%. How does this compare with resistance heating?**

(**Ans.** 5.4 kW)

Solution: $$\text{COP} = \frac{\text{desired effect}}{\text{input}}$$

$$(\text{COP})_{\text{ref.}} = (\text{COP})_{\text{H.P}} - 1$$

or $$6 = \frac{H}{W}$$ $\therefore (\text{COP})_{\text{H.P.}} = 6$

So input (W) = $\frac{H}{6}$

But motor efficiency 90% so

Electrical energy require (E) = $\frac{W}{0.9} = \frac{H}{0.9 \times 6}$

$$= 0.1852 \text{ H}$$
$$= 18.52\% \text{ of Heat (direct heating)}$$

$$H = \frac{100}{18.52} \frac{\text{kW}}{\text{kW of work}} = 5.3995 \text{ kW}$$

Q6.3 **Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerator?**

(**Ans.** 666.67 kJ)

If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

(**Ans.** 1.8 MJ)

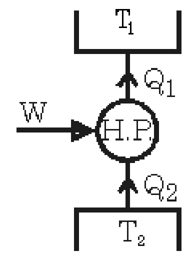
Solution: COP of the Ref. is 5

So for each MJ removed from the cold body we need work

$$= \frac{1\,MJ}{5} = 200\text{ kJ}$$

For 200 kJ work output of heat engine hair $\eta = 30\%$

We have to supply heat = $\frac{200\,\text{kJ}}{0.3} = 666.67$ kJ



Now

COP of H.P. = COP of Ref. + 1

= 5 + 1 = 6

Heat input to the H.E. = 1 MJ

∴ Work output (W) = 1 × 0.3 MJ = 300 kJ

That will be the input to H.P.

$\therefore\ (\text{COP})_{\text{H.P}} = \frac{Q_1}{W}$

$\therefore\ Q_1 = (\text{COP})_{\text{H.P.}} \times W = 6 \times 300\text{ kJ} = 1.8\text{ MJ}$

Q6.4 **An electric storage battery which can exchange heat only with a constant temperature atmosphere goes through a complete cycle of two processes. In process 1–2, 2.8 kWh of electrical work flow into the battery while 732 kJ of heat flow out to the atmosphere. During process 2–1, 2.4 kWh of work flow out of the battery.**

(a) Find the heat transfer in process 2–1.

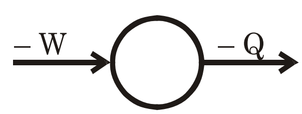
(b) If the process 1–2 has occurred as above, does the first law or the second law limit the maximum possible work of process 2–1? What is the maximum possible work?

(c) If the maximum possible work were obtained in process 2–1, what will be the heat transfer in the process?

(**Ans.** (a) – 708 kJ (b) Second law, $W_{2-1} = 9348$ kJ (c) $Q_{2-1} = 0$)

Solution: **From the first Law of thermodynamics**

(a) For process 1–2



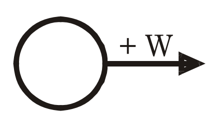
$Q_{1-2} = E_2 - E_1 + W_{1-2}$

$-732 = (E_2 - E_1) - 10080$

$[2.8\text{ kWh} = 2.8 \times 3600\text{ kJ}]$

$\therefore\ E_2 - E_1 = 9348$ kJ

For process 2–1



$Q_{21} = E_1 - E_2 + W_{21}$

$= -9348 + 8640$

$= -708$ kJ i.e. Heat flow out to the atmosphere.

(b) Yes Second Law limits the maximum possible work. As Electric energy stored in a battery is High grade energy so it can be completely converted to the work. Then,

W = 9348 kJ

(c) $Q_{21} = -9348 + 9348 = 0$ kJ

Q6.5 **A household refrigerator is maintained at a temperature of 2°C. Every time the door is opened, warm material is placed inside, introducing an average of 420 kJ, but making only a small change in the temperature of the refrigerator. The door is opened 20 times a day, and the refrigerator operates at 15% of the ideal COP. The cost of work is Rs. 2.50 per kWh. What is the monthly bill for this refrigerator? The atmosphere is at 30°C.**

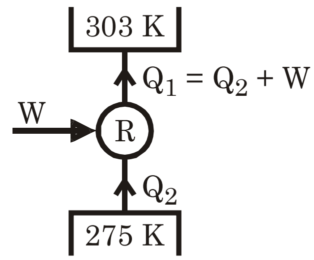
(**Ans.** Rs. 118.80)

Solution: Ideal COP of Ref. $= \dfrac{275}{30-2} = \dfrac{275}{28} = 9.82143$

Actual COP $= 0.15 \times \text{COP}_{\text{ideal}} = 1.4732$

Heat to be removed in a day

$(Q_2) = 420 \times 20$ kJ
$= 8400$ kJ



$\therefore$ Work required $= 5701.873$ kJ/day
$= 1.58385$ kWh/day

Electric bill per month $= 1.58385 \times 0.32 \times 30$ Rupees
$=$ Rs. 15.20

Q6.6 **A heat pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C. The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840°C and rejects heat to a reservoir at 60°C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, determine**

(a) The rate of heat supply from the 840°C source

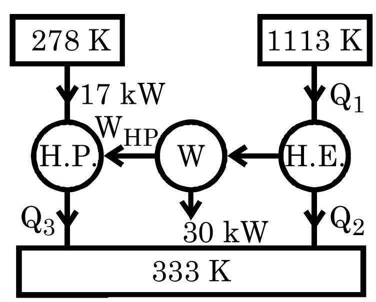
(b) The rate of heat rejection to the 60°C sink.

(**Ans.** (a) 47.61 kW; (b) 34.61 kW)

Solution:

COP of H.P.

$$= \frac{333}{333-278} = 6.05454$$



$$Q_3 = W_{H.P.} + 17$$

$$\therefore \quad \frac{W_{H.P.} + 17}{W_{H.P.}} = 6.05454$$

$$\therefore \quad \frac{17}{W_{H.P.}} = 5.05454$$

$$\therefore \quad W_{H.P.} = \frac{17}{5.05454} = 3.36 \text{ kW}$$

$\therefore$ Work output of the Heat engine

$W_{H.E.} = 30 + 3.36 = 33.36$ kW

$$\eta \text{ of the H.E.} = 1 - \frac{333}{1113} = 0.7$$

(a) $\therefore \quad \dfrac{W}{Q_1} = 0.7$

$\therefore \quad Q_1 = \dfrac{W}{0.7} = 47.61$ kW

(b) Rate of heat rejection to the 333 K
(i) From H.E. = $Q_1 - W = 47.61 - 33.36 = 14.25$ kW
(ii) For H.P. = $17 + 3.36 = 20.36$ kW
$\therefore$ Total = 34.61 kW

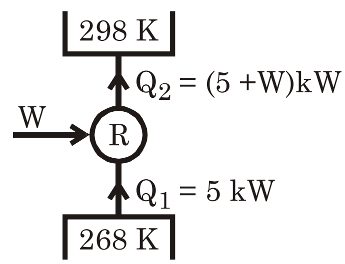
Q6.7 **A refrigeration plant for a food store operates as a reversed Carnot heat engine cycle. The store is to be maintained at a temperature of – 5°C and the heat transfer from the store to the cycle is at the rate of 5 kW. If heat is transferred from the cycle to the atmosphere at a temperature of 25°C, calculate the power required to drive the plant.**

(**Ans.** 0.56 kW)

Solution: $(\text{COP})_R = \dfrac{268}{298 - 268} = 8.933$

$= \dfrac{5\text{ kW}}{W}$

$\therefore \quad W = \dfrac{5}{8.933}\text{ kW} = 0.56$ kW

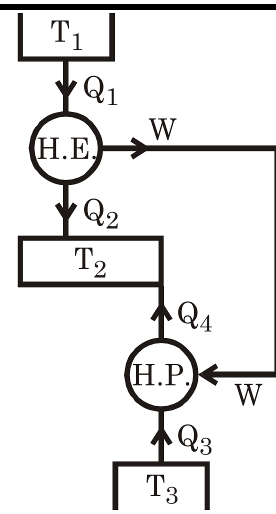


Q6.8 **A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.**

(**Ans.** 1.81)

Solution: For H.E.

$1 - \dfrac{Q_2}{Q_1} = 0.27$



$$\frac{Q_2}{Q_1} = 0.73$$

$$Q_2 = 0.73\ Q_1$$

$$W = Q_1 - Q_2 = 0.27\ Q_1$$

For H.P.

$$\frac{Q_4}{W} = 4$$

$\therefore$ $Q_4 = 4W = 1.08\ Q_1$

$\therefore$ $Q_2 + Q_4 = (0.73 + 1.08)\ Q_1 = 1.81\ Q_1$

$$\therefore \quad \frac{\text{Heat transfer to the circulating water}}{\text{Heat for to the Heat Engine}}$$

$$= \frac{1.81\ Q_1}{Q_1} = 1.81$$

Q6.9 **If 20 kJ are added to a Carnot cycle at a temperature of 100°C and 14.6 kJ are rejected at 0°C, determine the location of absolute zero on the Celsius scale.**

(**Ans.** – 270.37°C)

Solution: $\dfrac{Q_1}{Q_2} = \dfrac{\phi(t_1)}{\phi(t_2)}$ Let ϕ (t) = at + b

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{at_1 + b}{at_2 + b}$$

$$\text{or} \quad \frac{20}{14.6} = \frac{a \times 100 + b}{a \times 0 + b} = \frac{a}{b} \times 100 + 1$$

$$\therefore \quad \frac{a}{b} = 3.6986 \times 10^{-3}$$

For absolute zero, $Q_2 = 0$

$$\therefore \quad \frac{Q_1}{0} = \frac{a \times 100 + b}{a \times t + b}$$

or $a \times t + b = 0$

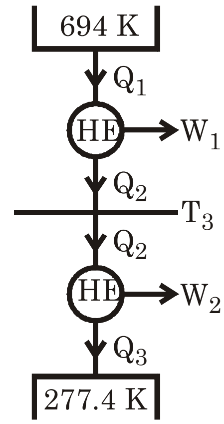
$$\text{or} \quad t = \frac{-b}{a} = -\frac{1}{3.6986 \times 10^{-3}} = -270.37^{\circ}\ \text{C}$$

Q6.10 **Two reversible heat engines *A* and *B* are arranged in series, A rejecting heat directly to *B.* Engine *A* receives 200 kJ at a temperature of 421°C from a hot source, while engine *B* is in communication with a cold sink at a temperature of 4.4°C. If the work output of *A* is twice that of *B,* find**

(a) The intermediate temperature between *A* and *B*
(b) The efficiency of each engine
(c) The heat rejected to the cold sink

(**Ans.** 143.4°C, 40% and 33.5%, 80 kJ)

Solution: $\dfrac{Q_1}{694} = \dfrac{Q_2}{T} = \dfrac{Q_1 - Q_2}{694 - T} = \dfrac{Q_3}{277.4} = \dfrac{Q_2 - Q_3}{T - 277.4}$



Hence $Q_1 - Q_2 = 2\ W_2$

$Q_2 - Q_3 = W_2$

$\therefore \quad \dfrac{2}{694 - T} = \dfrac{1}{T - 277.4}$

or $\quad 2T - 277.4 \times 2 = 694 - T$

or $\quad T = 416.27\text{ K} = 143.27^\circ\text{ C}$

(b) $\quad \eta_1 = 40\%$

$\eta_2 = 1 - \dfrac{277.4}{416.27} = 33.36\%$

(c) $\quad Q_2 = \dfrac{416.27}{694} \times 200\text{ kJ} = 119.96\text{ kJ}$;

$Q_1 = \dfrac{277.4}{416.27} \times 119.96 = 79.94\text{ kJ}$

Q6.11 **A heat engine operates between the maximum and minimum temperatures of 671°C and 60°C respectively, with an efficiency of 50% of the appropriate Carnot efficiency. It drives a heat pump which uses river water at 4.4°C to heat a block of flats in which the temperature is to be maintained at 21.1°C. Assuming that a temperature difference of 11.1°C exists between the working fluid and the river water, on the one hand, and the required room temperature on the other, and assuming the heat pump to operate on the reversed Carnot cycle, but with a COP of 50% of the ideal COP, find the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more wasteful?**

(**Ans.** 0.79 kJ/kJ heat input)

Solution: Carnot efficiency $(\eta) = 1 - \dfrac{273 + 60}{273 + 671} = 1 - \dfrac{333}{944} = 0.64725$

Actual $(\eta) = 0.323623 = 1 - \dfrac{Q_1'}{Q_1}$

$\therefore \quad \frac{Q_1'}{Q_1} = 0.6764$

Ideal COP

$$= \frac{305.2}{305.2 - 266.4} = 7.866$$

Actual COP

$$= 3.923 = \frac{Q_3}{W}$$

if $Q_3 = 1$ kJ

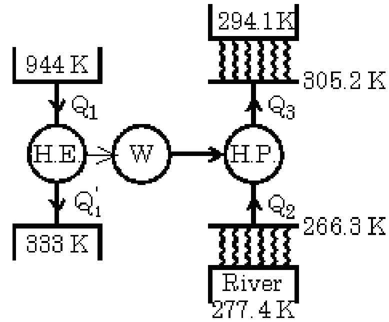
$$\therefore \quad W = \frac{Q_3}{3.923} = \frac{1}{3.923}$$

= 0.2549 kJ/kJ heat input to block

$W = Q_1 - Q_1' = Q_1 - 0.6764\ Q_1 = 0.2549$

$$Q_1 = \frac{0.2549}{(1 - 0.6764)}$$

= 0.7877 kJ/kJ heat input to block.



Q6.12 **An ice-making plant produces ice at atmospheric pressure and at 0°C from water. The mean temperature of the cooling water circulating through the condenser of the refrigerating machine is 18°C. Evaluate the minimum electrical work in kWh required to produce 1 tonne of ice (The enthalpy of fusion of ice at atmospheric pressure is 333.5 kJ/kg).**

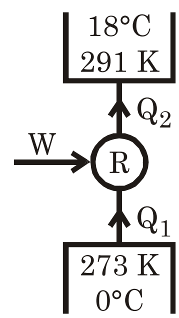
(**Ans.** 6.11 kWh)

Solution: Maximum (COP) $= \frac{273}{291 - 273} = 15.2$

$$\therefore \quad \frac{Q}{W_{min}} = 15.2$$

or $\quad W_{min} = \frac{Q}{15.2} = \frac{1000 \times 333.5}{15.2}$ kJ

= 21.989 MJ = 6.108 kWh

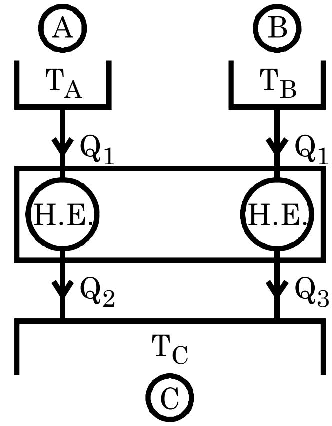


Q6.13 **A reversible engine works between three thermal reservoirs, A, B and C. The engine absorbs an equal amount of heat from the thermal reservoirs A and B kept at temperatures T_A and T_B respectively, and rejects heat to the thermal reservoir C kept at temperature T_C. The efficiency of the engine is α times the efficiency of the reversible engine, which works between the two reservoirs A and C. prove that**

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C}$$

Solution: η of H.E. between A and C

$$\eta_A = \left(1 - \frac{T_C}{T_A}\right)$$



η of our engine $= \alpha\left(1 - \frac{T_C}{T_A}\right)$

Here $Q_2 = \frac{Q_1}{T_A} \times T_C = Q_3 = \frac{Q_1}{T_B} \times T_C$

$\therefore$ Total Heat rejection

$$(Q_2 + Q_3) = Q_1 T_C\left(\frac{1}{T_A} + \frac{1}{T_B}\right)$$

Total Heat input = $2Q_1$

$$\eta \text{ of engine } = \left[1 - \frac{Q_1 T_c\left(\frac{1}{T_A} + \frac{1}{T_B}\right)}{2Q_1}\right]$$

$$\therefore \quad \alpha - \frac{\alpha T_C}{T_A} = 1 - \frac{T_C}{2T_A} - \frac{T_C}{2T_B}$$

Multiply both side by T_A and divide by T_C

or $$\alpha\frac{T_A}{T_C} - \alpha = \frac{T_A}{T_C} - \frac{1}{2} - \frac{1}{2}\frac{T_A}{T_B}$$

or $$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1-\alpha)\frac{T_A}{T_C} \text{ Proved}$$

Q6.14 **A reversible engine operates between temperatures T_1 and T (T_1 > T). The energy rejected from this engine is received by a second reversible engine at the same temperature T. The second engine rejects energy at temperature T_2 (T_2 < T).**
Show that:
(a) Temperature T is the arithmetic mean of temperatures T_1 and T_2 if the engines produce the same amount of work output
(b) Temperature T is the geometric mean of temperatures T_1 and T_2 if the engines have the same cycle efficiencies.

Solution: (a) If they produce same Amount and work

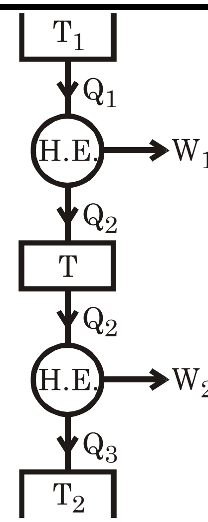
Then $W_1 = W_2$

or $\eta_1 Q_1 = \eta_2 Q_2$

or $$\left(1 - \frac{T}{T_1}\right)\left(\frac{T_1}{T}\right)Q_2 = \left(1 - \frac{T_2}{T}\right)Q_2$$

We know that $\frac{Q_1}{T_1} = \frac{Q_2}{T}$

or $$Q_1 = \frac{T_1}{T}Q_2$$



or $$\frac{T_1}{T} - 1 = 1 - \frac{T_2}{T}$$

or $$\frac{T_1 + T_2}{T} = 2$$

or $$T = \frac{T_1 + T_2}{2}$$

i.e., Arithmetic mean and T_1, T_2

(b) If their efficiency is same then

$$1 - \frac{T}{T_1} = 1 - \frac{T_2}{T}$$

or $$T = \sqrt{T_1 T_2}$$

(as T is + ve so –ve sign neglected)

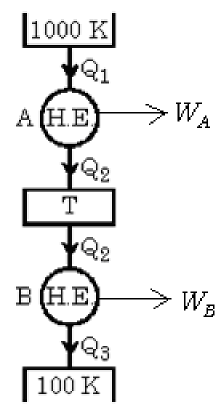
∴ T is Geometric mean of T_1 and T_2.

Q6.15 **Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at 1000 K and 100 K respectively. Engine A receives 1680 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine**

(a) The heat rejected by engine B
(b) The temperature at which heat is rejected by engine, A
(c) The work done during the process by engines, A and B respectively. If engines A and B deliver equal work, determine
(d) The amount of heat taken in by engine B
(e) The efficiencies of engines A and B

(**Ans.** (a) 168 kJ, (b) 316.2 K, (c) 1148.7, 363.3 kJ, (d) 924 kJ, (e) 45%, 81.8%)

Solution: As their efficiency is same so



$$\eta_A = \eta_B$$

or $$1 - \frac{T}{1000} = 1 - \frac{100}{T}$$

(b) $T = \sqrt{1000 \times 100} = 316.3\text{K}$

$$Q_2 = \frac{Q_1}{1000} \times T = \frac{1680 \times 316.3}{1000}$$

$$= 531.26 \text{ kJ}$$

(a) $$Q_3 = \frac{Q_2}{316.3} \times 100 = \frac{531.26 \times 100}{316.3}$$

$= 168$ kJ as (a)

(c) $W_A = Q_1 - Q_2 = (1880 - 531.26)$ kJ
$= 1148.74$ kJ

$W_B = (531.26 - 168)$ kJ
$= 363.26$ kJ

(d) If the equal work then $T = \dfrac{100 + 1000}{2} = 550$ K

$$\therefore\ Q_2 = \frac{Q_1}{1000} \times T = \frac{1680 \times 550}{1000} = 924 \text{ kJ}$$

(e) $\eta_A = 1 - \dfrac{550}{1000} = 0.45$

$\eta_B = 1 - \dfrac{100}{550} = 0.8182$

Q6.16 **A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 20°C. Heat transfer through the walls and roof is estimated to be 0.525 kJ/s per degree temperature difference between the inside and outside.**

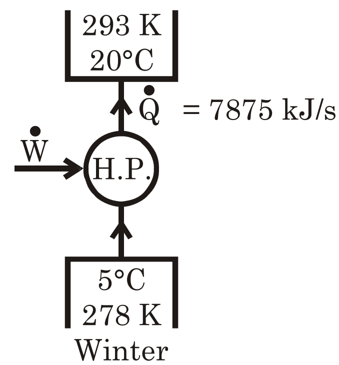
(a) If the outside temperature in winter is 5°C, what is the minimum power required to drive the heat pump?

(b) If the power output is the same as in part (a), what is the maximum outer temperature for which the inside can be maintained at 20°C?

(**Ans.** (a) 403 W, (b) 35.4°C)

Solution : (a) Estimated Heat rate

$= 0.525 \times (20 - 5)$ kJ/s = 7.875 kJ/s

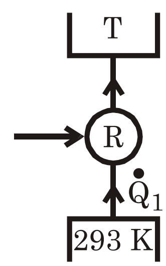


$$\text{COP} = \frac{293}{293 - 278} = 19.53$$

$$W_{min} = \frac{\dot{Q}}{(\text{COP})_{max}}$$

$$= \frac{7.875}{19.53} = 0.403 \text{ kW} = 403 \text{ W}$$

(b) Given $\dot{W}$ = 403 W



Heat rate $(\dot{Q}_1) = 0.525\,(T - 293)$ kW

$= 525(T - 293)$ W

$$\therefore\quad \text{COP} = \frac{525(T - 293)}{403} = \frac{293}{(T - 293)}$$

$$\text{or } (T - 293) = \frac{403 \times 293}{525} = 15$$

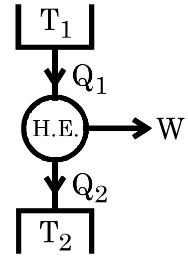
or $T = 308 \text{ K} = 35^\circ \text{ C}$

$\therefore$ Maximum outside Temperature = 35°C

Q6.17 **Consider an engine in outer space which operates on the Carnot cycle. The only way in which heat can be transferred from the engine is by radiation. The rate at which heat is radiated is proportional to the fourth power of the absolute temperature and to the area of the radiating surface. Show that for a given power output and a given T_1, the area of the radiator will be a minimum when**

$$\frac{\mathbf{T_2}}{\mathbf{T_1}} = \frac{\mathbf{3}}{\mathbf{4}}$$

Solution: Heat have to radiate = Q_2



$\therefore \quad Q_2 = \sigma A T_2^4$

From engine side

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{W}{T_1 - T_2}$$

$$\therefore \quad Q_2 = \frac{WT_2}{T_1 - T_2}$$

$$\therefore \quad \frac{WT_2}{T_1 - T_2} = \sigma A T_2^4$$

$$\text{or} \quad A = \frac{W}{\sigma T_2^4}\left\{\frac{T_2}{T_1 - T_2}\right\} = \frac{W}{\sigma}\left\{\frac{1}{T_1 T_2^3 - T_2^4}\right\}$$

For minimum Area

$$\frac{\partial A}{\partial T_2} = 0 \quad \text{or} \quad \frac{\partial}{\partial T_2}\{T_1 T_2^3 - T_2^4\} = 0$$

$$\text{or} \quad T_1 \times 3T_2^2 - 4T_2^3 = 0$$

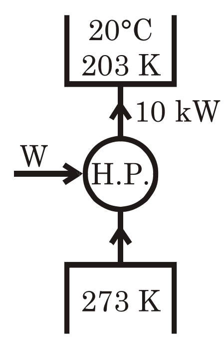
$$\text{or} \quad 3T_1 = 4T_2$$

$$\text{or} \quad \frac{T_2}{T_1} = \frac{3}{4} \text{ proved}$$

Q6.18 **It takes 10 kW to keep the interior of a certain house at 20°C when the outside temperature is 0°C. This heat flow is usually obtained directly by burning gas or oil. Calculate the power required if the 10 kW heat flow were supplied by operating a reversible engine with the house as the upper reservoir and the outside surroundings as the lower reservoir, so that the power were used only to perform work needed to operate the engine.**

(**Ans.** 0.683 kW)

Solution: COP of the H.P.



$$\frac{10}{W} = \frac{293}{293 - 273}$$

$$\text{or} \quad W = \frac{10 \times 20}{293} \text{ kW}$$

$$= 683 \text{ W only.}$$

Q6.19 **Prove that the COP of a reversible refrigerator operating between two given temperatures is the maximum.**

Solution: Suppose A is any refrigerator and B is reversible refrigerator and also assume

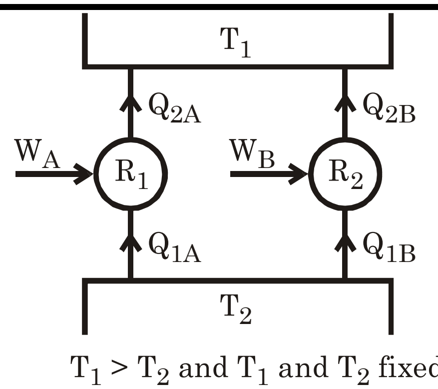
$$(COP)_A > (COP)_B$$

$$\text{and} \quad Q_{1A} = Q_{1B} = Q$$

or $\frac{Q_{1A}}{W_A} > \frac{Q_{1B}}{W_B}$

or $\frac{Q}{W_A} > \frac{Q}{W_B}$

or $W_A < W_B$

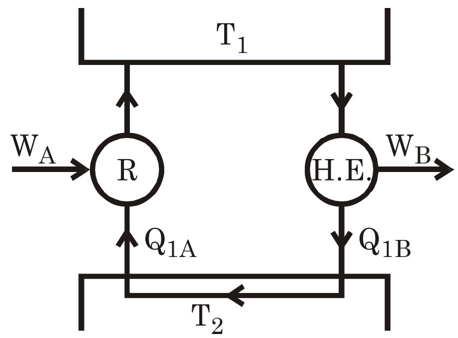


$T_1 > T_2$ and T_1 and T_2 fixed

Then we reversed the reversible refrigerator 'B' and then work output of refrigerator 'B' is W_B and heat rejection is $Q_{1B} = Q$ (same)

So we can directly use Q to feed for refrigerator and Reservoir 'T_2' is eliminated then also a net work output $(W_B - W_A)$ will be available. But it violates the Kelvin-Plank statement i.e. violates Second Law of thermodynamic so our assumption is wrong.

So $(COP)_R \geq (COP)_A$



Q6.20 **A house is to be maintained at a temperature of 20°C by means of a heat pump pumping heat from the atmosphere. Heat losses through the walls of the house are estimated at 0.65 kW per unit of temperature difference between the inside of the house and the atmosphere.**

(a) If the atmospheric temperature is – 10°C, what is the minimum power required driving the pump?

(b) It is proposed to use the same heat pump to cool the house in summer. For the same room temperature, the same heat loss rate, and the same power input to the pump, what is the maximum permissible atmospheric temperature?

(**Ans.** 2 kW, 50°C)

Solution: Same as 6.16

Q6.21 **A solar-powered heat pump receives heat from a solar collector at T_h, rejects heat to the atmosphere at T_a, and pumps heat from a cold space at T_c. The three heat transfer rates are Q_h, Q_a, and Q_c respectively. Derive an expression for the minimum ratio Q_h/Q_c, in terms of the three temperatures.**

If T_h = 400 K, T_a = 300 K, T_c = 200 K, Q_c = 12 kW, what is the minimum Q_h? If the collector captures 0.2 kW 1 m², what is the minimum collector area required?

(**Ans.** 26.25 kW, 131.25 m²)

Solution: $$W_{output} = \frac{Q_h}{T_h}(T_h - T_a)$$

$$W_{input} = \frac{Q_c}{T_c}(T_a - T_c)$$

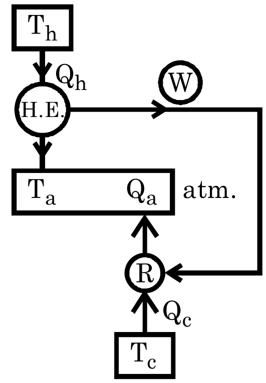
As they same

So $$\frac{Q_h}{Q_c} = \frac{T_h}{T_c} \times \frac{(T_a - T_c)}{(T_h - T_a)}$$

$$Q_h = 12 \times \frac{400}{200} \times \left\{\frac{300-200}{400-300}\right\} \text{kW}$$

$$= 24 \text{ kW}$$

$$\text{Required Area (A)} = \frac{2.4}{0.2} = 120 \text{ m}^2$$



Q6.22 **A heat engine operating between two reservoirs at 1000 K and 300 K is used to drive a heat pump which extracts heat from the reservoir at 300 K at a rate twice that at which the engine rejects heat to it. If the efficiency of the engine is 40% of the maximum possible and the COP of the heat pump is 50% of the maximum possible, what is the temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW?**

(**Ans.** 326.5 K, 86 kW)

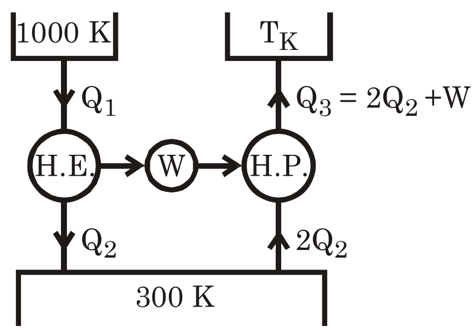
Solution:

$$\eta_{actual} = 0.4\left(1 - \frac{300}{1000}\right) = 0.28$$

$\therefore$ $W = 0.28\ Q_1$

$Q_2 = Q_1 - W = 0.72\ Q_1$

$Q_3 = 2\ Q_2 + W = 1.72\ Q_1$



$$\therefore \ (COP)_{actual} = \frac{1.72\,Q_1}{0.28\,Q_1}$$

$$= \frac{T}{T-300} \times (0.5)$$

or $6.143\ T - 300 \times 6.143 = T \times 0.5$

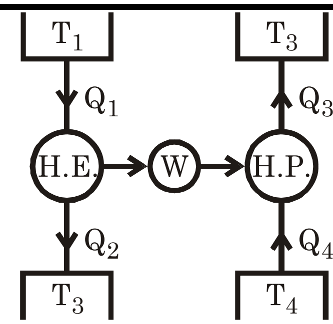
or $T = 326.58$ K

$Q_3 = 1.72 \times 50$ kW $= 86$ kW

Q6.23 **A reversible power cycle is used to drive a reversible heat pump cycle. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 at T_2. The heat pump abstracts Q_4 from the sink at T_4 and discharges Q_3 at T_3. Develop an expression for the ratio Q_4/Q_1 in terms of the four temperatures.**

$$\left(\textbf{Ans.}\ \frac{Q_4}{Q_1} = \frac{T_4(T_1 - T_2)}{T_1(T_3 - T_4)}\right)$$

Solution: For H.E.



Work output (W) = $\frac{Q_1}{T_1}(T_1 - T_2)$

For H.P.

Work input (W) = $\frac{Q_4}{T_4}(T_3 - T_4)$

$$\therefore \quad \frac{Q_1}{T_1}(T_1 - T_2) = \frac{Q_4}{T_4}(T_3 - T_4)$$

or $$\boxed{\frac{Q_4}{Q_1} = \frac{T_4}{T_1}\left\{\frac{T_1 - T_2}{T_3 - T_4}\right\}}$$

This is the expression.

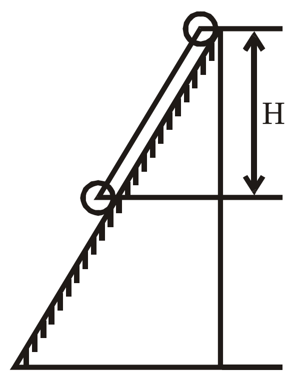
Q6.24 **Prove that the following propositions are logically equivalent:**
(a) A PMM2 is Impossible
(b) A weight sliding at constant velocity down a frictional inclined plane executes an irreversible process.

Solution: Applying First Law of Thermodynamics

$Q_{12} = E_2 - E_1 + W_{1.2}$

or $0 = E_2 - E_1 - mgh$

or $E_1 - E_2 = mgh$



7. Entropy

Some Important Notes

1. Clausius theorem: $\oint \left(\frac{đQ}{T}\right)_{rev.} = 0$

2. $S_f - S_i = \int_i^f \frac{đ Q_{rev.}}{T} = (\Delta S)_{\text{irrev. Path}}$

Integration can be performed only on a reversible path.

3. **Clausius Inequality:** $\oint \frac{đQ}{T} \le 0$

4. At the equilibrium state, the system is at the peak of the entropy hill. (Isolated)

5. TdS = dU + pdV

6. TdS = dH – Vdp

7. Famous relation S = K ln W

Where K = Boltzmann constant
W = thermodynamic probability.

8. General case of change of entropy of a Gas

$$S_2 - S_1 = m\left\{c_v \ln\frac{p_2}{p_1} + c_p \ln\frac{V_2}{V_1}\right\}$$

Initial condition of gas p_1, V_1, T_1, S_1 and
Final condition of gas p_2, V_2, T_2, S_2

Questions with Solution P. K. Nag

Q7.1. **On the basis of the first law fill in the blank spaces in the following table of imaginary heat engine cycles. On the basis of the second law classify each cycle as reversible, irreversible, or impossible.**

Cycle	*Temperature*		*Rate of Heat Flow*		*Rate of work*	*Efficiency*
	Source	*Sink*	*Supply*	*Rejection*	*Output*	
(a)	**327°C**	**27°C**	**420 kJ/s**	**230 kJ/s**	**…kW**	
(b)	**1000°C**	**100°C**	**…kJ/min**	**4.2 MJ/min**	**… kW**	**65%**
(c)	**750 K**	**300 K**	**…kJ/s**	**…kJ/s**	**26 kW**	**65%**
(d)	**700 K**	**300 K**	**2500 kcal/h**	**…kcal/h**	**1 kW**	**—**

(**Ans.** (a) Irreversible, (b) Irreversible, (c) Reversible, (d) Impossible)

Solution:

Cycle	Temperature		Rate of Heat Flow		Rate of work	Effici-ency	Remark
	Source	Sink	Supply	Rejection			
(a)	327°C	27°C	420 kJ/s	230 kJ/s	190kW	0.4523	η_{max} = 50%, irrev.possible
(b)	1000°C	100°C	12000 kJ/km	4.2 kJ/m	7800 kW	65%	η_{max}=70.7% irrev.possible
(c)	750 K	300 K	43.33 kJ/s	17.33 kJ/s	26 kW	60%	η_{max}= 60% rev. possible
(d)	700 K	300 K	2500 kcal/h	1640 kcal/h	1 kW	4.4%	η_{max}=57% irrev.possible

Q7.2 **The latent heat of fusion of water at 0°C is 335 kJ/kg. How much does the entropy of 1 kg of ice change as it melts into water in each of the following ways:**

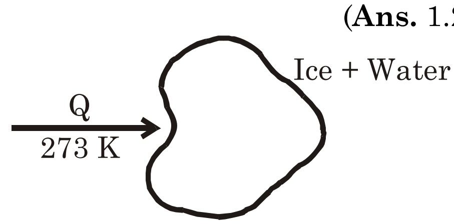
(a) Heat is supplied reversibly to a mixture of ice and water at 0°C.

(b) A mixture of ice and water at 0°C is stirred by a paddle wheel.

(**Ans.** 1.2271 kJ/K)

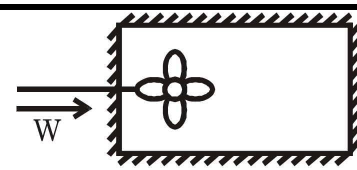
Solution : (a) $(\Delta S)_{\text{system}} = +\dfrac{1\times 335}{273}$ kJ/ K

$= 1.227$ kJ/K



(b) $(\Delta S)_{\text{system}}$

$$= \int_{273}^{273} m\, c_P \frac{dT}{T} = 0$$

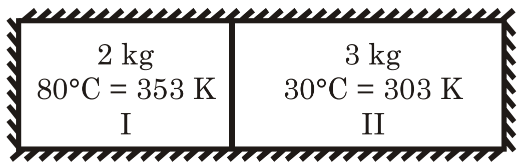


Q7.3 **Two kg of water at 80°C are mixed adiabatically with 3 kg of water at 30°C in a constant pressure process of 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process (c_p of water = 4.187 kJ/kg K).**

(**Ans.** 0.0576 kJ/K)

Solution: If final temperature of mixing is T_f then

$$2 \times c_P \ (353 - T_f) = 3 \times c_P \ (T_f - 303)$$



or $T_f = 323$ K

$$(\Delta S)_{\text{system}} = (\Delta S)_{\text{I}} + (\Delta S)_{\text{II}}$$

$$= \int_{353}^{323} m_1 c_P \frac{dT}{T} + \int_{303}^{323} m_1 c_P \frac{dT}{T}$$

$$= 2 \times 4.187 \ln\left(\frac{323}{353}\right) + 3 \times 4.187 \times \ln \frac{323}{303}$$

$$= 0.05915 \text{ kJ/K}$$

Q7.4 **In a Carnot cycle, heat is supplied at 350°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K.**

(a) If the cycle operates on a stationary mass of 1 kg of water, how much is the work done per cycle, and how much is the heat supplied?

(b) If the cycle operates in steady flow with a power output of 20 kW, what is the steam flow rate?

(**Ans.** (a) 465.12, 897.12 kJ/kg, (b) 0.043 kg/s)

Solution: If heat required for evaporation is Q kJ/kg then

(a) $$\frac{Q}{(350 + 273)} = 1.44$$

or Q = 897.12 kJ/kg

It is a Carnot cycle so $\eta = 1 - \dfrac{(273 + 27)}{(350 + 273)}$

$\therefore$ W = η.Q = 465.12 kJ

(b) $$P = \dot{m}W \text{ or } \dot{m} = \frac{P}{W} = \frac{20}{465.12} \text{ kg/s} = 0.043 \text{ kg/s}$$

Q7.5 **A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $đQ / T$.**

from these results show which case is irreversible, which reversible, and which impossible:
(a) 210 kJ/cycle rejected
(b) 105 kJ/cycle rejected
(c) 315 kJ/cycle rejected

(**Ans.** (a) Reversible, (b) Impossible, (c) Irreversible)

Solution: (a) $\oint \frac{dQ}{T} = \frac{+420}{(327+273)} - \frac{210}{(27+273)} = 0$

$\therefore$ Cycle is Reversible, Possible

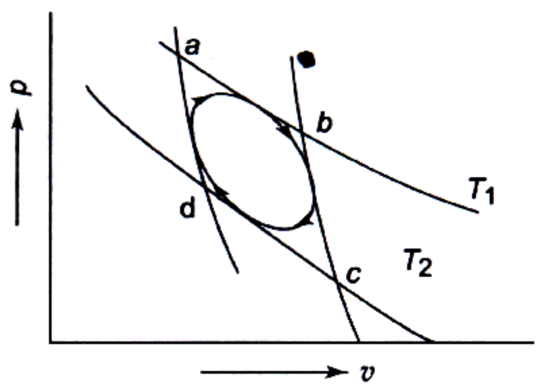
(b) $\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{105}{300} = 0.35$

$\therefore$ Cycle is Impossible

(c) $\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{315}{300} = -0.35$

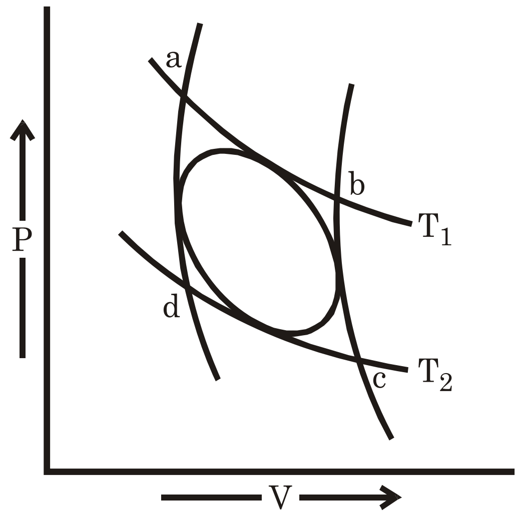
$\therefore$ Cycle is irreversible but possible.

Q7.6 **In Figure, *abed* represents a Carnot cycle bounded by two reversible adiabatic and two reversible isotherms at temperatures T_1 and T_2 ($T_1 > T_2$).**



The oval figure is a reversible cycle, where heat is absorbed at temperature less than, or equal to, T_1, and rejected at temperatures greater than, or equal to, T_2. Prove that the efficiency of the oval cycle is less than that of the Carnot cycle.

Solution:



Q7.7 **Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam**

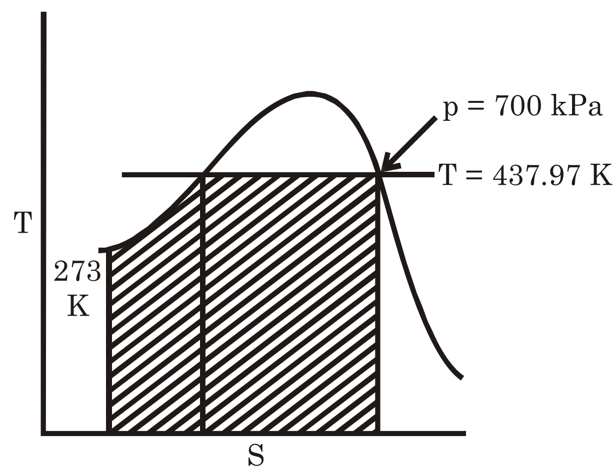
(**Ans.** 6.6967 kJ/kg K)

Solution: $(\Delta S)_{Water}$

$$= \int_{273}^{437.97} 1 \times 4187 \times \frac{dT}{T}$$

$$= 4.187 \ln\left(\frac{437.97}{273}\right) \text{kJ}/K$$

= 1.979 kJ/K



$(\Delta S)_{Eva\ pour}$

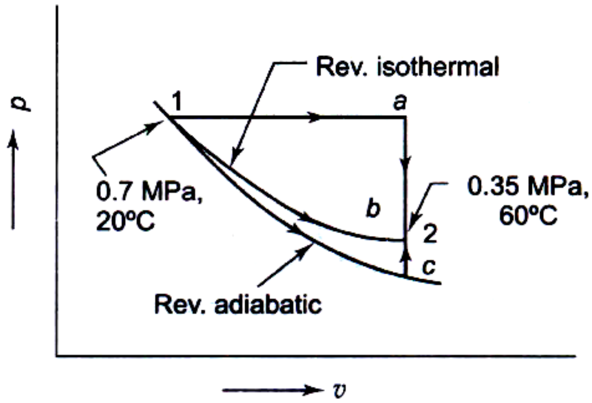
$$= \frac{1 \times 2066.3}{437.97} \text{kJ}/K$$

= 4.7179 kJ/K

$(\Delta s)_{\text{system}}$

= 6.697 kJ/kg – K

Q7.8 **One kg of air initially at 0.7 MPa, 20°C changes to 0.35 MPa, 60°C by the three reversible non-flow processes, as shown in Figure. Process 1: *a*-2 consists of a constant pressure expansion followed by a constant volume cooling, process 1: *b*-2 an isothermal expansion followed by a constant pressure expansion, and process 1: *c*-2 an adiabatic**



Expansion followed by a constant volume heating. Determine the change of internal energy, enthalpy, and entropy for each process, and find the work transfer and heat transfer for each process. Take c_p = 1.005 and c_v = 0.718 kJ/kg K and assume the specific heats to be constant. Also assume for air pv = 0.287 T, where p is the pressure in kPa, v the specific volume in m³/kg, and T the temperature in K.

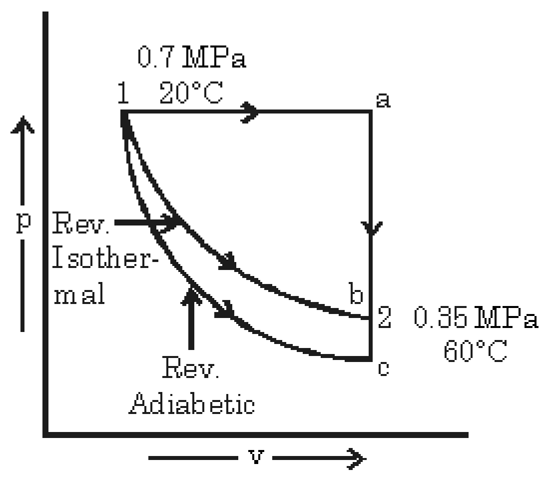
Solution: $p_1 = 0.7$ MPa = 700 kPa $\quad T_1 = 293$ K

$\therefore \quad v_1 = 0.12013$ m³/kg $\quad p_a = 700$ kPa

$\therefore \quad T_a = 666$ K $\quad v_a = 0.27306$ m³/kg

$p_2 = 350$ kPa $\quad T_2 = 333$ K

$\therefore \quad v_2 = 0.27306$ m³/kg



For process 1–a–2

$$Q_{1-a} = u_a - u_1 + \int_{v_1}^{v_a} p\,dV$$

$$= u_a - u_1 + 700(0.27306 - 0.12013)$$

$$= u_a - u_1 + 107$$

$$Q_{a-2} = u_2 - u_a + 0$$

$\therefore \quad u_a - u_1 = 267.86$ kJ/kg

$u_2 - u_a = -239$ kJ/kg

$$Q_{1-a} = \int_{T_1}^{T_a} c_P \, dT$$
$= 1.005 \times (666 - 293)$
$= 374.865$ kJ/kg

$$Q_{a-2} = \int_{T_a}^{T_2} c_v \, dT$$
$= 0.718\ (333 - 666)$
$= -239$ kJ/kg

(i) $\Delta u = u_2 - u_1 = 28.766$ kJ/kg

(ii) $\Delta h = h_2 - h_1 = u_2 - u_1 + p_2 v_2 - p_1 v_1$
$= 28.766 + 350 \times 0.27306 - 700 \times 0.12013 = 40.246$ kJ/kg

(iii) $Q = Q_2 + Q_1 = 135.865$ kJ/kg

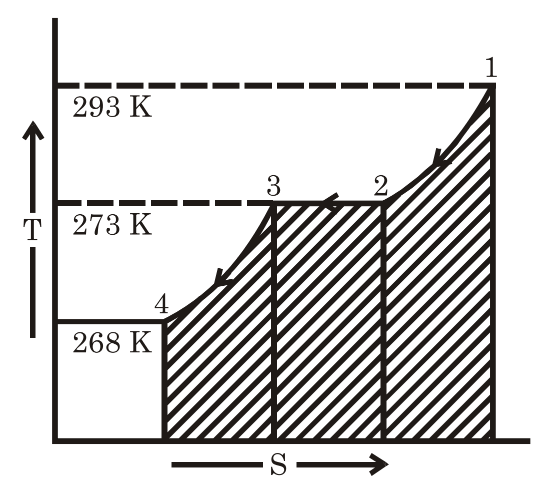
(iv) $W = W_1 + W_2 = 107$ kJ/kg

(v) $\Delta s = s_2 - s_1 = (s_2 - s_a) + (s_a - s_1)$
$$= C_v \ln\left(\frac{T_2}{T_a}\right) + C_P \ln\left(\frac{T_a}{T_1}\right)$$
$= 0.3275$ kJ/kg – K

Q7.9 **Ten grammes of water at 20°C is converted into ice at –10°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.2 J/gK and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 J/g, calculate the total entropy change of the system.**

(**Ans.** 16.02 J/K)

Solution:



$$S_2 - S_1 = \int_{293}^{273} \frac{m c_P \, dT}{T}$$
$$= 0.01 \times 4.2 \times \ln \frac{273}{293} \text{ kJ/}K$$
$= -0.00297$ kJ/K
$= -2.9694$ J/K
$$S_3 - S_2 = \frac{-mL}{T}$$
$$= \frac{-0.01 \times 335 \times 1000}{273}$$
$= -12.271$ J/K

$$S_4 - S_3 = \int_{273}^{268} \frac{m c_P \, dT}{T} = 0.01 \times \left(\frac{4.2}{2}\right) \times \ln\frac{268}{273} \text{ kJ}/K$$
$$= -0.3882 \text{ J/K}$$

$\therefore$ $S_4 - S_1 = -15.63$ J/K

$\therefore$ Net Entropy change = 15.63 J/K

Q7.10 **Calculate the entropy change of the universe as a result of the following processes:**

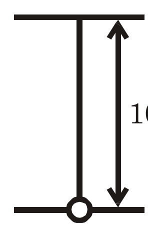
(a) A copper block of 600 g mass and with C_p of 150 J/K at 100°C is placed in a lake at 8°C.

(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(c) Two such blocks, at 100 and 0°C, are joined together.

(**Ans.** (a) 6.69 J/K, (b) 2.095 J/K, (c) 3.64 J/K)

Solution:



(a)
$$(\Delta S)_{\text{copper}} = \int_{373}^{281} m c_P \frac{dT}{T}$$
$$= 150 \ln\frac{281}{373} \text{ J}/K$$
$$= -42.48 \text{ J/K}$$

As unit of C_P is J/K there for

$\therefore$ It is heat capacity

i.e. $C_p = m c_p$

$$(\Delta S)_{\text{lake}} = \frac{C_p(100-8)}{281} \text{ J}/K$$
$$= \frac{150(100-8)}{281} \text{ J}/K = 49.11 \text{ J/K}$$
$$(\Delta S)_{\text{univ}} = (\Delta S)_{\text{COP}} + (\Delta S)_{\text{lake}} = 6.63 \text{ J/K}$$

(b) Work when it touch water = 0.600 × 9.81 × 100 J = 588.6 J

As work dissipated from the copper

$$(\Delta S)_{\text{copper}} = 0$$

As the work is converted to heat and absorbed by water then

$$(\Delta S)_{\text{lake}} = \frac{W = Q}{281} = \frac{588.6}{281} \text{ J}/K = 2.09466 \text{ J/K}$$

$\therefore$ $(\Delta S)_{\text{univ}} = 0 + 2.09466$ J/k = 2.09466 J/K

(c) Final temperature $(T_f) = \frac{100+0}{2} = 50^\circ$ C = 323 K

$$(\Delta S)_I = C_p \int_{T_1}^{T_f} \frac{dT}{T} \; ; \quad (\Delta S)_{II} = C_p \int_{T_2}^{T_f} \frac{dT}{T}$$

$$\therefore \; (\Delta S)_{\text{system}} = 150 \ln\left(\frac{T_f}{T_1}\right) + 150 \ln\left(\frac{T_f}{T_2}\right)$$

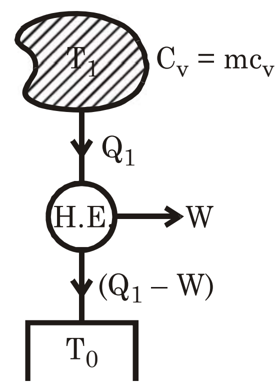
$$= 150\left\{\ln\frac{323}{373} + \ln\frac{323}{273}\right\} \text{J}/K = 3.638 \text{ J/K}$$

Q7.11 **A system maintained at constant volume is initially at temperature T_1, and a heat reservoir at the lower temperature T_0 is available. Show that the maximum work recoverable as the system is cooled to T_0 is**

$$W = C_v\left[(T_1 - T_0) - T_0 \ln\frac{T_1}{T_0}\right]$$

Solution:

For maximum work obtainable the process should be reversible



$$(\Delta S)_{\text{body}} = \int_{T_1}^{T_0} C_v \frac{dT}{T} = C_v \ln\left(\frac{T_0}{T_1}\right)$$

$$(\Delta S)_{\text{resoir}} = \frac{Q - W}{T_0}$$

$$(\Delta S)_{\text{cycle}} = 0$$

$$\therefore \quad (\Delta S)_{\text{univ.}} = C_v \ln\left(\frac{T_0}{T_1}\right) + \frac{Q-W}{T_0} \geq 0$$

$$\therefore \quad C_v \ln\left(\frac{T_0}{T_1}\right) + \frac{Q-W}{T_0} \geq 0$$

or $$C_v T_0 \ln\left(\frac{T_0}{T_1}\right) + Q - W \geq 0 \qquad \therefore \; Q = C_v(T_1 - T_0)$$

or $$W \leq Q + C_v T_0 \ln\left(\frac{T_0}{T_1}\right)$$

or $$W \leq C_v (T_1 - T_0) + C_v T_0 \ln\left(\frac{T_0}{T_1}\right)$$

or $$W \leq C_v\left\{(T_1 - T_0) + T_0 \ln\left(\frac{T_0}{T_1}\right)\right\}$$

$$\therefore \quad \text{Maximum work } W_{\max} = C_v\left\{(T_1 - T_0) + T_0 \ln\left(\frac{T_0}{T_1}\right)\right\}$$

Q7.12 **If the temperature of the atmosphere is 5°C on a winter day and if 1 kg of water at 90°C is available, how much work can be obtained. Take c_v, of water as 4.186 kJ/kg K.**

Solution: TRY PLEASE

Q7.13 **A body with the equation of state $U = CT$, where C is its heat capacity, is heated from temperature T_1 to T_2 by a series of reservoirs ranging from**

T_1 to T_2. The body is then brought back to its initial state by contact with a single reservoir at temperature T_1. Calculate the changes of entropy of the body and of the reservoirs. What is the total change in entropy of the whole system?

If the initial heating were accomplished merely by bringing the body into contact with a single reservoir at T_2, what would the various entropy changes be?

Solution: TRY PLEASE

Q7.14 **A body of finite mass is originally at temperature T_1, which is higher than that of a reservoir at temperature T_2. Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2, thus extracting heat Q from the body. If the engine does work W, then it will reject heat $Q–W$ *to* the reservoir at T_2. Applying the entropy principle, prove that the maximum work obtainable from the engine is**

$$W\,(\max) = Q - T_2\,(S_1 - S_2)$$

Where $S_1 - S_2$ is the entropy decrease of the body.

If the body is maintained at constant volume having constant volume heat capacity C_v = 8.4 kJ/K which is independent of temperature, and if T_1 = 373 K and T_2 = 303 K, determine the maximum work obtainable.

(**Ans.** 58.96 kJ)

Solution: Final temperature of the body will be T_2

$$\therefore \quad S_2 - S_1 = \int_{T_1}^{T_2} m\,c_v \frac{dT}{T} = m\,c_v \ln\left(\frac{T_2}{T_1}\right)$$

$[c_v$ = heat energy Cv$]$

$$(\Delta S)_{\text{reservoir}} = \frac{Q - W}{T_2} \qquad \therefore (\Delta S)_{\text{H.E.}} = 0$$

$$\therefore \quad (\Delta S)_{\text{univ.}} = (S_2 - S_1) + \frac{Q - W}{T_2} \ge 0$$

or $\quad T_2\,(S_2 - S_1) + Q - W \ge 0$

or $\quad W \le Q + T_2\,(S_2 - S_1)$

or $\quad W \le [Q - T_2\,(S_1 - S_2)]$

$\therefore \quad W_{max} = [Q - T_2\,(S_1 - S_2)]$

$$W_{max} = Q - T_2\,(S_1 - S_2)$$
$$= Q + T_2 C_v \ln\left(\frac{T_2}{T_1}\right)$$
$$= C_v\,(T_1 - T_2) + T_2\,C_V \ln\left(\frac{T_2}{T_1}\right)$$
$$= 8.4\left[373 - 303 + 303\ln\left(\frac{303}{373}\right)\right]$$

$= 58.99$ kJ

Q7.15 **Each of three identical bodies satisfies the equation *U* = *CT,* where *C* is the heat capacity of each of the bodies. Their initial temperatures are 200 K, 250 K, and 540 K. If *C* = 8.4 kJ/K, what is the maximum amount of work that can be extracted in a process in which these bodies are brought sto a final common temperature?**

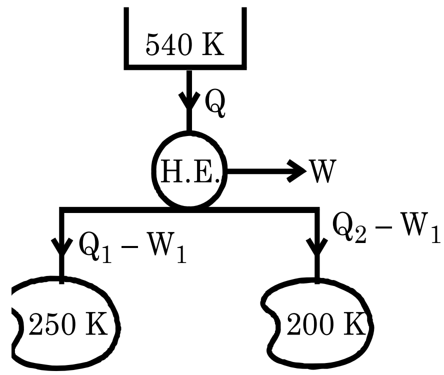
(**Ans.** 756 kJ)

Solution:

$U = CT$

Therefore heat capacity of the body is C = 8.4 kJ/K

Let find temperature will be (T_f)



$\therefore \quad W = W_1 + W_2$

$Q = Q_1 + Q_2$

$$(\Delta S)_{540\text{K body}} = C \ln \frac{T_f}{540} \text{ kJ/}K$$

$$(\Delta S)_{250\text{ K}} = C \ln \left(\frac{T_f}{250}\right)$$

$$(\Delta S)_{200\text{ K}} = C \ln \left(\frac{T_f}{200}\right)$$

$(\Delta S)_{\text{surrounds}} = 0 \qquad (\Delta S)_{\text{H.E.}} = 0$

$$\therefore \quad (\Delta S)_{\text{univ.}} = C \ln \left(\frac{T_f^3}{540 \times 250 \times 200}\right) \ge 0$$

For minimum T_f

$T_f^3 = 540 \times 250 \times 200$

$\therefore \; T_f = 300$ K

$\therefore \quad Q = 8.4(540 - 300) = 2016$ kJ

$Q_1 - W_1 = 8.4(300 - 250) = 420$ kJ

$Q_2 - W_2 = 8.4(300 - 200) = 840$ kJ

$\therefore \; Q_1 + Q_2 - (W_1 + W_2) = 1260$

or $\quad (W_1 + W_2) = 2016 - 1260$ kJ $= 756$ kJ

$\therefore \quad W_{max} = 756$ kJ

Q7.16 **In the temperature range between 0°C and 100°C a particular system maintained at constant volume has a heat capacity.**

$$C_v = A + 2BT$$

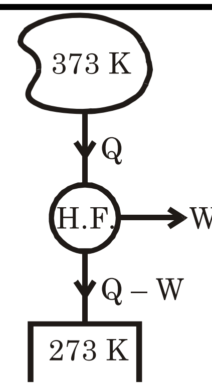
With $\quad$ ***A* = 0.014 J/K and *B* = 4.2 × 10^{-4} J/K^2**

A heat reservoir at 0°C and a reversible work source are available. What is the maximum amount of work that can be transferred to the reversible work source as the system is cooled from 100°C to the temperature of the reservoir?

(**Ans.** 4.508 J)

Solution:

Find temperature of body is 273 K



$$\therefore \qquad Q = \int_{373}^{273} C_v dT = AT + BT^2]_{373}^{273}$$

$$= -A(100) + B(273^2 - 373^2)\ J$$

$$= -28.532\ J \text{ (flow from the system)}$$

$$(\Delta S)_{\text{body}} = \int_{373}^{273} C_v \frac{dT}{T}$$

$$= \int_{373}^{273} \left(\frac{A + 2BT}{T}\right) dT$$

$$= A \ln \frac{273}{373} + 2B(273 - 373)\ J/K$$

$$= -0.08837\ J/K$$

$$(\Delta S)_{\text{res.}} = \frac{Q - W}{273}; \quad (\Delta S)_{\text{H.E.}} = 0 \qquad (\Delta S)_{\text{surrounds}} = 0$$

$$\therefore \qquad (\Delta S)_{\text{univ}} = -0.08837 + \frac{Q - W}{273} \geq 0$$

or $\quad -24.125 + Q - W \geq 0$

or $\quad W \leq Q - 24.125$

or $\quad W \leq (28.532 - 24.125)\ J$

or $\quad W \leq 4.407\ J$

$\quad W_{\max} = 4.407\ J$

Q7.17 Each of the two bodies has a heat capacity at constant volume

$$\boldsymbol{C_v = A + 2BT}$$

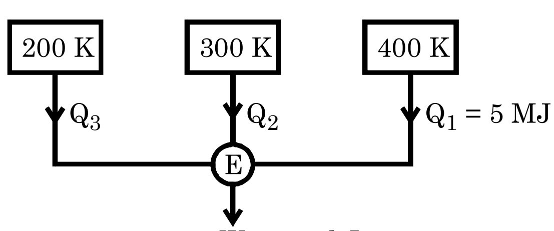
Where $A = 8.4$ J/K and $B = 2.1 \times 10^{-2}$ J/K²

If the bodies are initially at temperatures 200 K and 400 K and if a reversible work source is available, what are the maximum and minimum final common temperatures to which the two bodies can be brought? What is the maximum amount of work that can be transferred to the reversible work source?

(**Ans.** $T_{\min}$ = 292 K)

Solution: TRY PLEASE

Q7.18 A reversible engine, as shown in Figure during a cycle of operations draws 5 MJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.

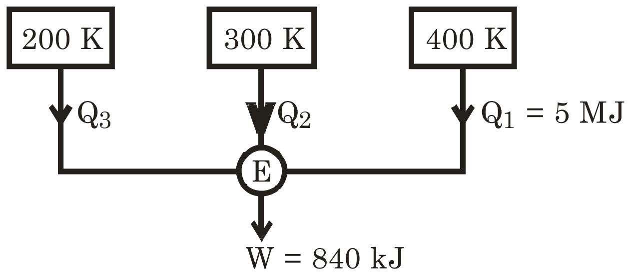


(**Ans.** Q_2 = + 4.98 MJ and Q_3 = – 0.82 MJ)

Solution: Let Q_2 and Q_3 both incoming i.e. out from the system

$\therefore$ $Q_2 \rightarrow$ +ve, $Q_3 \rightarrow$ +ve

$$(\Delta S)_{\text{univ}} = \frac{Q_3}{200} + \frac{Q_2}{300} + \frac{5000}{400} + (\Delta S)_{\text{H.E.}} + (\Delta S)_{\text{surrounds}} = 0$$



Or $$\frac{Q_3}{2} + \frac{Q_2}{3} + \frac{5000}{4} + 0 + 0 = 0$$

or $6\ Q_3 + 4\ Q_2 + 3 \times 5000 = 0$ … (i)

$Q_3 + Q_2 + 5000 - 840 = 0$ … (ii)

Heat balance

or $4\ Q_3 + 4\ Q_2 + 16640 = 0$ … (iii)

$\therefore$ (i) – (iii) gives

$2\ Q_3 = +1640$

$\therefore$ $Q_3 = +820$ kJ

(Here –ve sign means heat flow opposite to our assumption)

$\therefore$ $Q_2 = -4980$ kJ

Q7.19 **For a fluid for which *pv/T* is a constant quantity equal to *R,* show that the change in specific entropy between two states *A* and *B* is given by**

$$s_B - s_A = \int_{T_A}^{T_B} \left(\frac{C_p}{T}\right) dT - R \ln \frac{p_B}{p_A}$$

A fluid for which *R* is a constant and equal to 0.287 kJ/kg K, flows steadily through an adiabatic machine, entering and leaving through two adiabatic pipes. In one of these pipes the pressure and temperature are 5 bar and 450 K and in the other pipe the pressure and temperature are 1 bar and 300 K respectively. Determine which pressure and temperature refer to the inlet pipe.

(**Ans.** *A* is the inlet pipe)

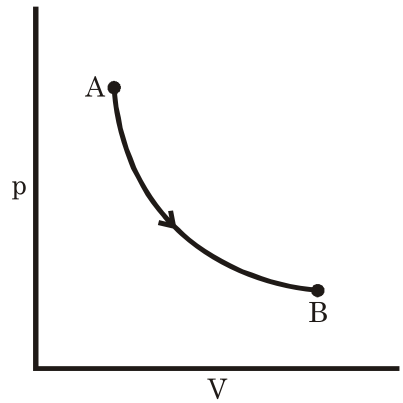
For the given temperature range, c_p is given by

$$Cp = a \ln T + b$$

Where *T* is the numerical value of the absolute temperature and *a* = 0.026 kJ/kg K, *b* = 0.86 kJ/kg K.

(**Ans.** $s_B - s_A$ = 0.0509 kJ/kg K. *A* is the inlet pipe.)

Solution:



$$dS = \frac{C_v\, dT}{T} + \frac{R}{V}\, dV$$

$$\frac{pV}{T} = R$$

$$\therefore \quad \frac{V}{T} = \frac{R}{p}$$

$$\cancel{d}Q = dH - Vdp$$

or $$TdS = dH - Vdp$$

or $$ds = \frac{C_P dT}{T} - \frac{Vdp}{T}$$

or $$ds = \frac{C_P dT}{T} - \frac{R}{p}\, dp$$

Intrigation both side with respect A to B

$$\int_{S_A}^{S_B} d\,s = \int_{T_A}^{T_B} \left(\frac{C_P}{T}\right) dT - R \int_{P_A}^{P_B} \frac{dp}{p}$$

or $$s_B - s_A = \left[\int_{T_A}^{T_B} \left(\frac{C_P}{T}\right) dT - R \ln \left(\frac{p_B}{p_A}\right)\right] \text{ proved}$$

$$s_B - s_A = \int_{450}^{300} \left(\frac{a \ln T + b}{T}\right) dT - 0.287 \times \ln\left(\frac{1}{5}\right)$$

$$= \left[a \frac{(\ln T)^2}{2} + b \ln T\right]_{450}^{300} - 0.287 \times \ln\left(\frac{1}{5}\right)$$

$$s_B - s_A = \frac{a}{2}\{(\ln 300)^2 - (\ln 450)^2\} + b \ln \frac{300}{450} - 0.287 \ln\left(\frac{1}{5}\right)$$

or $$s_B - s_A = 0.05094 \text{ kJ/kg} - \text{K}$$

$\therefore$ A is the inlet of the pipe

Q7.20 **Two vessels, *A* and *B*, each of volume 3 m³ may be connected by a tube of negligible volume. Vessel *a contains air at 0.7 MPa, 95 ° C*, while vessel *B* contains air at 0.35 MPa, 205°C. Find the change of entropy when *A* is connected to *B* by working from the first principles and assuming the**

mixing to be complete and adiabatic. For air take the relations as given in Example 7.8.

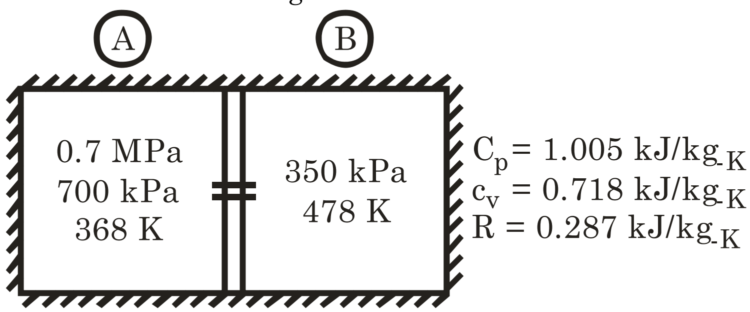
(**Ans.** 0.959 kJ/K)

Solution: Let the find temperature be (T_f)

$$\text{Mass of } (m_A) = \frac{p_A V_A}{RT_A}$$

$$= \frac{700 \times 3}{0.287 \times 368} \text{ kg}$$

$$= 19.88335 \text{ kg}$$



$$\text{Mass of gas } (m_B) = \frac{p_B V_B}{R\, T_B} = \frac{350 \times 3}{0.287 \times 478} = 7.653842 \text{ kg}$$

For adiabatic mixing of gas Internal Energy must be same

$$\therefore \quad u_A = m_A\, c_v\, T_A$$

$$= 19.88335 \times 0.718 \times 368 \text{ kJ} = 5253.66 \text{ kJ}$$

$$u_B = m_B\, c_v\, T_B$$

$$= 7.653842 \times 0.718 \times 478 \text{ kJ} = 2626.83 \text{ kJ}$$

$$U_{\text{mixture}} = \left(m_A c_v + m_B c_v\right) T_f$$

$$\text{Or} \quad T_f = 398.6 \text{ K}$$

If final pressure (p_f)

$$\therefore \quad p_f \times V_f = m_f\, RT_f$$

$$\therefore \quad p_f = \frac{27.5372 \times 0.287 \times 398.6}{6} \text{ kPa} = 525 \text{ kPa}$$

$$(\Delta S)_A = m_A\left[c_P \ln\frac{T_f}{T_A} - R\ln\left(\frac{p_f}{p_A}\right)\right] = 3.3277$$

$$(\Delta S)_B = m_B\left[c_P \ln\frac{T_f}{T_B} - R\ln\left(\frac{p_f}{p_B}\right)\right] = -2.28795 \text{ kJ/K}$$

$$\therefore \quad (\Delta S)_{\text{univ}} = (\Delta S)_A + (\Delta S)_B + 0 = 0.9498 \text{ kJ/K}$$

Q7.21 **(a) An aluminium block (*c_p* = 400 J/kg K) with a mass of 5 kg is initially at 40°C in room air at 20°C. It is cooled reversibly by transferring heat to a completely reversible cyclic heat engine until the block reaches 20°C. The 20°C room air serves as a constant temperature sink for the engine. Compute (i) the change in entropy for the block,**

(ii) the change in entropy for the room air, (iii) the work done by the engine.

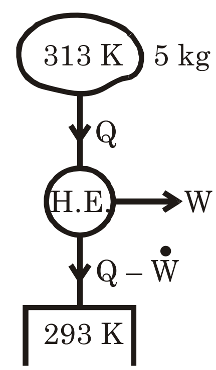
(b) If the aluminium block is allowed to cool by natural convection to room air, compute (i) the change in entropy for the block, (ii) the change in entropy for the room air (iii) the net the change in entropy for the universe.

(Ans. (a) – 134 J/K, + 134 J/K, 740 J;
(b) – 134 J/K, + 136.5 J/K, 2.5 J/K)

Solution:

(a) $(\Delta S)_{A1} = \int_{313}^{293} \frac{m c_P \, dT}{T}$

$5 \times 400 \times \ln \frac{293}{313} \text{ J/} K = -132.06 \text{ J/K}$



$(\Delta S)_{air} = \frac{Q - W}{293}$

And $Q = m \; c_p \; (313 - 293) = 40000$ J

As heat is reversibly flow then

$(\Delta S)_{Al} + (\Delta S)_{air} = 0$

or $-132.06 + 136.52 - \frac{W}{293} = 0$

or $W = 1.306$ kJ

(b) $(\Delta S)_{\Delta f}$ = Same for reversible or irreversible = –132.06 J/K

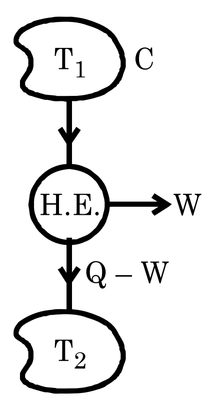
$(\Delta S)_{air} = \frac{4000}{293} = 136.52$ J/K

$(\Delta S)_{air} = +4.4587$ J/K

Q7.22 Two bodies of equal heat capacities *C* and temperatures T_1 and T_2 form an adiabatically closed system. What will the final temperature be if one lets this system come to equilibrium (a) freely? (b) Reversibly? (c) What is the maximum work which can be obtained from this system?

Solution:

(a) Freely $T_f = \frac{T_1 + T_2}{2}$



(b) Reversible

Let find temperature be T_f

the $(\Delta S)_{hot} = \int_{T_1}^{T_f} C \frac{dT}{T}$

$= C \ln \frac{T_f}{T_1}$

$(\Delta S)_{cold} = \int_{T_2}^{T_f} C \frac{dT}{T} = C \ln \left(\frac{T_f}{T_2} \right)$

$\therefore \; (\Delta S)_{univ.} = (\Delta S)_{hot} = (\Delta S)_{cold} = (\Delta S)_{surroundings}$

$= C \ln \frac{T_f}{T_1} + C \ln \frac{T_f}{T_2} = 0$

or $\quad T_f = \sqrt{T_1 T_2}$

$$\therefore \quad Q = C(T_1 - T_f)$$

$$Q - W = C(T_f - T_2)$$

$$- \quad + \qquad -$$

$$W = C(T_1 - T_f - T_f + T_2)$$

$$= C\{T_1 + T_2 - 2\ T_f\}$$

$$= C[T_1 + T_2 - 2\sqrt{T_1 T_2}]$$

Q7.23 **A resistor of 30 ohms is maintained at a constant temperature of 27°C while a current of 10 amperes is allowed to flow for 1 sec. Determine the entropy change of the resistor and the universe.**

(**Ans.** $(\Delta S)_{\text{resistor}} = 0$, $(\Delta S)_{\text{univ}} = 10$ J/K)

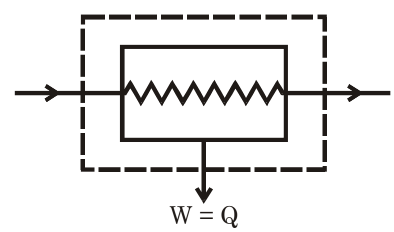
If the resistor initially at 27°C is now insulated and the same current is passed for the same time, determine the entropy change of the resistor and the universe. The specific heat of the resistor is 0.9 kJ/kg K and the mass of the resistor is 10 g.

(**Ans.** $(\Delta S)_{\text{univ}} = 6.72$ J/K)

Solution: As resistor is in steady state therefore no change in entropy. But the work = heat is dissipated to the atmosphere.

So $\quad (\Delta S)_{\text{atm}} = \dfrac{i^2 Rt}{T_{\text{atm}}}$

$$= \frac{10^2 \times 30 \times 1}{300} = 10 \text{ kJ/kg}$$



If the resistor is insulated then no heat flow to surroundings

So $\quad (\Delta S)_{\text{surroundings}} = 0$

And, Temperature of resistance (Δt)

$$= \frac{10^2 \times 30 \times 1}{900 \times 0.01} = 333.33^\circ \text{ C}$$

$\therefore \quad$ Final temperature $(T_f) = 633.33$ K

Initial temperature $(T_o) = 300$ K

$$\therefore \quad (\Delta S) = \int_{300}^{633.33} m\, c \frac{dT}{T}$$

$$= 0.01 \times 0.9 \times \ln\left(\frac{633.33}{300}\right) = 6.725 \text{ J/K}$$

$$(\Delta S)_{\text{univ}} = (\Delta S)_{\text{rev.}} = 6.725 \text{ J/K}$$

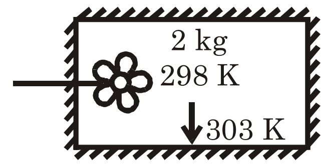
Q7.24 **An adiabatic vessel contains 2 kg of water at 25°C. By paddle-wheel work transfer, the temperature of water is increased to 30°C. If the specific heat of water is assumed constant at 4.187 kJ/kg K, find the entropy change of the universe.**

(**Ans.** 0.139 kJ/K)

Solution:

$(\Delta S)_{surr.} = 0$

$$(\Delta S)_{sys} = \int_{298}^{303} mc\frac{dT}{T}$$



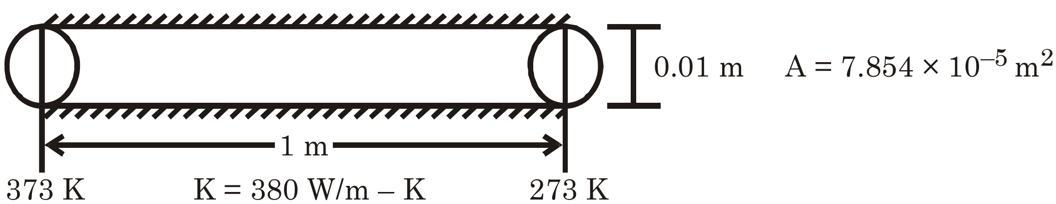
$$= 2 \times 4.187 \times \ln\frac{303}{298} = 0.13934 \text{ kJ/K}$$

$\therefore \quad (\Delta S)_{univ} = (\Delta S)_{sys} + (\Delta S)_{surr} = 0.13934 + 0 = 0.13934$ kJ/K

Q7.25 **A copper rod is of length 1 m and diameter 0.01 m. One end of the rod is at 100°C, and the other at 0°C. The rod is perfectly insulated along its length and the thermal conductivity of copper is 380 W/mK. Calculate the rate of heat transfer along the rod and the rate of entropy production due to irreversibility of this heat transfer.**

(**Ans.** 2.985 W, 0.00293 W/K)

Solution:



$$\dot{Q} = kA\frac{\Delta T}{\Delta x}$$

$$= 380 \times 7.854 \times 10^{-5} \times \frac{100}{1} \text{ W} = 2.9845 \text{ W}$$

At the 373 K end from surrounding $\dot{Q}$ amount heat is go to the system. So at this end

$$(\Delta \dot{S})_{charge} = -\frac{\dot{Q}}{373}$$

And at the 273 K and from system $\dot{Q}$ amount of heat is rejected to the surroundings.

$$\therefore \quad (\Delta \dot{S})_{charge} = \frac{\dot{Q}}{273}$$

$$\therefore \quad (\Delta \dot{S})_{univ.} = \frac{\dot{Q}}{273} - \frac{\dot{Q}}{373} = 0.00293 \text{ W/K}$$

Q7.26 **A body of constant heat capacity C_p and at a temperature T_i is put in contact with a reservoir at a higher temperature T_f. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to**

$$C_p\left[\frac{T_i - T_f}{T_f} - \ln\left(1 + \frac{T_i - T_f}{T_f}\right)\right]$$

Prove that entropy change is positive.

Given $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}..... -$ **{where** $x < 1$**}**

Solution:

Final temperature of the body will be T_f

$$\therefore \quad (\Delta S)_{\text{body}} = C_p \int_{T_i}^{T_f} \frac{dT}{T} = C_p \ln\left(\frac{T_f}{T_i}\right)$$

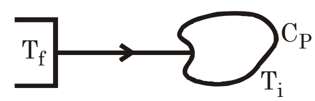
$$(\Delta S)_{\text{resoier}} = \frac{C_p(T_f - T_1)}{T_f}$$

$\therefore$ Total entropy charge

$$(\Delta S)_{\text{univ}} = C_p\left[\frac{T_f - T_i}{T_f} + \ln\frac{T_f}{T_i}\right]$$

$$= C_p\left[\frac{T_f - T_i}{T_f} - \ln\frac{T_i}{T_f}\right]$$

$$= C_p\left[\frac{T_f - T_i}{T_f} - \ln\left(1 + \frac{T_i - T_f}{T_f}\right)\right]$$



Let $\quad \frac{T_f - T_i}{T_f} = x \qquad$ as $T_f > T_i$

$$\therefore \quad \frac{T_f - T_i}{T_f} < 1$$

$$\therefore \quad (\Delta S)_{\text{in}} = C_P\{x - \ln(1 + x)\}$$

$$= C_p\left[x - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + ...\alpha\right]$$

$$= C_p\left[\frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} + ...\ \alpha\right]$$

$$= C_p\left[\frac{x^2(3 - 2x)}{6} + \frac{x^4(5 - 4x)}{20} + ...\ \alpha\right]$$

$\therefore \quad (\Delta S)_{\text{univ}}$ is +ve

Q7.27 **An insulated 0.75 kg copper calorimeter can containing 0.2 kg water is in equilibrium at a temperature of 20°C. An experimenter now places 0.05 kg of ice at 0°C in the calorimeter and encloses the latter with a heat insulating shield.**

(a) When all the ice has melted and equilibrium has been reached, what will be the temperature of water and the can? The specific heat of copper is 0.418 kJ/kg K and the latent heat of fusion of ice is 333 kJ/kg.

(b) Compute the entropy increase of the universe resulting from the process.

(c) What will be the minimum work needed by a stirrer to bring back the temperature of water to 20°C?

(**Ans.** (a) 4.68°C, (b) 0.00276 kJ/K, (c) 20.84 kJ)

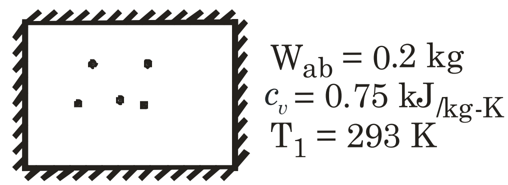
Solution:

Mass of ice = 0.05 kg

(a) Let final temperature be (T_f)

$\therefore\quad 0.75 \times 0.418 \times (293 - T_f) + 0.2 \times 4.187 \times (293 - T_f)$

$= 333 \times 0.05 + 0.05 \times 4.187 \times (T_f - 273)$



or $\quad 1.1509(293 - T_f) = 16.65 - 57.15255 + 0.20935\ T_f$

or $\quad 337.2137 - 1.1509\ T_f$

or $\quad T_f = 277.68\text{ K} = 4.68^\circ\text{ C}$

(b) $(\Delta S)_{system}$

$$= 0.75 \times 0.418 \times \ln\left(\frac{T_f}{293}\right) + 0.2 \times 4.187 \times \ln\left(\frac{T_f}{293}\right) + \frac{333 \times 0.05}{273} + 0.05 \times 4.187\ln\left(\frac{T_f}{273}\right)$$

$= 0.00275$ kJ/K = 2.75 J/K

(c) Work fully converted to heat so no Rejection.

$\therefore\quad W = C \times (20 - 4.68) = 20.84$ kJ

$\therefore\quad C = (\text{Heat capacity}) = 1.36025$

Q7.28 **Show that if two bodies of thermal capacities C_1 and C_2 at temperatures T_1 and T_2 are brought to the same temperature T by means of a reversible heat engine, then**

$$\ln T = \frac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2}$$

Solution:

$$(\Delta S)_1 = \int_{T_1}^{T} C_1 \frac{dT}{T} = C_1 \ln\left(\frac{T}{T_1}\right)$$

$$(\Delta S)_2 = \int_{T_2}^{T} C_2 \frac{dT}{T} = C_2 \ln\left(\frac{T}{T_2}\right)$$

$$(\Delta S)_{univ} = (\Delta S)_1 + (\Delta S)_2$$

For reversible process for an isolated system (ΔS) since.

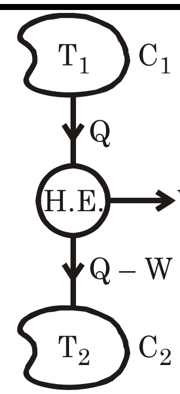
$$0 = C_1 \ln\left(\frac{T}{T_1}\right) + C_2 \ln\left(\frac{T}{T_2}\right)$$

or $\left(\dfrac{T}{T_1}\right)^{C_1}\left(\dfrac{T}{T_2}\right)^{C_2} = 1$

or $T^{C_1 + C_2} = T_1^{C_1}\, T_2^{C_2}$

or $(C_1 + C_2) \ln T = C_1 \ln T_1 + C_2 \ln T_2$

or $\ln T = \dfrac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2}$ Proved



Q7.29 **Two blocks of metal, each having a mass of 10 kg and a specific heat of 0.4 kJ/kg K, are at a temperature of 40°C. A reversible refrigerator receives heat from one block and rejects heat to the other. Calculate the work required to cause a temperature difference of 100°C between the two blocks.**

Solution: Mass = 10 kg

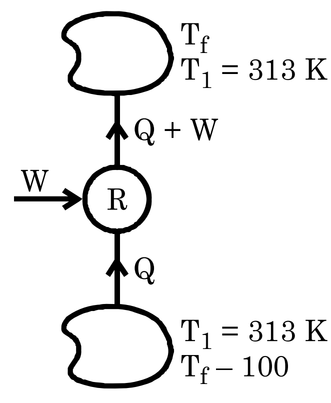
$C = 0.4$ kJ/kg – K

$T = 40^\circ$ C = 313 K

$\therefore \quad (\Delta S)_{hot} = mc \ln\left(\dfrac{T_f}{313}\right)$

$(\Delta S)_{cold} = mc \ln\left(\dfrac{T_f - 100}{313}\right)$

For minimum work requirement process must be reversible so $(\Delta S)_{univ} = 0$



$\therefore \quad \ln \dfrac{T_f(T_f - 100)}{(313)^2} = 0 = \ln 1$

or $\quad T_f^2 - 100\, T_f - 313^2 = 0$

or $\quad T_f = \dfrac{100 \pm \sqrt{100^2 + 4 \times 313^2}}{2}$

= 367 K or (–267)

$\therefore \quad Q + W = 10 \times 10.4 \times (367 - 313) = 215.87$ kJ

$Q = 10 \times 0.4 \times (313 - 267) = 184$ kJ

$\therefore \quad W_{min} = 31.87$ kJ

Q7.30 **A body of finite mass is originally at a temperature T_1, which is higher than that of a heat reservoir at a temperature T_2. An engine operates in infinitesimal cycles between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2. In this process there is a heat flow Q out of the body. Prove that the maximum work obtainable from the engine is $Q + T_2 (S_1 - S_2)$, where $S_1 - S_2$ is the decrease in entropy of the body.**

Solution: Try please.

Q7.31 **A block of iron weighing 100 kg and having a temperature of 100°C is immersed in 50 kg of water at a temperature of 20°C. What will be the change of entropy of the combined system of iron and water? Specific heats of iron and water are 0.45 and 4.18 kJ/kg K respectively.**

(**Ans.** 1.1328 kJ/K)

Solution: Let final temperature is t_f °C

$\therefore$ $100 \times 0.45 \times (100 - t_f) = 50 \times 4.18 \times (t_f - 20)$

$100 - t_f = 4.644\, t_f - 20 \times 4.699$

or $5.644\, t_f = 192.88$

or $t_f = 34.1732°$ C

$\therefore$ $t_f = 307.1732$ K

ENTROPY = $(\Delta S)_{\text{iron}} + (\Delta S)_{\text{water}}$

$$= 100 \times 0.45 \ln\left(\frac{307.1732}{373}\right) + 50 \times 4.180 \times ln\left(\frac{307.1732}{293}\right)$$

$= 1.1355$ kJ/K

Q7.32 **36 g of water at 30°C are converted into steam at 250°C at constant atmospheric pressure. The specific heat of water is assumed constant at 4.2 J/g K and the latent heat of vaporization at 100°C is 2260 J/g. For water vapour, assume $pV = mRT$ where R = 0.4619 kJ/kg K, and**

$$\frac{C_p}{R} = a + bT + cT^2, \text{ where } a = 3.634,$$

$$b = 1.195 \times 10^{-3}\ K^{-1} \text{ and } c = 0.135 \times 10^{-6}\ K^{-2}$$

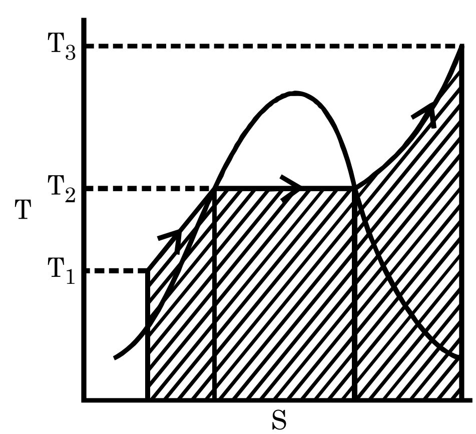
Calculate the entropy change of the system.

(**Ans.** 277.8 J/K)

Solution:

m = 36 g = 0.036 kg

$T_1 = 30°C = 303$ K
$T_2 = 373$ K
$T_3 = 523$ K



$(\Delta S)_{\text{Water}}$

$$= m\, c_P \ln\left(\frac{373}{303}\right) \text{kJ}/K$$

$= 0.03143$ kJ/K

$$(\Delta S)_{\text{Vaporization}} = \frac{mL}{T_2}$$

$$= \frac{0.036 \times 2260}{373}$$

$= 0.21812$ kJ/K

$$(\Delta S)_{\text{Vapor}} = \int_{373}^{523} m\, c_p \frac{dT}{T}$$

$$= mR \int_{373}^{523} (\frac{a}{T} + b + CT)\, dT$$

$$= mR\left[a \ln T + bT + \frac{CT^2}{2}\right]_{373}^{523}$$

$$= mR\left[a \ln \frac{523}{373} + b \times (523 - 373) + \frac{C}{2}(523^2 - 373^2)\right]$$

= 0.023556 kJ/kg

$(\Delta S)_{System} = (\Delta S)_{water} + (\Delta S)_{vaporization} + (\Delta S)_{vapor} = 273.1$ J/K

Q7.33 **A 50 ohm resistor carrying a constant current of 1 A is kept at a constant temperature of 27°C by a stream of cooling water. In a time interval of 1s**
(a) What is the change in entropy of the resistor?
(b) What is the change in entropy of the universe?

(**Ans.** (a) 0, (b) 0.167 J/K)

Solution: Try please.

Q7.34 **A lump of ice with a mass of 1.5 kg at an initial temperature of 260 K melts at the pressure of 1 bar as a result of heat transfer from the environment. After some time has elapsed the resulting water attains the temperature of the environment, 293 K. Calculate the entropy production associated with this process. The latent heat of fusion of ice is 333.4 kJ/kg, the specific heat of ice and water are 2.07 and 4.2 kJ/kg K respectively, and ice melts at 273.15 K.**

(**Ans.** 0.1514 kJ/K)

Solution: Try please.

Q7.35 **An ideal gas is compressed reversibly and adiabatically from state a to state b. It is then heated reversibly at constant volume to state c. After expanding reversibly and adiabatically to state d such that $T_b = T_d$, the gas is again reversibly heated at constant pressure to state e such that $T_e = T_c$. Heat is then rejected reversibly from the gas at constant volume till it returns to state a. Express T_a in terms of T_b and T_c. If T_b = 555 K and T_c = 835 K, estimate T_a. Take γ = 1.4.**

$$\left(\textbf{Ans. } T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}},\ 313.29\text{ K}\right)$$

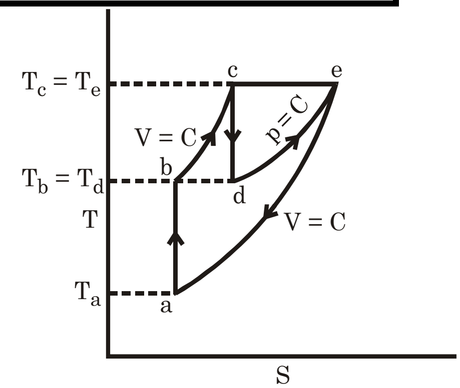
Solution:

$$(\Delta S)_{bc} = C_v \ln\left(\frac{T_c}{T_b}\right)$$

$$(\Delta S)_{de} = C_p \ln\left(\frac{T_e}{T_b}\right)$$

$$(\Delta S)_{ea} = C_v \ln\left(\frac{T_a}{T_c}\right)$$

$$(\Delta S)_{Cycles} = 0$$



$$\therefore \quad (C_p + C_v)\ln\left(\frac{T_c}{T_b}\right) + C_v \ln\left(\frac{T_a}{T_c}\right) = 0$$

or $$(\gamma + 1)\ln\left(\frac{T_c}{T_b}\right) + \ln\left(\frac{T_a}{T_c}\right) = \ln 1$$

or $$\left(\frac{T_c}{T_b}\right)^{\gamma+1}\left(\frac{T_a}{T_c}\right) = 1$$

Or $$(T_c)^{\gamma} . T_a . = T_b^{\gamma+1}$$

$$\therefore \quad T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}}$$

Given $T_b = 555$ K, $T_c = 835$ K, $\gamma = 1.4$ + Gas

$$T_a = \frac{(555)^{1.4+1}}{835^{1.4}} = 313.286 \text{ K}$$

Q7.36 **Liquid water of mass 10 kg and temperature 20°C is mixed with 2 kg of ice at – 5°C till equilibrium is reached at 1 atm pressure. Find the entropy change of the system. Given: c_p of water = 4.18 kJ/kg K, c_p of ice = 2.09 kJ/kg K and latent heat of fusion of ice = 334 kJ/kg.**

(**Ans.**190 J/K)

Solution: Try please.

Q7.37 **A thermally insulated 50-ohm resistor carries a current of 1 A for 1 s. The initial temperature of the resistor is 10°C. Its mass is 5 g and its specific heat is 0.85 J g K.**

(a) What is the change in entropy of the resistor?

(b) What is the change in entropy of the universe?

(**Ans.** (a) 0.173 J/K (b) 0.173 J/K)

Solution: Try please.

Q7.38 **The value of c_p for a certain substance can be represented by $c_p = a + bT$.**

(a) Determine the heat absorbed and the increase in entropy of a mass m of the substance when its temperature is increased at constant pressure from T_1 to T_2.

(b) Find the increase in the molal specific entropy of copper, when the temperature is increased at constant pressure from 500 to 1200 K. Given for copper: when T = 500 K, $c_p = 25.2 \times 10^3$ and when T = 1200 K, $c_p = 30.1 \times 10^3$ J/k mol K.

Ans. $\left(\text{(a) } m\left[a(T_2 - T_1) + \frac{b}{2}\left(T_2^2 - T_1^2\right), \; m\left[a \ln\frac{T_2}{T_1} + b(T_2 - T_2)\right]\right]; \text{ (b) } 24.7 \text{ kJ/k mol K} \right)$

Solution: $đQ = C_p \, dT$

$$\therefore \quad Q = m\int_{T_1}^{T_2} c_p \, dT$$

$$= m\int_{T_1}^{T_2} (a + bT)\, dT = m\left[aT + \frac{bT^2}{2}\right]_{T_1}^{T_2}$$

$$= m\left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)\right]$$

$$TdS = C_p\, dT$$

or $$dS = m\, c_p \frac{dT}{T}$$

or $$\int_{S_1}^{S_2} dS = m\int_1^2 c_p \frac{dT}{T} = m\int_{T_1}^{T_2} \frac{(a + bT)}{T}\, dT$$

$$(S_2 - S_1) = [a \ln T + bT]_{T_1}^{T_2} = m\left[a \ln \frac{T_2}{T_1} + b(T_2 - T_1)\right]$$

$\Rightarrow$ For a and b find

$$25.2 = a + b \times 500$$

$$30.1 = a + b \times 1200$$

$\therefore$ $b \times 700 = 4.9$ $\therefore$ $b = 0.007$ kJ/kg K $\therefore$ $a = 21.7$ kJ/kg – K

$$\therefore \quad S_2 - S_1 = \left[21.7 \ln\left(\frac{1200}{500}\right) + 0.007\,(1200 - 500)\right] \text{kJ}/K = 23.9 \text{ kJ/K}$$

8. Availability & Irreversibility

Some Important Notes

1. **Available Energy (A.E.)**

$$W_{max} = Q_1\left(1-\frac{T_0}{T_1}\right) = m\,c_P\int_{T_0}^{T}\left(1-\frac{T_0}{T}\right)dT$$

$$= (T_1 - T_0)\,\Delta S$$

$$= u_1 - u_2 - T_0\,(s_1 - s_2)$$

(**For closed system**), it is not ($\phi_1 - \phi_2$) because change of volume is present there.

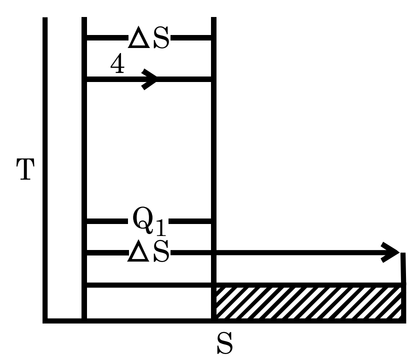
$$= h_1 - h_2 - T_0\,(s_1 - s_2)$$

(**For steady flow system**), it is (A_1 – A_2) as in steady state no change in volume is CONSTANT VOLUME (i.e. change in availability in steady flow)

2. **Decrease in Available Energy**

$$= T_0\,[\Delta S' - \Delta S]$$

Take $\Delta S'$ & ΔS both +Ve Quantity



3. **Availability function:**

$$A = h - T_0 s + \frac{V^2}{2} + gZ$$

Availability = maximum useful work

For steady flow

Availability $= A_1 - A_0 = (h_1 - h_0) - T_0\,(s_1 - s_0) + \frac{V_1^2}{2} + gZ$ $\quad(\therefore V_0 = 0, Z_0 = 0)$

$$\phi = u - T_0 s + p_0 V$$

For closed system

Availability $= \phi_1 - \phi_0 = u_1 - u_0 - T_0(s_1 - s_0) + p_0(V_1 - V_0)$

Available energy is maximum work obtainable not USEFULWORK.

4. **Unavailable Energy (U.E.)**

$$= T_0\,(S_1 - S_2)$$

5. Increase in unavailable Energy = Loss in availability

$$= T_0\,(\Delta S)_{univ.}$$

6. Irreversibility

$$I = W_{max} - W_{actual}$$
$$= T_0(\Delta S)_{univ.}$$

7. Irreversibility rate = $\dot{I}$ rate of energy degradation

$$S_{gen} = \int_1^2 \dot{m}\, dS$$
$$= \text{rate of energy loss } (\dot{W}_{lost})$$
$$= T_0 \times \dot{S}_{gen} \quad \text{for all processes}$$

8. $\mathbf{W_{actual}} \Rightarrow đQ = du + đW_{act}$ **this for closed system**

$$h_1 + \frac{V_1^2}{2} + gZ_1 + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2} + gZ_2 + \frac{đW_{act}}{dm}$$ **this for steady flow**

9. Helmholtz function, $F = U - TS$

10. Gibb's function, $G = H - TS$

11. Entropy Generation number (N_S) $= \dfrac{\dot{S}_{gen}}{\dot{m}\, c_P}$

12. Second law efficiency

$$\eta_{II} = \frac{\text{Minimum exergy intake to perform the given task } (X_{min})}{\text{Actual exergy intake to perform the given task (X)}} = \eta_1/\eta_{Carnot}$$

$\mathbf{X_{min}}$ = W, if work is involved

$= Q\left(1 - \dfrac{T_0}{T}\right)$ if Heat is involved.

13. To Calculate dS

i) Use $S_2 - S_1 = m\left[c_v \ln\frac{p_2}{p_1} + c_P \ln\frac{V_2}{V_1}\right]$

For closed system

$$TdS = dU + pdV$$

or $$dS = m\,c_v\frac{dT}{T} + \frac{p}{T}dV$$
$$= m\,c_v\frac{dT}{T} + mR\frac{dV}{V}$$
$$\int_1^2 dS = m\,c_v\int_1^2\frac{dT}{T} + mR\int_1^2\frac{dV}{V}$$

For steady flow system

$$TdS = dH - Vdp$$

or $$dS = m\,c_p\frac{dT}{T} - \frac{V}{T}dp \qquad pV = mRT$$
$$\int_1^2 dS = m\,c_p\int_1^2\frac{dT}{T} - mR\int_1^2\frac{dp}{p} \qquad \frac{V}{T} = \frac{mR}{p}$$

But Note that

$$TdS = dU + pdV$$

And $$TdS = dH - Vdp$$

Both valid for closed system only

14. In Pipe Flow Entropy generation rate

Due to lack of insulation it may be

$T_1 > T_2$ for hot fluid $T_1 < T_2$ for cold fluid

$$\dot{S}_{gen} = \dot{S}_{sys} - \frac{\dot{Q}}{T_0}$$

$$= \dot{m}(S_2 - S_1) - \frac{\dot{m}\, c_p (T_2 - T_1)}{T_0}$$

$\therefore$ Rate of Irreversibility $(\dot{I}) = T_0\, \dot{S}_{gen}$

15. Flow with friction

Decrease in availability $= \dot{m}\, RT_0 \times \dfrac{\Delta p}{p_1}$

Questions with Solution P. K. Nag

Q8.1 **What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if**

(a) A temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand, and the heat engine and the heat sink, on the other

(b) The source temperature drops by 50°C and the sink temperature rises by 50°C during the heat transfer process according to the linear law $\dfrac{\text{đ}Q}{dT} = \pm$ **constant?**

(**Ans.** (a) 11.2 kJ, (b) 5.25 kJ)

Solution:

Entropy change for this process

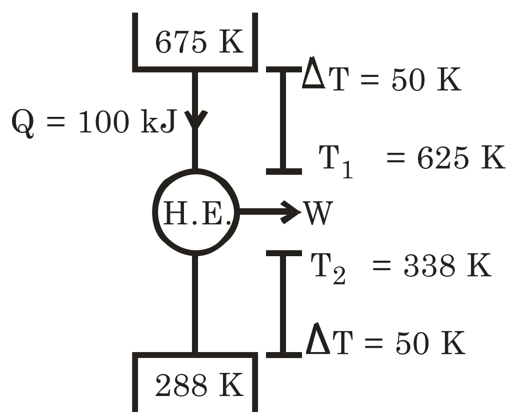
$$\Delta S = \frac{-100}{675} \text{ kJ}/K$$

$$= 0.14815 \text{ kJ/K}$$

$$W_{max} = (T - T_0)\,\Delta S$$

$$= (675 - 288)\,\Delta S = 57.333 \text{ kJ}$$

(a) Now maximum work obtainable



$$W'_{max} = 100\left(1 - \frac{338}{625}\right)$$

$$= 45.92 \text{ kJ}$$

$\therefore$ Loss of available work $= 57.333 - 45.92$

$= 11.413$ kJ

(b) Given $\dfrac{\text{đ}Q}{dT} = \pm$ constant

Let $\text{đ}Q = \pm\, mc_p \; dT$

$\therefore$ When source temperature is $(675 - T)$ and since temperature $(288 + T)$ at that time if $\text{đ}Q$ heat is flow then maximum. Available work from that $\text{đ}Q$ is $\text{đ}W$.

$$\therefore \; \text{đ}W_{max.} = \text{đ}Q\left(1 - \frac{288+T}{675-T}\right)$$

$$= \left(1 - \frac{288+T}{675-T}\right) m\, c_P \, dT$$

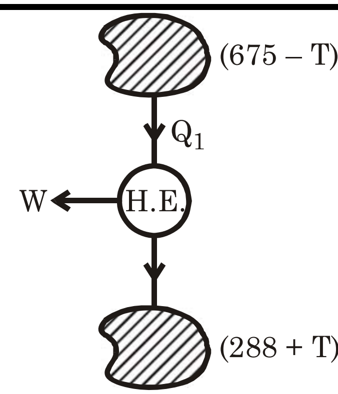
$$\therefore \quad W_{max} = m\, c_P \int_0^{50} \left(1 - \frac{288+T}{675-T}\right) dT$$

$$\left\{\frac{-288-T}{675-T} = \frac{-963+675-T}{675-T}\right\}$$

$$= m\,c_p \int_0^{50} \left\{1 + 1 - \frac{963}{675 - T}\right\} dT$$

$$= \quad m\,c_p \left\{2(50-0) + 963 \ln\left(\frac{675-50}{675-0}\right)\right\}$$

$$= 25.887 \; mc_p \quad kJ$$



$m\ c_p \ \times 50 = 100$ kJ

$= 51.773$ kJ

$\therefore \quad mc_p \ = 2$ kJ/K

$\therefore$ Loss of availability = (57.333 – 51.773) kJ

= 5.5603 kJ

Q 8.2 **In a steam generator, water is evaporated at 260°C, while the combustion gas (c_P = 1.08 kJ/kg K) is cooled from 1300°C to 320°C. The surroundings are at 30°C. Determine the loss in available energy due to the above heat transfer per kg of water evaporated (Latent heat of vaporization of water at 260°C = 1662.5 kJ/kg).**

(**Ans.** 443.6 kJ)

Solution: Availability decrease of gas

$$A_{gas} = h_1 - h_2 - T_0\,(s_1 - s_2)$$

$$= mc_p(T_1 - T_2) \quad - T_0\ mc_p \ln\left(\frac{T_1}{T_2}\right)$$

$$= m\,c_P\left[(T_1 - T_2) - T_0 \ln\frac{T_1}{T_2}\right]$$

$\therefore T_1 = 1573$ K; $T_2 = 593$ K; $T_0 = 303$ K

$= m \times 739.16$ kJ

Availability increase of water

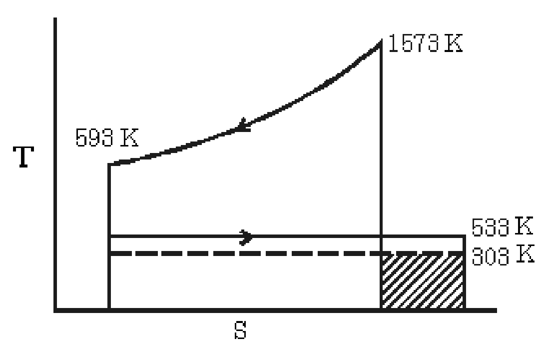
$A_w \quad = (T_1 - T_0)\,\Delta S$

$$= (T_1 - T_0) \times \frac{mL}{T_1}$$

$$= 1 \times 1662.5\left\{1 - \frac{303}{533}\right\}$$

$= 717.4$ kJ

For mass flow rate of gas (m)



$m_g\ c_{P_g}(T_2 - T_1) = m_w \times L$

$\therefore \quad m_g \times 1.08 \times (1300 - 320) = 1 \times 1662.5$

$\dot{m}_g = 1.5708$ kg/ of water of evaporator

$A_{gas} = 1161.1$ kJ

Loss of availability $= \dot{A}_{gas} - A_w$

= (1161.1 – 717.4) kJ

= 443.7 kJ

Q 8.3 **Exhaust gases leave an internal combustion engine at 800°C and 1 atm, after having done 1050 kJ of work per kg of gas in the engine (c_p of gas = 1.1 kJ/kg K). The temperature of the surroundings is 30°C.**

(a) How much available energy per kg of gas is lost by throwing away the exhaust gases?

(b) What is the ratio of the lost available energy to the engine work?

(**Ans.** (a) 425.58 kJ, (b) 0.405)

Solution: **Loss of availability**

(a) $= \int_{303}^{1073} m\, c_p\, dT\left(1 - \frac{T_0}{T}\right)$

$= 1 \times 1.1\left\{(1073 - 303) - 303 \ln\left(\frac{1073}{303}\right)\right\}$

$= 425.55$ kJ

(b) $r = \frac{425.55}{1050} = 0.40528$

Q 8.4 **A hot spring produces water at a temperature of 56°C. The water flows into a large lake, with a mean temperature of 14°C, at a rate of 0.1 m³ of water per min. What is the rate of working of an ideal heat engine which uses all the available energy?**

(**Ans.** 19.5 kW)

Solution: **Maximum work obtainable**

$$W_{max} = \int_{287}^{329} \dot{m}\, c_p\left(1 - \frac{287}{T}\right) dT$$

$$= \dot{V} \rho\, c_p\left\{(329 - 287) - 287 \ln\frac{329}{287}\right\}$$

$$= \frac{0.1}{60} \times 1000 \times 4.187\left\{(329 - 287) - 287 \ln\frac{329}{287}\right\} \text{ kW}$$

$= 19.559$ kW

Q8.5 **0.2 kg of air at 300°C is heated reversibly at constant pressure to 2066 K. Find the available and unavailable energies of the heat added. Take T_0 = 30°C and c_p = 1.0047 kJ/kg K.**

(**Ans.** 211.9 and 78.1 kJ)

Solution: **Entropy increase**

$$\Delta S = S_2 - S_1 = \int_{573}^{2066} m\, c_p \frac{dT}{T} = 0.2 \times 1.0047 \times \ln\frac{2066}{573} = 0.2577 \text{ kJ/K}$$

Availability increases

$$A_{increase} = h_2 - h_1 - T_0\,(s_2 - s_1)$$

$$= mc_p(T_2 - T_1) - T_0 \times 0.2577$$

$= 1250.24 - 78.084$

$= 1172.2$ kJ

Heat input $= m\, c_p\,(T_2 - T_1) = 1250.24$ kJ

Unavailable entropy = 78.086 kJ

Q8.6 **Eighty kg of water at 100°C are mixed with 50 kg of water at 60°C, while the temperature of the surroundings is 15°C. Determine the decrease in available energy due to mixing.**

(**Ans.** 236 kJ)

Solution: $m_1 = 80$ kg $\quad$ $m_2 = 50$ kg

$T_1 = 100° = 373$ K $\quad$ $T_2 = 60°C = 333$ K

$T_0 = 288$ K

Let final temperature $(T_f) = \dfrac{m_1T_1 + m_2T_2}{m_1 + m_2} = 357.62$ K

Availability decrease of 80 kg

$$A_{dec} = \int_{357.62}^{373} m\,c_p\,dT\left(1 - \frac{T_0}{T}\right)$$

$$= m\,c_p\left[(373 - 357.62) - 288\ln\left(\frac{373}{357.62}\right)\right]$$

$$= 1088.4 \text{ kJ}$$

Availability increase of 50 kg water

$$A_{in} = \int_{333}^{357.62} m\,c_p\left(1 - \frac{T_0}{T}\right)dT$$

$$= m\,c_p\left[(357.62 - 333) - 288\ln\left(\frac{357.62}{333}\right)\right]$$

$$= 853.6 \text{ kJ}$$

$\therefore$ Availability loss due to mixing

$= (1088.4 - 853.6)$ kJ

$= 234.8$ kJ

Q8.7 **A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is available for starting the car.**

Let compressed air be considered for doing an equivalent amount of work in starting the car. The compressed air is to be stored at 7 MPa, 25°C. What is the volume of the tank that would be required to let the compressed air have an availability of 5.2 MJ? For air, $pv = 0.287\,T$, where T is in K, p in kPa, and v in m³/kg.

(**Ans.** 0.228 m³)

Solution: Electrical Energy is high Grade Energy so full energy is available

$\therefore$ $A_{electric} = 5.2$ MJ $= 5200$ kJ

Availability of compressed air

$$A_{air} = u_1 - u_0 - T_0(s_1 - s_0)$$

$$= m\,c_v\,(T_1 - T_0) - T_0\,(s_1 - s_0)$$

$$(s_1 - s_0) = c_v\ln\frac{p_1}{p_0} + c_p\ln\frac{v_1}{v_0} \qquad = c_p\ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0}$$

$$\triangle W = T_0\,R\ln\frac{p_1}{p_0}$$

$= 298 \times 0.287 \times \ln\left(\dfrac{7000}{100}\right)$ $\qquad$ Here $T_1 = T_0 = 25°$ C $= 298$ K

$= 363.36$ kJ/kg $\qquad$ Let atm $p_r = 1$ bar $= 100$ kPa

Given $p_1 = 7$ MPa $= 7000$ kPa

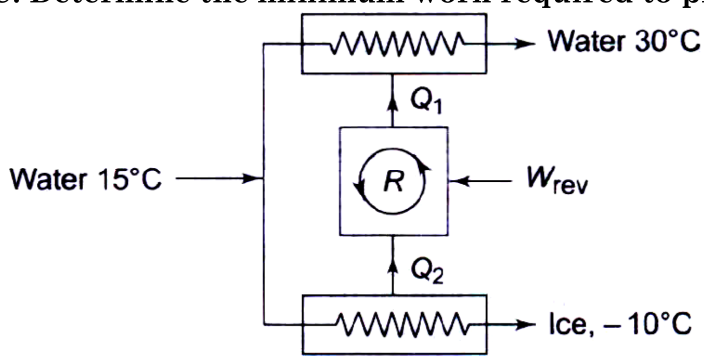
$\therefore$ Required mass of air = $\dfrac{5200}{363.36}$ kg = 14.311 kg

Specific volume of air at 7 MPa, 25ºC then

$$v = \frac{RT}{p} = \frac{0.287 \times 298}{7000} \text{ m}^3/\text{kg} = 0.012218 \text{ m}^3/\text{kg}$$

$\therefore$ Required storage volume (V) = 0.17485 m^3

Q8.8 **Ice is to be made from water supplied at 15°C by the process shown in Figure. The final temperature of the ice is – 10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C. Determine the minimum work required to produce 1000 kg of ice.**



Take c_p for water = 4.187 kJ/kg K, c_p for ice = 2.093 kJ/kg K, and latent heat of fusion of ice = 334 kJ/kg.

(**Ans.** 33.37 MJ)

Solution: Let us assume that heat rejection temperature is (T_0)

(i) Then for 15ºC water to 0º C water if we need W_R work minimum.

Then $(COP) = \dfrac{Q_2}{W_R} = \dfrac{T_2}{T_0 - T_2}$

or $W_R = Q_2 \dfrac{(T_0 - T_2)}{T_2}$

$$= Q_2 \left(\frac{T_0}{T_2} - 1\right)$$

When temperature of water is T if change is dT

Then $\cancel{d}Q_2 = -mc_p \, dT$

(heat rejection so –ve)

$$\therefore \quad \cancel{d}W_R = -m\,c_P\,dT\left(\frac{T_0}{T} - 1\right)$$

$$\therefore \quad W_{R_1} = -m\,c_P\int_{288}^{273}\left(\frac{T_0}{T} - 1\right)dT$$

$$= m\,c_P\left[T_0 \ln\frac{288}{273} - (288 - 273)\right]$$

$$= 4187\left[T_0 \ln\frac{288}{273} - 15\right] \text{kJ}$$

(ii) W_R required for 0º C water to 0 º C ice

$$W_{R_{II}} = Q_2\left(\frac{T_0}{T_2} - 1\right)$$
$$= mL\left(\frac{T_0}{T_2} - 1\right)$$
$$= 1000 \times 335\left(\frac{T_0}{273} - 1\right)$$
$$= 335000\left(\frac{T_0}{273} - 1\right) \text{kJ}$$

(iii) W_R required for 0º C ice to –10 º C ice.
When temperature is T if dT temperature decreases

$\therefore \quad \cancel{d}Q_2 = -mc_{p\ ice}\ dT$

$\therefore \quad \cancel{d}W_R = -m\,c_{p\,ice}\,dT\left(\frac{T_0}{T} - 1\right)$

$\therefore \quad W_{R_{II}} = m\,c_{p\,ice}\int_{263}^{273}\left(\frac{T_0}{T} - 1\right)dT = m\,c_{p\,ice}\left[T_0 \ln\frac{273}{263} - (273 - 263)\right]$

Let $c_{p,ice} = \frac{1}{2}\ c_{p,water} = \frac{4.187}{2}$ kJ/kg

$$= 1000 \times \frac{4.187}{2}\left[T_0 \ln\frac{273}{263} - 10\right]$$
$$= 2093.5\left[T_0 \ln\frac{273}{263} - 10\right] \text{kJ}$$

$\therefore$ Total work required

W_R = (i) + (ii) + (iii)

= [1529.2 T_0 – 418740] kJ

$\therefore$ W_R and T_0 has linear relationship

$\therefore \quad T_0 = \frac{15 + 30}{2}\,^\circ C$ = 22.5ºC = 295.5 K

$\therefore \quad W_R$ = 33138.6 kJ = 33.139 MJ

Q8.9 A pressure vessel has a volume of 1 m³ and contains air at 1.4 MPa, 175°C. The air is cooled to 25°C by heat transfer to the surroundings at 25°C. Calculate the availability in the initial and final states and the irreversibility of this process. Take p_0 = 100 kPa.

(**Ans.** 135 kJ/kg, 114.6 kJ/kg, 222 kJ)

Solution: Given T_i = 175ºC = 448 K; T_f = 25ºC = 298 K

V_i = 1 m³; V_f = 1 m³

p_i = 1.4 MPa = 1400 kPa; p_f = 931.25 kPa

Calculated Data:

p_0 = 101.325 kPa, T_0 = 298 K

c_p = 1.005 kJ/kg – K, c_V = 0.718 kJ/kg – K; R = 0.287 kJ/kg – K

$\therefore$ Mass of air (m) = $\frac{p_i V_i}{RT_i} = \frac{1400 \times 1}{0.287 \times 448}$ = 10.8885 kg

∴ Final volume $(V_0) = \dfrac{mRT_0}{p_0} = \dfrac{10.8885 \times 0.287 \times 298}{101.325} = 9.1907\ m^3$

∴ Initial availability

$A_i = \phi_1 - \phi_0$

$= u_1 - u_0 - T_0\ (s_1 - s_0) + p_0\ (V_1 - V_0)$

$$= mc_v(T_1 - T_0) - T_0\left\{mc_p \ln\frac{V_1}{V_0} + mc_v \ln\frac{p_1}{p_0}\right\} + p_0(V_1 - V_0)$$

$$= m\left[0.718(448 - 298) - 298\left\{1.005 \ln\frac{1}{9.1907} + 0.718 \ln\frac{1400}{101.325}\right\} + 101.325\,(1 - 9.1907)\right] kJ$$

$= 1458.58\ kJ = 133.96\ kJ/kg$

Final Availability

$A_f = \phi_f - \phi_0$

$$= m\,c_v(T_f - T_0) - T_0\left\{m\,c_P \ln\frac{V_f}{V_0} + m\,c_v \ln\frac{p_f}{p_0}\right\} + p_0(V_f - V_0)$$

$$\left[p_f = \frac{mRT_f}{V_f} = 931.25\ kPa \ \text{ and } T_f = T_0\right]$$

$$= 0 - T_0\ m\left\{c_P \ln\frac{V_f}{V_0} + c_v \ln\frac{p_f}{p_0}\right\} + p_0(V_f - V_0)$$

$= (2065.7 - 829.92)\ kJ$

$= 1235.8\ kJ = 113.5\ kJ/kg$

∴ Irreversibility = Loss of availability

$= (1458.5 - 1235.8)\ kJ = 222.7\ kJ$

Q8.10 **Air flows through an adiabatic compressor at 2 kg/s. The inlet conditions are 1 bar and 310 K and the exit conditions are 7 bar and 560 K. Compute the net rate of availability transfer and the irreversibility. Take T_0 = 298 K.**

(**Ans.** 481.1 kW and 21.2 kW)

Solution: Mass flow rate $(\dot{m})$ = 2 kg/s

p_i = 1 bar = 100 kPa p_f = 7 bar = 700 kPa T_0 = 298 K

T_i = 310 K T_f = 560 K

Calculated data:

$\dot{V}_i = \dfrac{\dot{m}\,RT_i}{p_i} = 1.7794\ m^3/s$ $\dot{V}_f = \dfrac{\dot{m}\,RT_f}{p_f} = 0.4592\ m^3/s$

Availability increase rate of air= $B_2 - B_1$

$= h_2 - h_1 - T_0\ (s_2 - s_1)$

$$= \dot{m}\,c_P(T_2 - T_1) - T_0\left\{m\,c_P \ln\frac{v_2}{v_1} + m\,c_v \ln\frac{p_2}{p_1}\right\}$$

$$= \dot{m}\left[c_P(T_2 - T_1) - T_0\left\{c_P \ln\frac{v_2}{v_1} + c_v \ln\frac{p_2}{p_1}\right\}\right]$$

= 2[251.25 – 10.682] kW

= 481.14 kW

Actual work required= $\dot{m}(h_2 - h_1)$

W = 2 × 251.25 kW = 502.5 kW

∴ Irreversibility = $W_{act.} - W_{min.}$

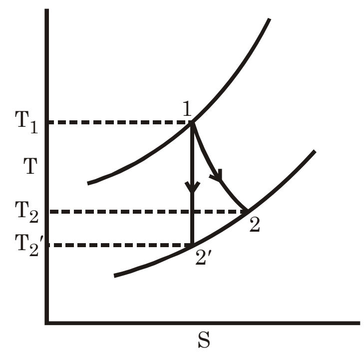
= (502.5 – 481.14) kW

= 21.36 kW

Q8.11 An adiabatic turbine receives a gas (c_p = 1.09 and c_v = 0.838 kJ/kg K) at 7 bar and 1000°C and discharges at 1.5 bar and 665°C. Determine the second law and isentropic efficiencies of the turbine. Take T_0 = 298 K.

(**Ans.** 0.956, 0.879)

Solution:



T_1 = 1273 K

$R = c_P - c_v = 0.252$

p_1 = 7 bar = 700 kPa

$$\therefore \quad v_1 = \frac{RT_1}{p_1} = \frac{(c_p - c_v)T_1}{p_1}$$

$$= \frac{0.252 \times 1273}{700}\ \text{m}^3/\text{kg}$$

= 0.45828 m³/kg

T_2 = 938 K $\qquad$ T_0 = 298 K

p_2 = 1.5 bar = 150 kPa

$$v_2 = \frac{RT_2}{p_2} = 1.57584\ \text{m}^3/\text{kg}$$

$W_{actual} = h_1 - h_2 = mc_P(T_1 - T_2)$

= 1 × 1.09 × (1273 – 938) kW = 365.15 kW

$$S_2 - S_1 = m\left[c_v \ln\frac{p_2}{p_1} + c_p \ln\frac{V_2}{V_1}\right]$$

$$= 1 \times \left\{0.838 \ln\frac{150}{700} + 1.09 \times \ln\frac{1.57584}{0.43828}\right\} \text{kW}/K$$

= 0.055326 kW/K

$$S_2 - S_2' = m\,c_p \ln\frac{T_2}{T_2'} = S_2 - S_1 = 0.055326$$

$$\therefore \quad 1 \times 1.09 \ln\frac{T_2}{T_2'} = 0.055326$$

$$\therefore \quad \frac{T_2}{T_2'} = 1.05207$$

$$\therefore \quad T_2' = \frac{T_2}{1.05207} = \frac{938}{(1.05207\ )} = 891.6\ \text{K}$$

Isentropic work = $h_1 - h'_2 = m\,c_p(T_1 - T'_2)$

$= 3 \times 1.09(1273 - 891.6)$ kW = 415.75 kW

$\therefore$ Isentropic efficiency = $\dfrac{365.15}{415.75}$ = 87.83%

Change of availability

$$\begin{aligned}\Delta A &= A_1 - A_2\\ &= h_1 - h_2 - T_0(S_1 - S_2)\\ &= mc_P(T_1 - T_2) + T_0(S_2 - S_1)\\ &= 1 \times 1.09\,(1273 - 938) + 298(0.055326)\text{ kW} = 381.64\text{ kW}\end{aligned}$$

$$\therefore\ \eta_{II} = \frac{\text{Minimum exergy required to perform the task}}{\text{Actual availability loss}}$$

$$= \frac{365.15}{381.64} = 95.7\%$$

Q8.12 **Air enters an adiabatic compressor at atmospheric conditions of 1 bar, 15°C and leaves at 5.5 bar. The mass flow rate is 0.01 kg/s and the efficiency of the compressor is 75%. After leaving the compressor, the air is cooled to 40°C in an after-cooler. Calculate**

(a) The power required to drive the compressor

(b) The rate of irreversibility for the overall process (compressor and cooler).

(**Ans.** (a) 2.42 kW, (b) 1 kW)

Solution:

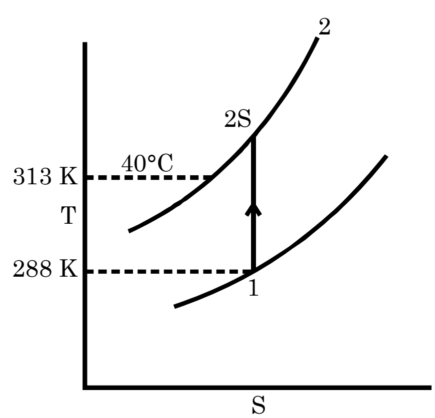
p_1 = 1 bar = 100 kPa

T_1 = 288 K

$\dot{m}$ = 0.01 kg/s

$v_1 = \dfrac{RT_1}{p_1}$ = 0.82656 m³/kg

p_2 = 5.5 bar = 550 kPa



For minimum work required to compressor is isentropic

$$W_{\text{isentropic}} = \frac{\gamma(p_2 V_2 - p_1 V_1)}{\gamma - 1}$$

$$= \frac{\gamma}{\gamma - 1} RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= \frac{1.4}{0.4} \times 0.287 \times 288\left[\left(\frac{550}{100}\right)^{\frac{0.4}{1.4}} - 1\right]\text{kJ/kg} \quad = 181.55\text{ kW/kg}$$

$\therefore$ **Actual work required**

$W_{act} = \dfrac{181.55}{0.75} = 242$ kJ/kg

(a) $\therefore$ Power required driving the compressor

$= \dot{m}\, W_{act} = 2.42$ kW

Extra work addede in 2′ to 2 is (242 – 181.55) = 60.85 kJ/kg

$\therefore$ If $C_p(T_2 - T_2') = 60.85$

or $T_2 = T_2' + \dfrac{60.85}{1.005} = 529.25$ K

$\therefore$ Availability loss due to cooling

$$= \int_{313}^{529.25} 1 \times 1.005 \left(1 - \frac{288}{T}\right) dT$$

$$= 1.005 \left\{(529.21 - 313) - 288 \ln\left(\frac{529.25}{313}\right)\right\} \text{ kJ/kg}$$

= 65.302 kJ/kg

$\therefore$ Total available energy loss

= (60.85 + 65.302) kJ/kg = 126.15 kJ/kg

$\therefore$ Power loss due to irreversibility = 1.2615 kW

Q8.13 In a rotary compressor, air enters at 1.1 bar, 21 ° C where it is compressed adiabatically to 6.6 bar, 250°C. Calculate the irreversibility and the entropy production for unit mass flow rate. The atmosphere is at 1.03 bar, 20°C. Neglect the K.E. changes.

(**Ans.** 19 kJ/kg, 0.064 kJ/kg K)

Solution:

p_1 = 1.1 bar = 110 kPa

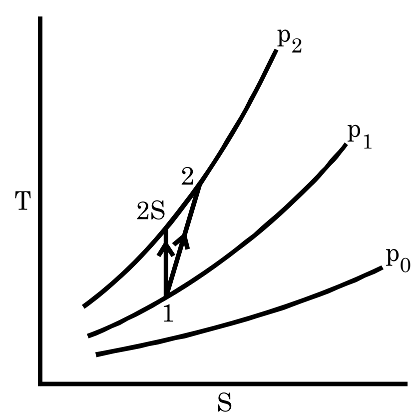
T_1 = 294 K

p_2 = 6.6 bar = 660 kPa

T_2 = 523 K

p_0 = 103 kPa

T_0 = 293 K



$$\Delta s = s_2 - s_1 = \int_1^2 \left(\frac{dh}{T} - v\frac{dp}{T}\right)$$

$$= \left[C_P \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}\right]$$

$$= \left[1.005 \ln\frac{523}{294} - 0.287 \ln\left(\frac{660}{110}\right)\right]$$

= 0.064647 kJ/kg – K = 64.647 J/kg – K

Minimum work required

W_{min} = Availability increase

$= h_2 - h_1 - T_0 (s_2 - s_1)$

$= mc_P (T_2 - T_1) - T_0 \Delta s$

= 1 × 1.005 (523 – 294) – 293 × 0.064647= 211.2 kJ/kg

Actual work required (W_{act}) = 230.145 kJ/kg

$\therefore$ Irreversibility = $T_0\ \Delta s$

$= 293 \times 0.064647 = 18.942$ kJ/kg

Q8.14 **In a steam boiler, the hot gases from a fire transfer heat to water which vaporizes at a constant temperature of 242.6°C (3.5 MPa). The gases are cooled from 1100 to 430°C and have an average specific heat, c_p = 1.046 kJ/kg K over this temperature range. The latent heat of vaporization of steam at 3.5 MPa is 1753.7 kJ/kg. If the steam generation rate is 12.6 kg/s and there is negligible heat loss from the boiler, calculate:**

(a) The rate of heat transfer
(b) The rate of loss of exergy of the gas
(c) The rate of gain of exergy of the steam
(d) The rate of entropy generation. Take T_0 = 21°C.

(**Ans.** (a) 22096 kW, (b) 15605.4 kW
(c) 9501.0 kW, (d) 20.76 kW/K)

Solution: (a) Rate of heat transfer = 12.6 × 1752.7 kW = 22.097 MW

If mass flow rate at gas is $\dot{m}_g$

Then $\dot{m}_g\, c_{P_g}\,(1100 - 430) = 22097$

or $\dot{m}_g = 31.53$ kg/s

$$\text{Loss of exergy of the gas} = \int_{703}^{1373} \dot{m}_g c_{P_g}\left(1 - \frac{294}{T}\right) dT$$

$$= \dot{m}_g\, c_{P_g}\left[(1373 - 703) - 294 \ln\left(\frac{1373}{703}\right)\right]$$

$= 15606$ kJ/s $= 15.606$ MW

$$\text{Gain of exergy of steam} = \dot{m}_w\, L_w\left(1 - \frac{294}{515.4}\right) = 9.497 \text{ MW}$$

$$\text{Rate of entropy gas} = \frac{\text{Irriversibility}}{T_0}$$

$= 20.779$ kW/K

Q8.15 **An economizer, a gas-to-water finned tube heat exchanger, receives 67.5 kg/s of gas, c_p = 1.0046 kJ/kg K, and 51.1 kg/s of water, c_p = 4.186 kJ/kg K. The water rises in temperature from 402 to 469 K, where the gas falls in temperature from 682 K to 470 K. There are no changes of kinetic energy and p_0 = 1.03 bar and T_0 = 289 K. Determine:**

(a) Rate of change of availability of the water
(b) The rate of change of availability of the gas
(c) The rate of entropy generation

(**Ans.** (a) 4802.2 kW, (b) 7079.8 kW, (c) 7.73 kW/K)

Solution: (a) Rate of charge of availability of water $= Q\left(1 - \frac{T_0}{T}\right)$

$$= \int_{402}^{469} \dot{m}_w\, c_{p\,w}\, dT\left(1 - \frac{289}{T}\right)$$

$$= 51.1 \times 4.186 \times \left[(469 - 402) - 289 \ln \frac{469}{402}\right] \text{kW}$$

$= 4.823$ MW (gain)

(b) Rate of availability loss of gas

$$= \int_{470}^{682} \dot{m}_g \, c_{Pg}\left(1 - \frac{289}{T}\right) dT$$

$$= 67.5 \times 1.0046 \left[(682 - 470) - 289 \ln \frac{682}{470}\right]$$

$= 7.0798$ MW

$\therefore$ (c) Rate of irreversibility $(\dot{I}) = 2.27754$ MW

$\therefore$ Entropy generation rate $\dot{S}_{gas} = \dfrac{\dot{I}}{T_0} = 7.8808$ kW/K

Q8.16 **The exhaust gases from a gas turbine are used to heat water in an adiabatic counter flow heat exchanger. The gases are cooled from 260 to 120°C, while water enters at 65°C. The flow rates of the gas and water are 0.38 kg/s and 0.50 kg/s respectively. The constant pressure specific heats for the gas and water are 1.09 and 4.186 kJ/kg K respectively. Calculate the rate of exergy loss due to heat transfer. Take T_0 = 35°C.**

(**Ans.** 12.5 kW)

Solution:

$T_{gi} = 260°$ C $= 533$ K $\qquad T_{wi} = 65°$C $= 338$ K

$T_{go} = 120°$C $= 393$ K $\qquad T_{wo} = 365.7$ K (Calculated)

$\dot{m}_g = 0.38$ kg/s $\qquad \dot{m}_w = 0.5$ kg/s

$c_{pg} = 1.09$ kJ/kg – K $\qquad c_{Pw} = 4.186$ kJ/kg – K

$T_o = 35°$ C $= 308$ K

To calculate T_{wo} from heat balance

$$\dot{m}_g \, c_{Pg}(T_{gi} - T_{go}) = \dot{m}_w \, c_{Pw}(T_{wo} - T_{wi})$$

$\therefore$ $T_{wo} = 365.7$ K

Loss rate of availability of gas

$$= \dot{m}_g \, c_{pg}\left[(533 - 393) - 308 \ln\left(\frac{533}{393}\right)\right] = 19.115 \text{ kW}$$

Rate of gain of availability of water

$$= \dot{m}_w \, c_{pw}\left[(365.7 - 338) - 308 \ln\left(\frac{365.7}{338}\right)\right] = 7.199 \text{ kW}$$

$\therefore$ Rate of exergy loss = 11.916 kW

Q8.17 **The exhaust from a gas turbine at 1.12 bar, 800 K flows steadily into a heat exchanger which cools the gas to 700 K without significant pressure drop. The heat transfer from the gas heats an air flow at constant pressure, which enters the heat exchanger at 470 K. The mass flow rate of air is twice that of the gas and the surroundings are at 1.03 bar, 20°C. Determine:**

(a) The decrease in availability of the exhaust gases.

(b) The total entropy production per kg of gas.

(c) What arrangement would be necessary to make the heat transfer reversible and how much would this increase the power output of

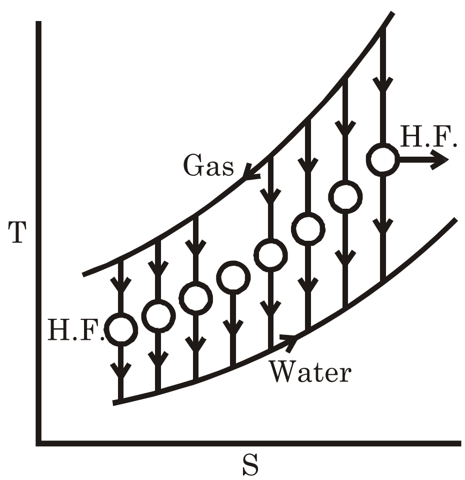
the plant per kg of turbine gas? Take c_p for exhaust gas as 1.08 and for air as 1.05 kJ/kg K. Neglect heat transfer to the surroundings and the changes in kinetic and potential energy.

(**Ans.** (a) 66 kJ/kg, (b) 0.0731 kJ/kg K, (c) 38.7 kJ/kg)

Solution: (a) Availability decrease of extra gases = $Q\left(1-\frac{T_0}{T}\right)$

$$= \int_{700}^{800} m c_p \left(1-\frac{293}{T}\right) dT = 1\times 1.08\left[(800-700) - 293 \ln\left(\frac{800}{700}\right)\right]$$

= 65.745 kJ/kg



(b) Exit air temperature T_{exit}

$$2 m\, c_{pa}\left(T_e - 470\right) = m \times c_{pg}\left(800 - 700\right)$$

or $T_e = 521.5$ K

∴ Availability increases

$$= 2\times 1.05 \times\left[(521.5-470) - 293 \ln \frac{521.5}{470}\right] = 44.257 \text{ kJ/kg}$$

∴ $\dot{S}_{gas}$ = 73.336 J/K of per kg gas flow

For reversible heat transfer

$(\Delta S)_{univ} = 0$

$(\Delta S)_{Gas} = -(\Delta S)_{water}$

$$m\times 1.08 \ln\frac{800}{700}$$

$$= -2 m \times 1.05 \times \ln\left(\frac{470}{T_o}\right)$$

or $\ln\frac{T_o}{470} = 0.068673$

∴ $T_o = 503.4$ K

∴ $Q_1 = m \times 1.08(800-700) = 108$ kJ/kg

$Q_2 = 2m \times 1.05\,(503.4 - 470) = 70.162$ kJ/kg of gas

$W = Q_1 - Q_2 = 37.84$ kJ/kg of gas flow [i.e. extra output]

Q8.18 **An air preheater is used to heat up the air used for combustion by cooling the outgoing products of combustion from a furnace. The rate of flow of the products is 10 kg/s, and the products are cooled from 300°C to**

200°C, and for the products at this temperature c_p = 1.09 kJ/kg K. The rate of air flow is 9 kg/s, the initial air temperature is 40°C, and for the air c_p = 1.005 kJ/kg K.

(a) What is the initial and final availability of the products?

(b) What is the irreversibility for this process?

(c) If the heat transfer from the products were to take place reversibly through heat engines, what would be the final temperature of the air?

What power would be developed by the heat engines? Take *To* = 300 K.

(**Ans.** (a) 85.97, 39.68 kJ/kg, (b) 256.5 kW,
(c) 394.41 K, 353.65 kW)

Solution: To calculate final air temperature (T_f)

$$\dot{m}_g c_{pg} (573 - 473) = \dot{m}_a c_{pa} (T_f - 313)$$

$$10 \times 1.09 (573 - 473) = 9 \times 1.005 (T_f - 313)$$

Or $T_f = 433.5$ K

(a) Initial availability of the product

$$= c_{pg}\left[(573-300) - 300 \ln \frac{573}{300}\right]$$

= 85.97 kJ/kg of product

Final availability

$$= c_{pg}\left[(473-300) - 300 \ln \frac{473}{300}\right] = 39.68 \text{ kJ/kg of product}$$

∴ Loss of availability = 46.287 kJ/kg of product

Availability gain by air

$$= c_{pg}\left[(433.5-313) - 300 \ln\left(\frac{433.5}{313}\right)\right] = 22.907 \text{ kJ/kg of air}$$

(b) ∴ Rate of irreversibility

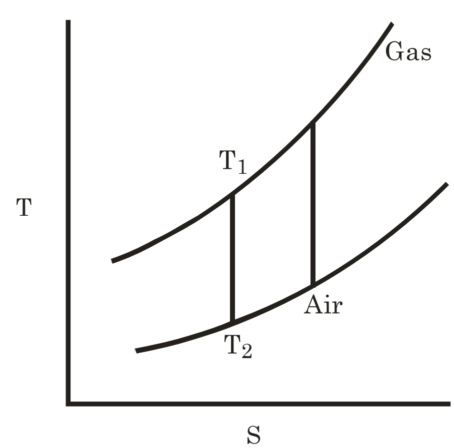
$$\dot{I} = (10 \times 46.287 - 22.907 \times 9) \text{ kW} = 256.7 \text{ kW}$$

(c) For reversible heat transfer

$$(\Delta S)_{Univ} = 0$$

$$\therefore (\Delta S)_{gas} + (\Delta S)_{air} = 0$$

or $(\Delta S)_{gas} = -(\Delta S)_{air}$



or $\dot{m}_g c_{pg} \ln\left(\frac{T_f}{T_i}\right)$

$$= \dot{m}_a c_{pa} \ln\left(\frac{T_f}{T_i}\right)$$

$$\text{or } 10 \times 1.09 \ln\left(\frac{473}{10 \times 1.09 \ln (473) 573}\right)$$

$$= -9 \times 1.005 \times \ln \frac{T_f}{313}$$

or $T_f = 394.4 = 399.4$ K

$$\therefore \quad \dot{Q}_1 = \dot{m}_g\, c_{pg}(T_i - T_f) = 1090 \text{ kJ}$$

$$Q_2 = \dot{m}_a\, c_{pa}(394.4 - 313) = 736.263 \text{ kJ}$$

$$\therefore \quad \dot{W} = \dot{Q}_1 - Q_2 = 353.74 \text{ kW output of engine.}$$

Q8.19 **A mass of 2 kg of air in a vessel expands from 3 bar, 70°C to 1 bar, 40°C, while receiving 1.2 kJ of heat from a reservoir at 120°C. The environment is at 0.98 bar, 27°C. Calculate the maximum work and the work done on the atmosphere.**

(**Ans.** 177 kJ, 112.5 kJ)

Solution:

Maximum work from gas

$$= u_1 - u_2 - T_0\,(s_1 - s_2)$$

$$= m\,c_v(T_1 - T_2)$$

$$- T_0\left[m\,c_P \ln \frac{T_1}{T_2} - mR \ln \frac{p_1}{p_2}\right]$$

$$= 2\left[0.718(343 - 313) - 300\left[1.005 \ln \frac{343}{313} - 0.287 \ln\left(\frac{3}{1}\right)\right]\right]$$

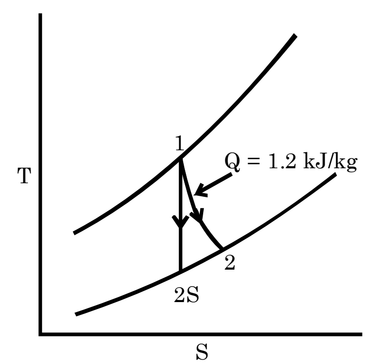
$= 177.07$ kJ

Work done on the atmosphere $= p_0\,(V_2 - V_1)$

$$= 98\left[mR\frac{T_0}{p_2} - mR\frac{T_1}{p_1}\right]$$

$$= 98\,mR\left[\frac{T_2}{p_2} - \frac{T_1}{p_1}\right]$$

$= 111.75$ kJ



Q8.20 **Air enters the compressor of a gas turbine at 1 bar, 30°C and leaves the compressor at 4 bar. The compressor has an efficiency of 82%. Calculate per kg of air**

(a) The work of compression

(b) The reversible work of compression

(c) The irreversibility. For air, use $\dfrac{T_{2s}}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\gamma-1/\gamma}$

Where T_{2s} is the temperature of air after isentropic compression and γ = 1.4. The compressor efficiency is defined as $(T_{2s} - T_1) / (T_2 - T_1)$, where T_2 is the actual temperature of air after compression.

(**Ans.** (a) 180.5 kJ/kg, (b) 159.5 kJ/kg (c) 21 kJ/kg)

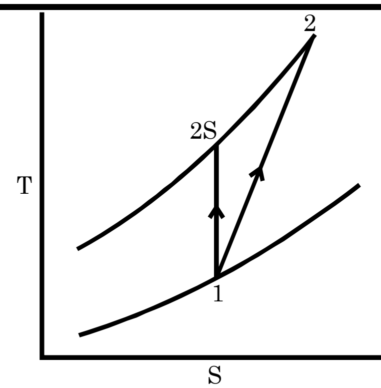
Solution:

$p_1 = 1 \text{ bar} = 100 \text{ kPa}$

$T_1 = 30^o \text{ C} = 303 \text{ K}$

$p_2 = 4 \text{ bar} = 400 \text{ kPa}$

$T_2 = ? \ \eta_{com} = 6.82$



(b) Minimum work required for compression is isentropic work

$$\therefore \quad W_R = \frac{\gamma}{\gamma-1} mRT \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

$$= \frac{1.4 \times 1 \times 0.287 \times 303}{(1.4-1)} \left\{ \left(\frac{400}{100}\right)^{\frac{0.4}{1.4}} - 1 \right\} = 147.92 \text{ kJ/kg}$$

(a) Actual work $= \dfrac{147.92}{0.82} = 180.4$ kJ/kg

$\therefore$ Extra work 32.47 kJ will heat the gas from T_2' to T_2

$$\frac{T_2'}{T_1} = \left(\frac{p_2'}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \therefore \ T_2' = 450.3 \text{ K}$$

$\therefore \quad 32.47 = mc_p \ (T_2 - T_2')$

$\therefore \quad T_2 = 482.6$ K

(c) Irreversibility (I) = (180.4 – 147.92) kJ/kg = 32.48 kJ/kg

Q8.21 A mass of 6.98 kg of air is in a vessel at 200 kPa, 27°C. Heat is transferred to the air from a reservoir at 727°C. Until the temperature of air rises to 327°C. The environment is at 100 kPa, 17°C. Determine
(a) The initial and final availability of air
(b) The maximum useful work associated with the process.

(**Ans.** (a) 103.5, 621.9 kJ (b) 582 kJ)

Solution: $p_1 = 200\text{kPa}$ $\qquad p_2 = \dfrac{mRT_2}{V_2} = 400$ kPa

$p_o = 100$ kPa

$T_1 = 300$ K $\qquad T_2 = 600$ K $\qquad T_o = 290$ K

$V_1 = \dfrac{mRT_1}{P_1} = 3.005 \text{ m}^3 \qquad V_2 = V_1 = 3.005 \text{ m}^3$

$V_o = 5.8095 \text{ m}^3 \qquad m = 6.98$ kg

(a) Initial availability

$A_i = u_1 - u_0 - T_0 (s_1 - s_0) + p_0 (V_1 - V_0)$

$$= m c_v (T_1 - T_0) - mT_0 \left[m c_p \ln \frac{T_1}{T_0} - R \ln \frac{p_1}{p_0} \right] + p_0 (V_1 - V_0)$$

$= 6.98 \times 0.718 (300 - 290) - 6.98 \times 290$

$$-6.98 \times 290\left[1.005 \ln \frac{300}{290} - 0.287 \ln \left(\frac{200}{100}\right)\right] + 100(3.005 - 5.8095)$$

$= 103.4$ kJ

Final availability

$$A_f = m\ c_v(T_2 - T_0) - mT_0\left[c_P \ln \frac{T_2}{T_0} - R \ln \frac{p_2}{p_0}\right] + p_0(V_2 - V_0)$$

$$= 6.98 \times 0.718(600 - 290) - 6.98 \times 290$$

$$\times \left[1.005 \ln \frac{600}{290} - 0.287 \ln \left(\frac{400}{100}\right)\right] + 100\,(3.005 - 5.8095)$$

$= 599.5$ kJ

(b) **Maximum useful work**

$$= u_2 - u_1 - T_0\,(s_2 - s_1) + p_0\,(V_2 - V_1)$$

$$= m c_v(T_2 - T_1) - T_0 m\left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}\right] + p_0(V_2 - V_1)$$

$$= 6.98 \times 0.718(600 - 300) - 300$$

$$\times 6.98\left[1.005 \ln \frac{600}{300} - 0.287 \ln \left(\frac{400}{200}\right)\right] + p_0 \times 0 \qquad = 461.35 \text{ kJ}$$

$\therefore \quad V_2 = V_1$

Heat transfer to the vessel

$$Q = \int m\, c_v\, dT \qquad p = \frac{mRT}{V}$$

$= m\ c_v\ (T_2 - T_1) = 6.98 \times 0.718 \times (600 - 300)$ kJ

$= 1503.402$ kJ

$\therefore$ Useful work loss of reservoir $= Q\left(1 - \frac{T_0}{T}\right)$

$$= 1503.402\left(1 - \frac{290}{1000}\right)$$

$= 1067.47$ kJ

Q8.22 **Air enters a compressor in steady flow at 140 kPa, 17°C and 70 m/s and leaves it at 350 kPa, 127°C and 110 m/s. The environment is at 100 kPa, 7°C. Calculate per kg of air**
(a) The actual amount of work required
(b) The minimum work required
(c) The irreversibility of the process

(**Ans.** (a) 114.4 kJ, (b) 97.3 kJ, (c) 17.1 kJ)

Solution: Minimum work required

$T_2 = 127°C = 400$ K $\qquad T_1 = 290$ K $\qquad T_0 = 280$ K

$$w = h_2 - h_1 - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2000}$$

$$= m c_P(T_2 - T_1) - mT_0\left[c_P \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}\right] + \frac{V_2^2 - V_1^2}{2000}$$

$$= 1 \times 1.005(400 - 290) - 1 \times 280\left[1.005 \ln \frac{400}{290} - 0.287 \ln \frac{350}{140}\right] + \frac{1.110^2 - 70^2}{2000} \text{ kJ}$$

$= 110.55 - 16.86 + 3.6 = 97.29$ kJ/kg

Actual work required

$= h_2 - h_1 + \dfrac{V_2^2 - V_1^2}{2000} = (110.55 + 3.6)$ kJ $= 114.15$ kJ

$\therefore$ Irreversibility of the process

$= T_0(s_2 - s_1) = T_0(\Delta S)_{univ} = 16.86$ kJ/kg

Q8.23 **Air expands in a turbine adiabatically from 500 kPa, 400 K and 150 m/s to 100 kPa, 300 K and 70 m/s. The environment is at 100 kPa, 17°C. Calculate per kg of air**

(a) The maximum work output
(b) The actual work output
(c) The irreversibility

(**Ans.** (a) 159 kJ, (b) 109 kJ, (c) 50 kJ)

Solution: Maximum work output

$$w = h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2000}$$

$$= C_P(T_1 - T_2) - T_0\left\{C_P \ln \frac{T_1}{T_2} - R \ln \frac{p_1}{p_2}\right\} + \frac{V_1^2 - V_2^2}{2000}$$

$$= 1.005(400 - 300) - 290\left\{1.005 \ln \frac{400}{200} - 0.287 \ln \frac{500}{100}\right\} + \frac{150^2 - 70^2}{2000}$$

$= 159.41$ kJ/kg

Actual output $= h_1 - h_2 + \dfrac{V_1^2 - V_2^2}{2000} = 100.5 + 8.8 = 109.3$ kJ/kg

The irreversibility **(I) = $T_0(\Delta S)_{univ}$** $= 50.109$ kJ/kg

Q8.24 **Calculate the specific exergy of air for a state at 2 bar, 393.15 K when the surroundings are at 1 bar, 293.15 K. Take $c_p = 1$ and $R = 0.287$ kJ/kg K.**

(**Ans.** 72.31 kJ/kg)

Solution: Exergy = Available energy

$= h_1 - h_2 - T_0 (s_1 - s_2)$

$$= C_p(T_1 - T_0) - T_0\left[C_p \ln \frac{T_1}{T_0} - R \ln \frac{p_1}{p_0}\right]$$

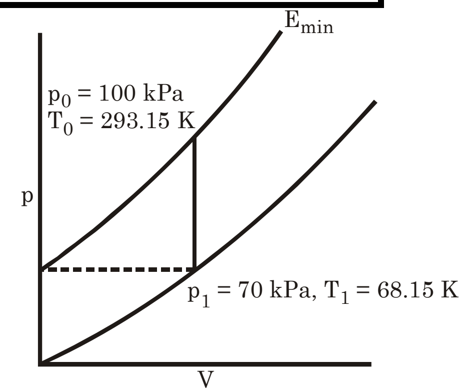
$$= 1 \times (393.15 - 293.15) - 293.15\left[1 \times \ln \frac{393.15}{293.15} - 0.287 \ln\left(\frac{2}{1}\right)\right] \text{ kJ/kg}$$

$= 72.28$ kJ/kg

Q8.25 **Calculate the specific exergy of CO_2 ($c_p = 0.8659$ and $R = 0.1889$ kJ/kg K) for a state at 0.7 bar, 268.15 K and for the environment at 1.0 bar and 293.15 K.**

(**Ans.** – 18.77 kJ/kg)

Solution:



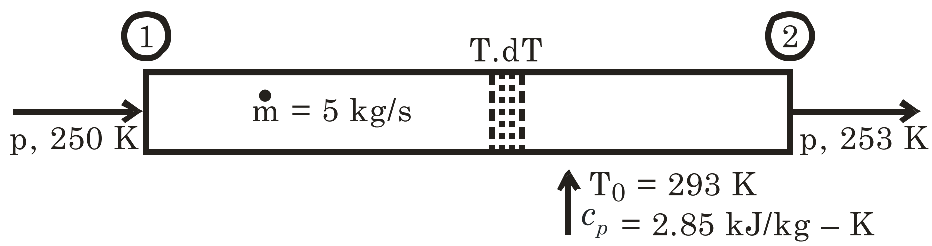
Exergy = Available energy

$$h_1 - h_0 - T_0\left(s_1 - s_2\right)$$
$$= C_p(T_1 - T_0) - T_0\left[C_p \ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0}\right]$$
$$= 0.8659\,(268.15 - 293.15) - 293.15 \times\left\{0.8659\ln\frac{268.15}{293.15} - 0.1889\ln\frac{70}{100}\right\}\text{kJ/ kg}$$
$$= -18.772 \text{ kJ/kg}$$

Q8.26 **A pipe carries a stream of brine with a mass flow rate of 5 kg/s. Because of poor thermal insulation the brine temperature increases from 250 K at the pipe inlet to 253 K at the exit. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) associated with the heat leakage. Take T_0 = 293 K and c_p = 2.85 kJ/kg K.**

(**Ans.** 7.05 kW)

Solution:



Entropy generation rate

$$\dot{S}_{gas} = \dot{S}_{sys} - \frac{\dot{Q}}{T_0}$$
$$= \dot{m}\,(S_2 - S_1) - \frac{\dot{m}\,c_P(253 - 250)}{T_0}$$
$$= \dot{m}\,c_P\left[\ln\frac{T_2}{T_1} - \frac{3}{T_0}\right]\text{kW/ }K$$
$$= 0.0240777 \text{ kW/K}$$

Where, $\dot{Q} = -\dot{m}\,c_p(253 - 250)$

–ve because $\dot{Q}$ flux from surroundings.

$$S_2 - S_1 = c_p \ln\frac{T_2}{T_1}$$

$\therefore$ $\dot{I}$ = rate of energy degradation
= rate of exergy loss

To $\dot{S}_{gen}$ = 293 × 0.0240777 kW = 7.0548 kW

Q8.27 **In an adiabatic throttling process, energy per unit mass of enthalpy remains the same. However, there is a loss of exergy. An ideal gas flowing at the rate *m* is throttled from pressure p_1 to pressure p_2 when the environment is at temperature T_0. What is the rate of exergy loss due to throttling?**

$$\left(\textbf{Ans.}\ \dot{I} = \dot{m} RT_0 \ln \frac{p_1}{p_2}\right)$$

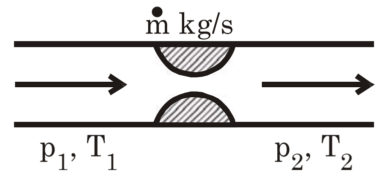
Solution:

Adiabatic throttling process $h_1 = h_2$

$\therefore$ Rate of entropy generation $(\dot{S}_{gen})$

$$\dot{S}_{gen} = (\dot{\Delta S})_{sys} + (\dot{\Delta S})_{surr.}$$
$$= (\dot{\Delta S})_{sys} + 0$$
$$= \dot{m}(S_2 - S_1)$$
$$= \dot{m}\ R \ln\left(\frac{p_1}{p_2}\right)$$

(as no heat interaction with surroundings)



$$TdS = dh - Vdp$$

or $dS = \frac{dh}{T} - V\frac{dp}{T} \qquad \frac{V}{T} = \frac{mR}{p}$

$$dS = 0 - mR\frac{dp}{p}$$

or $\quad S_2 - S_1 = -\int_1^2 mR\frac{dp}{p} = -mR \ln\frac{p_2}{p_1} = mR \ln\frac{p_1}{p_2}$

$\therefore$ Irreversibility rate $(\dot{I})$

$$= T_0 \times \dot{S}_{gen}$$
$$= T_0 \times mR \ln\left(\frac{p_1}{p_2}\right)$$
$$= mR\ T_0 \ln\left(\frac{p_1}{p_2}\right)$$

Q8.28. Air at 5 bar and 20°C flows into an evacuated tank until the pressure in the tank is 5 bar. Assume that the process is adiabatic and the temperature of the surroundings is 20°C.

(a) What is the final temperature of the air?

(b) What is the reversible work produced between the initial and final states of the air?

(c) What is the net entropy change of the air entering the tank?

(d) Calculate the irreversibility of the process.

(**Ans.** (a) 410.2 K, (b) 98.9 kJ/kg, (c) 0.3376 kJ/kg K, (d) 98.9 kJ/kg)

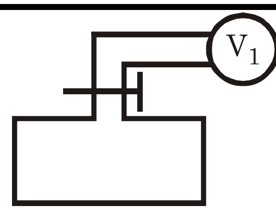
Solution: If m kg of air is entered to the tank then the enthalpy of entering fluid is equal to internal energy of tank fluid.

(a) $\quad h = v$

$\therefore\ C_pT_1 = C_vT_2$

or $\quad \mathbf{T_2} = \left(\frac{C_p}{C_v}\right)T_1 = \boldsymbol{\gamma}\ \mathbf{T_1}$

$= 1.4 \times 293\ K = 410.2\ K$



(b) Reversible work

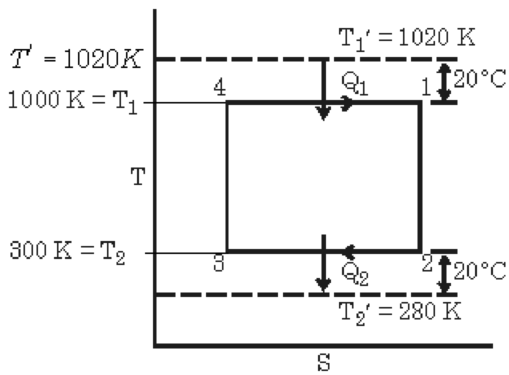
$$\begin{aligned} W &= pdV \text{ work} \\ &= p\,(V_2 - V_1) \\ &= p\,(0 - V_1) \\ &= pV_1 \end{aligned}$$

$$v_1 = \frac{RT_1}{p_1} = 0.168182\ m^3/kg$$

Q8.29 **A Carnot cycle engine receives and rejects heat with a 20°C temperature differential between itself and the thermal energy reservoirs. The expansion and compression processes have a pressure ratio of 50. For 1 kg of air as the working substance, cycle temperature limits of 1000 K and 300 K and T_0 = 280 K, determine the second law efficiency.**

(**Ans.** 0.965)

Solution:



Let Q_1 amount of heat is in input. Then actual Carnot cycle produces work

$$W = Q_1\left(1 - \frac{360}{1000}\right) = 0.7\ Q_1$$

If there is no temperature differential between inlet and outlet then from Q_1 heat input Carnot cycle produce work.

$$W_{max} = Q_1\left(1 - \frac{280}{1020}\right) = 0.72549\ Q_1$$

$\therefore$ Second law efficiency $(\eta_{II}) = \dfrac{W}{W_{max}} = \dfrac{0.7}{0.72549} = 0.965$

Q8.30 **Energy is received by a solar collector at the rate of 300 kW from a source temperature of 2400 K. If 60 kW of this energy is lost to the surroundings at steady state and if the user temperature remains constant at 600 K, what are the first law and the second law efficiencies? Take T_0 = 300 K.**

(**Ans.** 0.80, 0.457)

Solution: First law efficiency

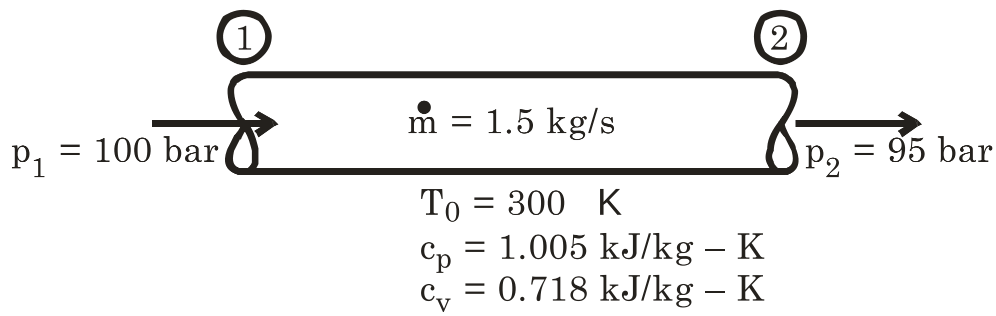
$$= \frac{300 - 60}{300} = 0.8$$

$$\text{Second law efficiency} = \frac{(300-60)\left(1-\frac{300}{600}\right)}{300\left(1-\frac{300}{2400}\right)} = 0.457$$

Q8.31 **For flow of an ideal gas through an insulated pipeline, the pressure drops from 100 bar to 95 bar. If the gas flows at the rate of 1.5 kg/s and has c_p = 1.005 and c_v = 0.718 kJ/kg-K and if T_0 = 300 K, find the rate of entropy generation, and rate of loss of exergy.**

(**Ans.** 0.0215 kW/K, 6.46 kW)

Solution:



Rate of entropy generation

$$\dot{S}_{gen} = (\Delta\dot{S})_{sys} - \frac{\dot{Q}}{T_0}$$

As it is insulated pipe so $\dot{Q} = 0$

$= (\Delta\dot{S})_{sys}$ $\qquad$ $TdS = dh - Vdp$

$= \dot{m}(S_2 - S_1)$ $\qquad$ Here $h_1 = h_2$ so $dh = 0$

$= \dot{m}\,R\ln\left(\frac{p_1}{p_2}\right)$ $\qquad$ $\therefore \quad TdS = -Vdp$

$= 1.5 \times 0.287 \times \ln\left(\frac{100}{95}\right)\text{kW/K}$ $\qquad$ $dS = -\frac{V}{T}dp$

$= 0.022082$ kW/K $\qquad$ $\int_1^2 dS = -mR\int_1^2 \frac{dp}{p} = mR\ln\frac{p_2}{p_1}$

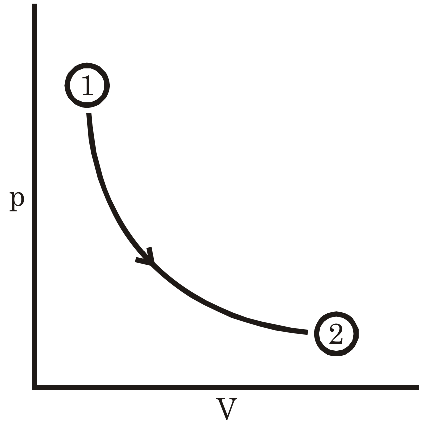
Rate of loss of exergy = Irreversibility rate $(\dot{I})$

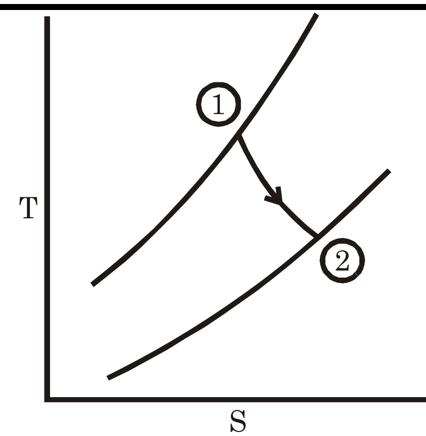
$\text{To}\ \dot{S}_{gen} = 300 \times 0.22082 = 6.6245$ kW

Q8.32 **The cylinder of an internal combustion engine contains gases at 2500°C, 58 bar. Expansion takes place through a volume ratio of 9 according to $pv^{1.38}$ = const. The surroundings are at 20°C, 1.1 bar. Determine the loss of availability, the work transfer and the heat transfer per unit mass. Treat the gases as ideal having R = 0.26 kl/kg-K and c_v = 0.82 kJ/kg-K.**

(**Ans.** 1144 kJ/kg, 1074 kJ/kg, – 213 kJ/kg)

Solution:





$p_1 = 58$ bar = 5800 kPa $\quad$ $p_2 = 279.62$ kPa (calculated)

$v_1 = 0.1243\ m^3/kg$ (calculating) $\quad$ $v_2 = 9v_1 = 1.11876\ m^3/kg$

$T_1 = 2500°C = 2773$ K $\quad$ $T_2 = 1203.2$ K (calculated)

$$\therefore\ v_1 = m\frac{RT_1}{p_1} = 0.1243\ m^3/kg$$

$p_0 = 1.1$ bar = 110 kPa $\quad$ R = 0.26 kJ/kg – K

$T_0 = 20°C = 293$ K $\quad$ W = 0.82 kJ/kg – K

$\therefore\ c_P = c_v + R = 1.08$ kJ/kg

$$\therefore\ \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \text{ or } p_2 = p_1\left(\frac{v_1}{v_2}\right)^{1.38} = \frac{p_1}{9^{1.38}}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \frac{1}{9^{0.38}}$$

$$\therefore\ T_2 = \frac{T_1}{9^{0.38}} = 1203.2 \text{ K}$$

$\Rightarrow$ Loss of availability

$$\therefore\ \phi_1 - \phi_2$$

$$= (u_1 - u_2) - T_0\ (s_1 - s_2) + p_0\,(v_1 - v_2)$$

$$= C_v(T_1 - T_2) - T_0\left[C_p \ln\frac{T_1}{T_2} - R\ln\frac{p_1}{p_2}\right] + p_0(v_1 - v_2)$$

$$= 0.82(2773 - 1203.2) - 293\left[1.08\ln\frac{2773}{1203.2} - 0.26\ln\left(\frac{5800}{279.62}\right)\right] + 110(0.1243 - 1.11876) \text{ kJ/kg}$$

$$= 1211 \text{ kJ/kg}$$

$$\text{Work transfer (W)} = \frac{p_1 v_1 - p_2 v_2}{n-1} = 1074 \text{ kJ/kg}$$

$$\cancel{d}Q = du + \cancel{d}W$$

$$\therefore\ Q_{1-2} = C_v(T_2 - T_1) + W_{1-2}$$

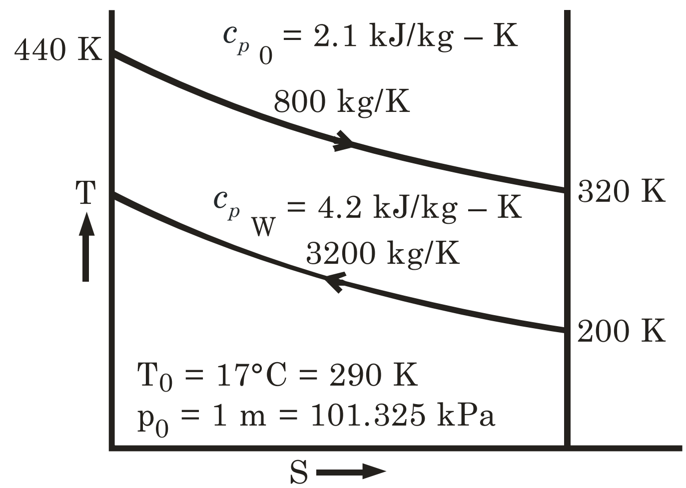
$$= -1287.2 + 1074 = -213.2 \text{ kJ/kg}$$

Q8.33 **In a counterflow heat exchanger, oil (c_p = 2.1 kJ/kg-K) is cooled from 440 to 320 K, while water (c_p = 4.2 kJ/kg K) is heated from 290 K to temperature *T*. The respective mass flow rates of oil and water are 800**

and 3200 kg/h. Neglecting pressure drop, KE and PE effects and heat loss, determine
(a) The temperature *T*
(b) The rate of exergy destruction
(c) The second law efficiency
Take T_0 = I7°C and p_0 = 1 atm.

(**Ans.** (a) 305 K, (b) 41.4 MJ/h, (c) 10.9%)

Solution:



From energy balance

(a) $\dot{m}_P\, c_P(440 - 320)$

$= \dot{m}_w\, c_{P_w}(T - 290)$

$\therefore$ T = 290 + 15 = 305 K

(b) $\dot{S}_{gen} = (\dot{\Delta S})_0 + (\dot{\Delta S})$

$$= \dot{m}_o c_{po} \ln \frac{T_{fo}}{T_{io}} + \dot{m}_w c_{pw} \ln \frac{T_{fw}}{T_{iw}}$$

$$\left[\frac{800}{3600} \times 2.1 \times \ln \frac{320}{440} + \frac{3200}{3600} \times 4.2 \times \ln \frac{305}{290}\right]$$

= 0.039663 kW/K = 39.6634 W/K

$\therefore$ Rate of energy destruction = $T_o \times \dot{S}_{gen}$ = 290 × 0.039663 kW

= 11.5024 kW = 41.4 MJ/K

(c) Availability decrease of oil

$= A_1 - A_2 = h_1 - h_2 - T_0\,(s_1 - s_2)$

$$= \dot{m}_0\, c_{p0}\left[(T_1 - T_2) - T_0 \ln \frac{T_1}{T_2}\right]$$

$$= \frac{800}{3600} \times 2.1 \times \left[(440 - 320) - 290 \ln \frac{440}{320}\right]$$

= 12.903 kW

Availability decrease of water

$A_1 - A_2 = h_1 - h_2 - T_0\,(s_1 - s_2)$

$$= \dot{m}_w\, c_{pw}\left[(T_1 - T_2) - T_0 \ln \frac{T_1}{T_2}\right]$$

$$= \frac{3200}{3600} \times 4.2 \times \left[(305 - 290) - 290 \ln \frac{305}{290}\right] \text{kW} = 1.4 \text{ kW}$$

$\therefore$ Second law efficiency $(\eta_{II}) = \dfrac{\text{Gain of availability}}{\text{Loss for that}} = \dfrac{1.4}{12.903}$ = 10.85%

9. Properties of Pure Substances

Some Important Notes

1. Triple point

On p-T diagram	**It is a Point**.
On p-V diagram	**It is a Line**
On T-s diagram	**It is a Line**
On U-V diagram	**It is a Triangle**

2. Triple point of water

T = 273.16 K = 0.01°C	p = 0.00612 bar = 4.587 mm of Hg	Entropy (S) = 0 Internal Energy (u) = 0 Enthalpy (h) = u + pV = Slightly positive

3. Triple point of CO_2

p $\simeq$ 5 atm	And T = 216.55 K = – 56.45° C that so why sublimation occurred.

3. Critical Point

For water $p_c = 221.2$ bar ≈ 225.5 kgf/cm^2
$T_c = 374.15°C \approx 647.15$ K
$v_c = 0.00317$ m^3/kg

At critical point

$h_{fg} = 0$; $v_{fg} = 0$; $S_{fg} = 0$

4. Mollier Diagram

Basis of the h-S diagram is $\left(\frac{\partial h}{\partial S}\right)_P = T$ $\left[\begin{array}{l} \because TdS = dh - vdp \\ \therefore \left(\frac{\partial h}{\partial S}\right)_p = T \end{array}\right]$

$\therefore$ The slope of an isobar on the h-s co-ordinates is equal to the absolute saturation temperature at that pressure. And for that isobars on Mollier diagram diverges from one another.

5. **Dryness friction**

$$x = \frac{m_v}{m_v + m_l}$$

6. $v = (1 - x)\ v_f + x\ v_g$ $\quad$ $v = v_f + x\ v_{fg}$

$u = (1 - x)\ u_f + x\ u_g$ $\quad$ $u = u_f + x\ u_{fg}$

$h = (1 - x)\ h_f + x\ h_g$ $\quad$ $h = h_f + x\ h_{fg}$

$s = (1 - x)\ s_f + x\ s_g$ $\quad$ $s = s_f + x\ s_{fg}$

7. **Super heated vapour:** When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure.

8. **Compressed liquid:** When the temperature of the liquid is less than the saturation temperature at the given pressure, the liquid is called compressed liquid.

9. **In combined calorimeter**

$x = x_1 \times x_2$ $\quad$ x_1 = from throttle calorimeter

x_2 = from separation calorimeter

Questions with Solution P. K. Nag

Q9.1 **Complete the following table of properties for 1 kg of water (liquid, vapour or mixture)**

	p (*bar*)	*t* (*°C*)	*v* (*m³/kg*)	*x* (*%*)	*Super-heat* (*°C*)	*h* (*kJ/kg*)	*s* (*kJ/kg K*)
(a)	—	35	25.22	—	—	—	—
(b)	—	—	0.001044	—	—	419.04	—
(c)	—	212.42	—	90	—	—	—
(d)	1	—	—	—	—	—	6.104
(e)	10	320	—	—	—	—	—
(f)	5	—	0.4646	—	—	—	—
(g)	4	—	0.4400	—	—	—	—
(h)	—	500	—	—	—	3445.3	—
(i)	20	—	—	—	50	—	—
(j)	15	—	—	—	—	—	7.2690

Solution:

	p bar	t°C	v m³/kg	x/%	Superheat 0°C	h kJ/kg	s kJ/ kg – K
a	0.0563	35	25.22	100	0	2565.3	8.353
b	1.0135	100°	0.001044	0	0	419.04	1.307
c	20	212.42	0.089668	90	0	2608.3	5.94772
d	1	99.6	1.343	79.27	0	2207.3	6.104
e	10	320	0.2676	100	140	3093.8	7.1978
f	5	238.8°C	0.4646	100	87.024	2937.1	7.2235
g	4	143.6	0.4400	95.23	0	2635.9	6.6502
h	40	500	0.0864	100	249.6	3445.3	7.090
i	20	212.4°C	0.1145	100	50	2932.5	6.600
j	15	400	0.203	100	201.70	3255.8	7.2690

Calculations: For (a) ………… For (b)

For (c) $v = v_f + x(v_g - v_f)$ $\quad h = h_f + x\, h_{fg}$ $\quad \Rightarrow s = s_f + x\, s_{fg}$

For (d) $s = s_f + x\, s_{fg}$ $\quad \therefore x = \dfrac{s - s_f}{s_{fg}} = 0.7927$ $\quad \therefore h = h_f + x\, h_{fg}$

$v = v_f + x(v_{fg} - v_f)$

For (e) $t_{sat} = 180°C$ $\quad v = 0.258 + \dfrac{20}{50}(0.282 - 0.258),$

$$h = 3051.2 + \frac{20}{50}(3157.8 - 3051.2)) = 3093.8$$

$$s = 7.123 + \frac{20}{50}(7.310 - 7.123) = 7.1978$$

For (f) $\quad t = 200 + \dfrac{0.4646 - 0.425}{0.476 - 0.425} \times 50 = 238.8°\ C$

$$h = 2855.4 + \frac{38.8}{50}(2960.7 - 2855.4) = 2937.1$$

$$s = 7.059 + \frac{38.8}{50}(7.271 - 7.059) = 7$$

(g) $0.4400 = 0.001084 + x(0.462 - 0.001084)$ $\therefore x = 09523$

$h = 604.7 + x \times 2133$, $s = 1.7764 + x \times 5.1179 = 6.6502$

(i) $t = 262.4°C$ $v = 0.111 + \frac{12.4}{50}(0.121 - 0.111)$,

$$h = 2902.5 + \frac{12.4}{50}(3023.5 - 2902.5) = 2932.5$$

$$s = 6.545 + \frac{12.9}{50}(6.766 - 6.545) = 6.600$$

Q9.2 **(a) A rigid vessel of volume 0.86 m³ contains 1 kg of steam at a pressure of 2 bar. Evaluate the specific volume, temperature, dryness fraction, internal energy, enthalpy, and entropy of steam.**

(b) The steam is heated to raise its temperature to 150°C. Show the process on a sketch of the *p–v* diagram, and evaluate the pressure, increase in enthalpy, increase in internal energy, increase in entropy of steam, and the heat transfer. Evaluate also the pressure at which the steam becomes dry saturated.

(**Ans.** (a) 0.86 m³/kg, 120.23°C, 0.97, 2468.54 k/kg, 2640.54 kJ/kg, 6.9592 kJ/kg K
(b) 2.3 bar, 126 kJ/kg, 106.6 kJ/kg, 0.2598 kJ/kg K, 106.6 kJ/K)

Solution: (a) → Specific volume = Volume/mass = $\frac{0.86\ \text{m}^3}{1\ \text{kg}}$ = 0.86 m³/kg

→ at 2 bar pressure saturated steam sp. Volume = 0.885 m³/kg
So it is wet steam and temperature is saturation temperature
= 120.2° C

→ $v = v_f + x(v_g - v_f)$ $\therefore x = \frac{v - v_f}{v_g - v_f}$

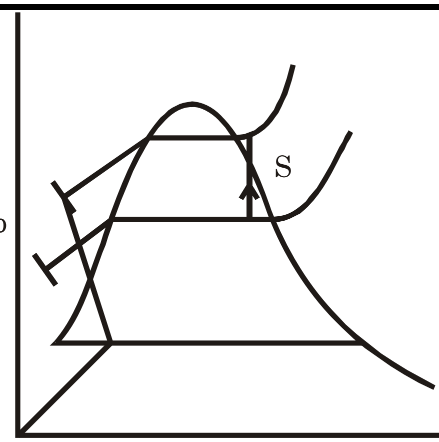
$$= \frac{0.86 - 0.001061}{0.885 - 0.001061} = 0.97172$$

→ Internal energy (u) = h – pv = 2644 – 200 × 0.86 = 2472 kJ/kg

→ Here $h = h_f + x\, h_{fg} = 504.7 + 0.97172 \times 2201.6 = 2644$ kJ/kg

→ $s = s_f + x\, s_{fg} = 1.5301 + 0.97172 \times 5.5967 = 6.9685$ kJ/kg – K

(b) $T_2 = 150°C = 423$ K
$v_2 = 0.86$ m³/kg



$$p_S = 2 + \frac{0.885 - 0.86}{0.885 - 0.846} + (2 + 0 - 2) = 2.0641 \text{ bar}$$

$$v_S = 0.86 \text{ m}^3/\text{kg}$$

$$T_S = \frac{0.0691}{0.1}(121.8 - 120.2) + 120.2 = 121.23^\circ \text{ C} = 394.23 \text{ K}$$

Path 2 – 5 are is super heated zone so gas law (obey)

$$\therefore \quad \frac{p_S v_1}{T_S} = \frac{p_2 v_2}{T_2} \qquad [\therefore v_2 = v_1]$$

$$\therefore \quad p_2 = \frac{T_2}{T_S} \times p_S = \frac{423}{394.23} \times 2.0641 = 2.215 \text{ bar}$$

From Molier diagram $p_s = 2.3$ bar, $h_2 = 2770$ kJ/kg, $s_2 = 7.095$

$$\therefore \quad \Delta h = 127 \text{ kJ/kg}, \ \Delta s = 0.1265 \text{ kJ/kg} - \text{K},$$

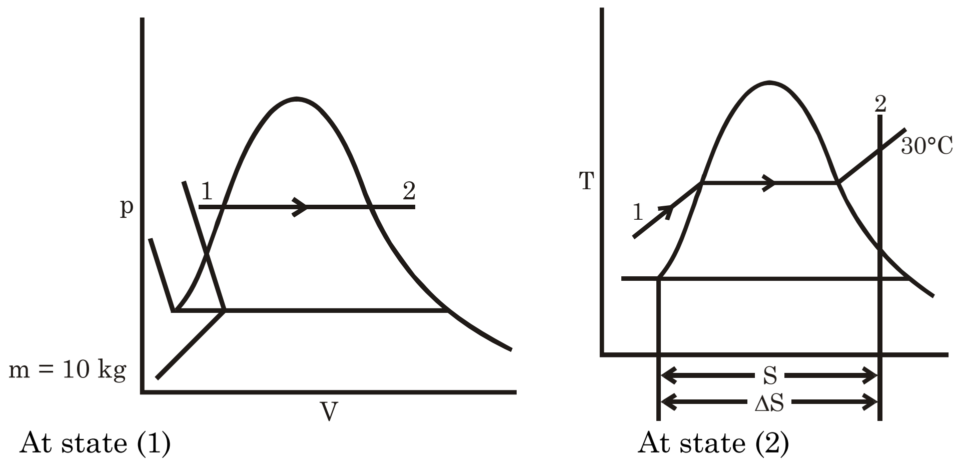
$$u_2 = h_2 - p_2 v_2 = 2580$$

$$\therefore \quad \Delta q = u_2 - u_1 = 107.5 \text{ kJ/kg}$$

Q9.3 **Ten kg of water at 45°C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C. Find the change in volume, enthalpy, internal energy and entropy.**

(**Ans.** 2.569 m³, 28627.5 kJ, 26047.6 kJ, 64.842 kJ/K)

Solution:



At state (1)

$p_1 = 10$ bar = 1000 kPa

$T_1 = 45$°C = 318 K

At state (2)

$p_2 = p_1 = 10$ bar

$T_2 = 300$°C

For Steam Table

$v_1 = 0.001010\ m^3/kg$ $\qquad$ $v_2 = 0.258\ m^3/kg$

$h_1 = 188.4\ kJ/kg$ $\qquad$ $u_2 = 2793.2\ kJ/kg$

$u_1 = h_1 - p_1v_1 = 187.39\ kJ/kg$ $\qquad$ $h_2 = 3051.2\ kJ/kg$

$s_1 = 0.693\ kJ/kg - K$ $\qquad$ $s_2 = 7.123\ kJ/kg - K$

$\therefore$ Change in volume $= m\,(v_2 - v_1) = 2.57\ m^3$

Enthalpy change $= m(h_2 - h_1) = 28.628\ MJ$

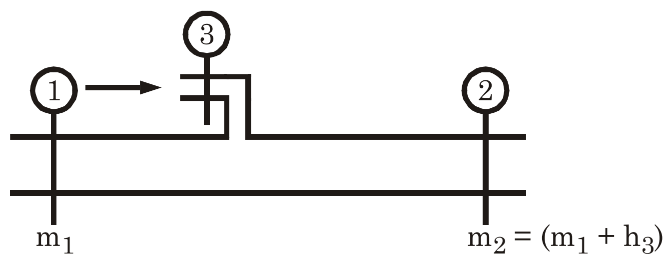
Internal Energy change $= m(u_2 - u_1) = 26.0581\ MJ$

Entropy change $= m\,(s_2 - s_1) = 64.3\ kJ/K$

Q9.4 **Water at 40°C is continuously sprayed into a pipeline carrying 5 tonnes of steam at 5 bar, 300°C per hour. At a section downstream where the pressure is 3 bar, the quality is to be 95%. Find the rate of water spray in kg/h.**

(**Ans.** 912.67 kg/h)

Solution:



$p_1 = 5$ bar $= 500$ kPa

$T_1 = 300°C$

$h_1 = 3064.2\ kJ/kg$

$p_2 = 3$ bar

$T_2 = 133.5°C$

$h_2 = 561.4 + 0.95 \times 2163.2 = 2616.44\ kJ/kg$

$T_3 = 40^o\ C$

$h_3 = 167.6\ kJ/kg$

$\therefore$ **For adiabatic steady flow**

$$\dot{m}_1h_1 + \dot{m}_3h_3 = \dot{m}_2\,(h_2) = (\dot{m}_1 + \dot{m}_3)\,h_2$$

$$\therefore \quad \dot{m}_1(h_1 - h_2) = \dot{m}_3(h_2 - h_3)$$

$$\therefore \quad \dot{m}_3 = \dot{m}_1\frac{(h_1 - h_2)}{(h_2 - h_3)}$$

$$= 5000 \times \left\{\frac{3064.2 - 2616.44}{2616.44 - 167.6}\right\} \text{kg/hr}$$

$$= 914.23 \text{ kg/hr.}$$

Q9.5 **A rigid vessel contains 1 kg of a mixture of saturated water and saturated steam at a pressure of 0.15 MPa. When the mixture is heated, the state passes through the critical point. Determine**

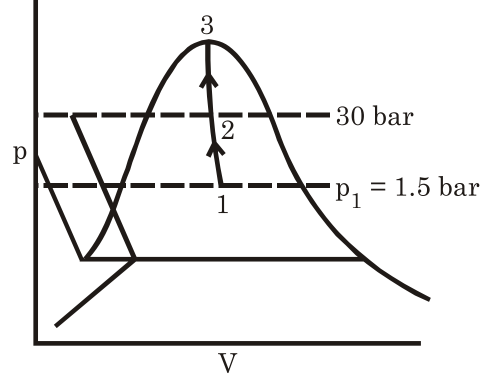
(a) The volume of the vessel

(b) The mass of liquid and of vapour in the vessel initially

(c) **The temperature of the mixture when the pressure has risen to 3 MPa**
(d) **The heat transfer required to produce the final state (c).**

(**Ans.** (a) 0.003155 m^3, (b) 0.9982 kg, 0.0018 kg, (c) 233.9°C, (d) 581.46 kJ/kg)

Solution:



It is a rigid vessel so if we

(a) Heat this then the process will be constant volume heating. So the volume of the vessel is critical volume of water = 0.00317 m^3

(b) $v = v_f + x(v_g - v_{fg})$ $\quad\therefore x = \dfrac{v - v_f}{v_g - v_f} = \dfrac{0.00317 - 0.001053}{1.159 - 0.001053}$

$\therefore$ Mass of vapour = 0.0018282 kg
$\therefore$ Mass of water = 0.998172 kg

(c) As it passes through critical point then at 3 MPa i.e. 30 bar also it will be wet steam 50 temperatures will be 233.8ºC.

(d) Required heat (Q) = $(u_2 - u_1)$

$= (h_2 - h_1) - (p_2v_2 - p_1v_1)$

$= (h_{2f} + x_2h_{fg_2}) - (h_{1f} + x_1h_{fg_1}) - p_2\{\ v_f + x_2(v_g - v_f)\ \}_2$
$+ p_1\{v_f + x_1(v_g - v_f)_1\}$

$v_2 = v_{f_2} + x_2(v_{g_2} - v_{f_2})$

$\therefore \quad x_2 = \dfrac{v_2 - v_{f_2}}{v_{g_2} - v_{f_2}} = \dfrac{0.00317 - 0.001216}{0.0666 - 0.001216} = 0.029885$

$\therefore$ Q = (1008.4 + 0.029885 × 1793.9)
– (467.1 + 0.0018282 × 2226.2) – 3000 (0.001216 + 0.029885 (0.0666 – 0.001216)) + 150(0.001053 + 0.001828 (1.159 – 0.0018282))
= 581.806 kJ/kg

Q9.6 **A rigid closed tank of volume 3 m^3 contains 5 kg of wet steam at a pressure of 200 kPa. The tank is heated until the steam becomes dry saturated. Determine the final pressure and the heat transfer to the tank.**

(**Ans.** 304 kPa, 3346 kJ)

Solution: $V_1 = 3\ m^3$

$m = 5$ kg

$\therefore \quad v_1 = \dfrac{3}{5} = 0.6\ m^3/kg$

$p_1 = 200$ kPa = 2 bar

$\therefore \quad x_1 = \dfrac{v_1 - v_f}{v_g - v_f} = \dfrac{(0.6 - 0.001061)}{(0.885 - 0.001061)} = 0.67758$

$h_1 = h_f + x_1 h_{fg} = 504.7 + 0.67758 \times 2201.6 = 1996.5$ kJ/kg

$u_1 = h_1 - p_1 v_1 = 1996.5 - 200 \times 0.6 = 1876.5$ kJ/kg

As rigid tank so heating will be cost vot heating.

$\therefore \quad v_{g_2} = 0.6\ m^3/kg$

From Steam Table $v_g = 0.606\ m^3/kg$ for p = 300 kPa
$v_g = 0.587\ m^3/kg$ for p = 310 kPa

$\therefore$ For V = 0.6 m³ $\quad p_2 = 300 \times \dfrac{10 \times 0.006}{0.019} = 303.16$ kPa

$\therefore \quad u_2 = h_2 - p_2 v_2 = 2725 - 303.16 \times 0.6 = 2543$ kJ/kg

$\therefore$ Heat supplied $Q = m(u_2 - u_1) = 3333$ kJ

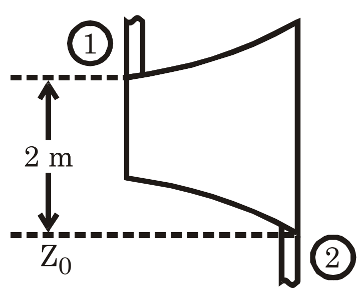
Q9.7 **Steam flows through a small turbine at the rate of 5000 kg/h entering at 15 bar, 300°C and leaving at 0.1 bar with 4% moisture. The steam enters at 80 m/s at a point 2 in above the discharge and leaves at 40 m/s. Compute the shaft power assuming that the device is adiabatic but considering kinetic and potential energy changes. How much error would be made if these terms were neglected? Calculate the diameters of the inlet and discharge tubes.**

(**Ans.** 765.6 kW, 0.44%, 6.11 cm, 78.9 cm)

Solution: $\dot{m} = 5000$ kg/hr $= \dfrac{5000}{3600}$ kg/s

$p_1 = 15$ bar

$t_1 = 300°$ C



$\therefore$ From Steam Table

$h_1 = 3037.6$ kJ/kg $\quad p_2 = 0.1$ bar

$V_1 = 80$ m/s $\quad x_2 = \dfrac{(100-4)}{100} = 0.96$

$Z_1 = (Z_0 + 2)$ m $\quad t_2 = 45.8°$ C

$v_1 = 0.169\ m^3/kg$ $h_2 = h_f + x_2\, h_{fg}$

$= 191.8 + 0.96 \times 2392.8 = 2489\ kJ/kg$

$V_2 = 40\ m/s,\ Z_2 = Z_0\ m$ $\therefore$ $v_2 = 14.083\ m^3/kg$

$$\therefore \quad \text{Work output (W)} = \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2000} + \frac{g(Z_1 - Z_2)}{2000}\right]$$

$$= \frac{5000}{3600}\left[(3037.6 - 2489) + \frac{80^2 - 90^2}{2000} + \frac{9.81(2)}{2000}\right] kW$$

$= 765.45\ kW$

If P.E. and K.E. is neglected the

$$W' = \dot{m}(h_1 - h_2) = 762.1\ kW$$

$$\therefore \quad \text{Error} = \frac{W - W'}{W} \times 100\% = 0.44\%$$

$$\text{Area at inlet } (A_1) = \frac{\dot{m}v_1}{V_1} = 0.002934\ m^2 = 29.34\ cm^2$$

$\therefore\ d_1 = 6.112\ cm$

$$\text{Area at outlet } (A_2) = \frac{\dot{m}v_2}{V_2} = 0.489\ m^2 \qquad \therefore\ d_2 = 78.9\ cm$$

Q9.8 **A sample of steam from a boiler drum at 3 MPa is put through a throttling calorimeter in which the pressure and temperature are found to be 0.1 MPa, 120°C. Find the quality of the sample taken from the boiler.**

(**Ans.** 0.951)

Solution : $p_1 = 3\ MPa = 30\ bar$

$p_2 = 0.1\ MPa = 1\ bar$

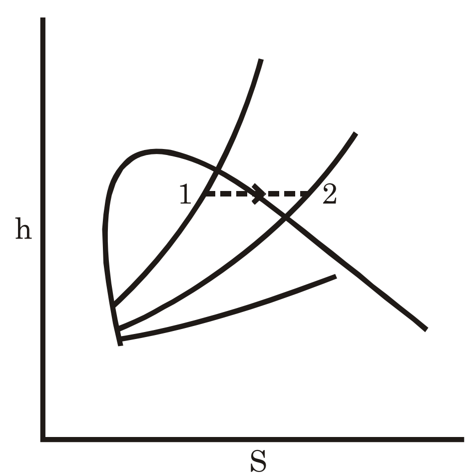
$t_2 = 120^\circ\ C$

$$h_2 = 2676.2 + \frac{20}{50}(2776.4 - 2676.2)$$

$= 2716.3\ kJ/kg$

$\therefore \quad h_1 = h_2$

$\therefore \quad h_1 = 2716.3$ at 30 bar



If dryness fraction is

$\therefore\ h_1 = h_{g_1} + x\, h_{fg_1}$

$$\therefore \quad x = \frac{h_1 - h_{f_1}}{h_{fg_1}}$$

$$= \frac{2716.3 - 1008.4}{1793.9} = 0.952$$

Q9.9 **It is desired to measure the quality of wet steam at 0.5 MPa. The quality of steam is expected to be not more than 0.9.**

(a) Explain why a throttling calorimeter to atmospheric pressure will not serve the purpose.

(b) Will the use of a separating calorimeter, ahead of the throttling calorimeter, serve the purpose, if at best 5 C degree of superheat is desirable at the end of throttling? What is the minimum dryness fraction required at the exit of the separating calorimeter to satisfy this condition?

(**Ans.** 0.97)

Solution: (a) After throttling if pressure is atm. Then minimum temperature required
$t = t_{sat} + 5°C = 100 + 5 = 105° C$
Then Enthalpy required

$$= 2676 + \frac{5}{50}(2776.3 - 2676) \text{ kJ/kg} = 2686 \text{ kJ/kg}$$

If at 0.5 MPa = 5 bar dryness fraction is < 0.9

$\therefore$ $h_{max} = h_f + 0.9\ h_{fg} = 640.1 + 0.9 \times 2107.4 = 2536.76$ kJ/kg

So it is not possible to give 5° super heat or at least saturation i.e. (2676 kJ/kg) so it is not correct.

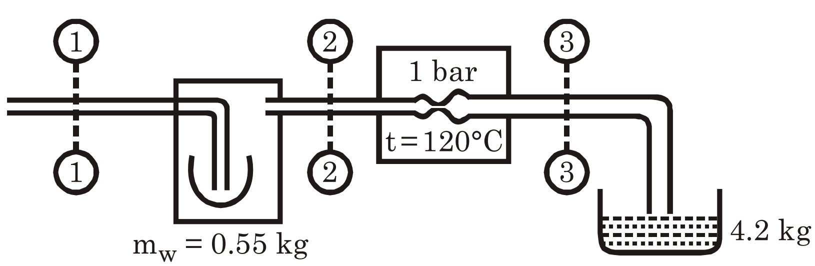
(b) Minimum dryness fraction required at the exit of the separating calorimeter (x) then

$h = h_f + x\ h_{fg}$ $\quad \therefore x = \dfrac{2686 - 640.1}{2107.4} = 0.971$

Q9.10 **The following observations were recorded in an experiment with a combined separating and throttling calorimeter:**
Pressure in the steam main–15 bar
Mass of water drained from the separator–0.55 kg
Mass of steam condensed after passing through the throttle valve –4.20 kg
Pressure and temperature after throttling–1 bar, 120°C
Evaluate the dryness fraction of the steam in the main, and state with reasons, whether the throttling calorimeter alone could have been used for this test.

(**Ans.** 0.85)

Solution: $p_1 = 15 \text{ bar} = p_2$
$T_1 = 198.3° \text{ C} = t_2$
$p_3 = 1 \text{ bar}, T_3 = 120° \text{ C}$
$\therefore$ $h_3 = 2716.3$ kJ/kg



$h_2 = h_{2f} + x_2 \times h_{fg_2} = 844.7 + x_2 \times 1945.2$

$\therefore$ $x_2 = 0.96216$ $\quad \therefore$ dry steam $= x_2 \times 4.2$

$\therefore$ Total dryness fraction (x)

$$= \frac{x_2 \times 4.2}{4.2 + 0.55} = \frac{0.96216 \times 4.2}{4.2 + 0.55} = 0.85$$

$h_1 = h_{f1} + x\, h_{fg1} = 844.7 + 0.85 \times 1945.2 = 2499.6$ kJ/kg

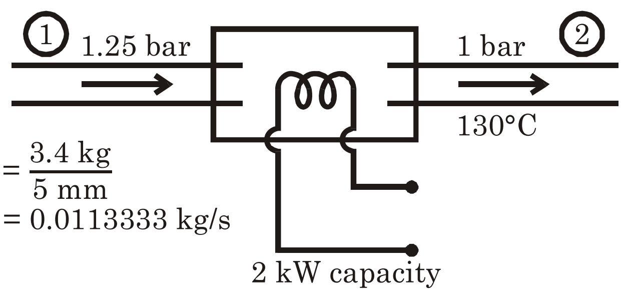
But at 1 bar minimum 5° super heat i.e. 105°C enthalpy is 2686 kJ/kg
So it is not possible to calculate only by throttling calorimeter.

Q9.11 Steam from an engine exhaust at 1.25 bar flows steadily through an electric calorimeter and comes out at 1 bar, 130°C. The calorimeter has two 1 kW heaters and the flow is measured to be 3.4 kg in 5 min. Find the quality in the engine exhaust. For the same mass flow and pressures, what is the maximum moisture that can be determined if the outlet temperature is at least 105°C?

(**Ans.** 0.944, 0.921)

Solution: $h_2 = 2676.2 + \frac{30}{50}(2776.4 - 2676.2) = 2736.3$ kJ/kg

$$\dot{m}\, h_1 = \dot{m}\, h_2 - \dot{Q}$$



$$\dot{m} = \frac{3.4 \text{ kg}}{5 \text{ mm}} = 0.0113333 \text{ kg/s}$$

$$h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2560 \text{ kJ/kg}$$

At 1.25 bar: from Steam Table
At 1.2 bar, $h_f = 439.4$ kJ/kg $h_{fg} = 2244.1$ kJ/kg
At 1.3 bar, hf = 449.2 kJ/kg $h_{fg} = 2237.8$ kJ/kg
At 1.25 bar $h_f = 444.3$ kJ/kg; $h_{fg} = 2241$ kJ/kg
If dryness fraction is x
Then $2560 = 444.3 + x \times 2241$
or $x = 0.9441$
If outlet temperature is 105° C then
$h_2 = 2686$ kJ/kg   (then from problem 9.9)

$$\therefore \quad h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2509.53 \text{ kJ/kg}$$

Then if dryness fraction is x_2 then
$2509 = 444.3 + x_2 \times 2241$ $\therefore\ x_2 = 0.922$ (min)

Q9.12 Steam expands isentropically in a nozzle from 1 MPa, 250°C to 10 kPa. The steam flow rate is 1 kg/s. Find the velocity of steam at the exit from the nozzle, and the exit area of the nozzle. Neglect the velocity of steam at the inlet to the nozzle.

The exhaust steam from the nozzle flows into a condenser and flows out as saturated water. The cooling water enters the condenser at 25°C and leaves at 35°C. Determine the mass flow rate of cooling water.

(**Ans.** 1224 m/s, 0.0101 m², 47.81 kg/s)

Solution: At inlet

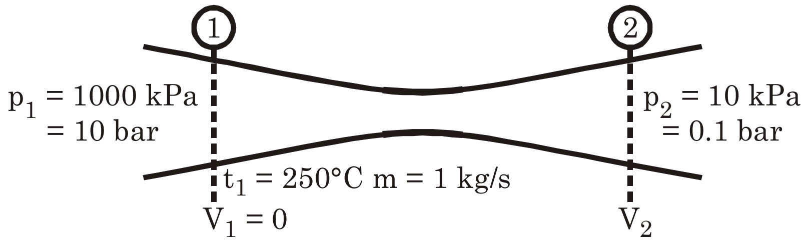
$h_1 = 2942.6$ kJ/kg $\qquad$ $t_2 = 45.8^\circ$ C

$s_1 = 6.925$ kJ/kg-K $\qquad$ $s_2 = 6.925$ kJ/kg-K

If dry fraction x

$h_{f2} = 191.8$ kJ/kg

$v_2 = 12.274$ m³/kg



$\therefore \qquad s_1 = s_2 = 0.649 + x \times 7.501 \qquad \therefore x = 0.8367$

$\therefore \qquad h_2 = 191.8 + 0.8367 \times 2392.8 = 2193.8$ kJ/kg

$\therefore \qquad V_2 = \sqrt{2000(h_1 - h_2)} = 1224$ m/s

$\therefore \qquad$ Outlet Area $= \dfrac{\dot{m} v_2}{V_2} = 100.3$ cm²

$\therefore \qquad$ If water flow rate is m kg/s

$\therefore \qquad 1 \times (2193.8 - 191.8) = m\ 4.187\ (35 - 25)$

$\therefore \qquad \dot{m} = 47.815$ kg/s

Q9.13 A reversible polytropic process, begins with steam at p_1 = 10 bar, t_1 = 200°C, and ends with p_2 = 1 bar. The exponent n has the value 1.15. Find the final specific volume, the final temperature, and the heat transferred per kg of fluid.

Solution: $p_1 = 10$ bar $= 1000$ kPa

$p_2 = 1$ bar $= 100$ kPa

$t_1 = 200^\circ$ C $= 473$ K

From Steam Table

$V_1 = 0.206$ m³/s

$h_1 = 2827.9$ kJ/kg

$$\therefore \qquad v_2 = \frac{p_1 v_1^n}{p_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} . v_1 = \left(\frac{10}{1}\right)^{\frac{1}{1.15}} \times 0.206 = 1.5256 \text{ m}^3/\text{kg}$$

As at 1 bar $v_g = 1.694$ m³/kg $\qquad \therefore$ then steam is wet

$\therefore \qquad 1.5256 = 0.001043 + x\ (1.694 - 0.001043)$

$\therefore \qquad x = 0.9$

Final temperature $= 99.6^\circ$ C

$= u_1 - u_2$

$= (h_1 - h_2) - (p_1 v_1 - p_2 v_2)$

$= (2827.9 - 2450.8) - (1000 \times 0.206 - 100 \times 1.5256)$
$= 323.7$ kJ/kg

$[h_2 = h_{f2} + x\, h_{fg2}]$
$= 417.5 + 0.9 \times 2257.9$
$= 2450.8$

Work done (W) $= \dfrac{p_1 v_1 - p_2 v_2}{n-1} = 356.27$ kJ/kg

$\therefore$ From first law of thermo dynamics

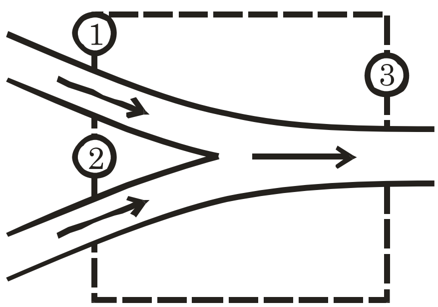
$Q_2 = (u_2 - u_1) + W_{1-2}$
$= (-323.7 + 356.27) = 32.567$ kJ/kg

Q9.14 **Two streams of steam, one at 2 MPa, 300°C and the other at 2 MPa, 400°C, mix in a steady flow adiabatic process. The rates of flow of the two streams are 3 kg/min and 2 kg/min respectively. Evaluate the final temperature of the emerging stream, if there is no pressure drop due to the mixing process. What would be the rate of increase in the entropy of the universe? This stream with a negligible velocity now expands adiabatically in a nozzle to a pressure of 1 kPa. Determine the exit velocity of the stream and the exit area of the nozzle.**

(**Ans.** 340°C, 0.042 kJ/K min, 1530 m/s, 53.77 cm²)

Solution: $p_2 = 2$ MPa $= 20$ bar
$t_2 = 400^\circ$ C
$\dot{m}_2 = 2$ kg/min
$h_2 = 3247.6$ kJ/kg
$s_2 = 7.127$ kJ/kg-K

$p_1 = 2$ MPa $= 20$ bar
$t_1 = 300°C$
$\dot{m}_1 = 3$ kg/min
For Steam table
$h_1 = 3023.5$ kJ/kg
$s_1 = 6.766$ KJ/kgK



$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5$ kg/min
$p_3 = 20$ bar
$s_3 = 6.766 + \dfrac{40}{50}(6.956 - 6.766)$
$= 6.918$ kJ/kg – K

For adiabatic mixing process

$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$\therefore$ $h_3 = 3113.14$ kJ/kg

$\therefore$ Final temperature (t) $= 300 + \dfrac{3113.14 - 3023.5}{3137 - 3023.5} \times 50 = 340^\circ$ C

Rate of increase of the enthalpy of the universe

$\dot{s}_{gen} = \dot{m}_3 S_3 - \dot{m}_1 S_1 - \dot{m}_2 S_2 = 0.038$ kJ/K – min

After passing through nozzle if dryness fraction is x then
$S_3 = S_{exit}$ or $6.918 = 0.106 + x \times 8.870$ $\therefore$ $x = 0.768$

$\therefore$ $h_e = 29.3 + 0.768 \times 2484.9 = 1937.7$ kJ/kg

$\therefore$ $V = \sqrt{2000\,(3113.14 - 1937.7)} = 1533.3$ m/s

Exit area of the nozzle = $\frac{\dot{m}v}{V}$ = 0.0054 m² = 54 cm²

Q9.15 **Boiler steam at 8 bar, 250°C, reaches the engine control valve through a pipeline at 7 bar, 200°C. It is throttled to 5 bar before expanding in the engine to 0.1 bar, 0.9 dry. Determine per kg of steam**

(a) The heat loss in the pipeline
(b) The temperature drop in passing through the throttle valve
(c) The work output of the engine
(d) The entropy change due to throttling
(e) The entropy change in passing through the engine.

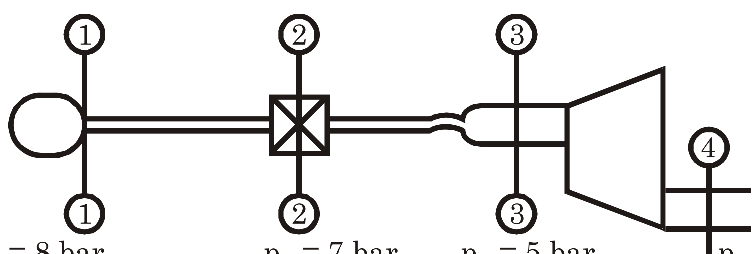
(**Ans.** (a) 105.3 kJ/kg, (b) 5°C, (c) 499.35 kJ/kg, (d) 0.1433 kJ/kg K, (e) 0.3657 kJ/kg K)

Solution: ∴ From Steam Table

$h_1 = 2950.1$ kJ/kg $\quad h_2 = 2844.8$

$h_3 = 2844.8 \quad h_4 = h_{fa} + x_a h_{fga} = 2345.3$ kJ/kg

∴ Heat loss in pipe line = $(h_1 - h_2) = 105.3$ kJ/kg



(b) In throttling process $h_2 = h_3$

From Steam Table

5 bar 151.8° C $\quad h_g = 2747.5$

5 bar 200° C $\quad h = 2855.4$

$$\therefore \quad t_3 = 200 - \frac{2855.4 - 2844.8}{2855.4 - 2747.5} \times (200 - 151.8)$$

$$= 200 - 4.74 = 195.26° \text{ C}$$

∴ $\Delta t = 4.74°$ C (drop)

(c) Work output for the engine (W) = $h_3 - h_4$

= (2844.8 – 2345.3) kJ/kg= 499.48 kJ/kg

(d) From Steam Table

$s_2 = 6.886$ kJ/kg – K

$$s_3 = 6.8192 + \frac{(195.26 - 151.8)}{(200 - 151.8)}(7.059 - 6.8192) \quad = 7.03542 \text{ kJ/kg} - \text{K}$$

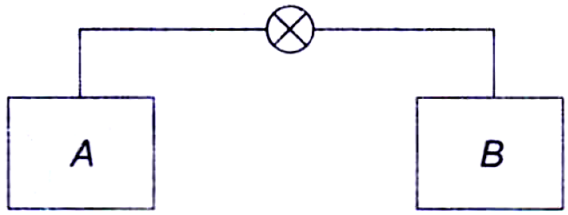
$$\therefore \quad \Delta s = s_3 - s_2 = 0.1494 \text{ kJ/kg} - \text{K}$$

(e) For Steam Table

$s_4 = s_{ga} + 0.9\, s_{fga} = 0.649 + 0.9 \times 7.501 = 7.4$ kJ/kg – K

$\Delta S = s_4 - s_3 = 0.3646$ kJ/kg – K

Q9.16 **Tank *A* (Figure) has a volume of 0.1 m³ and contains steam at 200°C, 10% liquid and 90% vapour by volume, while tank *B* is evacuated. The valve is then opened, and the tanks eventually come to the same pressure,**



Which is found to be 4 bar. During this process, heat is transferred such that the steam remains at 200°C. What is the volume of tank *B*?

(**Ans.** 4.89 m³)

Solution:

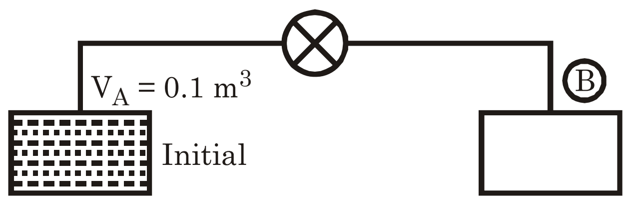
$t_1 = 200°C$

From Steam table

$p_a = 15.538$ bar

$V_f = 1.157 - 10^{-3}$

$V_g = 0.12736$



$\therefore$ Initial volume of liquid $= \frac{10}{100} \times 0.1\,m^3$

$m_f = 8.643$ kg

Initial mass of steam = (mg)

$= \dfrac{\frac{90}{100} \times 0.1}{0.12736}$ kg $= 0.70666$ kg

$\therefore$ Total mass = 9.3497 kg

After open the valve when all over per = 4 bar at 200ºC

Then sp. Volume = 0.534 m³/kg

$\therefore$ Total volume (V) = 9.3497 × 0.534 m³ = 4.9927 m³

$\therefore$ Volume of Tank B = $V - V_A$ = 4.8927 m³

Q9.17 **Calculate the amount of heat which enters or leaves 1 kg of steam initially at 0.5 MPa and 250°C, when it undergoes the following processes:**

(a) It is confined by a piston in a cylinder and is compressed to 1 MPa and 300°C as the piston does 200 kJ of work on the steam.

(b) It passes in steady flow through a device and leaves at 1 MPa and 300°C while, per kg of steam flowing through it, a shaft puts in 200 kJ of work. Changes in K.E. and P.E. are negligible.

(c) It flows into an evacuated rigid container from a large source which is maintained at the initial condition of the steam. Then 200 kJ of shaft work is transferred to the steam, so that its final condition is 1 MPa and 300°C.

(**Ans.** (a) –130 kJ (b) – 109 kJ, and (c) – 367 kJ)

Solution: **Initially:** $p_i = 0.5$ MPa = 5 bar; mass = 1 kg

$t_i = 250°C$

∴ From Steam Table $h_i = 2960.7$ kJ/kg

$u_i = 2729.5$ kJ/kg

$v_i = 0.474$ m³/kg

(a) After compression

p = 1 mPa = 10 bar

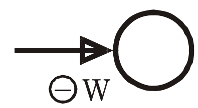
T = 300ºC

∴ From S.T. u = 2793.2 kJ/kg

h = 3051.2 kJ/kg

and $W_{input} = 200$ kJ

∴ From first law of thermodynamics



$$Q_{1-2} = m(u_2 - u_1) + W_{1-2}$$

$$[(2793.2 - 2729.5) - 200] \text{ kJ}$$

$$= [63.7 - 200 \text{ kJ}] = -136.3 \text{ kJ}$$

i.e. heat rejection to atm.

(b) For steady flow process

$$h_1 + \frac{V_1^2}{2} + gz_1 + \frac{\text{đ}Q}{dm}$$

$$= h_2 + \frac{V_2^2}{2} + gz_2 + \frac{\text{đ}W}{dm}$$

h or V_1, V_2, Z_1, Z_2 are negligible so

$$\frac{\text{đ}Q}{dm} = (h_2 - h_1) + \frac{\text{đ}W}{dm}$$

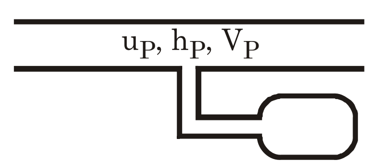
$$= (3051.2 - 2960.7) - 200$$

$$= -109.5 \text{ kJ/kg}$$

[heat rejection]

(c) Energy of the gas after filling

$E_1 = u_1$ kJ/kg = 2729.5 kJ/kg



Energy of the gas after filling

$E_2 = u_2 = 2793.2$ kJ/kg

∴ $\Delta E = E_2 - E_1$

= (2793.2 – 2729.5) kJ/kg

= 63.7 kJ/kg

There is a change in a volume of gas because of the collapse of the envelop to zero volume

$$W_1 = p_i(0 - v_i) = -p_i v_i = -500 \times 0.474 \text{ kJ/kg} = -237 \text{ kJ/kg}$$

∴ From first law of thermodynamic

$$Q = \Delta E + W_1 + W_2$$

$$= (63.7 - 237 - 200) \text{ kJ/kg} = -373.3 \text{ kJ/kg}$$

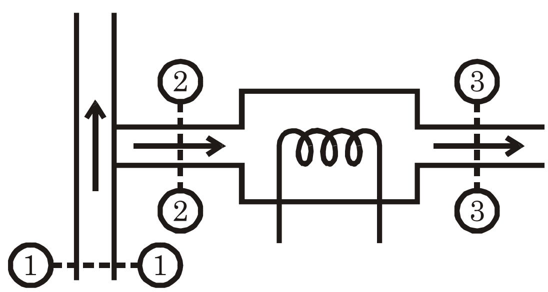
Q9.18 **A sample of wet steam from a steam main flows steadily through a partially open valve into a pipeline in which is fitted an electric coil. The valve and the pipeline are well insulated. The steam mass flow rate is 0.008 kg/s while the coil takes 3.91 amperes at 230 volts. The main pressure is 4 bar, and the pressure and temperature of the steam downstream of the coil are 2 bar and 160°C respectively. Steam velocities may be assumed to be negligible.**

(a) Evaluate the quality of steam in the main.

(b) State, with reasons, whether an insulted throttling calorimeter could be used for this test.

(**Ans.** (a) 0.97, (b) Not suitable)

Solution: (a)



$\dot{m}_2 = 0.008$ kg/s $\qquad \dot{Q} = i^2 R = \dfrac{3.91 \times 230}{1000}$ kW $= 0.8993$ kW

$p_2 = 4$ bar $\qquad p_3 = 2$ bar; $t_2 = 160$ºC

$t_2 = 143.6$º C $\qquad h_3 = 2768.8 + \dfrac{10}{50}(2870.5 - 2768.8)$ kJ/kg

$= 2789.14$ kJ/kg

From steady flow energy equation

$$\dot{m}\, h_2 + \dot{Q} = \dot{m}\, h_3 + 0 \ : h_2 = h_3 - \frac{\dot{Q}}{\dot{m}} = 2676.73 \text{ kJ/kg}$$

If dryness fraction of steam x then

$h_2 = h_{f2} + x\, h_{fg2}$

or $\quad 2676.73 = 604.7 + x \times 2133 \qquad \therefore x = 0.9714$

(b) For throttling minimum enthalpy required 2686 kJ/kg if after throttling 5ºC super heat and atm. Pressure is maintained as here enthalpy is less so it is not possible in **throttling calorimeter.**

Q9.19 **Two insulated tanks, *A* and *B*, are connected by a valve. Tank *A* has a volume of 0.70 m³ and contains steam at 1.5 bar, 200°C. Tank *B* has a volume of 0.35 m³ and contains steam at 6 bar with a quality of 90%. The valve is then opened, and the two tanks come to a uniform state. If there is no heat transfer during the process, what is the final pressure? Compute the entropy change of the universe.**

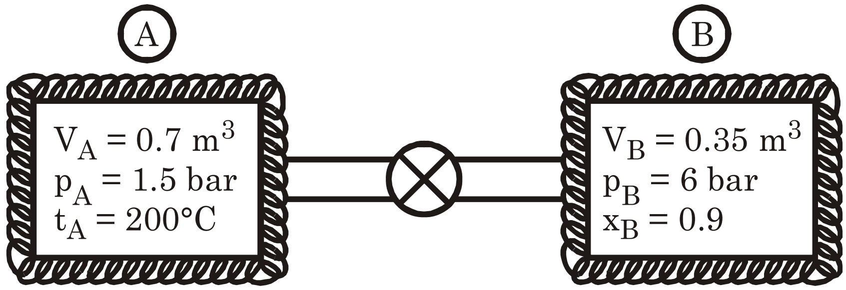
(**Ans.** 322.6 KPa, 0.1985 kJ/K)

Solution: **From Steam Table** $\qquad$ from Steam Table

Sp. Enthalpy $(h_A) = 2872.9$ kJ/kg $\qquad t_B = 158.8$º C

Sp. Vol $(v_A) = 1.193$ m³/kg $\qquad$ Sp. Enthalpy (h_B)

$= (670.4 + 0.9 \times 2085) = 2547$ kJ/kg



Sp. Internal energy (u) = 2656.2 kJ/kg =

Sp. Vol. $(v_B) = v_{Bf} + x\left(v_{Bg} - v_{Bf}\right) = 0.2836$ m³/kg

Sp. entropy (s) = 7.643 kJ/kg – K

Sp. in energy $(u_B) = u_f + x \times u_{fg} = 2376.7$ kJ/kg

Sp. entropy $(s_B) = 6.2748$ kJ/kg – K

$$u_{f_B} = h_{f_B} - p_{f_B} v_{f_B} = 670.4 - 600 \times 0.001101 = 669.74 \text{ kJ/kg}$$

$$u_{fg} = h_{fg} - p_{f_B}\left(v_g - v_f\right) \qquad m_B = \frac{V_B}{v_B} = 1.2341 \text{ kg}$$

$= 1896.7$ kJ/kg

$$\therefore\ m_A = \frac{V_A}{v_A} = 0.61242 \text{ kg}$$

$\therefore$ From First Law of thermodynamics

$U_1 = U_2$

$$\therefore\ m_A u_A + m_B u_B = (m_A + m_B)u \qquad \therefore\ u = 2469.4 \text{ kJ/kg}$$

And sp. volume of gas after mixing $= \dfrac{V_A + V_B}{m_A + m_B} = 0.5686$ m³/kg

Q9.20 **A spherical aluminum vessel has an inside diameter of 0.3 m and a 0.62 cm thick wall. The vessel contains water at 25°C with a quality of 1%. The vessel is then heated until the water inside is saturated vapour. Considering the vessel and water together as a system, calculate the heat transfer during this process. The density of aluminum is 2.7 g/cm³ and its specific heat is 0.896 kJ/kg K.**

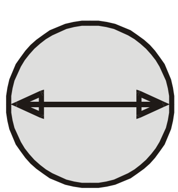
(**Ans.** 2682.82 kJ)

Solution: Volume of water vapour mixture (V) $= \dfrac{4}{3}\pi d_i^3 = 0.113097$ m³

Ext. volume $= \dfrac{4}{3}\pi d_o^3 = 0.127709$ m³

$\therefore$ Volume of $A_1 = 0.0146117$ m³

$\therefore$ Mass = 39.451 kg



$d_i = 0.3$ m

$d_o = d_i + 21 = 0.3 + \dfrac{0.62}{100} \times 2$ m $= 0.3124$ m

At 25° C; 1% quality

From Steam Table $p_1 = 0.0317$ bar

$$v_1 = 0.001003 + \frac{1}{100}(43.36) = 0.434603 \text{ m}^3/\text{kg}$$

$$h_1 = 104.9 + \frac{1}{100} \times 24212.3 = 129.323 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1 = 127.95 \text{ kJ/kg}$$

Mass of water and water vapour $= \dfrac{0.113097}{0.434603}$ kg $= 0.26023$ kg

Carnot volume heating until dry saturated

So then Sp. volume $v_g = 0.434603$ m^3/kg

For Steam Table

At 4.2 bar $v_g = 0.441$

At 4.4 bar $v_g = 0.423$

$$(p_f) = 4.2 + 0.2 \times \frac{0.441 - 0.434603}{0.441 - 0.423} = 4.27 \text{ bar}$$

Then $h_f = 2739.8 + \dfrac{0.07}{0.2}(2741.9 - 2739.8) = 2740.55$ kJ/kg

$t_f = 146^\circ$ C

$u_f = h_f - p_f v_f = 2555$ kJ/kg

$\therefore$ Heat required to water $= m(u_f - u_1)$

$= 0.26023(2555 - 127.95)$ kJ

$= 631.58$ kJ

Heat required for A_1

$= 39.451 \times 0.896 \times (146 - 25)$

$= 4277.2$ kJ

Total heat required = 4908.76 kJ

Q9.21 **Steam at 10 bar, 250°C flowing with negligible velocity at the rate of 3 kg/min mixes adiabatically with steam at 10 bar, 0.75 quality, flowing also with negligible velocity at the rate of 5 kg/min. The combined stream of steam is throttled to 5 bar and then expanded isentropically in a nozzle to 2 bar. Determine**

(a) The state of steam after mixing

(b) The state of steam after throttling

(c) The increase in entropy due to throttling

(d) The velocity of steam at the exit from the nozzle

(e) The exit area of the nozzle. Neglect the K.E. of steam at the inlet to the nozzle.

(**Ans.** (a) 10 bar, 0.975 dry, (b) 5 bar, 0.894 dry, (c) 0.2669 kJ/kg K, (d) 540 m/s, (e) 1.864 cm^2)

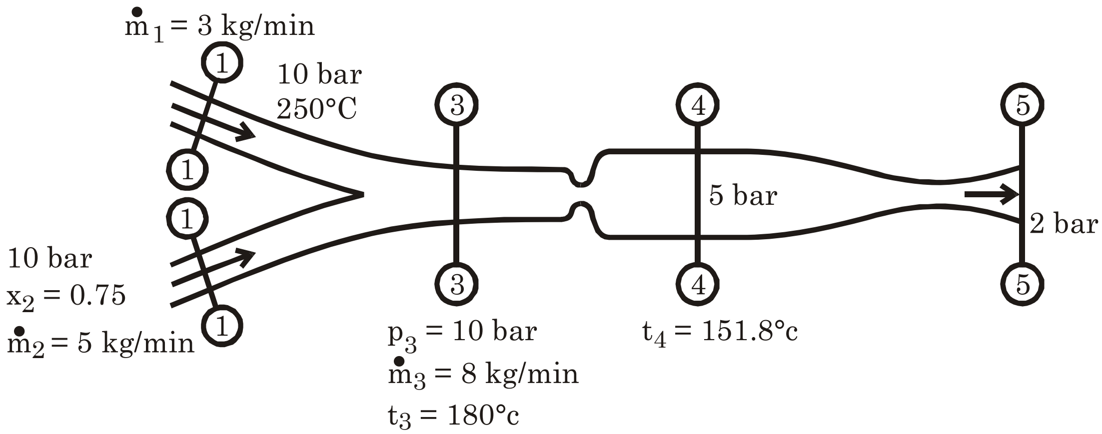
Solution: From Steam Table

$h_1 = 2942.6$ kJ/kg

$h_2 = 762.6 + 0.75 \times 2013.6 = 2272.8$ kJ/kg

$$\therefore \quad h_3 = \frac{3 \times 2942.6 + 5 \times 2272.8}{8}$$

$= 2524$ kJ/kg



(a) $h_3 = 762.6 + x_3 \times 2013.6$ or $x_3 = 0.87474$

(b) $h_4 = 2524 \text{ kJ/kg} = 640.1 + x_4 \times 2107.4$

$x_4 = 0.89395$

$s_4 = 1.8604 + x_4 \times 4.9588 = 6.2933 \text{ kJ/kg} - \text{K}$

$\therefore$ $s_5 = s_4$

$\therefore$ at 2 bar quality of steam

$6.2933 = 1.5301 + x_5 \times 5.5967$

$x_5 = 0.851$

$\therefore$ $h_5 = 504.7 + 0.851 \times 2201.6 = 2378.4 \text{ kJ/kg}$

(c) $s_3 = s_f + x_3 s_{fg} = 2.1382 + 0.89395 \times 4.4446 = 6.111451 \text{ kJ/kg}$

$\Delta s = s_4 - s_3 = 6.2933 - 6.11145 = 0.18185 \text{ kJ/kg} - \text{K}$

(d) $V = \sqrt{2000(2524 - 2378.4)} = 540 \text{ m/s}$

(e) $A = \dfrac{mv}{V}$

$$\Rightarrow \quad \frac{m \times x_5 \, . \, 0.885}{V} = \frac{8}{60} \times 0.851 \times \frac{0.885}{540} \text{ m}^2 = 1.86 \text{ cm}^2$$

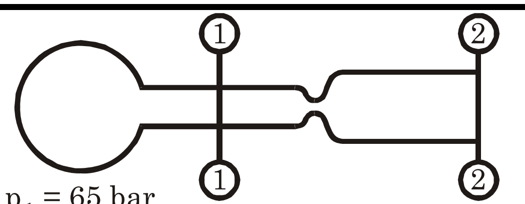
Q9.22 **Steam of 65 bar, 400°C leaves the boiler to enter a steam turbine fitted with a throttle governor. At a reduced load, as the governor takes action, the pressure of steam is reduced to 59 bar by throttling before it is admitted to the turbine. Evaluate the availabilities of steam before and after the throttling process and the irreversibility due to it.**

(**Ans.** $I = 21$ kJ/kg)

Solution: From Steam Table

$h_1 = 3167.65$ kJ/kg $\qquad h_2 = 3167.65$ kJ/kg

$s_1 = 6.4945$ kJ/kg-K $\qquad t_2 = 396.6^\circ$ C



$$s_2 = 6.333 + \frac{46.6}{50}(6.541 - 6.333)$$
$$= 6.526856$$

$\therefore\ \Delta s = s_4 - s_3 = 0.032356$ kJ/kg – K

Atmospheric Pressure $p_0 = 1$ bar

$T_0 = 25°$ C

$\therefore$ Availability before throttling

$$\psi = (h_1 - h_0) - T_0\,(s_1 - s_0) + \frac{V_1^2}{2} + g\,(2\,Z_0)$$

Same as example 9.14

Q9.23 **A mass of wet steam at temperature 165°C is expanded at constant quality 0.8 to pressure 3 bar. It is then heated at constant pressure to a degree of superheat of 66.5°C. Find the enthalpy and entropy changes during expansion and during heating. Draw the *T–s* and *h–s* diagrams.**

(**Ans.** – 59 kJ/kg, 0.163 kJ/kg K during expansion and 676 kJ/kg, 1.588 kJ/kg K during heating)

Solution: $p_1 = 7$ bar

$t_1 = 165°$ C

For Steam Table

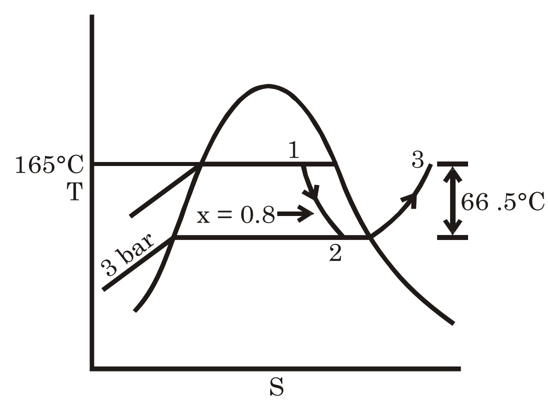
$h_1 = h_f + 0.8\, h_{fg}$

$= 2349$ kJ/kg

$s_1 = s_f + 0.8 \times s_{fg}$

$= 5.76252$ kJ/kg – K

For Steam Table at 3 bar



$t_2 = 133.5°$ C

$h_2 = 561.4 + 0.8 \times 2163.2 = 2292$ kJ

$s_2 = 1.6716 + 0.8 \times 5.3193$

$= 5.92704$ kJ/kg – K

$\therefore$ temperature of (3)

$t_3 = 200°C$

$\therefore$ $h_3 = 2865.6$ kJ/K

$s_3 = 7.311$ kJ/kg-K

$\therefore$ Enthalpy charge in expansion = $(h_1 - h_2) = 57$ kJ/kg

Entropy charge in expansion = $(s_2 - s_1) = 0.16452$ kJ/kg-K

Enthalpy charge in heating = $h_3 - h_2 = 573.6$ kJ/kg

Entropy charge in heating = $s_3 - s_2 = 1.38396$ kJ/kg – K

10. Properties of Gases and Gas Mixture

Some Important Notes

1. As $p \to 0$, or $T \to \infty$, the real gas approaches the **ideal gas behaviour.**

$\bar{R}$ = 8.3143 kJ/kmole-K

2. $Tds = du + pdv$

$Tds = dh - vdp$

3. $\gamma = 1 + \dfrac{2}{N}$

For mono-atomic gas N = 3 [N = **degrees of freedom]**

For di -atomic gas N = 5

For Tri-atomic gas N = 6

4. **Reversible adiabatic process**

$$pv^{\gamma} = C;\ \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

5. **For isentropic process**

$$u_2 - u_1 = \frac{RT_1}{\gamma-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right];\ h_2 - h_1 = \frac{\gamma}{\gamma-1}(RT_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

(i) For closed system $\displaystyle\int_1^2 pd\,v = \frac{p_1 v_1 - p_2 v_2}{\gamma-1}$

For steady flow $\displaystyle\int_1^2 v\,dp = \frac{\gamma}{\gamma-1}(p_1 v_1 - p_2 v_2)$

6. Isobaric process (p = C), n = 0, $pv^o = C$

Isothermal process (T = C), n = 1, $pv^1 = RT$

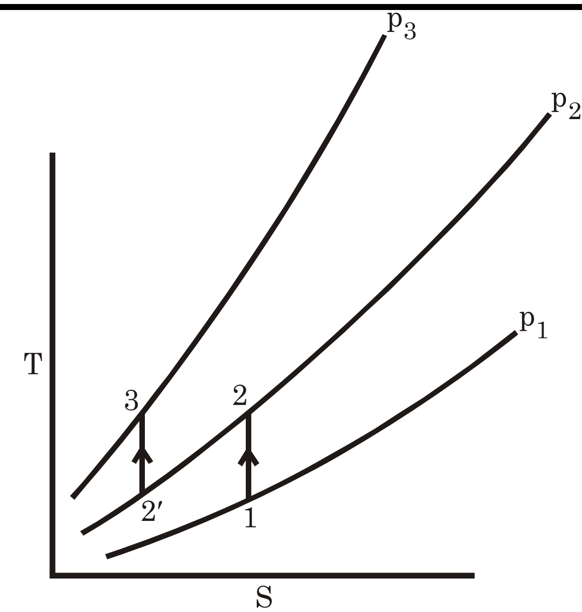
Isentropic process (s = C), n = γ, $pv^{\gamma} = C$

Isometric or isobaric process (V = C), n = ∞

7. For minimum work in multistage compressor, $p_2 = \sqrt{p_1 p_3}$

(i) Equal pressure ratio i.e. $\dfrac{p_2}{p_1} = \dfrac{p_3}{p_2}$

(ii) Equal discharge temperature $(T_2 = T_3)$



And (iii) Equal work for the two stages.

8. **Volumetric Efficiency** $(\eta_{vol}) = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

$$\text{Where, } C = \frac{\text{Clearance volume}}{\text{Piston displacement volume}}$$

9. **Equation of states for real gas**

(i) Van der Waals equation: $\left(p + \frac{a}{v^2}\right)(v - b) = RT$

or $p = \frac{RT}{v-b} - \frac{a}{v^2}$

or $\left(p_r + \frac{3}{v_r^2}\right)(3v_r - 1) = 8T_r$

(ii) Beattie Bridge man equation

$$p = \frac{RT\,(1-e)}{v^2}(v + B) - \frac{A}{v^2}$$

Where $A = A_0\left(1 - \frac{a}{v}\right);\ B = B_0\left(1 - \frac{b}{v}\right);\ e = \frac{C}{vT^3}$

'Does not' give satisfactory **results in the critical point region.**

(iii) Virial Expansions:

$$\frac{p\bar{v}}{\bar{R}T} = 1 + B'p + C'p^2 + D'p^3 + \ldots\ldots\ldots\ldots\ldots$$

Or $\frac{p\bar{v}}{\bar{R}T} = 1 + \frac{B}{\bar{v}} + \frac{C}{\bar{v}^2} + \frac{D}{\bar{v}^3} + \ldots\ldots\ \alpha$

10. $a = 3\,p_c v_c^2$; $b = \dfrac{v_c}{3}$; $R = \dfrac{8}{3}\dfrac{p_c v_c}{T_c}$; values of Z at critical **point 0.375 for Van der** Waal gas.

11. **Boyle temperature** $(T_B) = \dfrac{a}{bR}$

12. $\mu = x_1\mu_1 + x_2\mu_2 + \ldots\ldots. + x_c\mu_c$

$$R_m = \frac{m_1R_1 + m_2R_2 + \ldots\ldots.\, m_cR_c}{m_1 + m_2 + \ldots\ldots\ldots.\, m_c}$$

$$u_m = \frac{m_1u_1 + m_2u_2 + \ldots\ldots.\, m_cu_c}{m_1 + m_2 + \ldots\ldots\ldots.\, m_c}; \qquad h_m = \frac{m_1h_1 + m_2h_2 + \ldots\ldots.\, m_ch_c}{m_1 + m_2 + \ldots\ldots\ldots.\, m_c}$$

$$c_{pm} = \frac{m_1\,c_{P1} + m_2\,c_{P2} + \ldots\ldots.\, m_c\,c_{Pc}}{m_1 + m_2 + \ldots\ldots\ldots.\, m_c} \qquad c_{vm} = \frac{m_1\,c_{v1} + m_2\,c_{v2} + \ldots\ldots.\, m_c\,c_{vc}}{m_1 + m_2 + \ldots\ldots\ldots.\, m_c}$$

$$s_f - s_i = -\left[m_1R_1\ln\frac{p_1}{p} + m_2R_2\ln\frac{p_2}{p} + \ldots\ldots + m_cR_c\ln\frac{p_c}{p}\right]$$

13. **Gibbs function** $G = \bar{R}T\sum n_x\,(\phi_k + \ln p + \ln x_k)$

Questions with Solution P. K. Nag

Q.10·1 **What is the mass of air contained in a room 6 m × 9 m × 4 m if the pressure is 101.325 kPa and the temperature is 25°C?**

(**Ans.** 256 kg)

Solution: Given pressure (p) = 101.325 kPa
Temperature (T) = 25ºC = (25 + 273) K = 298 K
Volume (V) = 6 × 9 × 4 m³ = 216 m³
From equation of states
pV = mRT For air R = 0.287 kJ/kg – K, Gas constant mass is m kg

$$\therefore \quad m = \frac{pV}{RT} = \frac{101.325 \times 216}{0.287 \times 298}\text{ kg} = 255.9\text{ kg}$$

Q.10.2 **The usual cooking gas (mostly methane) cylinder is about 25 cm in diameter and 80 cm in height. It is changed to 12 MPa at room temperature (27°C).**
(a) Assuming the ideal gas law, find the mass of gas filled in the cylinder.
(b) Explain how the actual cylinder contains nearly 15 kg of gas.
(c) If the cylinder is to be protected against excessive pressure by means of a fusible plug, at what temperature should the plug melt to limit the maximum pressure to 15 MPa?

Solution: Given diameter (D) = 25 cm = 0.25 m
Height (H) = 80 cm = 0.8 m

$\therefore$ Volume of the cylinder $(V) = \dfrac{\pi D^2}{4} \times H = 0.03927\text{ m}^3$

Gas pressure (p) = 12 MPa = 12000 kPa

Temperature (T) = 27° C = 300 K

(a) Mass of gas filled in the cylinder

$$m = \frac{pV}{RT} \quad \left[\text{Here R = Gas constant} = \frac{\bar{R}}{M} = \frac{8.3143}{16}\text{ kJ/kg – K} = 0.51964\right]$$
$$= 3.023 \text{ kg}$$

(b) In cooking gas main component is ethen and it filled in 18 bar pressure. At that pressure it is not a gas it is liquid form in atmospheric temperature so its weight is amount 14 kg.

(c) Let temperature be T K, then pressure, p = 15 MPa = 15000 kPa

$$\therefore \quad T = \frac{pV}{mR} = \frac{15000 \times 0.03927}{3.023 \times 0.51964} = 375 \text{ K} = 102^\circ \text{ C}$$

Q.10.3 A certain gas has c_P = 0.913 and c_V = 0.653 kJ/kg K. Find the molecular weight and the gas constant *R* of the gas.

Solution: Gas constant, R = $c_p - c_v$ = (0.913 – 653) kJ/kg – K = 0.26 kJ/kg – K

If molecular weight,(M)kJ/kg – mole

Then $\bar{R}$ = MR $\quad \therefore M = \frac{\bar{R}}{R} = \frac{8.3143}{0.26}$ kJ/kg – mole = 31.98 kJ/kg – mole

Q.10.4 From an experimental determination the specific heat ratio for acetylene (C_2H_2) is found to 1.26. Find the two specific heats.

Solution: Gas constant of acetylene (C_2H_2) (R) = $\frac{\bar{R}}{M} = \frac{8.3143}{26}$ kJ/kg – K = 0.3198 kJ/kg – K

As adiabatic index (γ) = 1.26 then

$$c_p = \frac{\gamma}{\gamma - 1} R = 1.55 \text{ kJ/kg – K}$$

and

$$c_v = \frac{R}{\gamma - 1} = 1.23 \text{ kJ/kg – K}$$

Q.10.5 Find the molal specific heats of monatomic, diatomic, and polyatomic gases, if their specific heat ratios are respectively 5/3, 7/5 and 4/3.

Solution: Mono-atomic: $c_p = \frac{\gamma}{\gamma - 1}\bar{R}$ = 20.79 kJ/kg – mole – K;

$$c_v = \frac{\bar{R}}{\gamma - 1} = 12.47 \text{ kJ/kg – mole – K}$$

Di-atomic: $c_p = \frac{\gamma}{\gamma - 1}\bar{R}$ = 29.1 kJ/kg – mole – K;

$$c_v = \frac{\bar{R}}{\gamma - 1} = 20.79 \text{ kJ/kg – mole – K}$$

Polyatomic: $c_p = \frac{\gamma}{\gamma - 1}\bar{R} = 33.26$ kJ/kg – mole – K;

$c_v = 24.94$ kJ/kg – mole – K

Q.10.6 **A supply of natural gas is required on a site 800 m above storage level. The gas at - 150°C, 1.1 bar from storage is pumped steadily to a point on the site where its pressure is 1.2 bar, its temperature 15°C, and its flow rate 1000 m^3 /hr. If the work transfer to the gas at the pump is 15 kW, find the heat transfer to the gas between the two points. Neglect the change in K.E. and assume that the gas has the properties of methane (CH_4) which may be treated as an ideal gas having γ = 1.33 (g = 9.75 m/s^2).**

(**Ans.** 63.9 kW)

Solution: Given: At storage $(p_1) = 1.1$ bar = 110 kPa

$(T_1) = -150°C = 123$ K

$p_3 = 1.2$ bar = 120 kPa

$T_3 = 288$ K

Flow rate $(\dot{V}_3) = 1000$ m³/m $= \frac{5}{18}$ m³/s

Gas constant $(R) = \frac{\bar{R}}{16} = 0.51964$ kJ/kg – K

$\therefore \quad c_p = \frac{\gamma}{\gamma - 1} R = 2.0943$ kJ/kg

$\therefore \; p_3 \dot{V}_3 = \dot{m} RT_3 \qquad \therefore \; \dot{m} = \frac{p\dot{V}_3}{RT_3} = 0.22273$ kg/s

Pump work $\left(\frac{đW}{dt}\right) = -15$ kW

$\therefore$ From steady flow energy equation

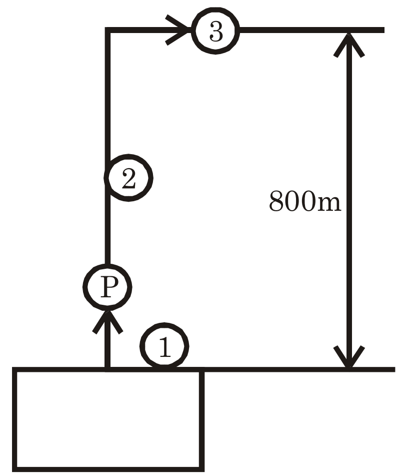
$$\dot{m}(h_1 + 0 + gZ_1) + \frac{đQ}{dt} = \dot{m}(h_3 + 0 + gZ_3) + \frac{đW}{dt}$$

$$\therefore \quad \frac{đQ}{dt} = \dot{m}\left[(h_3 - h_1) + g\frac{(Z_3 - Z_1)}{1000}\right] + \frac{đW}{dt}$$

$$= \dot{m}\left[c_P(T_3 - T_1) + g\frac{\Delta Z}{1000}\right] + \frac{đW}{dt}$$

$$= 0.22273\left[2.0943 \times (288 - 123) + \frac{9.75 \times 800}{1000}\right] + (-15)$$

= 63.7 kJ/s = 63.7 kW (heat given to the system)



Q.10.7 **A constant volume chamber of 0.3 m^3 capacity contains 1 kg of air at 5°C. Heat is transferred to the air until the temperature is 100°C. Find the work done, the heat transferred, and the changes in internal energy, enthalpy and entropy.**

Solution: Constant volume (V) = 0.3 m^3

$T_2 = 100^\circ C = 373\ K$

Mass (m) = 1 kg

$\therefore \quad p_2 = \frac{p_1}{T_1} \times T_2 = 357\ kPa$

$T_1 = 5^\circ\ C = 278\ K$

$\therefore \quad p_1 = \frac{mRT_1}{V} = 265.95\ kPa$

Work done = $\int pdV = 0$

Heat transferred $Q = \int du + \int \cancel{d}W = \int \cancel{d}W = m c_v \int dT = m\ c_v (T_2 - T_1) = 68.21\ kJ$

Change in internal Energy = $\int du = 68.21\ kJ$

Change in Enthalpy = $\int dh = m\ c_P (T_2 - T_1) = 95.475\ kJ$

Change in Entropy = $\int ds = s_2 - s_1 = m\ c_p \ln \frac{V_2}{V_1} + m\ c_v \ln \frac{p_2}{p_1}$

$= m\ c_v \ln \frac{p_2}{p_1} = 1 \times 0.718 \times \ln \frac{357}{265.95}$

$= 0.2114$ kJ/kg – K

Q.10.8 **One kg of air in a closed system, initially at 5°C and occupying 0.3 m^3 volume, undergoes a constant pressure heating process to 100°C. There is no work other than *pdv* work. Find (a) the work done during the process, (b) the heat transferred, and (c) the entropy change of the gas.**

Solution:

$T_1 = 278\ K$

$V_1 = 0.3\ m^3$

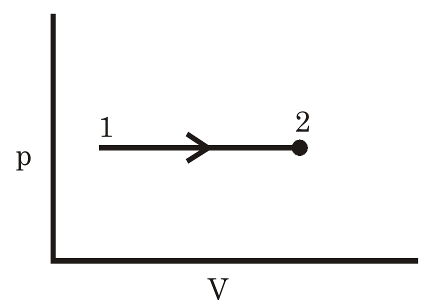
$m = 1\ kg$

$\therefore \quad p_1 = 265.95\ kPa$

$T_2 = 100^\circ\ C = 373\ K$

$p_2 = 265.95\ kPa$

$\therefore \quad V_2 = \frac{mRT_2}{p_2} = 0.40252\ m^3$



(a) Work during the process

$(W_{12}) = \int_1^2 p\, dV = p(V_2 - V_1) = 27.266\ kJ$

(b) Heat transferred $Q_{1-2} = u_2 - u_1 + W_{12}$

$= mc_v (T_2 - T_1) + W_{1-2} = 95.476\ kJ$

(c) Entropy change of the gas

$s_2 - s_1 = mc_p \ln \frac{V_2}{V_1} + mc_v \ln \frac{p_2}{p_1}$

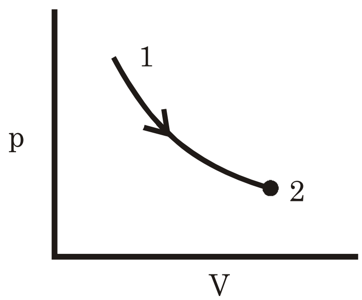
$= m c_p \ln \frac{v_2}{v_1} = 0.29543$ kJ/kg – K

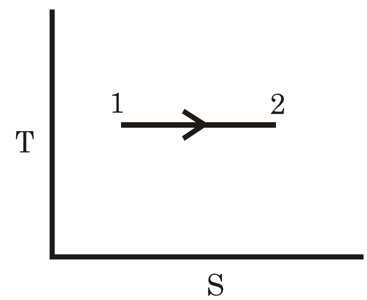
Q.10.9 **$0.1\,m^3$ of hydrogen initially at 1.2 MPa, 200°C undergoes a reversible isothermal expansion to 0.1 MPa. Find (a) the work done during the process, (b) the heat transferred, and (c) the entropy change of the gas.**

Solution: $V_1 = 0.1\ m^3$

$p_1 = 1.2\ MPa = 1200\ kPa$

$T_1 = 473\ K$





$$R = \frac{\bar{R}}{M} = \frac{8.3143}{2}\ kJ/kg - K = 4.157\ kJ/kg - K$$

$$m = \frac{p_1 V_1}{RT_1} = 0.06103\ kg$$

Reversible isothermal expansion

So $T_2 = T_1 = 473\ K$

Enthalpy change $(\Delta h) = m\, c_p (T_2 - T_1) = 0$

And $p_2 = 0.1\ MPa = 100\ kPa$

Heat transferred $(Q) = \Delta u + \Delta W$

$$= u_2 - u_1 + \int_1^2 p\, dV$$

$$= 0 + RT \int \frac{dV}{V}$$

$$= RT \ln\left(\frac{V_2}{V_1}\right)$$

$$= 4.157 \times 473 \times \ln\left(\frac{1.2}{0.1}\right)$$

$$= 4886\ kJ$$

$\therefore \quad V_2 = \frac{mRT_2}{p_2} = 1.2\ m^3$

$pV = RT$

$\therefore \quad p = \frac{RT}{V}$

Work done $(W) = \int_1^2 p\, dV = 4886\ kJ$

Entropy change, $s_2 - s_1 = mc_p \ln \frac{V_2}{V_1} + mc_v \ln \frac{p_2}{p_1}$

$$= 0.06103 \left[14.55 \ln\left(\frac{1.2}{0.1}\right) + 10.4 \ln\left(\frac{100}{1200}\right)\right]$$

$$= 0.6294\ kJ/kg - K$$

For H_2 diatomic gas ($\gamma = 1.4$)

$$c_p = \frac{\gamma}{\gamma - 1} R = 14.55\ kJ/kg - K; \quad c_v = \frac{R}{\gamma - 1} = 10.4\ kJ/kg - K$$

Q.10.10 **Air in a closed stationary system expands in a reversible adiabatic process from 0.5 MPa, 15°C to 0.2 MPa. Find the final temperature, and per kg of air, the change in enthalpy, the heat transferred, and the work done.**

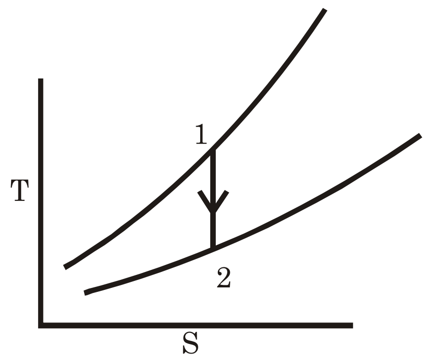
Solution: $p_1 = 0.5$ MPa = 500 kPa

$T_1 = 15°C = 288$ K

Let mass is 1 kg

$$\therefore \quad v_1 = \frac{1 \times R \times T_1}{p_1} = 0.1653 \text{ m}^3/\text{kg}$$

$p_2 = 0.2$ MPa = 200 kPa



$$\therefore \quad p_1 v_1^\gamma = p_2 v_2^\gamma :$$

$$v_2 = v_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.31809 \text{ m}^3/\text{kg}$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 222 \text{ K}$$

Change of Enthalpy

$$(\Delta H) = mc_p(T_2 - T_1) = -66.33 \text{ kJ/kg}$$

The Heat transferred (Q) = 0

The work done

$$(W) = \int_1^2 p\, dv = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

$$= 47.58 \text{ kJ/kg}$$

Q.10.11 **If the above process occurs in an open steady flow system, find the final temperature, and per kg of air, the change in internal energy, the heat transferred, and the shaft work. Neglect velocity and elevation changes.**

Solution: Final temperature will be same because then also $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ valid.

i.e. $T_2 = 222$ K

Change in internal energy = $\Delta u = mc_v(T_2 - T_1) = -47.4$ kJ/kg

Shaft work $$(W) = -\int_1^2 v\, dp = \frac{\gamma}{\gamma - 1}[p_1 v_1 - p_2 v_2] = +66.33 \text{ kJ/kg}$$

Heat transferred:

$$h_1 + 0 + 0 + \frac{đQ}{dm} = h_2 + 0 + 0 + \frac{đW}{dm}$$

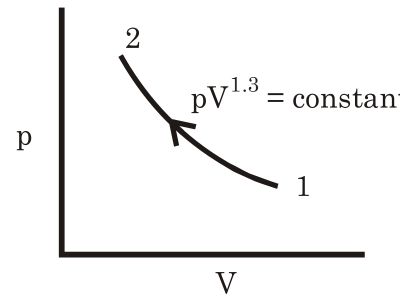
$$\therefore \quad \frac{đQ}{dm} = (h_2 - h_1) + \frac{đW}{dm} = -66.33 + 66.33 = 0$$

[As it is reversible adiabatic so $đQ = 0$]

Q.10.12 **The indicator diagram for a certain water-cooled cylinder and piston air compressor shows that during compression $pv^{1.3}$ = constant. The compression starts at 100 kPa, 25°C and ends at 600 kPa. If the process is reversible, how much heat is transferred per kg of air?**

Solution:

$$p_1 = 100 \text{ kPa}$$
$$T_1 = 298 \text{ K}$$
$$\therefore\ v_1 = \frac{RT_1}{p_1} = 0.8553 \text{ m}^3/\text{kg}$$



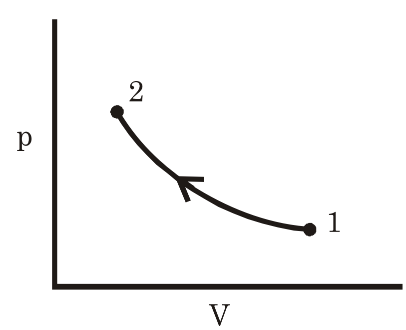
$$p_2 = 600 \text{ kPa}$$
$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 0.21554 \text{ m}^3/\text{kg}$$
$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 451 \text{ K}$$
$$h_1 + 0 + 0 + \frac{đQ}{dm} = h_2 + 0 + 0 + \frac{đW}{dm}$$
$$\therefore\ \frac{đQ}{dm} = (h_2 - h_1) + \frac{đW}{dm}$$
$$\therefore\ \frac{đW}{dm} = \frac{n[p_1 v_1 - p_2 v_2]}{n-1}$$
$$= -189.774 \text{ kJ/kg}$$
$$= C_p(T_2 - T_1) - 189.774$$
$$= 153.765 - 189.774$$
$$= -36 \text{ kJ/kg}$$

[Heat have to be rejected]

Q.10.13 An ideal gas of molecular weight 30 and $\gamma = 1.3$ occupies a volume of 1.5 m^3 at 100 kPa and 77°C. The gas is compressed according to the law $pv^{1.25}$ = constant to a pressure of 3 MPa. Calculate the volume and temperature at the end of compression and heating, work done, heat transferred, and the total change of entropy.

Solution:

$$R = \frac{\bar{R}}{30} = 0.27714 \text{ kJ/kg} - \text{K}$$
$$\gamma = 1.3;\ n = 1.25$$
$$\therefore\ c_v = \frac{R}{\gamma - 1} = 0.9238 \text{ kJ/kg} - \text{K}$$
$$c_p = \gamma\frac{R}{\gamma - 1} = 1.2 \text{ kJ/kg} - \text{K}$$
$$p_1 = 100 \text{ kPa};\ V_1 = 1.5 \text{ m}^3;\ T_1 = 350 \text{ K}$$
$$p_2 = 3 \text{ MPa} = 3000 \text{ kPa}$$



$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 0.09872 \text{ m}^3$$
$$m = \frac{p_1V_1}{RT_1} = 1.5464 \text{ kg}$$
$$\therefore\ T_2 = \frac{p_2V_2}{mR} = 691 \text{ K}$$
$$\therefore$$

Work done $(W_{1-2}) = \int_1^2 pdV$

$\therefore p_1 V_1^n = p V^n = p_2 V_2^n$

$$= p_1 V_1^n \int_1^2 \frac{dV}{V^n} = \frac{p_1 V_1^n}{-n+1}\left[\frac{1}{V_2^{n-1}} - \frac{1}{V_1^{n-1}}\right]$$

$$= \frac{p_2 V_2 - p_1 V_1}{1-n} = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{100 \times 1.5 - 3000 \times 0.09872}{1.25 - 1} \text{ kJ} = -584.64 \text{ kJ}$$

Heat transfer $Q = u_2 - u_1 + W_{1-2}$

$= mc_v(T_2 - T_1) + W_{1-2}$

$= [1.5464 \times 0.9238\ (691 - 350) - 584.64]$ kJ

$= -97.5$ kJ

$$\Delta S = S_2 - S_1 = \left[mc_v \ln\frac{p_2}{p_1} + mc_P \ln\frac{V_2}{V_1}\right]$$

$= -0.19$ kJ/K

Q.10.14 **Calculate the change of entropy when 1 kg of air changes from a temperature of 330 K and a volume of 0.15 m^3 to a temperature of 550 K and a volume of 0.6 m^3.**
If the air expands according to the law, pv^n = constant, between the same end states, calculate the heat given to, or extracted from, the air during the expansion, and show that it is approximately equal to the change of entropy multiplied by the mean absolute temperature.

Solution: Try please.

Q.10.15 **0.5 kg of air, initially at 25°C, is heated reversibly at constant pressure until the volume is doubled, and is then heated reversibly at constant volume until the pressure is doubled. For the total path, find the work transfer, the heat transfer, and the change of entropy.**

Solution: Try please.

Q.10.16 **An ideal gas cycle of three processes uses Argon (Mol. wt. 40) as a working substance. Process 1-2 is a reversible adiabatic expansion from 0.014 m^3, 700 kPa, 280°C to 0.056 m^3. Process 2-3 is a reversible isothermal process. Process 3-1 is a constant pressure process in which heat transfer is zero. Sketch the cycle in the *p-v* and *T-s* planes, and find (a) the work transfer in process 1-2, (b) the work transfer in process 2-3, and (c) the net work of the cycle. Take $\gamma = 1.67$.**

Solution: Try please.

Q.10.17 **A gas occupies 0.024 m^3 at 700 kPa and 95°C. It is expanded in the non-flow process according to the law $pv^{1.2}$ = constant to a pressure of 70 kPa**

after which it is heated at constant pressure back to its original temperature. Sketch the process on the p-v and T-s diagrams, and calculate for the whole process the work done, the heat transferred, and the change of entropy. Take c_p = 1.047 and c_V = 0.775 kJ/kg K for the gas.

Solution: Try please.

Q.10.18 **0.5 kg of air at 600 kPa receives an addition of heat at constant volume so that its temperature rises from 110°C to 650°C. It then expands in a cylinder poly tropically to its original temperature and the index of expansion is 1.32. Finally, it is compressed isothermally to its original volume. Calculate (a) the change of entropy during each of the three stages, (b) the pressures at the end of constant volume heat addition and at the end of expansion. Sketch the processes on the *p-v* and *T-s* diagrams.**

Solution: Try please.

Q.10.19 **0.5 kg of helium and 0.5 kg of nitrogen are mixed at 20°C and at a total pressure of 100 kPa. Find (a) the volume of the mixture, (b) the partial volumes of the components, (c) the partial pressures of the components, (d) the mole fractions of the components, (e) the specific heats c_p and c_V of the mixture, and (f) the gas constant of the mixture.**

Solution: Try please.

Q.10.20 **A gaseous mixture consists of 1 kg of oxygen and 2 kg of nitrogen at a pressure of 150 kPa and a temperature of 20°C. Determine the changes in internal energy, enthalpy and entropy of the mixture when the mixture is heated to a temperature of 100°C (a) at constant volume, and (b) at constant pressure.**

Solution: Try please.

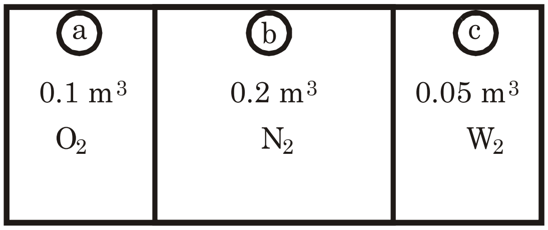
Q.10.21 **A closed rigid cylinder is divided by a diaphragm into two equal compartments, each of volume 0.1 m^3. Each compartment contains air at a temperature of 20°C. The pressure in one compartment is 2.5 MPa and in the other compartment is 1 MPa. The diaphragm is ruptured so that the air in both the compartments mixes to bring the pressure to a uniform value throughout the cylinder which is insulated. Find the net change of entropy for the mixing process.**

Solution: Try please.

Q.10.22 **A vessel is divided into three compartments (a), (b), and (c) by two partitions. Part (a) contains oxygen and has a volume of 0.1 m^3, (b) has a volume of 0.2 m^3 and contains nitrogen, while (c) is 0.05 m^3 and holds CO_2. All three parts are at a pressure of 2 bar and a temperature of 13°C. When the partitions are removed and the gases mix, determine the change of entropy of each constituent, the final pressure in the vessel and the partial pressure of each gas. The vessel may be taken as being completely isolated from its surroundings.**

(**Ans.** 0.0875, 0.0783, 0.0680 kJ/K; 2 bar; 0.5714, 1.1429, 0.2857 bar.)

Solution:



$p = 2$ bar $= 200$ kPa
$T = B°$ C $= 286$ K

After mixing temperature of the mixture will be same as before 13ºC = 286 K and also pressure will be same as before 2 bar = 200 kPa. But total volume will be

$V = V_a + V_b + V_c$
$= (0.1 + 0.2 + 0.05) = 0.35$ m^3

$$\therefore \qquad m_a = \frac{pV_a}{R_aT} = \frac{200\times 0.1}{\frac{8.3143}{32}\times 286}\text{ kg } = 0.26915\text{ kg}$$

$$m_b = \frac{pV_a}{R_bT} = \frac{200\times 0.2}{\frac{8.3143}{28}\times 286}\text{ kg } = 0.471\text{ kg}$$

$$m_c = \frac{pV_c}{R_cT} = \frac{200\times 0.05}{\frac{8.319}{44}\times 286}\text{ kg } = 0.18504\text{ kg}$$

$$\therefore \qquad \Delta S = S_2 - S_1 = mc_P \ln\frac{T_2}{T_1} - mR\ln\frac{p_2}{p_1} \qquad \text{Here } T_2 = T_1 \text{ so } \left[\because \frac{p_2}{p_1} = \frac{v_1}{v_2}\right]$$

$$(S_2 - S_1)_{O_2} = m_{O_2}R_{O_2}\ln\frac{V_o}{V_{O_2}} = 0.26915\times\frac{8.3143}{32}\times\ln\left(\frac{9.35}{0.1}\right)$$

$$= 0.087604\text{ kJ/K}$$

$$(S_2 - S_1)_{N_2} = m_{N_2}R_{N_2}\ln\left(\frac{V}{V_{n_2}}\right) = 0.471\times\frac{8.3143}{32}\times\ln\left(\frac{0.35}{0.2}\right) = 0.078267\text{ kJ/K}$$

$$(S_2 - S_1)_{CO_2} = m_{CO_2}R_{CO_2}\ln\left(\frac{V}{V_{CO_2}}\right) = 0.18504\times\frac{8.3143}{44}\times\ln\left(\frac{0.35}{0.05}\right) = 0.06804\text{ kJ/K}$$

Partial pressure after mixing

Mole fraction of $\qquad O_2(x_{O_2}) = \frac{0.1}{0.35}$

Mole fraction of $\qquad N_2(x_{N_2}) = \frac{0.2}{0.35}$

Mole fraction of $\qquad CO_2, \left(x_{O_2}\right) = \frac{0.05}{0.35}$

[$\because$ At same temperature and pressure same mole of gas has same]

$\therefore$ Partial pressure of $\qquad O_2; (p_{O_2}) = x_{O_2}\times p = \frac{0.1}{0.35}\times 200 = 57.143$ kPa

Partial pressure of $\qquad N_2; (p_{N_2}) = x_{N_2}\times p = \frac{0.2}{0.35}\times 200 = 114.29$ kPa

Partial pressure of $\qquad CO_2; (p_{CO_2}) = x_{CO_2}\times p = \frac{0.05}{0.35}\times 200 = 28.514$ kPa

Q.10.23 **A Carnot cycle uses 1 kg of air as the working fluid. The maximum and minimum temperatures of the cycle are 600 K and 300 K. The maximum pressure of the cycle is 1 MPa and the volume of the gas doubles during the isothermal heating process. Show by calculation of net work and heat supplied that the efficiency is the maximum possible for the given maximum and minimum temperatures.**

Solution: Try please.

Q.10.24 **An ideal gas cycle consists of three reversible processes in the following sequence: (a) constant volume pressure rise, (b) isentropic expansion to *r* times the initial volume, and (c) constant pressure decrease in volume. Sketch the cycle on the p-v and T'-s diagrams. Show that the efficiency of the cycle is**

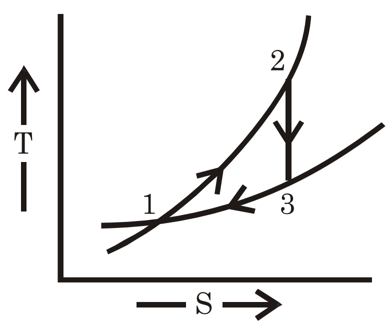
$$\eta_{cycle} = \frac{\mathbf{r^{\gamma} - 1 - \gamma(r-1)}}{\mathbf{r^{\gamma} - 1}}$$

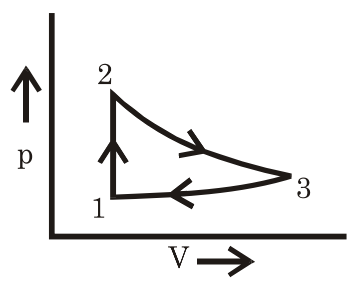
Evaluate the cycle efficiency when y = $\mathbf{\frac{4}{3}}$ and r = 8.

(**Ans.** ($\eta = 0.378$))

Solution: For process 1 – 2 constant volume heating

$$Q_{1-2} = \Delta u + pdv$$
$$= mc_v\ \Delta T + pdv$$
$$= mc_v\ \Delta T = mc_v(T_2 - T_1)$$





$Q_{2-3} = 0$ as isentropic expansion.

$Q_{3-1} = mc_P\,\Delta T = mc_P(T_3 - T_1)$

$\therefore$ Efficiency $= 1 - \dfrac{\text{heat rejection}}{\text{heat addition}}$

$$= 1 - \frac{mc_p(T_3 - T_1)}{mc_v(T_2 - T_1)} = 1 - \gamma\frac{\left(\dfrac{T_3}{T_1} - 1\right)}{\left(\dfrac{T_2}{T_1} - 1\right)}$$

Here $\dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$ as $V_1 = V_2$ $\qquad\qquad \therefore \dfrac{T_2}{T_1} = \dfrac{p_2}{p_1} = r^{\gamma}$

And $p_2 v_2^{\gamma} = p_3 v_3^{\gamma}$ $\qquad$ or $\qquad \dfrac{p_2}{p_3} = \left(\dfrac{v_3}{v_2}\right)^{\gamma} = r^{\gamma}$ as $p_3 = p_1$ then

And $\dfrac{p_3 v_3}{T_3} = \dfrac{p_2 v_2}{T_1}$ $\qquad\qquad \dfrac{p_2}{p_1} = r^{\gamma}$

or $\frac{T_3}{T_1} = \frac{v_3}{v_1} = \frac{v_3}{v_2} = r$ $\therefore \eta = 1 - \frac{\gamma(\gamma-1)}{r^{\gamma}-1} = \frac{r^{\gamma}-1-\gamma(r-1)}{r^{\gamma}-1}$ Proved

If $\gamma = \frac{4}{3}$ and r = 8 then $\eta_{cycle} = \frac{r^{\gamma}-1-\gamma(r-1)}{r^{\gamma}-1}$

$$\eta = 1 - \frac{\frac{4}{3}(8-1)}{(8^{\frac{4}{3}}-1)} = 0.37778$$

Q.10.25 **Using the Dietetic equation of state**

$$P = \frac{RT}{v-b}.\exp\left(-\frac{a}{RT_v}\right)$$

(a) Show that

$$p_c = \frac{a}{4e^2b^2}, v_c = 2b, T_c = \frac{a}{4Rb}$$

(b) Expand in the form

$$pv = RT\left(1 + \frac{B}{v} + \frac{C}{v^2} + ...\right)$$

(c) Show that

$$T_B = \frac{a}{bR}$$

Solution : Try please.

Q.10.26 **The number of moles, the pressures, and the temperatures of gases *a, b,* and c are given below**

Gas	m (kg mol)	P (kPa)	t (0C)
N_2	1	350	100
CO	3	420	200
O_2	2	700	300

If the containers are connected, allowing the gases to mix freely, find (a) the pressure and temperature of the resulting mixture at equilibrium, and (b) the change of entropy of each constituent and that of the mixture.

Solution : Try please.

Q.10.27 **Calculate the volume of 2.5 kg moles of steam at 236.4 atm. and 776.76 K with the help of compressibility factor versus reduced pressure graph. At this volume and the given pressure, what would the temperature be in K, if steam behaved like a van der Waals gas?**
The critical pressure, volume, and temperature of steam are 218.2 atm., 57 cm^3/g mole, and 647.3 K respectively.

Solution : Try please.

Q.10.28 **Two vessels, *A* and *B,* each of volume 3 m^3 may be connected together by a tube of negligible volume. Vessel a contains air at 7 bar, 95°C while B**

contains air at 3.5 bar, 205°C. Find the change of entropy when A is connected to B. Assume the mixing to be complete and adiabatic.

(**Ans.** 0.975 kJ/kg K)

Solution: $V_A = V_B = 3m^3$

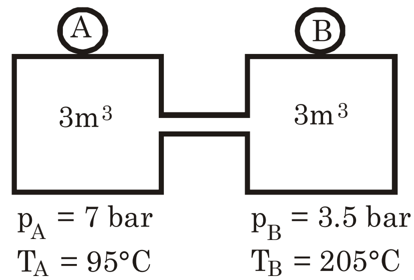
$p_A = 7 \text{ bar} = 700 \text{ kPa}$

$T_A = 95°C = 368 \text{ K}$

$p_B = 3.5 \text{ bar} = 350 \text{ kPa}$

$T_B = 205°C = 478 \text{ K}$

$\therefore \quad m_A = \dfrac{p_A V_A}{RT_A} = 19.883 \text{ kg} \qquad m_B = \dfrac{p_B V_B}{RT_B} = 7.6538 \text{ kg}$



In case of Adiabatic mixing for closed system Internal energy remains constant.

$\therefore \quad U_A + U_B = U$

or $\quad m_A c_v \,.\, T_A + m_B c_v \,.\, T_B = (m_A + m_B) c_v T$

$\therefore \quad T = \dfrac{m_A T_A + m_B T_B}{m_A + m_B} = 398.6 \text{ K}$

After mixing partial for of A

$$\left[\begin{array}{l}\text{Total pressure} \\ \therefore p = \dfrac{mRT}{V} = 525.03 \text{ kPa}\end{array}\right]$$

$p_{Af} = \dfrac{m_A RT}{V} = 379.1 \text{ kPa}$

$p_{Bf} = \dfrac{m_B RT}{V} = 145.93 \text{ kPa}$

$\Delta S_A = S_{Af} - S_A = m_A c_p \ln \dfrac{T}{T_A} - m_A R \ln \dfrac{p_{Af}}{p_A}$

$= 5.0957 \text{ kJ/K}$

$\Delta s_{Bf} - s_B = m_B c_P \ln \dfrac{T}{T_B} - m_B R \ln \dfrac{p_{Bf}}{p_B}$

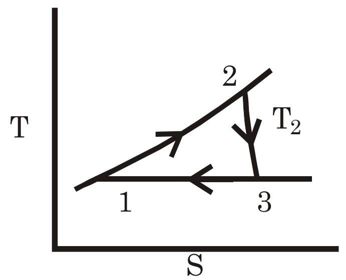
$= 0.52435 \text{ kJ/kg}$

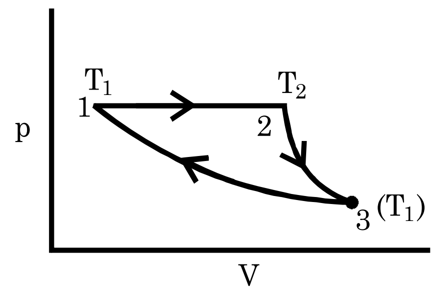
$\therefore \quad \Delta S_{univ} = \Delta S_A + \Delta S_B = 5.62 \text{ kJ/K}$

Q.10.29 **An ideal gas at temperature T_1 is heated at constant pressure to T_2 and then expanded reversibly, according to the law pv^n = constant, until the temperature is once again T_1 What is the required value of n if the changes of entropy during the separate processes are equal?**

$\left(\textbf{Ans.} \left(n = \dfrac{2\gamma}{\gamma + 1}\right)\right)$

Solution: Let us mass of gas is 1 kg





Then

$$s_2 - s_1 = 1 \times c_P \ln \frac{v_2}{v_1} + c_v \ln \frac{p_2}{p_1} = \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}\right] = c_p \ln \frac{T_2}{T_1} = \frac{\gamma R}{\gamma - 1} \times \ln \frac{T_2}{T_1}$$

or $$s_3 - s_2 = c_p \ln \frac{T_3}{T_2} - R \ln \frac{p_3}{p_2}$$

Hence $T_3 = T_1$ and $\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}$ $\therefore \left(\frac{p_3}{p_2}\right) = \left(\frac{T_3}{T_2}\right)^{\frac{n-1}{n}}$

$$= c_P \ln \frac{T_1}{T_2} - R \frac{n}{n-1} \ln \left(\frac{T_1}{T_2}\right) = \left[\frac{\gamma R}{\gamma - 4} - \frac{nR}{n-1}\right] \ln \frac{T_1}{T_2} = R\left(\ln \frac{T_2}{T_1}\right)\left[\frac{n}{n-1} - \frac{\gamma}{\gamma - 1}\right]$$

$\therefore$ As $s_2 - s_1 = s_3 - s_2$

$$\therefore \quad \frac{\gamma R}{\gamma - 1} \ln \frac{T_2}{T_1} = R\left(\ln \frac{T_2}{T_1}\right) \times \left[\frac{n}{n-1} - \frac{\gamma}{\gamma - 1}\right]$$

or $$2\frac{\gamma}{\gamma - 1} = \frac{n}{n-1}$$

or $$2n\gamma - 2\gamma = n\gamma - n$$

or $$n\ (\gamma n) = 2\gamma$$

or $$n = \left(\frac{2\gamma}{\gamma + 1}\right) \text{proved}$$

Q.10.30 **A certain mass of sulphur dioxide (SO_2) is contained in a vessel of 0.142 m^3 capacity, at a pressure and temperature of 23.1 bar and 18°C respectively. A valve is opened momentarily and the pressure falls immediately to 6.9 bar. Sometimes later the temperature is again 18°C and the pressure is observed to be 9.1 bar. Estimate the value of specific heat ratio.**

(**Ans.** 1.29)

Solution: Mass of SO_2 before open the valve $S = 32$ $O \to 16 \times 2 = 64$

$$m_1 = \frac{pV}{R_{SO_2} T} = \frac{2310 \times 0.142}{\frac{8.3143}{64} \times 291} = 8.6768 \text{ kg}$$

$R_{SO_2} = 0.12991$ kJ/kg-K

Mass of SO_2 after closing the valve

$$m_2 = \frac{910 \times 0.142}{R_{SO_2} \times 291} = 3.4181 \text{ kg}$$

If intermediate temperature is T then

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \text{or} \quad \frac{9.1 \times 0.142}{291} = \frac{6.9 \times 0.142}{T}$$

or T = 220.65 K

As valve is opened momentarily term process is adiabatic

$$\text{So} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \frac{220.65}{291} = \left(\frac{6.9}{23.1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{or} \quad \left(1 - \frac{1}{\gamma}\right) = \frac{\ln\left(\frac{220.65}{299}\right)}{\ln\left(\frac{6.9}{23.1}\right)} = 0.22903$$

$$\text{or} \quad \frac{1}{\gamma} = 1 - 0.22903 = 0.77097$$

$$\therefore \quad \gamma = 1.297$$

Q.10.31 A gaseous mixture contains 21% by volume of nitrogen, 50% by volume of hydrogen, and 29% by volume of carbon-dioxide. Calculate the molecular weight of the mixture, the characteristic gas constant *R* for the mixture and the value of the reversible adiabatic index γ. (At 10ºC, the c_p values of nitrogen, hydrogen, and carbon dioxide are l.039, 14.235, and 0.828 kJ/kg K respectively.)

A cylinder contains 0.085 m^3 of the mixture at 1 bar and 10°C. The gas undergoes a reversible non-flow process during which its volume is reduced to one-fifth of its original value. If the law of compression is $pv^{1.2}$ = constant, determine the work and heat transfer in magnitude and sense and the change in entropy.

(**Ans.** 19.64 kg/kg mol, 0.423 kJ/kg K, 1.365, –16 kJ, – 7.24 kJ, – 0.31 kJ/kg K)

Solution : Volume ratio = 21: 50: 29

$\therefore$ Mass ratio = 21 × 28: 50 × 2: 29 × 44

Let m_{N_2} = 21 × 28 kg, m_{H_2} = 50 × 2 kg, m_{N_2} = 29 × 44 kg

= 588 kg = 100 kg = 1276 kg

$$\therefore \quad R_{mix} = \frac{m_{N_2}R_{N_2} + m_{H_2}R_{H_2} + m_{CO_2}R_{CO_2}}{m_{N_2} + m_{H_2} + m_{CO_2}} \quad \left[\begin{matrix} R_{N_2} = \frac{\bar{R}}{28} \\ R_{H_2} = \frac{\bar{R}}{2}, R_{CO_2} = \frac{\bar{R}}{44} \end{matrix}\right]$$

$$= \frac{21 \times \bar{R} + 50 \times \bar{R} + 29\bar{R}}{21 \times 28 + 50 \times 2 + 29 \times 44}$$

= 0.42334 kJ/kg – K

$$c_{p\ Mix} = \frac{m_{N_2}C_{p_{N_2}} + m_{H_2}C_{p_{H_2}} + m_{CO_2}C_{p_{CO_2}}}{m_{N_2} + m_{H_2} + m_{CO_2}} \quad [m_{N_2} + m_{H_2} + m_{CO_2} = 1964]$$

$$= \frac{21 \times 28 \times 1.039 + 100 \times 14.235 + 0.828 \times 1276}{588 + 100 + 1276} = 1.5738 \text{ kJ/kg} - \text{K}$$

$$c_{VN_2} = 1.039 - \frac{\bar{R}}{28} = 0.74206$$

$$c_{VH_2} = 14.235 - \frac{\bar{R}}{2} = 10.078$$

$$c_{VCO_2} = 0.828 - \frac{\bar{R}}{44} = 0.63904$$

$$\therefore \quad c_{v\ \text{Mix}} = \frac{588 \times 0.74206 + 100 \times 10.078 + 1276 \times 0.63904}{588 + 100 + 1276} = 1.1505 \text{ kJ/kg} - \text{K}$$

$$c_{v\ \text{mix}} = c_{P\ \text{mix}} - R_{\text{mix}} = 1.5738 - 0.42334 = 1.1505 \text{ kJ/kg} - \text{K}$$

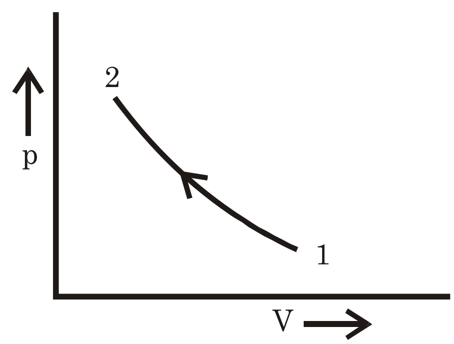
$$\therefore \quad \gamma_{\text{mix}} = \frac{c_{p\ \text{mix}}}{c_{v\ \text{mix}}} = 1.368$$

Given

$\Rightarrow$ $p_1 = 1 \text{ bar} = 100 \text{ kPa}$ $\qquad$ $p_2 = 690 \text{ kPa}$ (Calculated)

$V_2 = 0.085 \text{ m}^3$ $\qquad$ $V_2 = \dfrac{v_1}{5} = 0.017 \text{ m}^3$

$T_1 = 10^\circ \text{ C} = 283 \text{ K}$ $\qquad$ $T_2 = 390.5 \text{ K}$ (Calculated)



$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n = 5^{1.2}$$

$$\therefore \quad p_2 = 100 \times 5^{1.2} \text{ kPa}$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \therefore T_2 = 390.5 \text{ K}$$

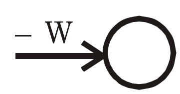
$$W = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$\left[\because W = \int_1^2 p dV = C \int_1^2 \frac{dV}{V^4}\right]$$

$$= \frac{100 \times 0.085 - 690 \times 0.017}{1.2 - 1}$$

$= -16.15$ kJ

[i.e. work have to be given to the system)



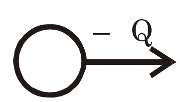
$$Q = u_2 - u_1 + W$$

$$m = \frac{p_1 V_1}{RT_1} = 0.070948 \text{ kg}$$

$= mc_v(T_2 - T_1) + W$

$= (8.7748 - 16.15)$ kJ

$= -7.3752$ kJ



[i.e. Heat flow through system]

Charge of entropy

$$\Delta S = S_2 - S_1 = mc_P \ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right)$$

$$= m\left[1.5738\ln\left(\frac{390.5}{283}\right) - 0.42334\times\ln\left(\frac{690}{100}\right)\right]\text{ kJ/K}$$

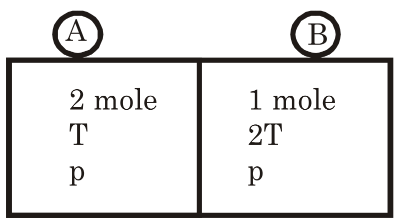
$$= -0.022062\text{ kJ/K} = -22.062\text{ J/K}$$

Q.10.32 **Two moles of an ideal gas at temperature T and pressure p are contained in a compartment. In an adjacent compartment is one mole of an ideal gas at temperature 2Tand pressure p. The gases mix adiabatically but do not react chemically when a partition separating: the compartments are withdrawn. Show that the entropy increase due to the mixing process is given by**

$$\mathbf{R\left(\ln\frac{27}{4} + \frac{\gamma}{\gamma-1}\ln\frac{32}{27}\right)}$$

Provided that the gases are different and that the ratio of specific heat γ is the same for both gases and remains constant.
What would the entropy change be if the mixing gases were of the same Species?

Solution: $V_A = \dfrac{n\bar{R}T}{p} = \dfrac{2\bar{R}T}{p}$ $\qquad V_B = \dfrac{n\bar{R}\,2T}{p} = \dfrac{2\bar{R}T}{p}$



After mixing if final temperature is T_f then

$T_f = \dfrac{2\times T + 1\times 2T}{2+1} = \dfrac{4}{3}T$ $\qquad p_f = \dfrac{n\bar{R}T_f}{V_f} = \dfrac{3\times\bar{R}\times\frac{9}{3}T\times p}{4RT}$

$\therefore$ Final pressure = p

Temperature = $\dfrac{4}{3}T$ and Volume = $V_A + V_B = \dfrac{4\bar{R}T}{p}$

After mixing Partial Pressure of A = $p_{fA} = \dfrac{2}{3}p$ $\qquad c_{PA} = \dfrac{\gamma}{\gamma-1}\bar{R}$

Partial pressure of B = $p_{fB} = \dfrac{1}{3}p$

$$\therefore \quad (\Delta S)_A = n_A\left[c_{p_A}\ln\frac{T_f}{T_A} - \bar{R}\ln\frac{p_{fA}}{p_A}\right]$$

$$= 2R\left[\frac{\gamma}{\gamma-1}\ln\frac{4}{3} - \ln\frac{2}{3}\right]$$

$$(\Delta S)_B = n_B\left[c_{p_B}\ln\frac{T_f}{T_B} - \bar{R}\ln\frac{p_{fB}}{p_B}\right]$$

$$= \bar{R}\left[\frac{\gamma}{\gamma-1}\ln\frac{2}{3} - \ln\frac{1}{3}\right]$$

$$\therefore \quad (\Delta S)_{univ} = (\Delta S)_A + (\Delta S)_B$$

$$= \bar{R}\left[\left(\ln\frac{9}{4} + \ln 3\right) + \frac{\gamma}{\gamma-1}\left(\ln\frac{16}{9} + \ln\frac{2}{3}\right)\right]$$

$$= \bar{R}\left[\ln\frac{27}{4} + \frac{\gamma}{\gamma-1}\ln\frac{32}{27}\right] \text{ Proved.}$$

Q.10.33 **n_1 moles of an ideal gas at pressure p_1 and temperature T are in one compartment of an insulated container. In an adjoining compartment, separated by a partition, are n_2 moles of an ideal gas at pressure p_2 and temperature T. When the partition is removed, calculate (a) the final pressure of the mixture, (b) the entropy change when the gases are identical, and (c) the entropy change when .the gases are different. Prove that the entropy change in (c) is the same as that produced by two independent free expansions.**

Solution: Try please.

Q.10.34 **Assume that 20 kg of steam are required at a pressure of 600 bar and a temperature of 750°C in order to conduct a particular experiment. A 140-litre heavy duty tank is available for storage.**
Predict if this is an adequate storage capacity using:
(a) The ideal gas theory,
(b) The compressibility factor chart,
(c) The van der Waals equation with a = 5.454 $(\text{litre})^2$ atm/ $(\text{g mol})^2$, b = 0.03042 litres/gmol for steam,
(d) The Mollier chart
(e) The steam tables.
Estimate the error in each.

Solution: Try please.

Q.10.35 **Estimate the pressure of 5 kg of CO_2 gas which occupies a volume of 0.70 m^3 at 75°C, using the Beattie-Bridgeman equation of state.**
Compare this result with the value obtained using the generalized compressibility chart. Which is more accurate and why?
For CO_2 with units of atm, litres/g mol and K, A_o = 5.0065, a = 0.07132, B_o = 0.10476, b = 0.07235, C * 10^{-4} = 66.0.

Solution: Try please.

Q.10.36 **Measurements of pressure and temperature at various stages in an adiabatic air turbine show that the states of air lie on the line $pv^{1.25}$ =**

constant. If kinetic and gravitational potential energy is neglected, prove that the shaft work per kg as a function of pressure is given by the following relation

$$\mathbf{W = 3.5 p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{1/5}\right]}$$

Take γ for air as 1.4.

Solution: **Using S.F.E.E.**

$$Q - W + \Delta\left[\frac{V_2^2}{2} + gZ\right] = h_2 - h_1$$

or $$Q - W = mc_p(T_2 - T_1)$$

$$= \frac{\gamma}{\gamma - 1} mR(T_2 - T_1) \qquad \therefore\ p_1v_1 = mRT_1 \qquad p_2v_2 = mRT_2$$

$$= \frac{\gamma p_1 v_1}{\gamma - 1}\left[\frac{p_2v_2}{p_1v_1} - 1\right]$$

$$= \frac{\gamma}{\gamma - 1} p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Here adiabatic process $\therefore\ Q \to 0$ and as

So $$W = \frac{\gamma}{\gamma - 1} \times p_1 v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$\gamma = 1.4$ and n = 1.25

$$W = 3.5\, p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{5}}\right] \text{ proved}$$

Q.10.37 **A mass of an ideal gas exists initially at a pressure of 200 kPa, temperature 300 K, and specific volume 0.5 m^3/kg. The value of r is 1.4. (a) Determine the specific heats of the gas. (b) What is the change in entropy when the gas is expanded to pressure 100 kPa according to the law $pv^{1.3}$ = const? (c) What will be the entropy change if the path is $pv^{1.5}$ = const. (by the application of a cooling jacket during the process)? (d) What is the inference you can draw from this example?**

(**Ans.** (a) 1.166,0.833 kJ/kg K, (b) 0.044 kJ/kg K (c) - 0.039 kJ/kg K
(d) Entropy increases when $n < \gamma$ and decreases when $n > \gamma$)

Solution: Given
$p_1 = 200$ kPa
$T_1 = 300$ K
$v_1 = 0.5$ m³/kg
$\gamma = 1.4$

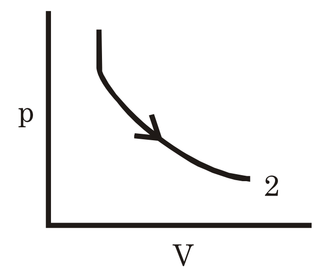
(a) Gas constant(R) = $\frac{p_1v_1}{T_1} = \frac{200 \times 0.5}{300} = 0.33333$ kJ/kg – K

$\therefore$ Super heat at constant Pressure

$$c_p = \frac{\gamma}{\gamma-1}\ R = \frac{1.4}{1.4-1}\times 0.33333 = 1.1667 \text{ kJ/kg}-\text{K}$$

$$c_V = c_p - R = 0.83333 \text{ kJ/kg}-\text{K}$$

(b) Given $p_2 = 100$ kPa



$$v_2 = v_1\times\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}} = 0.85218 \text{ m}^3\text{/kg}$$

$$\therefore\ s_2 - s_1 = c_p \ln\frac{V_2}{V_1} + c_V \ln\frac{p_2}{p_1}$$

$$= 1.1667\times\ln\left(\frac{0.85218}{0.5}\right) + 0.83333\times\ln\left(\frac{100}{200}\right) \text{kJ/kg}-\text{K}$$

$$= 0.044453 \text{ kJ/kg}-\text{K} = 44.453 \text{ J/kg}-\text{K}$$

(c) If path is $pv^{1.5} = C$. Then

$$v_2 = v_1\times\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.5}} = 0.7937 \text{ m}^3\text{/kg}$$

$$\therefore\quad s_2 - s_1 = 1.1667\times\ln\left(\frac{0.7937}{0.5}\right) + 0.83333\ln\left(\frac{150}{200}\right)\text{kJ/kg}-\text{K}$$

$$= -0.03849 \text{ kJ/kg}-\text{K}$$

(d) $n > \gamma$ is possible if cooling arrangement is used and ΔS will be –ve

Q.10.38 **(a) A closed system of 2 kg of air initially at pressure 5 atm and temperature 227°C, expands reversibly to pressure 2 atm following the law $pv^{1.25}$ = const. Assuming air as an ideal gas, determine the work done and the heat transferred.**

(**Ans.** 193 kJ, 72 kJ)

(b) If the system does the same expansion in a steady flow process, what is the work done by the system?

(**Ans.** 241 kJ)

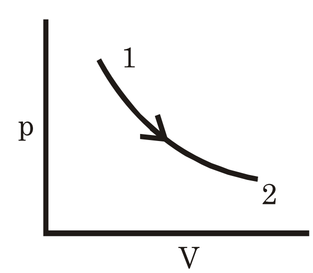
Solution: Given

$m = 2$ kg

$p_1 = 5$ atm $= 506.625$ kPa

$T_1 = 277°$ C $= 550$ K

$p_2 = 2$ atm $= 202.65$ kPa



$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 457.9 \text{ K}$$

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1-T_2)}{n-1}$$

$$= \frac{2\times 0.287(550-457.9)}{1.25-1} = 211.46 \text{ kJ}$$

Reversible polytropic process

Heat transfer

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$

$$= mc_v(T_2 - T_1) + W_{1-2}$$

$= 2 \times 0.718\ (457.9 - 550) + W = -132.26 + 211.46 = 79.204$ kJ

(b) For steady flow reversible polytropic process

$$W = h_1 - h_2$$

$$= \frac{n}{n-1}[p_1 V_1 - p_2 V_2] = \frac{mR}{n-1}[T_1 - T_2] = 264.33 \text{ kJ}$$

Q.10.39 Air contained in a cylinder fitted with a piston is compressed reversibly according to the law $pv^{1.25}$ = const. The mass of air in the cylinder is 0.1 kg. The initial pressure is 100 kPa and the initial temperature 20°C. The final volume is 1/ 8 of the initial volume. Determine the work and the heat transfer.

(**Ans.** – 22.9 kJ, –8.6 kJ)

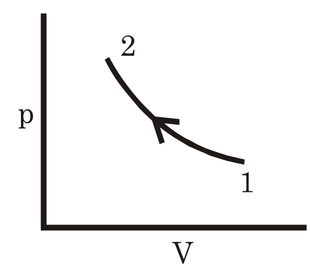
Solution: It is a reversible polytropic process

$m = 0.1$ kg $\quad p_2 = 1345.4$ kPa

$p_1 = 100$ kPa $\quad T_2 = 492.77$ K

$\therefore \quad V_1 = \frac{mRT_1}{P_1} \quad V_2 = 0.010511\ m^3$

$= 0.084091\ m^3$



$$\therefore \quad p_2 = p_1\left(\frac{V_1}{V_2}\right)^{1.25} = 100 \times 8^{1.25}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad W_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{100 \times 0.084091 - 1345.4 \times 0.010511}{1.25 - 1}$$

$$= -22.93 \text{ kJ}$$

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$

$$= mc_v(T_2 - T_1) + W_{1-2}$$

$$= 0.2 \times 0.718 \times (492.77 - 293) - 22.93$$

$$= -8.5865 \text{ kJ}$$

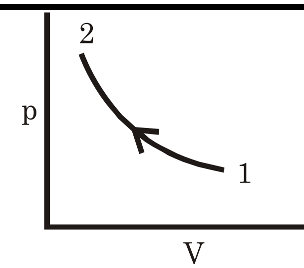
Q.10.40 Air is contained in a cylinder fitted with a frictionless piston. Initially the cylinder contains 0.5 m^3 of air at 1.5 bar, 20°C. The air is Then compressed reversibly according to the law pv^n = constant until the final pressure is 6 bar, at which point the temperature is 120°C. Determine: (a) the polytropic index *n,* (b) the final volume of air, (c) the work done on the air and the heat transfer, and (d) the net change in entropy.

(**Ans.** (a) 1.2685, (b) 0.1676 m^3 (c) –95.3 kJ, –31.5 kJ, (d) 0.0153 kJ/K**)**

Solution: Given $p_1 = 1.5$ bar $= 150$ kPa

$T_1 = 20$ºC $= 293$ K

$V_1 = 0.5\ m^3$



$$\therefore\ m = \frac{p_1 V_1}{RT_1} = 0.89189 \text{ kg}$$

$p_2 = 6$ bar $= 600$ kPa

$T_2 = 120°$ C $= 393$ K

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\text{or } \left(1 - \frac{1}{n}\right) = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)} = 0.2118$$

$$\therefore\ n = 1.2687$$

(a) The polytropic index, $n = 1.2687$

(b) Final volume of air $(V_1) = \frac{mRT_2}{p_2} = \frac{0.189 \times 0.287 \times 393}{600}$ m³ $= 0.16766$ m³

(c) $$W_{1-2} = \int_1^2 pdV = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{150 \times 0.5 - 600 \times 0.16766}{1.2687 - 1} \text{ kJ} = -95.263 \text{ kJ}$$

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$
$$= mc_v(T_2 - T_1) + W_{1-2}$$
$$= 0.89189 \times 0.718(393 - 293) + W_{1-2}$$
$$= -31.225 \text{ kJ}$$

(d) $$\Delta s = s_2 - s_1 = m\left[c_v \ln\frac{p_2}{p_1} + c_P \ln\frac{V_2}{V_1}\right]$$

$$= 0.89189\left[0.718 \ln\left(\frac{600}{150}\right) + 1.005 \times \ln\left(\frac{0.16766}{0.5}\right)\right] = -0.091663 \text{ kJ/K}$$

Q.10.41 The specific heat at constant pressure for air is given by

$$\mathbf{c_p = 0.9169 + 2.577 + 10^{-4}\ T - 3.974 * 10^{-8}\ T^2\ kJ/kg\ K}$$

Determine the change in internal energy and that in entropy of air when it undergoes a change of state from 1 atm and 298 K to a temperature of 2000 K at the same pressure.

(**Ans.** 1470.4 kJ/kg, 2.1065 kJ/kg K)

Solution: $p_1 = p_2 = 1$ atm $= 101.325$ kPa

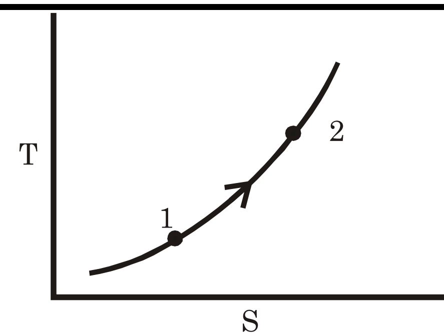
$T_1 = 298$ K; $T_2 = 2000$ K

$c_p = 0.9169 + 2.577 \times 10^{-4}\ T - 3.974$

$\times 10^{-3}\ T^2$ kJ/kg – K

$$\Delta u = u_2 - u_1 = \int m\, c_v\, dT$$

$$= \int m(c_p - R)\, dT$$
$$= \int mc_p\, dT - mR \int dT$$
$$= 1 \times \int_{298}^{2000} (0.9169 + 2.577 \times 10^{-4}\, T - 3.974 \times 10^{-8}\, T^2)\, dT - 1 \times 0.287 \int_{298}^{2000} dT \quad kJ/kg$$



= (1560.6 + 503.96 – 105.62 – 488.47) kJ/kg
= 1470.5 kJ/kg

$\therefore \quad Tds = dh - vdp$

or $\quad Tds = mc_p dT - v\,dp$

$$\therefore \quad \int_1^2 dS = m \int_{298}^{2000} c_P \frac{dT}{T}$$

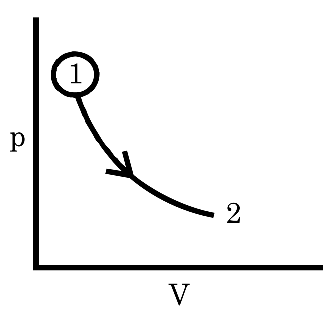
$$\therefore\ s_2 - s_1 = 0.9169 \times \ln \frac{2000}{298} + 2.577 \times 10^{-4} (2000 - 298) - 3.974 \times 10^{-8} \times \left(\frac{2000^2 - 298^2}{2}\right)$$

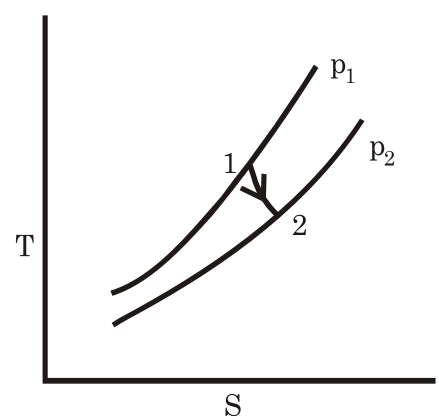
= 2.1065 kJ/kg – K

Q.10.42 A closed system allows nitrogen to expand reversibly from a volume of 0.25 m^3 to 0.75 m^3 along the path $pv^{1.32}$ = const. The original pressure of the gas is 250 kPa and its initial temperature is 100°C.
(a) Draw the p-v and T-s diagrams.
(b) What are the final temperature and the final pressure of the gas?
(c) How much work is done and how much heat is transferred?
(d) What is the Entropy change of nitrogen?

(**Ans.** (b) 262.44 K, 58.63 kPa,
(c) 57.89 kJ, 11.4 kJ, (d) 0.0362 kJ/K)

Solution: Given $p_1 = 250$ kPa
$V_1 = 0.25\ m^3$
$T_1 = 100°C = 373$ K





$$\therefore \quad m = \frac{p_1 v_1}{RT_1} = 0.563\ kg = 0.5643\ kg$$

$$R_{N_2} = \frac{8.3143}{28} = 0.29694 \text{ kJ/kg}$$

$$p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^n = 58.633 \text{ kPa}$$

$$V_2 = 0.75 \text{ m}^3$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 262.4 \text{ K}$$

$$W = \frac{p_1 V_1 - p_2 V_2}{n-1} = \frac{250 \times 0.25 - 58.633 \times 0.75}{(1.32-1)} = 57.891 \text{ kJ}$$

$$Q = u_2 - u_1 + W = mc_v(T_2 - T_1) + W$$

$$= 0.5643 \times 0.7423\,(262.4 - 373) + W$$

$$c_v = \frac{R}{\gamma - 1} = 0.7423$$

$$= 11.56 \text{ kJ} \qquad \gamma = 1 + \frac{2}{5} = 1.4$$

$$c_p = \frac{\gamma}{\gamma-1} R = \frac{1.4}{1.4-1} \times 0.29694 = 1.04 \text{ kJ/kg} - \text{K}$$

$$\therefore \quad \Delta s = s_2 - s_1 = m\left[c_P \ln\frac{V_2}{V_1} + c_V \ln\frac{p_2}{p_1}\right]$$

$$= 0.5643\left[1.04 \times \ln\left(\frac{0.75}{0.25}\right) + 0.7423 \times \ln\left(\frac{58.633}{250}\right)\right] \text{kJ/K}$$

$$= 0.0373 \text{ kJ/kg} - \text{K}$$

Q.10.43 Methane has a specific heat at constant pressure given by c_p = 17.66 + 0.06188 T kJ/kg mol K when 1 kg of methane is heated at constant volume from 27 to 500°C. If the initial pressure of the gas is 1 atm, calculate the final pressure, the heat transfer, the work done and the change in entropy.

(**Ans**. 2.577 atm, 1258.5 kJ/kg, 2.3838 kJ/kg K)

Solution: Given $p_1 = 1 \text{ atm} = 101.325 \text{ kPa}$

$T_1 = 27°\text{C} = 300 \text{ K}$

$m = 1 \text{ kg}$

$$\therefore \quad V_1 = \frac{mRT_1}{p_1}$$

$$= 1.5385 \text{ m}^3 = V_2$$

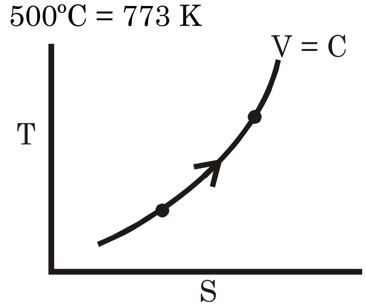
$$R = \frac{\bar{R}}{16}$$

$$= 0.51964 \text{ kJ/kg} - \text{K}$$

$p_2 = 261 \text{ kPa}$

$V_2 = 1.5385$

$T_2 = 500°\text{C} = 773 \text{ K}$



(i) Find pressure $(p_2) = \dfrac{mRT_2}{V_2}$

$$= 261 \text{ kPa} \approx 2.577 \text{ atm}$$

(ii) Heat transfer $Q = \int mc_v\, dT$

$$= m\int [c_P - R]\, dT$$

$$= 1\times\int_{300}^{773}(1.1038+3.8675\times10^{-3}-0.51964)\,dT$$

$$=0.58411(773-300)+3.8675\times10^{-3}\,\frac{(773^2-300^2)}{2}$$

$$c_P = \frac{17.66}{16}+\frac{0.06188}{16}T\text{ kJ/kg}-K$$

$$= 1.1038 + 3.8675 \times 10^{-3}\text{ T} = 1257.7\text{ kJ/kg}$$

(iii) Work done = $\int_1^2 pdV = 0$

$\therefore$ $Tds = du = mc_v\,dT$

$$ds = mc_v\frac{dT}{T} = m(c_p - R)\frac{dT}{T}$$

$$\therefore \int_1^2 dS = \int_{300}^{773}\left(\frac{1\times0.58411+3.8675\times10^{-3}\,T}{T}\right)dT$$

$$s_2 - s_1 = 0.58411\ln\frac{773}{300}+3.8675\times10^{-3}(773-300) = 2.3822\text{ kJ/kg}-K$$

Q.10.44 Air is compressed reversibly according to the law $pv^{1.25}$ = const. from an initial pressures of 1 bar and volume of 0.9 m^3 to a final volume of 0.6 m^3. Determine the final pressure and the change of entropy per kg of air.

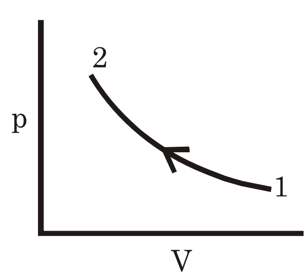
(**Ans.** 1.66 bar, –0.0436 kJ/kg K)

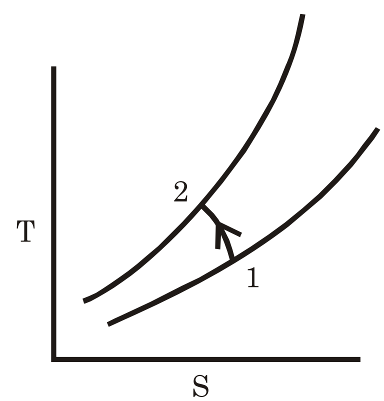
Solution: $p_1 = 1$ bar

$V_1 = 0.9\ m^3$

$V_2 = 0.6\ m^3$

$$\therefore \quad p_2 = p_1\left(\frac{V_1}{V_2}\right)^{1.25} = 1.66\text{ bar}$$





$$\Delta s = s_2 - s_1 = \left(c_p\ln\frac{V_2}{V_1}+c_v\ln\frac{p_2}{p_1}\right)$$

$$= 1.005\times\ln\left(\frac{0.6}{0.9}\right)+0.718\times\ln\left(\frac{1.66}{1}\right)\text{kJ/kg}-K$$

$$= -0.043587\text{ kJ/kg}-K$$

Q.10.45 In a heat engine cycle, air is isothermally compressed. Heat is then added at constant pressure, after which the air expands isentropically to

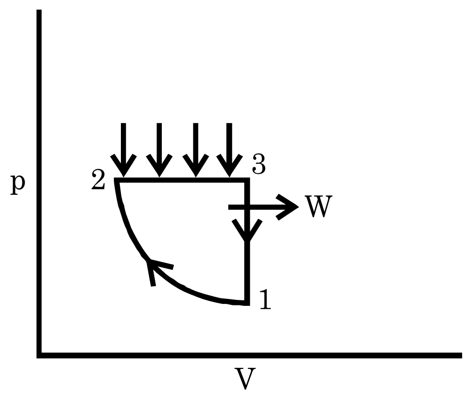
its original state. Draw the cycle on p-v and T'-s coordinates. Show that the cycle efficiency can be expressed in the following form

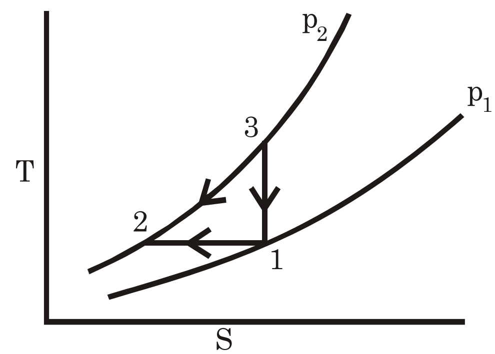
$$\eta = 1 - \frac{(\gamma - 1)\ln r}{\gamma\left[r^{\gamma-1/\gamma} - 1\right]}$$

Where *r* is the pressure ratio, p_2/p_1. Determine the pressure ratio and the cycle efficiency if the initial temperature is 27°C and the maximum temperature is 327°C.

(**Ans.** 13.4, 32.4%)

Solution: Heat addition $(Q_1) = Q_{2-3} = mc_p(T_3 - T_2)$





Heat rejection $(Q_2) = mRT_1 \ln\left(\frac{p_2}{p_1}\right)$

$$\therefore \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{RT_1 \ln\left(\frac{p_2}{p_1}\right)}{C_p(T_3 - T_2)} \qquad \text{Here, } \frac{p_2}{p_1} = r$$

$$= 1 - \frac{\gamma - 1}{\gamma}\frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\frac{T_3}{T_1} - 1\right)} \qquad \therefore \quad \frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$= 1 - \frac{\gamma - 1}{\gamma}\frac{\ln r}{(r^{\frac{\gamma-1}{\gamma}} - 1)} \quad \text{Proved}$$

$\Rightarrow$ If initial temperature $(T_1) = 27°C = 300\ K = T_2$

And $T_3 = 327°C = 600\ K$

$$\therefore \quad r = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{600}{300}\right)^{\frac{1.4}{1.4-1}} = 11.314$$

$$\therefore \quad \eta = 1 - \frac{(1.4-1)}{(1.4)} \times \frac{\ln(11.314)}{\left[(11.314)^{\frac{1.4-1}{1.4}} - 1\right]} = 0.30686$$

Q.10.46 **What is the minimum amount of work required to separate 1 mole of air at 27°C and 1 atm pressure (assumed composed of 1/5 O_2 and 4/5 N_2) into oxygen and nitrogen each at 27°C and 1 atm pressure?**

(**Ans.** 1250 J)

Solution: Total air is 1 mole

So $O_2 = \frac{1}{5}$ mole $= 0.0064$ kg

$N_2 = \frac{4}{5}$ mole $= 0.0224$ kg

Mixture, pressure = 1 atm, temperature = 300 K

Partial pressure of $O_2 = \frac{1}{5}$ atm

Partial pressure of $N_2 = \frac{4}{5}$ atm

Minimum work required is isothermal work

$$= m_{O_2} R_{O_2} T_{1_{O_2}} \ln \frac{p_{f_{O_2}}}{p_{1_{O_2}}} + m_{N_2} R_{N_2} T_{12} \ln\left(\frac{p_{f\,N_2}}{p_{1\,N_2}}\right)$$

$$= 0.0064 \times \frac{8.3143}{32} \times 300 \ln (5) + 0.0224 \times \frac{8.3143}{28} \times 300 \ln\left(\frac{5}{4}\right)$$

$$= 1.248 \text{ kJ} = 1248 \text{ J}$$

Q.10.47 **A closed adiabatic cylinder of volume 1 m^3 is divided by a partition into two compartments 1 and 2. Compartment 1 has a volume of 0.6 m^3 and contains methane at 0.4 MPa, 40°C, while compartment 2 has a volume of 0.4 m^3 and contains propane at 0.4 MPa, 40°C. The partition is removed and the gases are allowed to mix.**

(a) When the equilibrium state is reached, find the entropy change of the universe.

(b) What are the molecular weight and the specific heat ratio of the mixture?
The mixture is now compressed reversibly and adiabatically to 1.2 MPa. Compute

(c) the final temperature of the mixture,

(d) The work required per unit mass, and

(e) The specific entropy change for each gas. Take c_p of methane and propane as 35.72 and 74.56 kJ/kg mol K respectively.

(**Ans.** (a) 0.8609 kJ/K, (b) 27.2,1.193 (c) 100.9°C, (d) 396 kJ, (e) 0.255 kJ/kg K)

Solution: After mixing $p_f = 400$ kPa

$T_f = 313$ K

But partial pressure of (p_{1f})

1	2
$V_1 = 0.6\ m^3$	$V_2 = 0.4\ m^3$
$p_1 = 400$ kPa	$p_2 = 400$ kPa
$T_1 = 313$ K	$T_2 = 313$ K
CH_4	C_3H_8

$$CH_4 = \frac{0.6}{1} \times 400 = 240 \text{ kPa}$$

$$\therefore \quad p_{2f} = 0.4 \times 400 = 160 \text{ kPa}$$

(a)
$$(\Delta S)_{CH_4} = m_{CH_4}\left[c_{P_{CH_4}} \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right]$$
$$= m_{CH_4} R_{CH_4} \ln\left(\frac{p_i}{p_f}\right)$$
$$= \frac{p_1 V_1}{R_{CH_4} T_1} \times R_{CH_4} \ln\left(\frac{p_i}{p_f}\right)$$
$$= \frac{p_1 V_1}{T_1} \times R_{CH_4} \ln\left(\frac{p_i}{p_{f_1}}\right)$$
$$(\Delta S)_{C_3H_8} = \frac{p_2 V_2}{T_2} \times \ln\frac{p_i}{p_{f_2}}$$
$$(\Delta S)_{Univ} = (\Delta S)_{CH_4} + (\Delta S)_{C_3H_8}$$
$$= \frac{400\times 0.6}{313}\ln\left(\frac{400}{240}\right) + \frac{400\times 0.4}{313}\ln\left(\frac{400}{160}\right) \text{ kJ/K}$$
$$= 0.86 \text{ kJ/K}$$

(b) Molecular weight
$$xM = x_1M_1 + x_2M_2$$
$$\therefore \quad M = \frac{x_1}{x}M_1 + \frac{x_2}{x}\times M_2 = 0.6 \times 16 + 0.4 \times 44 = 27.2$$
$$c_{p\,mix} = \frac{n_1c_{p_1} + n_2c_{p_2}}{n_1+n_2} = \frac{0.6\times 35.72 + 0.4\times 74.56}{1} = 51.256 \text{ kJ/kg}$$
$$R_{mix} = \bar{R} = 8.3143$$
$$\therefore \quad c_{v\,mix} = c_{P\,mix} - \bar{R} = 42.9417$$
$$\therefore \quad \gamma_{mix} = \frac{c_{P\,mix}}{c_{v\,mix}} = \frac{51.256}{42.9417} = 1.1936$$

Q.10.48 An ideal gas cycle consists of the following reversible processes: (i) isentropic compression, (ii) constant volume heat addition, (iii) isentropic expansion, and (iv) constant pressure heat rejection. Show that the efficiency of this cycle is given by

$$\eta = 1 - \frac{1}{r_k^{\gamma-1}}\left[\frac{\gamma\left(a^{1/\gamma}-1\right)}{a-1}\right]$$

Where r_k is the compression ratio and *a* is the ratio of pressures after and before heat addition.

An engine operating on the above cycle with a compression ratio of 6 starts the compression with air at 1 bar, 300 K. If the ratio of pressures after and before heat addition is 2.5, calculate the efficiency and the m.e.p. of the cycle. Take

$\gamma = 1.4$ and $c_v = 0.718$ kJ/kg K.

(**Ans.** 0.579, 2.5322 bar)

Solution: $Q_{2-3} = u_3 - u_2 + pdV = mc_v(T_3 - T_2)$

$$Q_{1-4} = mc_p(T_4 - T_1)$$

$$\therefore \quad \eta = 1 - \frac{m c_p (T_4 - T_1)}{m c_v (T_3 - T_2)}$$

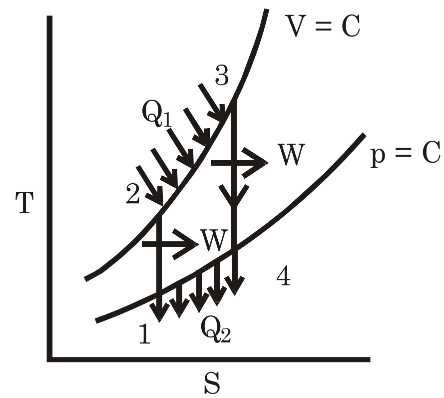
$$= 1 - \gamma\left(\frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = r_k^{\gamma - 1}$$

$$\therefore \quad T_2 = T_1 \times r_k^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{p_3}{p_2} = 9$$



$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

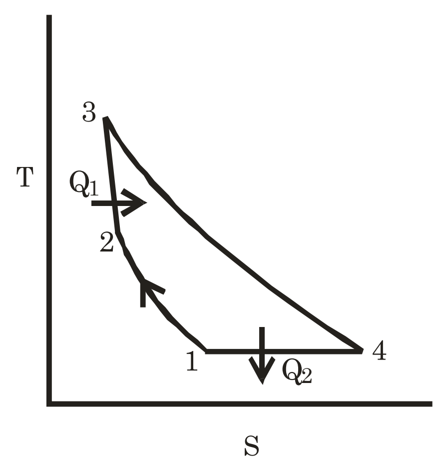
$$\therefore \frac{T_3}{T_4} = (a \times r)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = a^{\gamma} \qquad \therefore \frac{p_3}{p_2} = \frac{T_3}{T_2} = a,$$

$$\therefore \quad T_3 = aT_2 = aT_1 r_k^{\gamma-1}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2} \times \frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(\left(\frac{V_2}{V_1}\right)^{\gamma} \times \frac{1}{a}\right)^{\frac{\gamma-1}{\gamma}}$$



$$= (r_k^{-\gamma}\, a^{-1})^{\frac{r-1}{r}}$$

$$\therefore \quad T_4 = r_k^{1-\gamma} . a^{\frac{1-\gamma}{\gamma}} . T_3$$

$$= r_k^{1-\gamma} . a^{\frac{1}{\gamma}-1} . a \times T_1 . r_k^{\gamma-1}$$

$$= a^{\frac{1}{\gamma}} . T_1$$

$$\therefore \quad \eta = 1 - \frac{\gamma(a^{\frac{1}{\gamma}}\, T_1 - T_1)}{(aT_1 r_k^{\gamma-1} - T_1 r_k^{\gamma-1})}$$

$$= 1 - \frac{[\gamma(a^{\frac{1}{\gamma}} - 1)]}{r_k^{\gamma-1}(a-1)} \text{ Proved.}$$

Given p_1 = 1 bar = 100 kPa

T_1 = 300K, $r_k = 6$,

a = 2.5, $\gamma = 1.4$

$$\therefore \quad \eta = 1 - \frac{1.4(2.5^{\frac{1}{1.4}} - 1)}{6^{1.4-1}(2.5-1)} = 0.57876$$

$$Q_1 = mc_v(T_3 - T_2)$$
$$= m\,c_v(aT_1r_k^{\gamma-1} - T_1r_k^{\gamma-1})$$
$$= m\,c_v\,T_1r_k^{\gamma-1}(a-1)$$
$$= m \times 0.718 \times 300 \times 6^{0.4} \times (2.5 - 1)$$
$$= 661.6 \text{ m kJ}$$
$$\therefore W = \eta\, Q_1 = 382.9 \text{ m kJ}$$

For $V_4 =$; $T_4 = 2.5^{\frac{1}{1.4}} \times 300 = 577.25$ K

$$p_4 = p_1 = 100 \text{ kPa}$$

$$V_4 = \frac{mRT_4}{p_4} = \frac{m \times 0.287 \times 577.25}{100} \text{ m}^3$$
$$= 1.6567 \text{ m m}^3$$

$$V_1 = \frac{m\,R\,T_1}{p_1} = \frac{m \times 0.287 \times 300}{100}$$
$$= 0.861\text{m m}^3$$

$$\therefore \quad V_4 - V_1 = 0.7957 \text{ m}^3$$

Let m.e.p. is p_m then

$$p_m(V_4 - V_1) = W$$

$$p_m = \frac{382.9 \times m}{0.7957\ m} \text{ kPa}$$

$$= 481.21 \text{ kPa} = 4.8121 \text{ bar}$$

Q10.49 **The relation between u, p and v for many gases is of the form u = a + bpv where *a* and *b* are constants. Show that for a reversible adiabatic process pv^{γ} = constant, where**

$$\gamma = (b + 1)/b.$$

Solution: Try please.

Q10.50 **(a) Show that the slope of a reversible adiabatic process on p-v coordinates is**

$$\frac{dp}{dv} = \frac{1}{kv}\frac{c_p}{c_v} \text{ where } k = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(b) Hence, show that for an ideal *gas,* pv^{γ} = constant, for a reversible adiabatic process.

Solution: Try please.

Q10.51 **A certain gas obeys the Clausius equation of state p (v – b) = RT and has its internal energy given by u = c_VT. Show that the equation for a reversible adiabatic process is $p\,(v-b)^{\gamma}$ = constant, where $\gamma = c_p / c_v$.**

Solution: Try please.

Q10.52 **(a) Two curves, one representing a reversible adiabatic process undergone by an ideal gas and the other an isothermal process by the same gas, intersect at the same point on the p-v diagram. Show that the ratio of the slope of the adiabatic curve to the slope of the isothermal curve is equal to γ.**

(b) Determine the ratio of work done during a reversible adiabatic process to the work done during an isothermal process for a gas having γ = 1.6. Both processes have a pressure ratio of 6.

Solution: Try please.

Q10.53 **Two containers *p* and *q* with rigid walls contain two different monatomic gases with masses m_p and m_q, gas constants R_p and R_q, and initial temperatures T_p and T_q respectively, are brought in contact with each other and allowed to exchange energy until equilibrium is achieved. Determine:**

(a) the final temperature of the two gases and

(b) the change of entropy due to this energy exchange.

Solution: Try please.

Q10.54 **The pressure of a certain gas (photon gas) is a function of temperature only and is related to the energy and volume by p(T) = (1/3) (U/V). A system consisting of this gas confined by a cylinder and a piston undergoes a Carnot cycle between two pressures P_1 and P_2.**

(a) Find expressions for work and heat of reversible isothermal and adiabatic processes.

(b) Plot the Carnot cycle on p-v and T- s diagrams.

(c) Determine the efficiency of the cycle in terms of pressures.

(d) What is the functional relation between pressure and temperature?

Solution: Try please.

Q10.55 **The gravimetric analysis of dry air is approximately: oxygen = 23%, nitrogen = 77%. Calculate:**

(a) The volumetric analysis,

(b) The gas constant,

(c) The molecular weight,

(d) the respective partial pressures,

(e) The specific volume at 1 atm, 15°C, and

(f) How much oxygen must be added to 2.3 kg air to produce . A mixture which is 50% oxygen by volume?

(**Ans.** (a) 21% O_2, 79% N_2, (b) 0.288 kJ/kg K, (d) 21 kPa for O_2' (e) 0.84 m^3/kg, (f) 1.47 kg)

Solution: By gravimetric analysis O_2: N_2 = 23: 77

(a) $\therefore$ By volumetric analysis O_2: N_2 $= \frac{23}{32}:\frac{77}{28}$

$$= 0.71875: 2.75$$

$$= 0.71875 \times \frac{(100)}{(0.71875-2.75)} : \frac{2.75 \times 100}{2.75}$$

$$= 20.72: 79.28$$

(b) Let total mass = 100 kg

$\therefore$ $O_2 = 23$ kg, $N_2 = 77$ kg

$$\therefore \quad R = \frac{23 \times R_{O_2} + 77 \times R_{N_2}}{23 + 77},$$

$$= \frac{23 \times \frac{8.3143}{32} + 77 \times \frac{8.3143}{28}}{23 + 77}$$

$= 0.2884$ kJ/kg – K

(c) For molecular weight (μ)

$$x\mu = x_1 \mu_1 + x_2 \mu_2$$

$$\text{or} \quad \mu = \frac{x_1}{x} \times \mu_1 + \frac{x_2}{x} \mu_2$$

$= 2072 \times 32 + 0.7928 \times 28 = 28.83$

(d) Partial pressure of $O_2 = x_{O_2} \times p$

$= 0.2072 \times 101.325$ kPa $= 20.995$ kPa

Partial pressure of $N_2 = x_{N_2} \times p = 0.7928 \times 101.325$ kPa $= 80.33$ kPa

(e) Sp. volume, $v = \frac{RT}{\rho} = \frac{0.2884 \times 288}{101.325} \text{ m}^3/\text{kg} = 0.81973 \text{ m}^3/\text{kg}$

Density $\rho = \rho_1 + \rho_2$

$$\therefore \quad \frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2} = \frac{p_{O2}}{R_{O_2} \times 288} + \frac{p_{N_2}}{R_{N_2} \times 288}$$

$$= \frac{0.2072 \times 101.325 \times 32}{8.3143 \times 2.88} + \frac{0.7928 \times 101.325 \times 28}{8.3143 \times 288}$$

$\therefore$ $v = 0.81974$ m³/kg

(f) In 2.3 kg of air $O_2 = 2.3 \times 0.23$ kg $= 0.529$ kg

$\therefore$ $N_2 = 2.3 \times 0.77 = 1.771$ kg $= 63.25$ mole

For same volume we need same mole O_2

Total $O_2 = 63.25 \times \frac{32}{1000} \text{ kg} = 2.024$ kg

$\therefore$ Oxygen must be added = (2.024 – 0.529) kg = 1.495 kg

11. Thermodynamic Relations

Some Important Notes

Some Mathematical Theorem

Theorem 1. If a relation exists among the variables x, y and z, then z may be expressed as a function of x and y, or

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

then dz = M dx + N dy.

Where z, M and N are functions of x and y. Differentiating M partially with respect to y, and N with respect to x.

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x.\partial y}$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial y.\partial x}$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

This is the condition of exact (or perfect) differential.

Theorem 2. If a quantity f is a function of x, y and z, and a relation exists among x, y and z, then f is a function of any two of x, y and z. Similarly any one of x, y and z may be regarded to be a function of f and any one of x, y and z. Thus, if

x = x (f, y)

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df + \left(\frac{\partial x}{\partial y}\right)_f dy$$

Similarly, if

y = y (f, z)

$$dy = \left(\frac{\partial y}{\partial f}\right)_z df + \left(\frac{\partial y}{\partial z}\right)_f dz$$

Substituting the expression of dy in the preceding equation

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df + \left(\frac{\partial x}{\partial y}\right)_f \left[\left(\frac{\partial y}{\partial f}\right)_z df + \left(\frac{\partial y}{\partial z}\right)_f dz\right]$$

$$= \left[\left(\frac{\partial x}{\partial f}\right)_y + \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial f}\right)_z\right] df + \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f dz$$

Again

$$dx = \left(\frac{\partial x}{\partial f}\right)_z df + \left(\frac{\partial x}{\partial z}\right)_f dz$$

$$\left(\frac{\partial x}{\partial z}\right)_f = \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1$$

Theorem 3. Among the variables x, y, and z any one variable may be considered as a function of the other two. Thus

x = x(y, z)

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$$

Similarly,

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

$$= \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial x}\right)_y dx$$

$$= \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + dx$$

$$\therefore \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial x}{\partial z}\right)_y = 0$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x = -1$$

Among the thermodynamic variables p, V and T. The following relation holds good

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

Maxwell's Equations

A pure substance existing in a single phase has only two independent variables. Of the eight quantities p, V, T, S, U, H, F (Helmholtz function), and G (Gibbs function) any one may be expressed as a function of any two others.

For a pure substance undergoing an infinitesimal reversible process

(a) dU = TdS - pdV

(b) dH = dU + pdV + VdP = TdS + Vdp

(c) dF = dU - TdS - SdT = - pdT - SdT

(d) dG = dH - TdS - SdT = Vdp - SdT

Since U, H, F and G are thermodynamic properties and exact differentials of the type

dz = M dx + N dy, then

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Applying this to the four equations

$$\left(\frac{\partial T}{\partial V}\right)_s = -\left(\frac{\partial p}{\partial S}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial S}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial p}\right)_T$$

These four equations are known as **Maxwell's equations.**

Questions with Solution (IES & IAS)

(i) Derive: $dS = C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dV$ [IAS - **1986**]

Let entropy S be imagined as a function of T and V.

Then $S = S(T, V)$

or $dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

multiplying both side by T

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

Since $T\left(\frac{\partial S}{\partial T}\right)_V = C_V$, heat capacity at constant volume

and $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$ by Maxwell's equation

$\therefore$ $TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

dividing both side by T

$dS = C_V \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_V dV$ proved

(ii) Derive:

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$ **[IES-1998]**

Let entropy S be imagined as a function of T and p.

Then $S = S(T, p)$

or $dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$

multiplying both side by T

$TdS = T\left(\frac{\partial S}{\partial T}\right)_p dT + T\left(\frac{\partial S}{\partial p}\right)_T dp$

Since $T\left(\frac{\partial S}{\partial T}\right)_p = C_p$, heat capacity at constant pressure

and $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$ by Maxwell's equation

$\therefore$ $TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$ proved.

(iii) Derive:

$$TdS = C_V dT + T\frac{\beta}{k}dV = C_p dT - TV\beta\ dp = \frac{k\ C_v\ dp}{\beta} + \frac{C_p}{\beta V}dV$$

[IES-2001]

We know that volume expansivity (β) = $\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

and isothermal compressibility (k) = $-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

$\therefore$ From first TdS equation

$TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

$\frac{\beta}{k} = -\frac{\left(\frac{\partial V}{\partial T}\right)_p}{\left(\frac{\partial V}{\partial p}\right)_T} = -\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T$

As $\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_V \cdot \left(\frac{\partial p}{\partial V}\right)_T = -1$

$\therefore$ $-\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

or $\frac{\beta}{k} = \left(\frac{\partial p}{\partial T}\right)_V$

$\therefore$ $TdS = C_V dT + T\cdot\frac{\beta}{k}\cdot dV$ proved

From second TdS relation

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$

as $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

$\therefore \left(\frac{\partial V}{\partial T}\right)_p = V\beta$

$\therefore TdS = C_p dT - TV\beta dp$ proved

Let S is a function of p, V

$\therefore S = S(p, V)$

$\therefore dS = \left(\frac{\partial S}{\partial p}\right)_V dp + \left(\frac{\partial S}{\partial V}\right)_p dV$

Multiply both side by T

$$TdS = T\left(\frac{\partial S}{\partial p}\right)_V dp + T\left(\frac{\partial S}{\partial V}\right)_p dV$$

or $TdS = T\left(\frac{\partial S}{\partial T}\cdot\frac{\partial T}{\partial p}\right)_V dp + T\left(\frac{\partial S}{\partial T}\cdot\frac{\partial T}{\partial V}\right)_p dV$

or $TdS = T\left(\frac{\partial S}{\partial T}\right)_V \cdot \left(\frac{\partial T}{\partial p}\right)_V dp + T\left(\frac{\partial S}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial V}\right)_p dV$

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$ and $C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$\therefore TdS = C_v\left(\frac{\partial T}{\partial p}\right)_V dp + C_p\left(\frac{\partial T}{\partial V}\right)_p dV$

From first $\frac{\beta}{k} = \left(\frac{\partial p}{\partial T}\right)_V$ or $\frac{k}{\beta} = \left(\frac{\partial T}{\partial p}\right)_V$

$\therefore TdS = C_v \frac{k}{\beta} dp + C_p\left(\frac{\partial T}{\partial V}\right)_p dV$

$\therefore \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

$\therefore \left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{\beta V}$

$\therefore TdS = \frac{C_v\, k\, dp}{\beta} + \frac{C_p}{\beta V} dV$ proved.

(iv) Prove that

$$C_p - C_v = -T\left(\frac{\partial V}{\partial T}\right)_p^2 \cdot \left(\frac{\partial p}{\partial V}\right)_T$$ **[IAS-1998]**

We know that

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$$

or $$\left(C_p - C_v\right)dT = T\left(\frac{\partial V}{\partial T}\right)_p dp + T\left(\frac{\partial p}{\partial T}\right)_V dV$$

or $$dT = \frac{T\left(\frac{\partial V}{\partial T}\right)_p dp}{C_p - C_V} + \frac{T\left(\frac{\partial p}{\partial T}\right)_V dV}{C_p - C_V} \qquad ---(i)$$

since T is a function of p, V

$$T = T(p, V)$$

or $$dT = \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p dV \qquad ---(ii)$$

comparing (i) & (ii) we get

$$\frac{T\left(\frac{\partial V}{\partial T}\right)_p}{C_p - C_V} = \left(\frac{\partial T}{\partial p}\right)_V \qquad \text{and} \qquad \frac{T\left(\frac{\partial p}{\partial T}\right)_V}{C_p - C_V} = \left(\frac{\partial T}{\partial V}\right)_p$$

both these give

$$C_p - C_V = T\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_V$$

Here $$\left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial p}\right)_T = -1 \quad \text{or} \quad \left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T$$

$$\because \qquad C_p - C_V = -T\left(\frac{\partial V}{\partial T}\right)_p^2 \cdot \left(\frac{\partial p}{\partial V}\right)_T \qquad \text{proved.} \qquad \text{...............Equation(A)}$$

This is a very important equation in thermodynamics. It indicates the following important facts.

(a) Since $\left(\frac{\partial V}{\partial T}\right)_p^2$ is always positive, and $\left(\frac{\partial p}{\partial V}\right)_T$ for any substance is negative. ($C_p - C_v$) is always positive. Therefore, C_p is always greater than C_v.

(b) As $T \to 0K, C_p \to C_v$ or at absolute zero, $C_p = C_v$.

(c) When $\left(\frac{\partial V}{\partial T}\right)_p = 0$ (e.g for water at 4ºC, when density is maximum. Or specific volume minimum). $C_p = C_v$.

(d) For an ideal gas, pV = mRT

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{mR}{P} = \frac{V}{T}$$

and $$\left(\frac{\partial p}{\partial V}\right)_T = -\frac{mRT}{V^2}$$

$$\therefore \qquad C_p - C_v = mR$$

$$or\ c_p - c_v = R$$

Equation (A) may also be expressed in terms of **volume expansively (β)** defined as

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

and **isothermal compressibility (k_T)** defined as

$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

$$C_p - C_v = \frac{TV\left[\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p\right]^2}{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T}$$

$$C_p - C_v = \frac{TV\beta^2}{k_T}$$

(v) Prove that

$$\frac{\beta}{k} = \left(\frac{\partial p}{\partial T}\right)_V \quad \text{and} \quad C_p - C_V = \left\{p + \left(\frac{\partial U}{\partial V}\right)_T\right\}\left(\frac{\partial V}{\partial T}\right)_p$$

Hence show that

$$C_p - C_v = \frac{\beta^2 TV}{k}$$ [IES-2003]

Here $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

$k = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

$$\therefore \quad \frac{\beta}{k} = -\frac{\left(\frac{\partial V}{\partial T}\right)_p}{\left(\frac{\partial V}{\partial p}\right)_T} = -\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T$$

we know that $\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_V \cdot \left(\frac{\partial p}{\partial V}\right)_T = -1$

or $-\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

$\therefore \quad \frac{\beta}{k} = \left(\frac{\partial p}{\partial T}\right)_V$ proved.

$\Rightarrow$ From Tds relations

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dP = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$$

$$\therefore \quad (C_p - C_v)dT = T\left(\frac{\partial V}{\partial T}\right)_p dP + T\left(\frac{\partial p}{\partial T}\right)_v dV$$

$$\text{or} \quad dT = \frac{T\left(\frac{\partial V}{\partial T}\right)_p}{C_p - C_V} dP + \frac{T\left(\frac{\partial p}{\partial T}\right)_V}{C_p - C_V} dV \quad ---(i)$$

Since T is a function of (p, V)

$$T = T(p, V)$$

$$\therefore \quad dT = \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p dV \quad ---(ii)$$

Compairing (i) & (ii) we get

$$\frac{T\left(\frac{\partial V}{\partial T}\right)_p}{C_p - C_V} = \left(\frac{\partial T}{\partial p}\right)_V \quad \text{and} \quad \frac{T\left(\frac{\partial p}{\partial T}\right)_V}{C_p - C_V} = \left(\frac{\partial T}{\partial V}\right)_p$$

$$\therefore \quad C_p - C_V = T\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial T}\right)_V$$

$$\text{as} \quad dU = đQ - pdV$$

$$\therefore \quad dU = TdS - pdV$$

$$\text{or} \quad \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

$$\text{or} \quad \left(\frac{\partial U}{\partial V}\right)_T + p = T\left(\frac{\partial S}{\partial V}\right)_T$$

From Maxwell's Third relations

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\therefore \quad C_p - C_V = T\left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial T}\right)_V = \left\{p + \left(\frac{\partial U}{\partial V}\right)_T\right\}\left(\frac{\partial V}{\partial T}\right)_p$$

(vi) Prove that

Joule – Thomson co-efficient $\mu = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{C_p}\left[\frac{\partial}{\partial T}\left(\frac{V}{T}\right)\right]_p$ **[IES-2002]**

The numerical value of the slope of an isenthalpic on a T – p diagram at any point is called the Joule – Kelvin coefficient.

$$\therefore \quad \mu = \left(\frac{\partial T}{\partial p}\right)_h$$

Here $dH = TdS + Vdp$

as $\quad TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$

$\therefore \quad dH = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp + Vdp = C_p dT - \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]dp$

if $\quad H = \text{cost.} \quad \therefore \quad dH = 0$

so $\quad C_p (dT)_h - \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right](dp)_h = 0$

$\therefore \quad \left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] = \frac{T^2}{C_p}\left[\frac{1}{T}\cdot\left(\frac{\partial V}{\partial T}\right)_p - \frac{V}{T^2}\right] = \frac{T^2}{C_p}\left[\frac{\partial}{\partial T}\left(\frac{V}{T}\right)\right]_p$

(vii) Derive Clausius – Clapeyron equation

$\left(\frac{dp}{dT}\right) = \frac{h_{fg}}{T(v_g - v_f)}$ and $\frac{dp}{p} = \frac{h_{fg}}{RT^2}dT$ **[IES-2000]**

$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ Maxwells equation

When saturated liquid convert to saturated vapour at constant temperature. During the evaporation, the pr. & T is independent of volume.

$$\therefore \quad \left(\frac{dp}{dT}\right)_{sat} = \frac{s_g - s_f}{v_g - v_f}$$

$$s_g - s_f = s_{fg} = \frac{h_{fg}}{T}$$

$$\text{or} \quad \left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{T(v_g - v_f)}$$

→ It is useful to estimate properties like h from other measurable properties.

→ At a change of phage we may find h_{fg} i.e. latent heat.

At very low **pre**ssure $v_g \approx v_{fg}$ as v_f very small

$$pv_g = RT \quad \text{or} \quad v_g = \frac{RT}{p}$$

$$\therefore \quad \frac{dp}{dT} = \frac{h_{fg}}{T\cdot v_g} = \frac{h_{fg}}{T\cdot\frac{RT}{p}} = \frac{h_{fg}\cdot p}{RT^2}$$

$$\text{or} \quad \frac{dp}{p} = \frac{h_{fg}}{R}\cdot\frac{dT}{T^2}$$

$$\text{or} \quad \ln\left(\frac{p_2}{p_1}\right) = \frac{h_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

→ Knowing vapour pressure p_1 at temperature T_1, we may find out p_2 at temperature T_2.

Joule-Kelvin Effect or Joule-Thomson coefficient

The value of the specific heat c_p can be determined from p–v–T data and the Joule–Thomson coefficient. The ***Joule–Thomson coefficient*** μ_J is defined as

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$

Like other partial differential coefficients introduced in this section, the Joule–Thomson coefficient is defined in terms of thermodynamic properties only and thus is itself a property. The units of μ_J are those of temperature divided by pressure.

A relationship between the specific heat c_p and the Joule–Thomson coefficient μ_J can be established to write

$$\left(\frac{\partial T}{\partial p}\right)_h \left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p = -1$$

The first factor in this expression is the Joule–Thomson coefficient and the third is c_p. Thus

$$c_p = \frac{-1}{\mu_J \left(\partial p / \partial h\right)_T}$$

With $\left(\partial h / \partial p\right)_T = 1 / \left(\partial p / \partial h\right)_T$ this can be written as

$$c_p = -\frac{1}{\mu_J}\left(\frac{\partial h}{\partial p}\right)_T$$

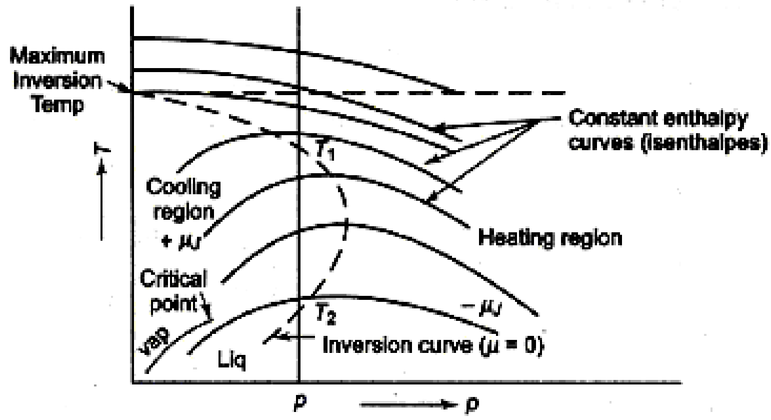
The partial derivative $\left(\partial h / \partial p\right)_T$, called the ***constant-temperature coefficient,*** can be eliminated. The following expression results:

$$c_p = \frac{1}{\mu_J}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]$$

allows the value of c_p at a state to be determined using p–v–T data and the value of the Joule–Thomson coefficient at that state. Let us consider next how the Joule–Thomson coefficient can be found experimentally.

The numerical value of the slope of an isenthalpic on a T-p diagram at any point is called the *Joule-Kelvin coefficient* and is denoted by μ_J. Thus the locus of all points at which μ_J is zero is the inversion curve. The region inside the inversion curve where μ_J is positive is called the *cooling region* and the region outside where μ_J is negative is called the *heating region*. So,

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$



Isenthalpic Curves and the Inversion Curve

Energy Equation

For a system undergoing an infinitesimal reversible process between two equilibrium states, the change of internal energy is

$$dU = TdS - pdV$$

Substituting the first TdS equation

$$dU = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_V dV - pdV$$

$$= C_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]dV$$

$$if \quad U = (T, V)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

This is known as energy equation. Two application of the equation are given below-

(a) For an ideal gas, $p = \frac{n\overline{R}T}{V}$

$$\therefore \left(\frac{\partial p}{\partial T}\right)_V = \frac{n\overline{R}}{V} = \frac{p}{T}$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T.\frac{p}{T} - p = 0$$

U does not change when V changes at T = C.

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T = 0$$

$$\text{since} \quad \left(\frac{\partial p}{\partial V}\right)_T \neq 0, \left(\frac{\partial U}{\partial p}\right)_T = 0$$

U does not change either when p changes at T = C. So the internal energy of an ideal gas is a function of temperature only.

Another important point to note is that for an ideal gas

$$pV = n\overline{R}T \text{ and } T\left(\frac{\partial p}{\partial T}\right)_V - p = 0$$

Therefore

$$dU = C_v\, dT$$

holds good for an ideal gas in any process (even when the volume changes). But for any other substance

$$dU = C_v\, dT$$

is true only when the volume is constant and dV = 0

Similarly

$$dH = TdS + Vdp$$

and $$TdS = C_p\, dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$

$$\therefore dH = C_p dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_p\right]dp$$

$$\therefore \left(\frac{\partial H}{\partial p}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_p$$

As shown for internal energy, it can be similarly proved from Eq. shown in above that the enthalpy of an ideal gas is not a function of either volume or pressure.

$$\left[i.e\left(\frac{\partial H}{\partial p}\right)_T = 0 \text{ and } \left(\frac{\partial H}{\partial V}\right)_T = 0\right]$$

but a function of temperature alone.

Since for an ideal gas, $pV = n\bar{R}T$

and $$V - T\left(\frac{\partial V}{\partial T}\right)_p = 0$$

the relation $dH = C_p\, dT$ is true for any process (even when the pressure changes.)

However, for any other substance the relation $dH = C_p\, dT$ holds good only when the pressure remains constant or dp = 0.

(b) Thermal radiation in equilibrium with the enclosing walls processes an energy that depends only on the volume and temperature. The energy density (u), defined as the ratio of energy to volume, is a function of temperature only, or

$$u = \frac{U}{V} = f(T) only.$$

The electromagnetic theory of radiation states that radiation is equivalent to a photon gas and it exerts a pressure, and that the pressure exerted by the black body radiation in an enclosure is given by

$$p = \frac{u}{3}$$

Black body radiation is thus specified by the pressure, volume and temperature of the radiation.

since.

$$U = uV \text{ and } p = \frac{u}{3}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = u \text{ and} \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT}$$

By substituting in the energy Eq.

$$u = \frac{T}{3}\frac{du}{dT} - \frac{u}{3}$$

$$\therefore \frac{du}{u} = 4\frac{dT}{T}$$

or $\quad \ln u = \ln T^4 + \ln b$

or $\quad u = bT^4$

where b is a constant. This is known as the **Stefan - Boltzmann Law.**

Since $U = uV = VbT^4$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_v = 4VbT^3$$

and $$\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT} = \frac{4}{3}bT^3$$

From the first TdS equation

$$TdS = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dV$$

$$= 4VbT^3 dT + \frac{4}{3}bT^4.dV$$

For a reversible isothermal change of volume, the heat to be supplied reversibly to keep temperature constant.

$$Q = \frac{4}{3}bT^4 \Delta V$$

For a reversible adiabatic change of volume

$$\frac{4}{3}bT^4 dV = -4VbT^3 dT$$

$$or \quad \frac{dV}{V} = -3\frac{dT}{T}$$

$$or \quad \mathrm{VT}^3 = const$$

If the temperature is one-half the original temperature. The volume of black body radiation is to be increased adiabatically eight times its original volume so that the radiation remains in equilibrium with matter at that temperature.

Gibbs Phase Rule

Gibbs Phase Rule determines what is expected to define the state of a system

$$\mathbf{F = C - P + 2}$$

F = Number of degrees of freedom (i.e.., no. of properties required)

C = Number of components P = Number of phases

e.g., Nitrogen gas C = 1; P = 1. Therefore, F = 2

- To determine the state of the nitrogen gas in a cylinder two properties are adequate.
- A closed vessel containing water and steam in equilibrium: P = 2, C = 1
- Therefore, F = 1. If any one property is specified it is sufficient.
- A vessel containing water, ice and steam in equilibrium
- P = 3, C = 1 therefore F = 0. The triple point is uniquely defined.

Question: **Which one of the following can be considered as property of a system?**

$(a)\int pdv$ $\quad (b)\int vdp$ $\quad (c)\int\left(\frac{dT}{T}+\frac{p.dv}{v}\right)$ $\quad (d)\int\left(\frac{dT}{T}-\frac{v.dp}{T}\right)$

Given: p = pressure, T = Temperature, v = specific volume [IES-1993]

Solution: P is a function of v and both are connected by a line path on p and v coordinates. Thus $\int pdv$ and $\int vdp$ are not exact differentials and thus not properties.

If X and Y are two properties of a system, then dx and dy are exact differentials. If the differential is of the form Mdx + Ndy, then the test for exactness is $\left[\frac{\partial M}{\partial y}\right]_x = \left[\frac{\partial N}{\partial x}\right]_y$

Now applying above test for

$$\int\left(\frac{dT}{T}+\frac{p.dv}{v}\right), \left[\frac{\partial(1/T)}{\partial v}\right]_T = \left[\frac{\partial(p/v)}{\partial T}\right]_v = \left[\frac{\partial(RT/v^2)}{\partial T}\right]_v \; or \; 0 = \frac{R}{v^2}$$

This differential is not exact and hence is not a point function and hence $\int\left(\frac{dT}{T}+\frac{p.dv}{v}\right)$ is not a point function and hence not a property.

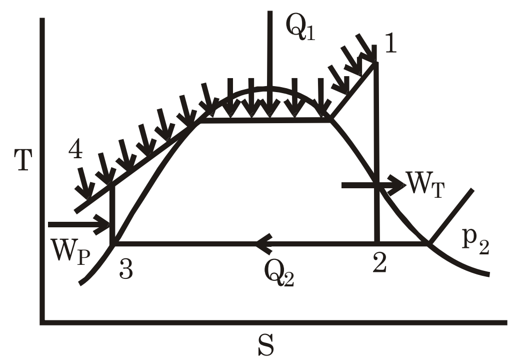
And for $\int\left(\frac{dT}{T}-\frac{v.dp}{T}\right) \left[\frac{\partial(1/T)}{\partial p}\right]_T = \left[\frac{\partial(-v/T)}{\partial T}\right]_P = \left[\frac{\partial(-R/P)}{\partial T}\right]_P \; or \; 0 = 0$

Thus $\int\left(\frac{dT}{T}-\frac{v.dp}{T}\right)$ is exact and may be written as ds, where s is a point function and hence a property

12. Vapour Power Cycles

Some Important Notes

A. Rankine Cycle



For 1 kg of fluid using S.F.E.E.

(i) $h_4 + Q_1 = h_1$

or $\boxed{\mathbf{Q_1 = h_1 - h_4}}$

(ii) $h_1 = W_T + h_2$

or $\boxed{\mathbf{W_T = h_1 - h_2}}$

(iii) $h_3 + W_P = h_4$

or $\boxed{\mathbf{W_P = h_4 - h_3}}$

About pump: The pump handles liquid water which is incompressible.

For reversible Adiabatic Compression Tds = dh – vdp where ds = 0

$\therefore$ $dh = vdp$ as v = constant

$\Delta h = v\Delta p$

or $h_4 - h_3 = v(p_1 - p_2) = W_P$

(iv) $\boxed{\mathbf{W_P = h_4 - h_3 = v(p_1 - p_2)\ kJ/kg}}$ Where v in m³/kg and p in kPa

B. Rankine Cycle efficiency:

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - N_P}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

C. Steam rate = $\mathbf{\dfrac{3600}{W_T - W_P} \dfrac{kg}{kWh}}$

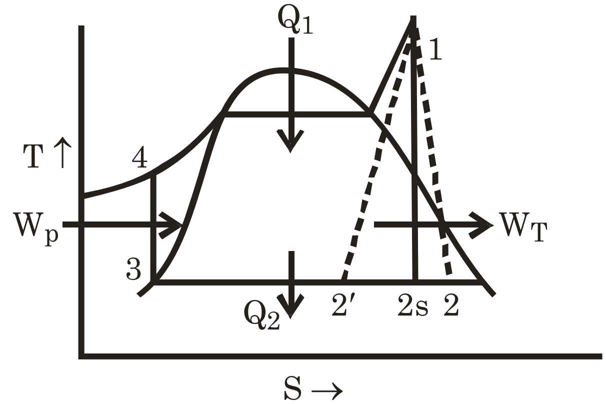
D. Heat Rate = Steam rate × Q_1 = $\dfrac{3600\, Q_1}{W_T - W_P}\dfrac{kJ}{kWh} = \dfrac{3600}{\eta}\dfrac{kJ}{kWh}$

E. About Turbine Losses: If there is heat loss to the surroundings, h_2 will decrease, accompanied by a decrease in entropy. If the heat loss is large, the end state of steam from the turbine may be 2′.(figure in below).

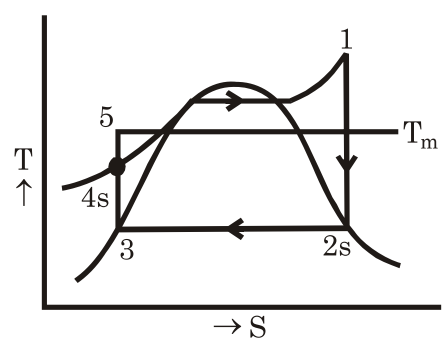
It may so happen that the entropy increase due to frictional effects just balances the entropy decrease due to heat loss, with the result that the initial and final entropies of steam in the expansion process are equal, but the expansion is neither adiabatic nor reversible.

F. Isentropic Efficiency:

$$\boxed{\eta_{isen} = \frac{h_1 - h_2}{h_1 - h_{2s}}} = \frac{\text{Actual Enthalpy drop}}{\text{isentropic enthalpy drop}}$$



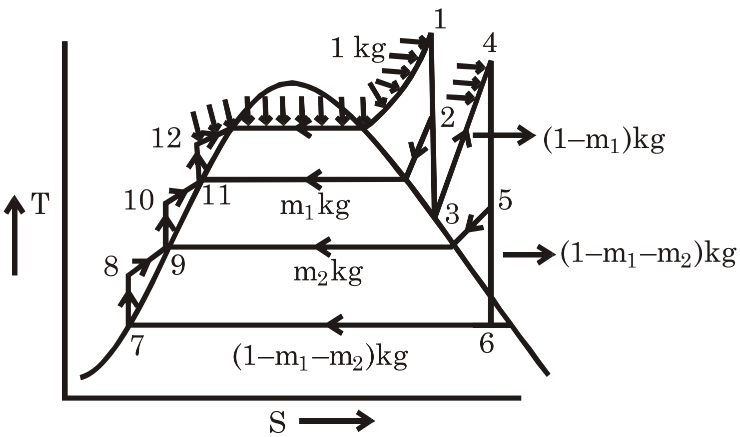
G. Mean temperature of heat addition:



$$Q_1 = h_1 - h_{4s} = T_m(s_1 - s_{4s})$$

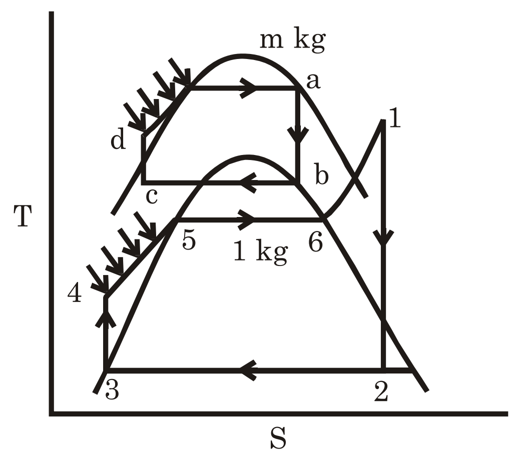
$$\therefore \quad \boxed{T_m = \frac{h_1 - h_{4s}}{s_1 - s_{4s}}}$$

H. For Reheat – Regenerative Cycle:



$W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$ kJ/kg
$W_P = (1 - m_1 - m_2)(h_8 - h_7) + (1 - m_1)(h_{10} - h_9) + 1(h_{12} - h_{11})$ kJ/kg
$Q_1 = (h_1 - h_{12}) + (1 - m_1)(h_4 - h_3)$ kJ/kg
Energy balance of heater 1 and 2
$m_1 h_2 + (1 - m_1) h_{10} = 1 \times h_{11}$ ………… For calculation of m_1
And $m_2 h_5 + (1 - m_1 - m_2) h_8 = (1 - m_1) h_9$ ……… For calculation of m_2.

I. For Binary vapour Cycles:



$W_T = m(h_a - h_b) + (h_1 - h_2)$ kJ/kg of steam

$W_P = m(h_d - h_c) + (h_4 - h_3)$ kJ /kg of steam

$Q_1 = m(h_a - h_d) + (h_1 - h_6) + (h_5 - h_4)$ kJ /kg of steam.

Energy balance in mercury condenser-steam boiler
$m(h_b - h_c) = (h_6 - h_5)$
$\therefore \quad m = \dfrac{h_6 - h_5}{h_b - h_c}$ kg of Hg/kg of H_2O i.e. ≈ 8 kg

J. Efficiency of Binary vapour cycle:

$$1-\eta = (1-\eta_1)(1-\eta_2)\,........\,(1-\eta_n)$$

$\therefore$ For two cycles

$$\boxed{\eta = n_1 + n_2 - n_1 n_2}$$

K. Overall efficiency of a power plant

$$\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{turbine\ (mean)} \times \eta_{generator}$$

Questions with Solution P. K. Nag

Q. 12.1 for the following steam cycles find

(a) W_T in kJ/kg **(b) W_p in kJ/kg,**

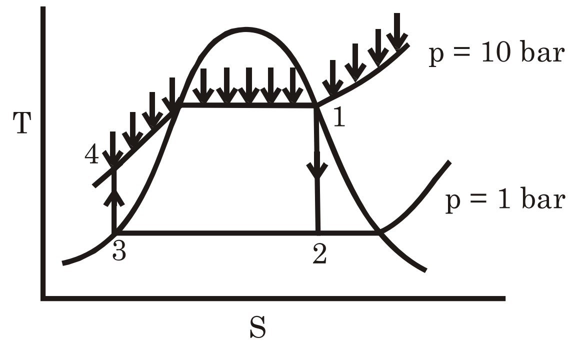
(c) Q_1 in kJ/kg, **(d) cycle efficiency,**

(e) steam rate in kg/kW h, and

(f) moisture at the end of the turbine process. Show the results in tabular form with your comments.

Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	1 bar	Ideal Rankine Cycle
-do-	-do-	Neglect W_p
-do-	-do-	Assume 75% pump and Turbine efficiency
-do-	0.1 bar	Ideal Rankine Cycle
10 bar, $300°c$	-do-	-do-
150 bar, $600°c$	-do-	-do-
-do-	-do-	Reheat to $600°C$ maximum intermediate pressure to limit end moisture to 15%
-do-	-do-	-do- but with 85% tur- bine efficiencies
10 bar, saturated	0.1 bar	Isentropic pump process ends on satura
Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	0.1 bar	-do- but with 80% machine efficiencies
-do-	-do-	Ideal regenerative cycle
-do-	-do-	Single open heater at $110°c$
-do-	-do-	Two open heaters at $90°c$ and $135°c$
-do-	-do-	-do- but the heaters are closed heaters

Solution: Boiler outlet: 10 bar, saturated
Condenser: 1 bar
Ideal Rankine Cycle



From Steam Table

$h_1 = 2778.1$ kJ/kg

$s_1 = 6.5865$ kJ/kg-K

$\therefore\quad s_2 = s_1 = 6.5865 = 1.3026 + x\,(7.3594 - 1.3026)$

$\therefore\quad x = 0.8724$

$\therefore\quad h_2 = 417.46 + 0.8724 \times 2258 = 2387.3$ kJ/kg

$h_3 = 417.46$ kJ/kg

$h_4 = h_3 + W_P$

$W_P = 1.043 \times 10^{-3}\,[1000 - 100]$ kJ/kg $= 0.94$ kJ/kg

$\therefore\quad h_4 = 418.4$ kJ/kg

(a) $W_T = h_1 - h_2 = (2778.1 - 2387.3)$ kJ/kg $= 390.8$ kJ/kg

(b) $W_P = 0.94$ kJ/kg

(c) $Q_1 = (h_1 - h_4) = (2778.1 - 418.4)$ kJ/kg $= 2359.7$ kJ/kg

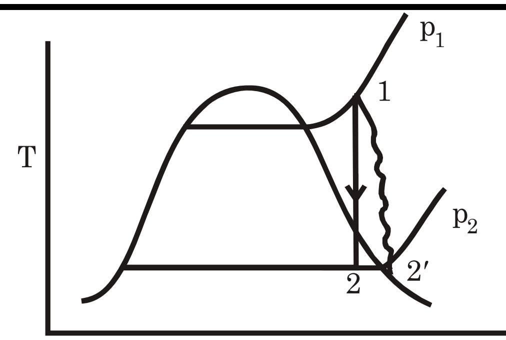
(d) $$\text{Cycle efficiency } (\eta) = \frac{W_{net}}{Q_1} = \frac{W_T - N_P}{Q_1} = \frac{390.8 - 0.94}{2359.7}$$
$$= 16.52\%$$

(e) $$\text{Steam rate} = \frac{3600}{W_{net}}\ \text{kJ/kWh} = \frac{3600}{390.8 - 0.94} = 9.234\ \text{kg/kWh}$$

(f) Moisture at the end of turbine process
$= (1 - x) = 0.1276 \cong 12.76\%$

Q.12.2 **A geothermal power plant utilizes steam produced by natural means underground. Steam wells are drilled to tap this steam supply which is available at 4.5 bar and 175°C. The steam leaves the turbine at 100 mm Hg absolute pressure. The turbine isentropic efficiency is 0.75. Calculate the efficiency of the plant. If the unit produces 12.5 MW, what is the steam flow rate?**

Solution: $p_1 = 4.5$ bar $\qquad T_1 = 175°C$
From super heated STEAM TABLE.



At 4 bar		at 5 bar	
150°C	200°C	152°C	200°C
h = 2752.8	h = 2860.5	h = 2748.7	h = 2855.4
s = 6.9299	s = 7.1706	s = 6.8213	s = 7.0592

$\therefore$ at 4 bar 175°C at 5 bar, 175°C

$$h = 2752.8 + \frac{1}{2}(2860.5 - 2752.8)$$

$$h = 2748.7 + \left(\frac{175-152}{200-152}\right)(2855.4 - 2748.7)$$

$= 2806.7$ kJ/kg $= 2800$ kJ/kg

$$s = 6.9299 + \frac{1}{2}(7.1706 - 6.9299) \quad s = 6.8213 + \frac{23}{48}(7.0592 - 6.8213)$$

$= 7.0503$ kJ/kg – K $= 6.9353$

$\therefore$ at 4.5 bar 175°C

$$h_1 = \frac{2806.7 + 2800}{2} = 2803.4 \text{ kJ/kg}$$

$$s_1 = \frac{7.0503 + 6.9353}{2} = 6.9928 \text{ kJ/kg} - \text{K}$$

Pressure 100 mm Hg

$$= \frac{100}{1000}\,\text{m} \times (13.6 \times 10^3)\,\text{kg/m}^3 \times 9.81 \text{ m/s}^2$$

$= 0.13342$ bar $= 13.342$ kPa

Here also entropy 6.9928 kJ/kg – K

So from S. T.	At 10 kPa		at 15 kPa
$h_f = 191.83$	$s_f = 0.6493$	$s_f = 0.7549$	$h_f = 225.94$
$h_{fg} = 2392.8$	$s_g = 8.1502$	$s_g = 8.0085$	$h_{fg} = 2373.1$

$\therefore$ at 13.342 kPa [Interpolation]

$$s_f = 0.6493 + \left(\frac{15-13.342}{15-10}\right)(0.7549 - 0.6493) = 0.68432 \text{ kJ/kg} - \text{K}$$

$$s_g = 8.1502 + \left(\frac{15-13.342}{15-10}\right)(8.0085 - 8.1502) = 8.1032 \text{ kJ/kg} - \text{K}$$

$\therefore$ If dryness fraction is x then

$6.9928 = 0.68432 + x\,(8.1032 - 0.68432)$

$\therefore$ $x = 0.85033$

$\therefore$ At 13.342 kPa

$$h_f = 191.83 + \left(\frac{15-13.342}{15-10}\right)(225.94 - 191.83) = 203.14 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 + \left(\frac{15-13.342}{15-10}\right)(2373.1 - 2392.8) = 2386.3 \text{ kJ/kg}$$

$$h_{2s} = h_f + x\, h_{fg} = 203.14 + 0.85033 \times 2386.3 = 2232.3 \text{ kJ/kg}$$

$$\eta_{isentropic} = \frac{h_1 - h_{2'}}{h_1 - h_{2s}}$$

$$\therefore \quad h_1 - h_{2'} = \eta_{isentropic} \times (h_1 - h_{2s})$$

$$\therefore \quad h_2' = h_1 - \eta_{isentropic}\,(h_1 - h_{2s})$$

$$= 2803.4 - 0.75\,(2803.4 - 2232.3) = 2375 \text{ kJ/kg.}$$

$\therefore$ Turbine work (W_T) = $h_1 - h_{2'}$ = (2803.4 – 2373) %

= 428.36 kJ/kg

$\therefore$ Efficiency of the plant = $\frac{W_T}{h_1} = \frac{428.36}{2803.4} \approx 0.1528 = 25.28\%$

If mass flow rate is $\dot{m}$ kg/s

$$\dot{m}.\, W_T = 12.5 \times 10^3$$

or $\dot{m} = \frac{12.5\times10^3}{428.36} = 29.18$ kg/s

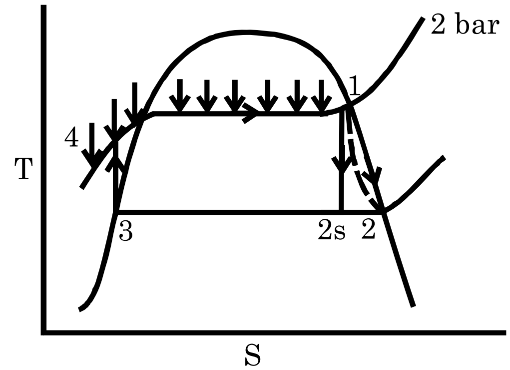
Q.12.3 **A simple steam power cycle uses solar energy for the heat input. Water in the cycle enters the pump as a saturated liquid at 40°C, and is pumped to 2 bar. It then evaporates in the boiler at this pressure, and enters the turbine as saturated vapour. At the turbine exhaust the conditions are 40°C and 10% moisture. The flow rate is 150 kg/h. Determine (a) the turbine isentropic efficiency, (b) the net work output (c) the cycle efficiency, and (d) the area of solar collector needed if the collectors pick up 0.58 kW/m^2.**

(**Ans**. (c) 2.78%, (d) 18.2 m^2)

Solution: From Steam Table T_1 = 120.23°C = 393.23 K

h_1 = 2706.7 kJ/kg

s_1 = 7.1271 kJ/kg – K



At 40°C saturated pressure 7.384 kPa

h_f = 167.57 h_{fg} = 2406.7

$s_f = 0.5725$ $s_g = 8.2570$

$\therefore$ $h_2 = h_f + 0.9 \times 2406.7 = 2333.6$ kJ/kg

For h_{2s} if there is dryness fraction x

$7.1271 = 0.5725 + x \times (8.2570 - 0.5725)$

$\therefore$ $x = 0.853$

$\therefore$ $h_{2s} = 167.57 + 0.853 \times 2406.7 = 2220.4$ kJ/kg

(a) $\therefore$ Isentropic efficiency, $\eta_{isentropic} = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$

$$= \frac{2706.7 - 2333.6}{2706.7 - 2220.4} = 76.72\%$$

(b) Net work output $W_T = h_1 - h_2 = 373.1$ kJ/kg

$\therefore$ Power = 15.55 kW i.e. $(W_T - W_P) \times \dot{m}$

Pump work, $W_P = v(p_1 - p_2)$

$= 1.008 \times 10^{-3} (200 - 7.384)$ kJ/kg $= 0.1942$ kJ/kg

$\therefore$ $h_3 = 167.57$ kJ/kg, $h_a = 167.76$ kJ/kg

$\therefore$ $Q_1 = (h_1 - h_4) = (2706.7 - 167.76)$ kJ/kg $= 2539$ kJ/kg

$\therefore$ $\eta_{cycle} = \dfrac{W_T - W_P}{Q_1} = \dfrac{373.1 - 0.1942}{2539} = 14.69\ \%$

Required area $A = \dfrac{Q_1 \times \dot{m}}{\text{collection picup}}$

$$= \frac{2539 \times 150}{0.58 \times 3600} = 182.4 \text{ m}^2$$

Q.12.4 **In a reheat cycle, the initial steam pressure and the maximum temperature are 150 bar and 550°C respectively. If the condenser pressure is 0.1 bar and the moisture at the condenser inlet is 5%, and assuming ideal processes, determine (a) the reheat pressure, (b) the cycle efficiency, and (c) the steam rate.**

(**Ans.** 13.5 bar, 43.6%,2.05 kg/kW h)

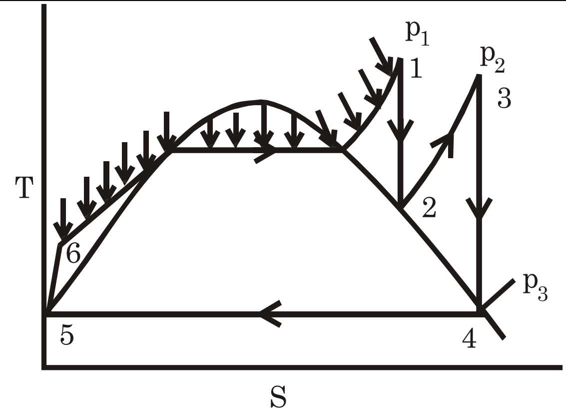
Solution : From Steam Table at 150 bar 550°C

$h_1 = 3448.6$ kJ/kg

$s_1 = 6.520$ kJ/kg – K

At $p_3 = 0.1$ bar $T = 45.8$°C

$h_f = 191.8$ kJ/kg $h_{fg} = 2392.8$ kJ/kg



$\therefore$ $h_4 = h_f + x\, h_{fg} = 191.8 + 0.95 \times 2392.8 = 2465$ kJ/kg

$s_f = 0.649$: $S_{fg} = 7.501$

$s_4 = s_f + x\, s_{fg} = 0.649 + 0.95 \times 7.501 = 7.775$ kJ/kg – K

From Molier Diagram at 550°C and 7.775 entropy, 13.25 bar

From S.T. at 10 bar 550°C s = 7.8955

$\therefore$ 15 bar 550°C s = 7.7045

$$\therefore \quad 7.775 = 7.8955 + \left(\frac{p-10}{15-10}\right)(7.7045 - 7.8955)$$

$-0.1205 = (p - 10)(-0.0382)$

$\therefore$ $p - 10 = 3.1544 \Rightarrow p = 13.15$ bar

$\therefore$ from Molier Dia. At 13 bar 550°C

$h_3 = 3580$ kJ/kg

$h_2 = 2795$ kJ/kg $t_2 = 195$°C

$h_5 = 191.8$ kJ/kg

$W_P = v_5\left(p_1 - p_3\right) = 0.001010\,(15000 - 10)$ kJ/kg

$= 1.505$ kJ/kg

$\therefore$ $h_6 = h_5 + W_P = 193.3$ kJ/kg

$\therefore$ $W_T = (h_1 - h_2) + (h_3 - h_4) = 1768.6$ kJ/kg

$W_P = 1.50$ kJ/kg

$\therefore$ $W_{net} = 1767.5$ kJ/kg

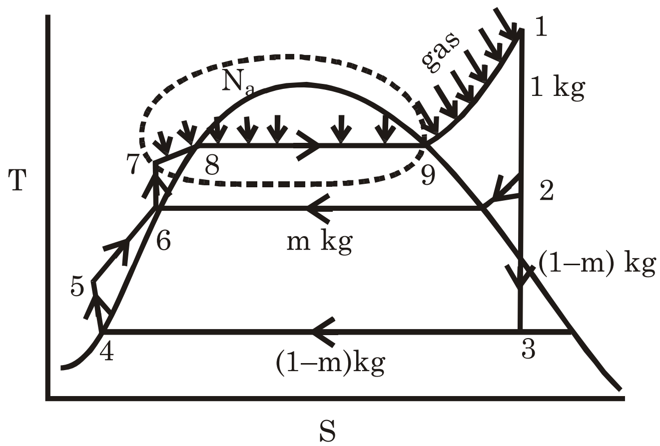
$Q = (h_1 - h_6) + (h_3 - h_2) = 4040.3$ kJ/kg

$$\therefore \quad \eta_{cycle} = \frac{W_{net}}{Q} = \frac{1767.5}{4040.3} \times 100\,\% = 43.75\%$$

$$\text{Steam rate} = \frac{3600}{W_T - W_P} = \frac{3600}{1767.5}\ \text{kg/kWh} = 2.0368 \text{ kg/kWh}$$

Q.12.5 **In a nuclear power-plant heat is transferred in the reactor to liquid sodium. The liquid sodium is then pumped to a heat exchanger where heat is transferred to steam. The steam leaves this heat exchanger as saturated vapour at 55 bar, and is then superheated in an external gas-fired super heater to 650°C. The steam then enters the turbine, which has one extraction point at 4 bar, where steam flows to an open feed water heater. The turbine efficiency is 75% and the condenser temperature is 40°C. Determine the heat transfer in the reactor and in the super heater to produce a power output of 80 MW.**

Solution: From Steam Table at 55 bar saturated state
h_9 = 2789.9 kJ/kg



From super heated S.T. at 55 bar 650°C

at 50 bar 600°C,	700°C	∴ By calculation at 650°C
h = 3666.5	h = 3900.1	h = 3783.3
s = 7.2589	s = 7.5122	s = 7.3856

At 60 bar 600°	C = 700°C	∴ by calculation
h = 3658.4	h = 3894.2	h = 3776.3
s = 7.1677	s = 7.4234	s = 7.2956

∴ at 55 bar 650°C (by interpolation)
h_1 = 3770.8 kJ/kg
s_1 = 7.3406 kJ/kg

For h_2, at 4 bar where S = 7.3406

At 200°C	at 250°C if temp is t
s = 7.172,	s = 7.379
h = 2860.5	h = 2964.2

Then $h_2 = 2860.5 + \left(\dfrac{7.3406 - 7.171}{7.379 - 7.171}\right) \times (2964.2 - 2860.5) = 2945$ kJ/kg

For h_3, at point 3 at 40°C
$h_f = 167.6$, $h_{fg} = 2406.7$ $s_f = 0.573$ $s_{fg} = 7.685$

If dryness fraction is x then

$$0.573 + x \times 7.685 = 7.3406$$

$$\therefore \quad x = 0.8806$$

$$\therefore \quad h_3 = 167.6 + 0.8806 \times 2406.7 = 2287 \text{ kJ/kg}$$

$$h_4 = h_f = 167.6 \text{ kJ/kg}$$

$$W_{P_{4-5}} = v_4 (p_2 - p_3) = 0.001010 \times (400 - 7.38) \text{ kJ/kg}$$

$$= 0.397 \text{ kJ/kg} \approx 0.4 \text{ kJ/kg}$$

$$\therefore \quad h_5 = h_4 + W_{P_{4-5}} = 168 \text{ kJ/kg}$$

h_6 = 604.7 kJ/kg [at 4 bar saturated liquid]

$W_{P_{6-7}} = v_6 \left(p_1 - p_2\right) = 0.001084\ (5500 - 400) = 5.53$ kJ/kg

$\therefore \quad h_7 = h_6 + W_{P_{6-7}} = 610.23$ kJ/kg

From heater energy balance

$(1 - m)\ h_5 + mh_2 = h_5 \qquad \Rightarrow m = 0.1622$ kg

$\therefore \quad W_T = [(h_1 - h_2) + (1 - m)\ (h_2 - h_3) \times 0.75 = 1049.8$ kJ/kg ;

$W_{net} = W_T - W_{P_{4-5}} - W_{P_{6-7}} = 1043.9$ kJ/kg

$\therefore \quad$ Steam flow rate $(\dot{m}) = \dfrac{80 \times 10^3}{1049.8}$ kg/s $= 76.638$ kg/s

$\therefore \quad$ Heat transfer in heater $= \dot{m}(h_9 - h_7)$

$= 76.638(2789.9 - 610.23) = 167.046$ MW

Heat transfer in super heater $= \dot{m}(h_1 - h_9)$

$= 76.638(3779.8 - 2789.9) = 75.864$ MW

Q.12.6 **In a reheat cycle, steam at 500°C expands in a h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a l.p. turbine to 40°C. If the maximum moisture content at the turbine exhaust is limited to 15%, find (a) the reheat pressure, (b) the pressure of steam at the inlet to the h.p. turbine, (c) the net specific work output, (d) the cycle efficiency, and (e) the steam rate. Assume all ideal processes.**
What would have been the quality, the work output, and the cycle efficiency without the reheating of steam? Assume that the other conditions remain the same.

Solution: Try please.

Q.12.7 **A regenerative cycle operates with steam supplied at 30 bar and 300°C and -condenser pressure of 0.08 bar. The extraction points for two heaters (one Closed and one open) are at 3.5 bar and 0.7 bar respectively. Calculate the thermal efficiency of the plant, neglecting pump work.**

Solution: Try please.

Q.12.8 **The net power output of the turbine in an ideal reheat-regenertive cycle is 100 MW. Steam enters the high-pressure (H.P.) turbine at 90 bar, 550°C. After .expansion to 7 bar, some of the steam goes to an open heater and the balance is reheated to 400°C, after which it expands to 0.07 bar. (a) What is the steam flow rate to the H.P. turbine? (b) What is the total pump work? (c) Calculate the cycle efficiency. (d) If there is a $10°c$ rise in the temperature of the cooling 'water, what is the rate of flow of the cooling water in the condenser? (e) If the velocity of the steam flowing from the turbine to the condenser is limited to a maximum of 130 *m/s,* find the diameter of the connecting pipe.**

Solution: Try please.

Q.12.9 **A mercury cycle is superposed on the steam cycle operating between the boiler outlet condition of 40 bar, 400°C and the condenser temperature**

of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute (a) kg of mercury circulated per kg of water, and (b) the efficiency of the combined cycle.
The property values of saturated mercury are given below

p (bar)	T($^\circ C$)	h_f (kJ/kg) h_g	s_f (kJ/kg k) s_g	v_f (m^3/kg) v_g
10	515.5	72.23 363.0	0.1478 0.5167	80.9×10^{-6} 0.0333
0.2	277.3	38.35 336.55	0.0967 0.6385	77.4×10^{-6} 1.163

Solution: Try please.

Q.12.10 **In an electric generating station, using a binary vapour cycle with mercury in the upper cycle and steam in the lower, the ratio of mercury flow to steam flow is 10 : 1 on a mass basis. At an evaporation rate of 1,000,000 kg/h for the mercury, its specific enthalpy rises by 356 kJ/kg in passing through the boiler. Superheating the steam in the boiler furnace adds 586 kJ to the steam specific enthalpy. The mercury gives up 251.2 *kJ/kg* during condensation, and the steam gives up 2003 *kJ/kg* in its condenser. The overall boiler efficiency is 85%. The combined turbine metrical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries require 5% of the energy generated by the units. Find the overall efficiency of the plant.**

Solution: Try please

Q.12.11 **A sodium-mercury-steam cycle operates between l000°C and 40°C. Sodium rejects heat at 670°C to mercury. Mercury boils at 24.6 bar and rejects heat at 0.141 bar. Both the sodium and mercury cycles are saturated. Steam is formed at 30 bar and is superheated in the sodium boiler to 350°C. It rejects heat at 0.0 8 bar. Assume isentropic expansions, no heat losses, and no generation and neglect pumping work. Find (a) the amounts of sodium and mercury used per kg of steam, (b) the heat added and rejected in the composite cycle per kg steam, (c) the total work done per kg steam. (d) the efficiency of the composite cycle, (e) the efficiency of the corresponding Carnot cycle, and (f) the work, heat added, and efficiency of a supercritical pressure steam (single fluid) cycle operating at 250 bar and between the same temperature limits.**

For mercury, at 24.6 bar, h_g = 366.78 kJ/kg

s_g= 0.48kJ/kg$K$ and at 0.141 bar, s_j=0.09

And s_g= 0.64kJ/kg K, h_j=36.01 and h_g=330.77 kJ/kg

For sodium, at 1000°C, h_g = 4982.53 kJ/kg

At turbine exhaust = 3914.85 kJ/kg

At 670°C, h_f = 745.29 kJ/kg

For a supercritical steam cycle, the specific enthalpy and entropy at the turbine inlet may be computed by extrapolation from the steam tables.

Solution: Try please.

Q.12.12 **A textile factory requires 10,000 kg/h of steam for process heating at 3 bar saturated and 1000 kW of power, for which a back pressure turbine of 70% internal efficiency is to be used. Find the steam condition required at the inlet to the turbine.**

Solution: Try please.

Q.12.13 **A 10,000 kW steam turbine operates with steam at the inlet at 40 bar, 400°C and exhausts at 0.1 bar. Ten thousand kg/h of steam at 3 bar are to be extracted for process work. The turbine has 75% isentropic efficiency throughout. Find the boiler capacity required.**

Solution: Try please.

Q.12.14 **A 50 MW steam plant built in 1935 operates with steam at the inlet at 60 bar, 450°C and exhausts at 0.1 bar, with 80% turbine efficiency. It is proposed to scrap the old boiler and put in a new boiler and a topping turbine of efficiency 85% operating with inlet steam at 180 bar, 500°C. The exhaust from the topping turbine at 60 bar is reheated to 450°C and admitted to the old turbine. The flow rate is just sufficient to produce the rated output from the old turbine. Find the improvement in efficiency with the new set up. What is the additional power developed?**

Solution: Try please.

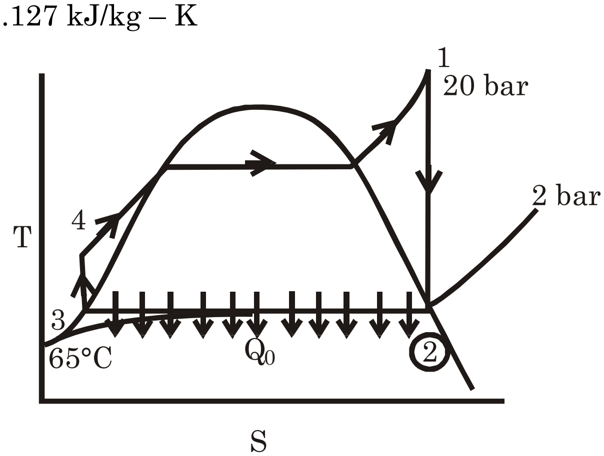
Q.12.15 **A steam plant operates with an initial pressure at 20 bar and temperature 400°C, and exhausts to a heating system at 2 bar. The condensate from the heating system is returned to the boiler plant at 65°C, and the heating system utilizes for its intended purpose 90% of the energy transferred from the steam it receives. The turbine efficiency is 70%. (a) What fraction of the energy supplied to the steam plant serves a useful purpose? (b) If two separate steam plants had been set up to produce the same useful energy, one to generate heating steam at 2 bar, and the other to generate power through a cycle working between 20 bar, 400°C and 0.07 bar, what fraction of the energy supplied would have served a useful purpose?**

(**Ans**. 91.2%, 64.5%)

Solution: From S.T. at 20 bar 400°C

$h_1 = 3247.6$ kJ/kg

$s_1 = 7.127$ kJ/kg – K



At 2 bar

$s_f = 1.5301,\ s_{fg} = 5.5967$

$s_g = 7.127$ kJ/kg – K so at point (2)

Steam is saturated vapour

So $h_2 = 2706.3$ kJ/kg

At 2 bar saturated temperature is 120.2°C but 65°C liquid

So $h_3 = h_2 - C_P\,\Delta T$

$= 504.7 - 4.187 \times (120.2 - 65) = 273.6$ kJ/kg

$W_{P_{3-4}} = v_3\left(p_1 - p_2\right) = 0.001 \times (2000 - 200) = 1.8$ kJ/kg

$\therefore\ h_4 = 275.4$ kJ/kg

$\therefore$ Heat input (Q) = $h_1 - h_4 = (3247.6 - 275.4) = 2972.2$ kJ/kg

Turbine work = $(h_1 - h_2)\,\eta = (3247.6 - 2706.3) \times 0.7$ kJ/kg

$= 378.9$ kJ/kg

Heat rejection that utilized $(Q_0) = (h_2 - h_3)\,\eta$

$= (2706.3 - 273.6) \times 0.9 = 2189.4$ kJ/kg

$\therefore$ Net work output $(W_{net}) = W_T - W_P = 378.9 - 1.8$

$= 377.1$ kJ/kg

$\therefore$ Fraction at energy supplied utilized

$$= \frac{W_{net} + Q_0}{Q_1} = \frac{377.1 + 2189.4}{2972.2} \times 100\%$$

$= 86.35\%$

(b) At 0.07 bar

$s_f = 0.559,\ s_{fg} = 7.717$

$\therefore$ Dryness fraction x, $0.559 + x \times 7.717 = 7.127$

$\therefore\quad x = 0.85137$

$\therefore\quad h_2 = 163.4 + 0.85137 \times 2409.1 = 2214.4$ kJ/kg

$h_3 = 163.4$ kJ/kg

$W_P = 0.001007 \times (2000 - 7) = 2$ kJ/kg

$\therefore\quad h_4 = 165.4$ kJ/g.

$\therefore\quad W_T = (h_1 - h_2) \times 0.7 = 723.24$ kJ/kg

$W_{net} = W_T - W_P = 721.24$ kJ/kg

Here heat input for power = $(h_1 - h_4) = 3082.2.$ kJ/kg

For same 377.1 kg power we need 0.52285 kg of water

So heat input = 1611.5 kJ for power

Heat input for heating = $\dfrac{2189.4}{0.9}$ kJ = 2432.7 kJ

$\therefore$ Fraction of energy used = $\dfrac{377.1 + 2189.4}{1611.5 + 2432.7} \times 100\%$

$= 63.46\%$

Q.12.16 In a nuclear power plant saturated steam at 30 bar enters a h.p. turbine and expands isentropically to a pressure at which its quality is 0.841. At this pressure the steam is passed through a moisture separator which removes all the liquid. Saturated vapour leaves the separator and is

expanded isentropically to 0.04 bar in I.p. turbine, while the saturated liquid leaving the separator is returned via a feed pump to the boiler. The condensate leaving the condenser at 0.04 bar is also returned to the boiler via a second feed pump. Calculate the cycle efficiency and turbine outlet quality taking into account the feed pump term. Recalculate the same quantities for a cycle with the same boiler and condenser pressures but without moisture separation.

(**Ans**. 35.5%, 0.S24; 35%; 0.716)

Solution: Form Steam Table at 30 bar saturated

$h_1 = 2802.3$ kJ/kg

$s_1 = 6.1837$

From Molier diagram

$h_2 = 2300$ kJ/kg

$p_r = 2.8$ bar

From S.T. $h_g = 2721.5$ kJ/kg, $s_g = 7.014$ kJ/kg K

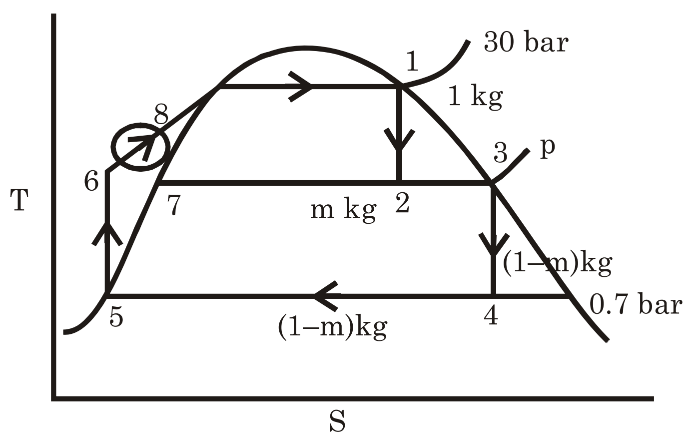
$t = 131.2°C$

$h_f = 551.4$ kJ/kg

From 0.04 bar S.T

$s_f = 0.423$ kJ/kg, $s_{fg} = 8.052$ kJ/kg

$h_f = 121.5$ kJ/kg, $h_{fg} = 2432.9$ kJ/kg



$\therefore$ If dryness fraction is x the

$7.014 = 0.423 + x \times 8.052 \quad \Rightarrow \quad x = 0.8186$

$\therefore \quad h_4 = h_f + x\, h_{fg} = 2113$ kJ/kg

$W_{P_{5-6}} = 0.001(3000 - 4) = 3$ kJ/kg

$W_{P_{7-8}} = 0.001071(3000 - 280) = 2.9$ kJ/kg

So $h_1 - 2802.3$ kJ/kg $\qquad h_5 = 121.5$ kJ/kg

$h_2 - 2380$ kJ/kg $\qquad h_6 = 124.5$ kJ/kg

$h_3 - 2721.5$ kJ/kg $\qquad h_7 = 551.4$ kJ/kg

$h_4 - 2113$ kJ/kg $\qquad h_8 = 554.3$ kJ/kg

$\therefore \quad m = 1 - 0.841 = 0.159$ kg of sub/ kg of steam

$\therefore \quad W_T = (h_1 - h_2) + (1 - m)(h_3 - h_4) = 934$ kJ/kg

$W_P = m \times W_{P_{7-8}} + (1 - m)\, W_{P_{5-6}} = 2.98 \text{ kJ/kg} \approx 3 \text{ kJ/kg}$

$\therefore \quad W_{net} = 931$ kJ/kg

Heat supplied (Q) = $m(h_1 - h_8) + (1 - m)(h_1 - h_6) = 2609.5$ kJ/kg

$\therefore \eta = \frac{931}{2609.5} \times 100\% = 35.68\%$ with turbine exhaust quality 0.8186

If No separation is taking place, Then is quality of exhaust is x

Then $6.1837 = 0.423 + x \times 8.052 \quad \Rightarrow x = 0.715$

$\therefore \quad h_4 = h_f + x \times h_{fg} = 1862$ kJ/kg

$\therefore \quad W_T = h_1 - h_4 = 941.28$ kJ/kg

$W_P = W_{P_{5-6}} = 3$ kJ/kg

$\therefore \quad W_{net} = 938.28$ kJ/kg

$\therefore$

Heat input, $Q = h_1 - h_6 = 2677.8$ kJ/kg

$\therefore \quad \eta = \frac{938.28}{2677.8} \times 100\% = 35\%$

Q.12.17 **The net power output of an ideal regenerative-reheat steam cycle is 80MW. Steam enters the h.p. turbine at 80 bar, 500°C and expands till it becomes saturated vapour. Some of the steam then goes to an open feedwater heater and the balance is reheated to 400°C, after which it expands in the I.p. turbine to 0.07 bar. Compute (a) the reheat pressure, (b) the steam flow rate to the h.p. turbine, and (c) the cycle efficiency. Neglect pump work.**

(**Ans.** 6.5 bar, 58.4 kg/s, 43.7%)

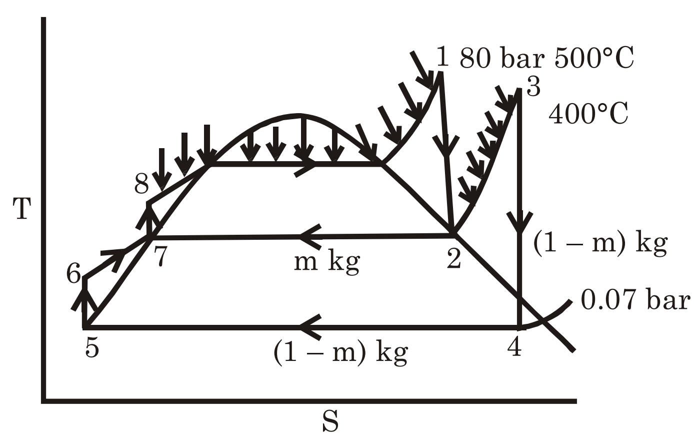
Solution: From S.T of 80 bar 500°C

$h_1 = 3398.3$ kJ/kg

$s_1 = 6.724$ kJ/kg – K

$s_2 = 6.725$ at 6.6 bar so

Reheat pr. 6.6 bar



$\therefore \quad h_2 = 2759.5$ kJ/kg

$h_3 = 3270.3 + 0.6(3268.7 - 3270.3) = 3269.3$ kJ/kg

$s_3 = 7.708 + 0.6(7.635 - 7.708)$

$= 7.6643$ kJ/kg – K

At 0.07 bar

$h_f = 163.4, \quad h_{fg} = 2409.1$

$h_f = 0.559$, $s_{fg} = 7.717$

$\therefore$ If quality is x then

$7.6642 = 0.559 + x \times 7.717 \Rightarrow x = 0.9207$

$\therefore h_4 = 163.4 + 0.9207 \times 2409.1 = 2381.5$ kJ/kg

$h_5 = 163.4$ kJ/kg $\approx h_6$, $h_7 = 686.8$ kJ/kg $\approx h_8$

$\therefore$ From Heat balance of heater

$m \times h_2 + (1 - m)\, h_6 = h_7$ $\therefore$ m = 0.2016 kg/kg of steam at H.P

$\therefore (1 - m) = 0.7984$

$W_T = h_1 - h_2 + (1 - m)(h_3 - h_4) = 1347.6$kJ/kg

W_P neglected

$Q = (h_1 - h_8) + (1 - m)(h_3 - h_2) = 3118.5$ kJ/kg at H.P

$\therefore$ (a) Reheat pr. 6.6 bar

(b) Steam flow rate at H.P = $\dfrac{80 \times 10^3}{1347.6}$ kg/s = 59.36 kg/s

(c) Cycle efficiency (η) = $\dfrac{W}{Q} = \dfrac{1347.6}{3118.5} \times 100\% = 43.21\%$

Q.12.18 **Figure shows the arrangement of a steam plant in which steam is also required for an industrial heating process. The steam leaves boiler *B* at 30 bar, 320°C and expands in the H.P. turbine to 2 bar, the efficiency of the H.P. turbine being 75%. At this point one half of the steam passes to the process heater *P* and the remainder enters separator S which removes all the moisture. The dry steam enters the L.P. turbine at 2 bar and expands to the condenser pressure 0.07 bar, the efficiency of the L.P. turbine being 70%. The drainage from the**

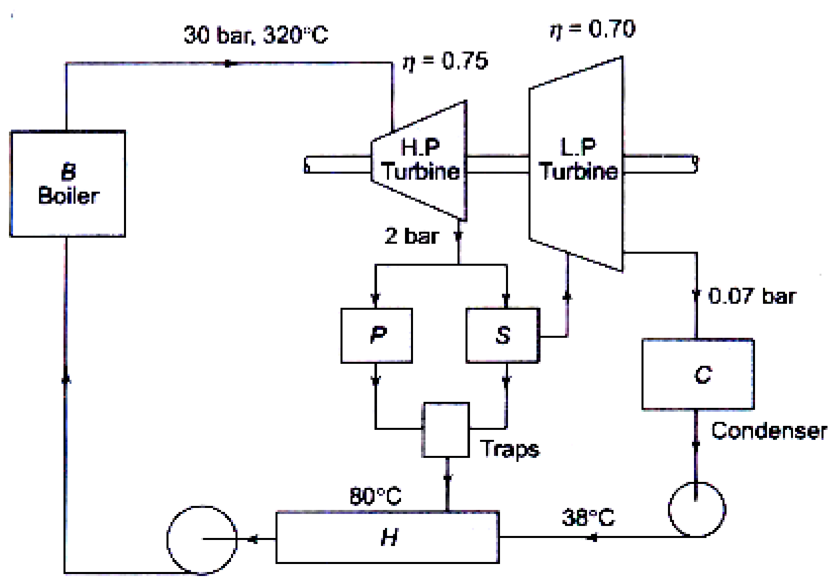


Fig. 12.51

Separator mixes with the condensate from the process heater and the combined flow enters the hotwell H at 50°C. Traps are provided at the exist from *P* and S. A pump extracts the condensate from condenser C and this enters the hot well at 38°C. Neglecting the feed pump work and radiation loss, estimate the temperature of water leaving the hotwell which is at atmospheric pressure. Also calculate, as percentage of heat transferred in the boiler, (a) the heat transferred in the process heater, and (b) the work done in the turbines.

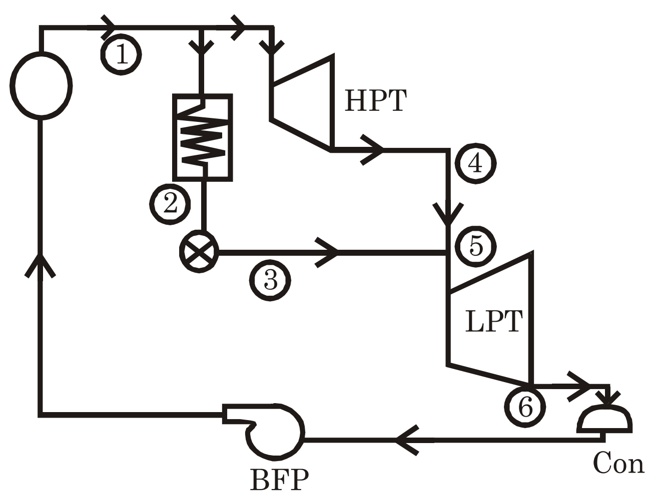
Solution: Try please.

Q.12.19 **In a combined power and process plant the boiler generates 21,000 kg/h of steam at a pressure of 17 bar, and temperature $230°C$. A part of the steam goes to a process heater which consumes 132.56 kW, the steam leaving the process heater 0.957 dry at 17 bar being throttled to 3.5 bar. The remaining steam flows through a H.P. turbine which exhausts at a pressure of 3.5 bar. The exhaust steam mixes with the process steam before entering the L.P. turbine which develops 1337.5 kW. At the exhaust the pressure is 0.3 bar, and the steam is 0.912 dry. Draw a line diagram of the plant and determine (a) the steam quality at the exhaust from the H.P. turbine, (b) the power developed by the H.P. turbine, and (c) the isentropic efficiency of the H.P. turbine.**

(**Ans.** (a) 0.96, (b) 1125 kW, (c) 77%)

Solution: Given steam flow rate

$\dot{m} = 21000 \text{ kg/h} = \frac{35}{6} \text{ kg/s}$



From Steam Table at 17 bar 230°C

15 bar 200°C	250°C
h = 2796.8	2923.3
s = 6.455	6.709

$\therefore$ at 230°C

$$h = 2796.8 + \frac{30}{50}(2623.3 - 2796.8) = 2872.7 \text{ kJ/kg}$$

$$s = 6.455 + \frac{30}{50}(6.709 - 6.455) = 6.6074 \text{ kJ/kg}$$

20 bar 212.4°C 250°C

h = 2797.2 h = 2902.5

s = 6.3366 s = 6.545

$\therefore$ at 230°C

$$h = 2797.2 + \frac{17.6}{37.6}(2902.5 - 2797.2) = 2846.4 \text{ kJ/kg}$$

$$s = 6.3366 + \frac{17.6}{37.6}(6.545 - 6.3366) = 6.434 \text{ kJ/kg}$$

$\therefore$ at 17 bar 230°C

$$h_1 = 2872.7 + \frac{2}{5}(2846.5 - 2872.7) = 2862.2 \text{ kJ/kg}$$

$$s_1 = 6.6074 + \frac{2}{5}(6.434 - 6.6074) = 6.5381 \text{ kJ/kg}$$

$h_2 = 871.8 + 0.957 \times 1921.5 = 2710.7$ kJ/kg $\approx h_3$

$h_4 = ?$

$\therefore$ Mass flow through process heater

$$= (\dot{m}_1) = \frac{132.56}{h_1 - h_2} = 0.97597 \text{ kg/s} = 3513.5 \text{ kg/h}$$

$\therefore$ Mass flow through HPT

= 17486.5 kJ/kg = 4.8574 kg/s

$\therefore$ $21000\, h_5 = 17486.5\, h_4 + 3513.5\, h_3$ ... (i)

$h_6 = 289.3 = 0.912 \times 2336.1 = 2419.8$ kJ/kg

$$W_T = \dot{m}(h_5 - h_6)$$

$$\therefore \quad h_5 = \frac{W_T}{\dot{m}} + h_6 = \left(\frac{1337.5 \times 3600}{21000} + 2419.8\right) = 2649.1 \text{ kJ/kg}$$

$\therefore$ From (i) $h_4 = 2636.7$ kJ/kg

At 3–5 bar $h_g = 2731.6$ kJ/kg so it is wet is quality x

(a) $\therefore$ $2636.7 = 584.3 + x \times 2147.4 \Rightarrow x = 0.956$

(b) $W_{HPT} = \dot{m}_2 (h_1 - h_4)$

$$= \frac{17486.5}{3600}(2862.2 \times 2636.7) \text{ kJ/kg} = 1095 \text{ kW}$$

(c) At 3.5 bar, $s_f - 1.7273$, $s_{fg} = 5.2119$ quality is isentropic

$6.5381 = 1.7273 + x \times 5.2119$ $x = 0.923$

$\therefore$ $h_{4s} = 584.3 + 0.923 \times 2147.4 = 2566.4$ kJ/kg

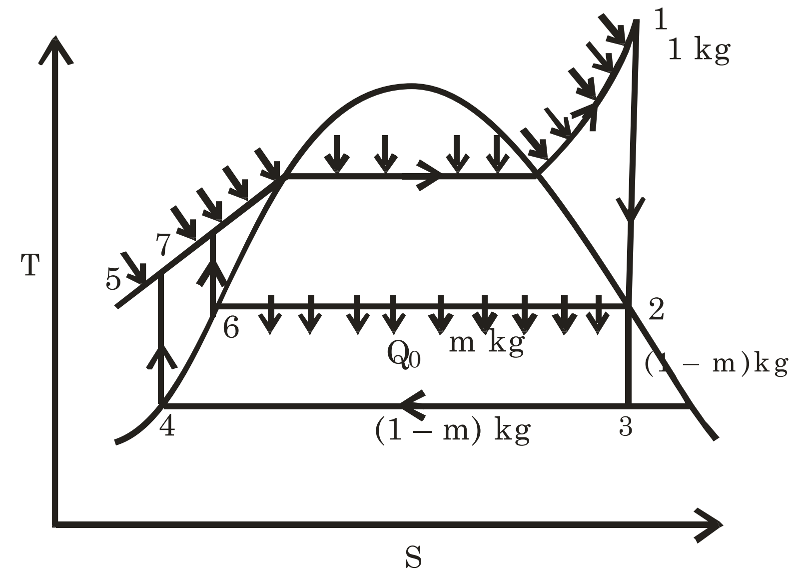
$$\therefore \quad \eta_{isen.} = \frac{h_1 - h_4}{h_1 - h_{4s}} = \frac{2862.2 - 2636.7}{2862.2 - 2566.4} \times 100\% = 76.24\%$$

Q.12.20 **In a cogeneration plant, the power load is 5.6 MW and the heating load is 1.163 MW. Steam is generated at 40 bar and 500°C and is expanded isentropically through a turbine to a condenser at 0.06 bar. The heating load is supplied by extracting steam from the turbine at 2 bar which condensed in the process heater to saturated liquid at 2 bar and then pumped back to the boiler. Compute (a) the steam generation capacity of the boiler in tonnes/h, (b) the heat input to the boiler in MW, and (c) the heat rejected to the condenser in MW.**

(**Ans.** (a) 19.07 t/h, (b) 71.57 MW, and (c) 9.607 MW)

Solution: From steam table at 40 bar 500°C

$h_1 = 3445.3$ kJ/kg
$s_1 = 7.090$ kJ/kg



$\rightarrow$ at 2 bar

$$s_f = 1.5301,\ s_{fg} = 5.5967$$

$\therefore \quad 7.090 = 1.5301 + x \times 5.5967$

$\therefore \quad x = 0.9934$

$$h_2 = 504.7 + 0.9934 \times 2201.6$$
$$= 2691.8 \text{ kJ/kg}$$

$\rightarrow$ at 0.06 bar

$$s_f = 0.521,\ s_{fg} = 7.809$$

$\therefore \quad 7090 = 0521 + x \times 7.809 \Rightarrow x = 0.8412$

$\therefore \quad h_3 = 151.5 + 0.8412 \times 2415.9 = 2183.8$ kJ/kg

$h_4 = 151.5$ kJ/kg

$h_6 = 504.7$ kJ/kg

$W_{P_{4-5}} = 0.001006 \times (4000 - 6) = 4$ kJ/kg

so $\quad h_5 = h_4 + W_P = 155.5$ kJ/kg

$W_{P_{6-7}} = 0.001061 \times (4000 - 100) = 4$ kJ/kg

so $\quad h_7 = h_6 + W_P = 508.7$kJ/kg

For heating load $Q_o = h_2 - h_6 = (2691.8 - 504.7)$ kJ/kg
$= 2187.1$ kJ/kg

For $W_T = (h_1 - h_2) + (1 - m)(h_2 - h_3)$
$= 753.5 + (1 - m)\, 508$
$= 1261.5 - 508\, m$

$\therefore \quad W_{net} = W_T - W_{P_{4-5}}(1 - m) - m\, W_{P_{6-7}}$
$= (1257.5 - 508\, m)$ kJ/kg

If mass flow rate at '1' of steam is w kg/s then

$w\,(1257.5 - 508m) = 5600$ ...(i)

$w_m \times 2187.1 = 1163$ ...(ii)

From (i) & (ii) w = 4.668 kg/s = 16.805 Ton/h

$\therefore \quad m = 0.11391$ kg/kg of the generation

(a) Steam generation capacity of boiler = 16.805 t/h

(b) Heat input to the boiler
$= W[(1 - m)(h_1 - h_5) + m(h_1 - h_7)] = 15.169$ MW

(c) Heat rejection to the condenser
$= (1 - m)(h_3 - h_4) = 8.406$ MW

Q.12.21 **Steam is supplied to a pass-out turbine at 35 bar, 350°C and dry saturated process steam is required at 3.5 bar. The low pressure stage exhausts at 0.07 bar and the condition line may be assumed to be straight (the condition line is the locus passing through the states of steam leaving the various stages of the turbine). If the power required is 1 MW and the maximum process load is 1.4 kW, estimate the maximum steam flow through the high and low pressure stages. Assume that the steam just condenses in the process plant.**

(**Ans.** 1.543 and 1.182 kg/s)

Solution: Form Steam Table 35 bar 350°C

$$s_1 = \frac{6.743 + 6.582}{2} = 6.6625 \text{ kJ/kg}$$

$$h_1 = \frac{3115.3 + 3092.5}{2} = 3103.9 \text{ kJ/kg}$$

at 3.5 bar $s_f = 1.7273$ $s_{fg} = 5.2119$

$\therefore$ if condition of steam is x_1

$6.6625 = 1.7273 + x_1 \text{ x } 5.2119$

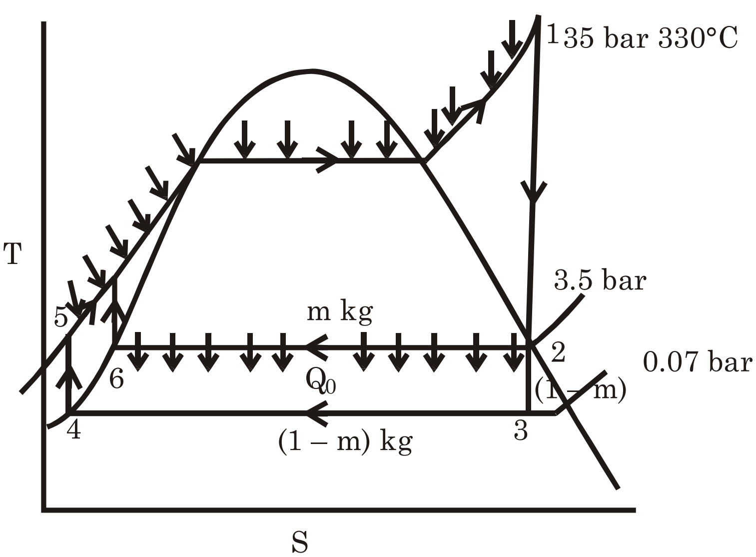
$x_1 = 0.9469$

$\therefore$ $h_2 = 584.3 + 0.9469 \times 2147.4 = 2617.7$ kJ/kg

At 0.07 bar

$s_f = 0.559$ $s_{fg} = 7.717$

$\therefore$ $6.6625 = 0.559 + x \times 7.717 \Rightarrow x_2 = 0.7909$



$\therefore$ $h_3 = 163.4 + 07909 \times 2409.1 = 2068.8$ kJ/kg

$h_4 = 163.4$ kJ/kg $\therefore$ $h_6 = 584.3$ kJ/kg

$W_{P_{4-5}} = 0.001007\,(3500 - 7) = 3.5$ kJ/kg

$\therefore \quad h_5 = h_4 + W_{P_{4-5}} = 166.9$ kJ/kg

$W_{P_{6-7}} = 0.001079\,(3500 - 350) = 3.4$ kJ/kg

$\therefore \quad h_7 = h_6 + W_{P_{6-7}} = 587.7$ kJ/kg

Let boiler steam generation rate = w kg/s

$\therefore \quad W_T = w\,[(h_1 - h_2) + (1 - m)(h_2 - h_3)]$

$W_{net} = w\,[(h_1 - h_2) + (1 - m)(h_2 - h_3) - (1 - m)\,3.5 - m \times 3.4]$ kW

$= w\,[486.2 + 545.4 - 542\,m]$

$= w\,[1031.6 - 542\,m]$ kW

$Q = mw\,[h_2 - h_6] = mw\,(2033.4)$ kW

Here $w\,[1031.6 - 542m] = 1000$ … (i)

$mw \times 2033.4 = 1400$ ...(ii)

$\therefore \quad w = 1.3311$ kg/s ;

$m = 0.51724$ kg/kg of steam at H.P

$\therefore$ AT H.P flow 1.3311 kg/s

At L.P flow $= (1 - m)\,w = 0.643$ kg/s

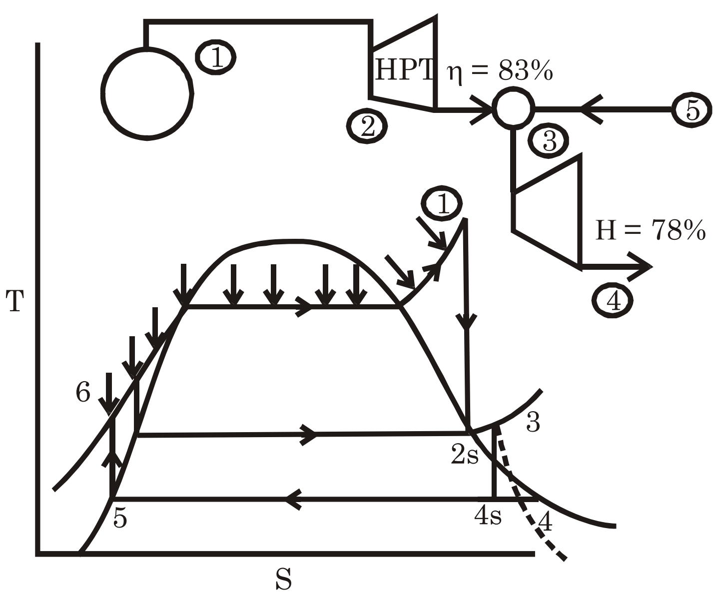
Q.12.22 **Geothermal energy from a natural geyser can be obtained as a continuous supply of steam 0.87 dry at 2 bar and at a flow rate of 2700 kg/h. This is utilized in a mixed-pressure cycle to augment the superheated exhaust from a high pressure turbine of 83% internal efficiency, which is supplied with 5500 kg/h of steam at 40 bar and 500°C. The mixing process is adiabatic and the mixture is expanded to a condenser pressure of 0.10 bar in a low pressure turbine of 78% internal efficiency. Determine the power output and the thermal efficiency of the plant.**

(**Ans.** 1745 kW, 35%)

Solution : From Steam Table 40 bar 500°C

$h_1 = 3445.3$ kJ/kg

$s_1 = 7.090$ kJ/kg – K



At 2 bar

$h_f = 504.7$ kJ/kg, $h_{fg} = 2201.6$ kJ/kg

$s_f = 1.5301$ kJ/kg, $s_{fg} = 5.5967$ kJ/kg

$7.090 = 1.4301 + x_1 \times 4.5967$

$\therefore \quad x_1 = 0.99342$

$\therefore\ h_{2s} = 504.7 + 0.99342 \times 2201.6$ kJ/kg $= 2691.8$ kJ/kg

$$\eta_{isen.} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$\therefore \quad h_2 - h_1 - \eta_{in}(h_1 - h_{2s})$

$= 3445.3 - 0.83\,(3445.3 - 2691.8) = 2819.9$ kJ/kg

From molier diagram $\quad s_2 = 7.31$ kJ/kg – K

Adiabatic mixing $\quad h_5 = 504.7 + 0.87 \times 2201.6 = 2420$ kJ/kg

$\therefore\ h_2 \times 5500 + h_5 \times 2700 = h_3 \times (5500 + 2700)$

$\therefore\ h_3 = 2688.3$ kJ/kg from molier dia at 2 bar 2688.3 kJ/kg

quality of steam x_3

Then $504.7 + x_2 \times 2201.6 = 2688.3 \Rightarrow x_3 = 0.9912$

$\therefore\ s_3 = 1.5301 + 0.9918 \times 5.5967 = 7.081$ kJ/kg – K

at 0.1 bar

$s_f = 0.649 + s_{fg} = 7.501$

$\therefore\ x_4 \times 7.501 + 0.649 = 7.081 \quad \Rightarrow x_4 = 0.8575$

$\therefore \quad h_{4s} = 191.8 + 0.8575 \times 2392.8 = 2243.6$ kJ/kg

$$\therefore \quad W_{T_{H.P}} = \frac{5500}{3600}(h_1 - h_2) = 955.47 \text{ kW}$$

$$W_{T_{L.P}} = \frac{8200}{3600}(h_3 - h_{4s}) \times 0.78 = 790 \text{ kW } 790.08 \text{ kW}$$

$\therefore \quad W_T = 1745.6$ kW

$$W_P = \frac{5500}{3600} \times 0.001010 \times (4000 - 10) = 6.16 \text{ kW}$$

$\therefore \quad W_{net} = 1739.44$ kW

$h_5 = 191.8$ kJ/kg, $h_6 = h_5 + W_P = 195.8$ kJ/kg

$$\therefore \text{ Heat input} = \frac{5500}{3600}(h_1 - h_6) = 4964.5 \text{ kW}$$

$$\therefore \quad \eta = \frac{1739.44}{4964.5} \times 100\% = 35.04\%$$

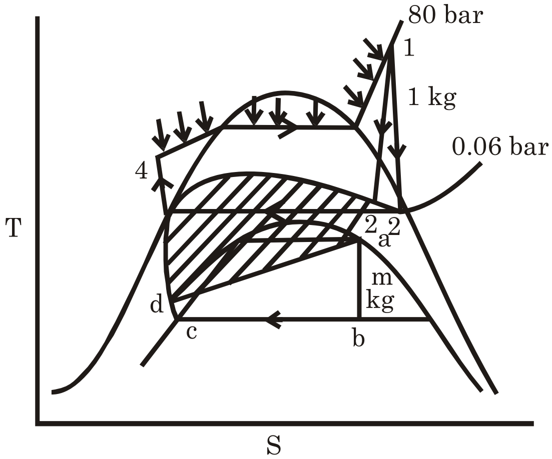
Q.12.23 **In a study for a space projects it is thought that the condensation of a working fluid might be possible at - 40°C. A binary cycle is proposed, using Refrigerant 12 as the low temperature fluid, and water as the high temperature fluid. Steam is generated at 80 bar, 500°C and expands in a turbine of 81% isentropic efficiency to 0.06 bar, at which pressure it is condensed by the generation of dry saturated refrigerant vapour at 30°C from saturated liquid at -40°C. The isentropic efficiency of the R-12 turbine is 83%. Determine the mass ratio of R-12 to water and the efficiency of the cycle. Neglect all losses.**

(**Ans.** 10.86; 44.4%.)

Solution : at 80 bar 500°C

$h_1 = 3398.3$ kJ/kg

$s_1 = 6.724$ kJ/kg – K



at 0.06 bar if quality is x, then

$6.724 = 0.521 + x_2 \times 7.809$

$\therefore \quad x_2 = 0.79434$

$\therefore \quad h_{2s} = 151.5 + 0.79434 \times 2415.9 = 2070.5$ kJ/kg

$h_3 = 151.5$

$W_P = 0.001006\ (8000 - 6) = 8$ kJ/kg

$\therefore \quad h_4 = 159.5$ kJ/kg

$\therefore \quad W_T = (h_1 - h_{2s}) \times \eta = (3398.3 - 2070.5) \times 0.81 = 1075.5$ kJ/kg

$\therefore \quad W_{net} = W_T - W_P = 1067.5$ kJ/kg

$Q_1 = h_1 - h_4 = 3238.8$ kJ/kg

$Q_2 = h_2 - h_3 = h_1 - \eta\ (h_1 - h_{2s}) - h_3 = 2171.3$ kJ/kg

For R-12

at 30°C saturated vapour

$h_a = 199.6$ kJ/kg, p = 7.45 bar $s_g = 0.6854$ kJ/kg – K

at 40°C

$s_f = 0$, $\quad s_{fg} = 0.7274$, $\quad \therefore$ if dryness x_b then

$x_b \times 0.7274 = 0.6854 \qquad \Rightarrow x_b = 0.94226$

$\therefore \quad s_f = 0$, $h_{fg} = 169.0$ kJ/kg

$\therefore \quad h_b - 0.94226 \times 169 = 159.24$ kJ/kg

$h_C = 0$

$$W_P = V_c\ (p_a - p_c) = \left(\frac{0.66 + 0.77}{2}\right) \times\ 10^{-3}\,(745 - 64.77)\ x = 0.4868 \text{ kJ/kg}$$

$\therefore \quad h_b = h_C + W_P = 0.4868$ kJ/kg

$\therefore \quad$ Heat input $= m\ (h_a - h_d)$

$= m\ (199.6 - 0.4868) = 199.11\ m = 2171.3$

$\therefore \quad m = 10.905$ kg of R-12/kg of water

Power output $W_{T_R} = m\ (h_a - h_b) \times \eta$

$= 10.905 \times (199.6 - 159.24) \times 0.83 = 365.\ 3$ K

$\therefore \quad W_{net} = 364.8$ kJ/kg of steam

$\therefore \quad W_{output} = W_{net}\ H_2O + W_{net}\ R12$

$= (1067.5 + 364.8) = 1432.32$ kJ/kg

$$\therefore \quad \eta = \frac{W_{output}}{\text{Heat input}} = \frac{1432.32}{3238.8} \times 100\% = 44.22\%$$

Q.12.24 **Steam is generated at 70 bar, 500°C and expands in a turbine to 30 bar with an isentropic efficiency of 77%. At this condition it is mixed with twice its mass of steam at 30 bar, 400°C. The mixture then expands with an isentropic efficiency of80% to 0.06 bar. At a point in the expansion where me pressure is 5 bar, steam is bled for feedwater heating in a direct contact heater, which raises the feed water to the saturation temperature of the bled steam. Calculate the mass of steam bled per kg of high pressure steam and the cycle efficiency. Assume that the L.P. expansion condition line is straight.**

(**Ans.** 0.53 kg; 31.9%)

Solution : From Steam Table 70 bar 500°C

$h_1 = 3410.3$ kJ/kg

$s_1 = 6.798$ kJ/kg – K

s_1 at 30 bar 400°C

$h_3' = 3230.9$ kJ/kg

$s_3' = 6.921$ kJ/kg

From Molier diagram $h_{2s} = 3130$ kJ/kg

$\therefore \quad h_2 = h_1 - \eta_{isentropic} \times (h_1 - h_{2s})$

$= 3410.3 - 0.77\ (3410.3 - 3130) = 3194.5$ kJ/kg

For adiabatic mixing

$1 \times h_2 + 2 \times h_3' = 3 \times h_3$

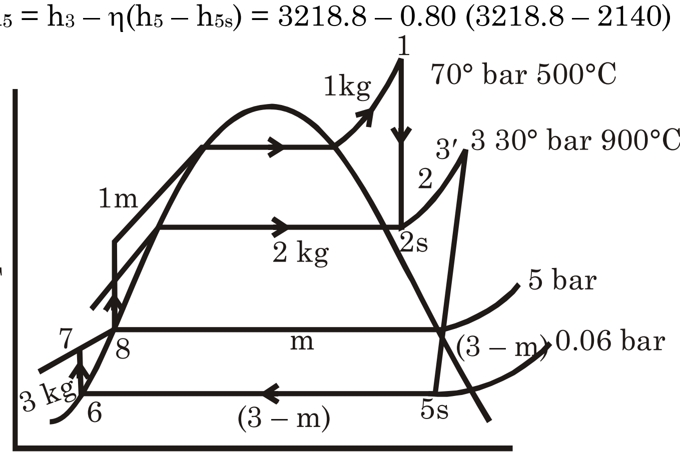
$h_3 = 3218.8$ kJ/kg

$s_3 = 6.875$ kJ/kg (From Molier diagram)

$h_4' = 2785$ kJ/kg

$\therefore \quad h_{5s} = 2140$ kJ/kg

$\therefore \quad h_5 = h_3 - \eta(h_5 - h_{5s}) = 3218.8 - 0.80\ (3218.8 - 2140)$ kJ/kg



From S.L in H.P. $h_4 = 2920$ kJ/kg

From Heat balance & heater

$m \times h_4 + (3 - m)h_7 = 3h_g \qquad h_7 = h_6 + W_P$

$= 151.5 + 0.001006 \times (500 - 6) \approx 0.5$ kJ/kg

$\therefore \quad m \times 2920 + (3 - m) \times 152 = 3 \times 640.1 = 152$ kJ/kg

$m = 0.529$ kg $\qquad h_g = 640.1$

$W_{T_{H.P}} = 1 \times (h_1 - h_2) = (3410.3 - 3194.5)$ kJ/kg $= 215.8$ kJ/kg

$W_{T_{L.P}} = 3\,(h_3 - h_4) + (3 - m)\,(h_4 - h_5)$

$= 3\,(3218.8 - 2920) + (3 - 0.529)\,(2920 - 2355.8)$

$= 2290.5$ kJ/kg of steam H.P

$W_P = (3 - m)\,(h_7 - h_6) + 2 \times 0.001(3000 - 500)$
$+ 1 \times 0.001\,(7000 - 500) = 12.74$ kJ/kg of H.P

$\therefore\ W_{net} = (215.8 + 2290.5 - 12.74)$ kJ/kg & H.P steam
$= 2493.6$ kJ/kg of H.P steam

Heat input $Q_1 = (h_1 - h_{10}) + 2\,(h_{3'} - h_9)$

$\therefore \quad h_{10} + h_8 + W_{P_{8-10}} = 646.6$ kJ/kg

$= (3410.3 - 646.6) + 2\,(3230.9 - 642.6) \qquad h_9 = h_8 + W_{P_{8-9}}$

$= 7940.3$ kJ/kg of H.P steam $\qquad = 642.6$ kJ/kg

$$\therefore \quad \eta_{cycle} = \frac{2493.6}{7940.3} \times 100\% = 31.4\%$$

Q.12.25 **An ideal steam power plant operates between 70 bar, 550°C and 0.075 bar. It has seven feed water heaters. Find the optimum pressure and temperature at which each of the heaters operate.**

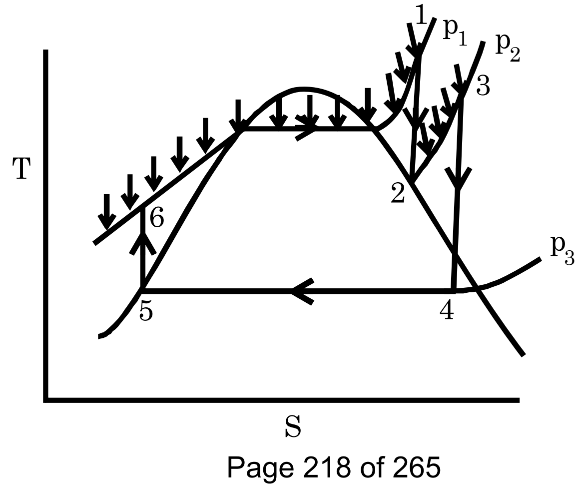
Solution: Try please.

Q.12.26 **In a reheat cycle steam at 550°C expands in an h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a I.p. turbine to 40°C. If the moisture content at turbine exhaust is given to be 14.67%, find (a) the reheat pressure, (b) the pressure of steam at inlet to the h.p. turbine, (c) the net work output per kg, and (d) the cycle efficiency. Assume all processes to be ideal.**

(**Ans**. (a) 20 bar, (b) 200 bar, (c) 1604 kJ/kg, (d) 43.8%)

Solution: From S.T. at 40°C, 14.67% moisture

$\therefore \quad x = 0.8533$



$p_3 = 0.0738$ bar

$h_f = 167.6$ kJ/kg; $h_{fg} = 2406.7$ kJ/kg

$\therefore$ $h_4 = 167.6 + 0.85322 \times 2406.7$

$= 2221.2$ kJ/kg

$s_f = 0.573$ kJ/kg – K $\quad$ $s_{fg} = 7.685$ kJ/kg – K

$s_4 = 0.573 + 0.8533 \times 7.685 = 7.1306$ kJ/kg

$\therefore$ at 400°C and $s_4 = 7.1306$ From Steam Table

(a) Pr = 20 bar, $\quad$ $h_3 = 3247.6$ kJ/kg

At 20 bar saturation

$h_2 = 2797.2$ kJ/kg $\quad \therefore \quad S_2 = 6.3366$ kJ/kg – K

(b) at 550°C and 6.3366 kJ/kg – k

From Steam Table

Pr = 200 bar

$\therefore$ $h_1 = 3393.5$ kJ/kg

$\therefore$ $h_5 = 167.6$ kJ/kg $\quad$ $W_P = 0.001 \times (20000 - 7.38)$ kJ/kg

$\therefore$ $h_6 = h_5 + W = 187.6$ kJ/kg $\quad$ = 20 kJ/kg

$\therefore$ $W_T = (h_1 - h_2) + (h_3 - h_4) = 1622.7$ kJ/kg

(c) $\therefore$ $W_{net} = W_T - W_P = 1602.7$ kJ/kg

Heat input $Q_1 = (h_1 - h_6) + (h_3 - h_2)$

$= (3393.5 - 187.6) + (3247.6 - 2797.2)$ kJ/kg

$= 3656.3$ kJ/kg

(d) $\therefore$ $\eta = \dfrac{1602.7}{3656.3} \times 100\% = 43.83\ \%$

Q.12.27 **In a reheat steam cycle, the maximum steam temperature is limited to 500°C. The condenser pressure is 0.1 bar and the quality at turbine exhaust is 0.8778. Had there been no reheat, the exhaust quality would have been 0.7592. Assuming ideal processes, determine (a) reheat pressure, (b) the boiler pressure, (c) the cycle efficiency, and (d) the steam rate.**

(**Ans.** (a) 30 bar, (b) 150 bar, (c) 50.51%, (d) l.9412 kg/kWh)

Solution: From 0.1 bar (saturated S.T.)

$s_f = 0.649$ kJ/kg – K

$s_{fg} = 7.501$ kJ/kg – K

$\therefore$ $s_4 = s_3 = 0.649 + 0.8778 \times 7.501 = 7.233$ kJ/kg – K

$s_4'' = s_1 = 0.649 + 0.7592 \times 7.501 = 6.344$ kJ/kg – K

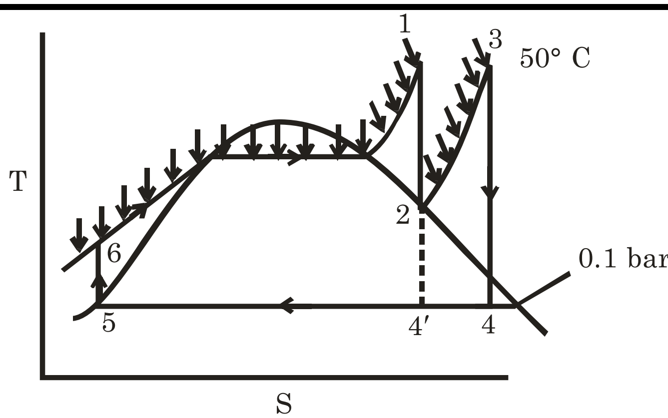
$h_4 = 191.8 + 0.8778 \times 2392.8 = 2292.2$ kJ/kg

From super heated steam turbine at 500°C 7.233 kJ/kg

$p_3 = 30$ bar

$\therefore$ $h_3 = 3456.5$ kJ/kg, $h_2 = 2892.3$ kJ/kg

From Molier diagram



At 500°C and 6.344 kJ/kg – K

p_1 = 150 bar, h_2 = 3308.6 kJ/kg – K

h_5 = 191.8 kJ/kg, W_P = 0.001010 (15000 – 10) =15.14 kJ/kg

$\therefore$ h_6 = 206.94 kJ/kg

$\therefore$ $W_T = (h_1 - h_2) + (h_3 - h_4)$ = 1580.6 kJ/kg

$W_{net} = W_T - W_P$ = 1565.46 kJ/kg

$Q_1 = (h_1 - h_6) + (h_3 - h_2)$ = 3665.86 kJ/kg – K

$$\therefore \quad \eta = \frac{1565.45}{3665.86} \approx 42.7\%$$

Q.12.28 **In a cogeneration plant, steam enters the h.p. stage of a two-stage turbine at 1 MPa, 200°C and leaves it at 0.3 MPa. At this point some of the steam is bled off and passed through a heat exchanger which it leaves as saturated liquid at 0.3 MPa. The remaining steam expands in the I.p. Stage of the turbine to 40 kPa. The turbine is required to produce a total power of 1 MW and the heat exchanger to provide a heating rate of 500 kW. Calculate the required mass flow rate of steam into the h.p. stage of the turbine. Assume (a) steady condition throughout the plant, (b) velocity and gravity terms to be negligible, (c) both turbine stages are adiabatic with isentropic efficiencies of 0.80.**

(**Ans.** 2.457 kg/s)

Solution: From S.T at 1 MPa 200°C

i.e 10 bar 200°C

h_1 = 2827.9 kJ/kg

s_1 = 6.694 kJ/kg-K

At 3 bar

s_f = 1.6716 s_{fg} = 5.3193

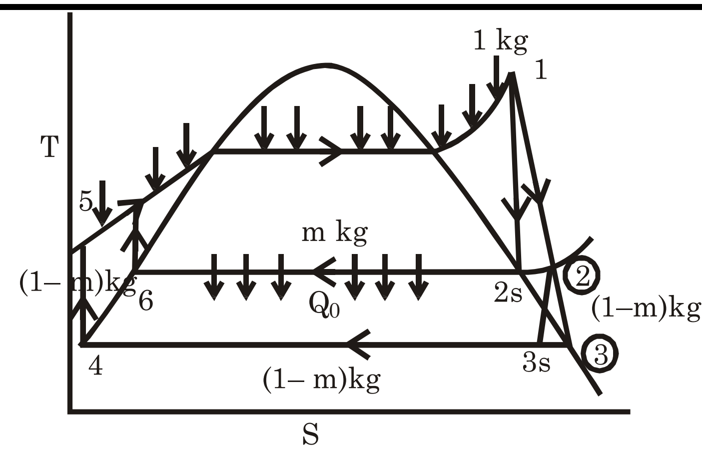
$\therefore$ 6.694 = 1.6716 + x′ × 5.3193

$\therefore$ x′ = 0.9442

$\therefore$ h_{2s} = 561.4 + 0.9242 × 2163 = 2603.9 kJ/kg

$\therefore$ $h_2 = h_1 - \eta(h_1 - h_{2s})$ = 2827.9 – 0.8 (2827.9 – 2603.0) kJ/kg

= 2648.7 kJ/kg



This is also wet so

$s_2 = 1.6716 + x_2' \times 5.319.3$ $[2648.7 = 561.4 + x_2 \times 2163.2]$

$= 6.8042$ kJ/kg – K

If at 3s condition of steam is x_3 then 40 kPa = 0.4 bar

$6.8042 = 1.0261 + x_3 \times 6.6440 \Rightarrow x_3 = .8697$

$\therefore$ $h_{3s} = 317.7 + 0.8697 \times 2319.2 = 2334.4$ kJ/kg

$\therefore$ $h_3 = h_2 - \eta\ (h_2 - h_{3s}) = 2397.3$ kJ/kg

$h_4 = 317.7$ kJ/kg

$W_{P_{4s}} = 0.001 \times (1000 - 40) \approx 1$ kJ/kg

$\therefore$ $h_5 = 318.7$ kJ/kg

$h_6 = 562.1$ kJ/kg, $W_{P_{6-7}} = 0.001 \times (1000 - 300)$

$= 0\ 0.7$ kJ/kg

$h_7 = 562.1$ kJ/kg

$\therefore$ $W_T = (h_1 - h_2) + (1 - m)\ (h_2 - h_3)$

$\therefore$ $W_{net} = (h_1 - h_2) + (1 - m)\ (h_2 - h_3) - (1 - m)\ 1 - m \times 0.7$

$= (429.6 - 251.1\ m)$ kJ/kg of steam of H.P

Process Heat $= m \times (h_2 - h_6) = m \times 2087.3$

$\therefore$ If mass flow rate of w then

$w\ (429.6 - 251.1\ m) = 1000$

$mw \times 2087.3 = 500$

$\therefore$ $w = 2.4678$ kg/s

$\therefore$ required mass flow rate in H.P = 2.4678 kg/s

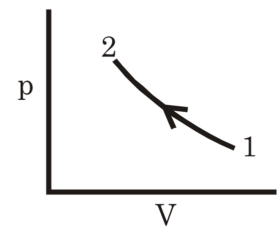
13. Gas Power Cycles

Some Important Notes

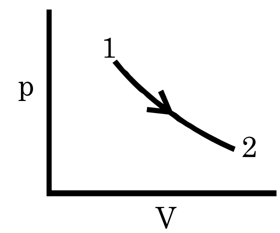
1. Compression ratio, (r_c)

$$r_c = \frac{\text{Volume at the begining of compression } (V_1)}{\text{Volume at the end of compression } (V_2)}$$

$$\therefore \quad r_c = \frac{V_1}{V_2} = \frac{\text{bigger term}}{\text{smaller term}}$$



2. Expansion ratio,(r_e)



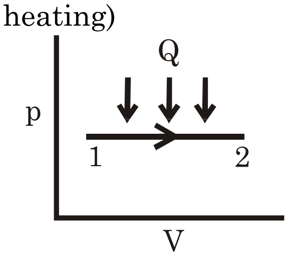
$$r_e = \frac{\text{Volume at the end of expansion } (V_2)}{\text{Volume at the begining of expansion } (V_1)}$$

$$\therefore \; r_e = \frac{V_2}{V_1} = \frac{\text{bigger term}}{\text{smaller term}}$$

3. Cut-off ratio, $\rho = \dfrac{\text{volume after heat addition } (v_2)}{\text{volume before heat addition } (v_1)}$

(For constant Pressure heating)

$$\therefore \qquad \rho = \frac{V_2}{V_1} = \frac{\text{bigger term}}{\text{smaller term}}$$



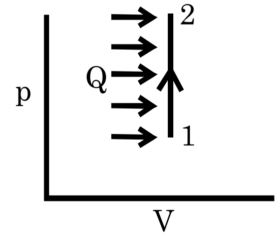
Relation $\boxed{r_c = r_e \,.\, \rho}$

4. Constant volume pressure ratio,

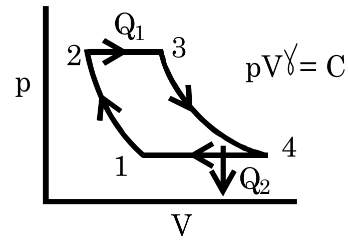
$$\propto = \frac{\text{Pressure after heat addition}}{\text{Pressure before heat addition}}$$

[For constant volume heating)

$\therefore \quad \alpha = \frac{p_2}{p_1} = \frac{\text{bigger term}}{\text{smaller term}}$



5. Pressure ratio, $r_P = \frac{\text{Pressure after compresion or before expansion}}{\text{Pressure before compresion or after expansion}} \left(\frac{p_2}{p_1}\right)$



$$\therefore r_P = \frac{p_2}{p_1}$$

6. **Carnot cycle:** The large back work (i.e compressor work) is a big draw back for the Carnot gas cycle, as in the case of the Carnot Vapour cycle.

7. **Stirling Cycle**: comparable with Otto.

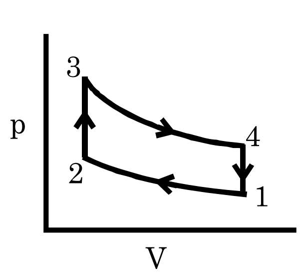
8. **Ericsson Cycle**: comparable with Brayton cycle.

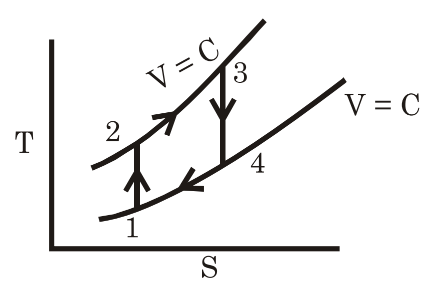
9. **The regenerative, stirling and Ericsson cycles** have the same efficiency as the carnot cycle, but much less back work.

10. **Air standards cycles**

a. **Otto cycle (1876)**

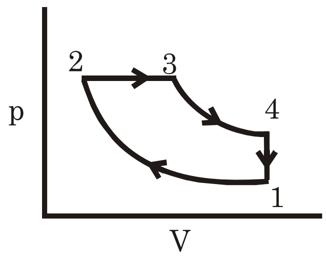
$$\eta = 1 - \frac{1}{r_c^{\gamma - 1}}$$

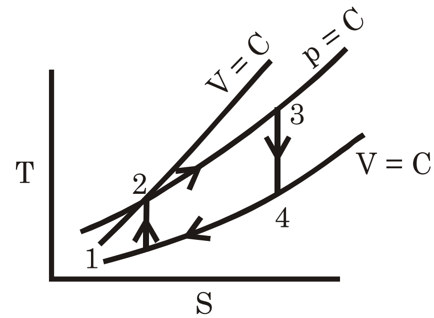




For W$_{max}$; $r_c = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{1}{2(\gamma - 1)}}$

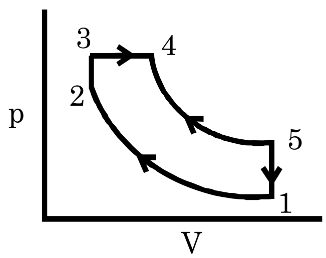
b. Diesel cycle (1892)

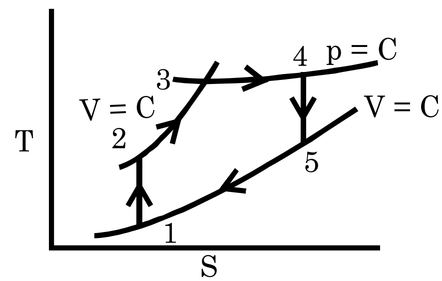




$$\eta = 1 - \frac{(\rho^{\gamma} - 1)}{r_c^{\gamma-1} \cdot \gamma(\rho - 1)}$$

C. Dual or Limited pressure or mixed cycle





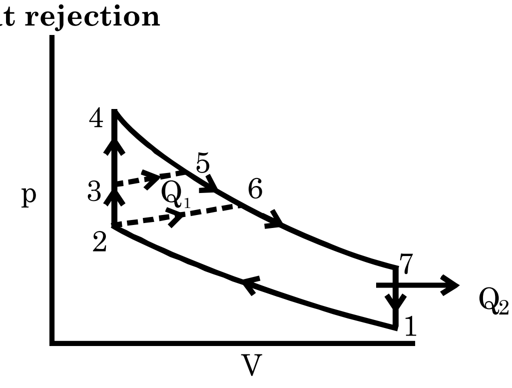
$$\eta = 1 - \frac{(\alpha\rho^{\gamma} - 1)}{r_c^{\gamma-1}\,[(\alpha - 1) + \alpha\gamma(\rho - 1)]}$$

Where $\alpha = \frac{p_3}{p_2}$

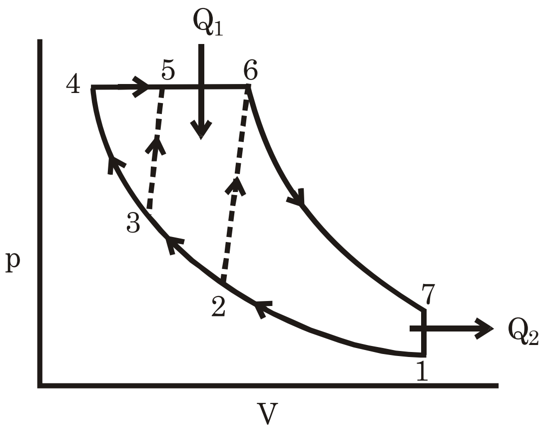
Comparison of Otto, Diesel and Dual cycle

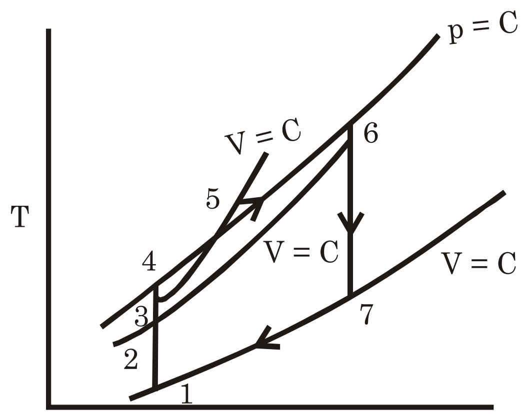
a. With same compression ratio and heat rejection

$$\therefore \quad \eta_{\text{otto}} > \eta_{\text{Dual}} > \eta_{\text{Diesel}}$$



b. For the Same maximum Pressure and Temperature (also heat rejection same)

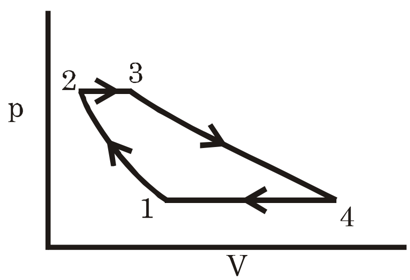


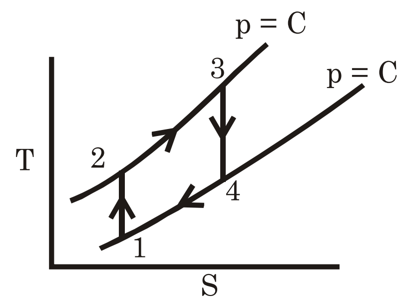


$$\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

11. Brayton cycle

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{r_P^{\frac{\gamma-1}{\gamma}}}$$





∴ Brayton cycle efficiency depends on either compression ratio (r_c) or Pressure ratio r_p

*** For same compression ratio** $\left[\eta_{\text{Otto}} = \eta_{\text{Brayton}}\right]$

a. For Maximum efficiency $(r_p)_{\max} = \left(\frac{T_{\max}}{T_{\min}}\right)^{\frac{\gamma}{(\gamma-1)}}$

$$\therefore \quad \eta_{\max} = \eta_{\text{Carnot}} = 1 - \frac{T_{\min}}{T_{\max}}$$

b. For Maximum work

(i) $(r_p)_{\text{opt.}} = \left(\frac{T_{\max}}{T_{\min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$

$\therefore \quad \eta_{cycle} = 1 - \sqrt{\dfrac{T_{min}}{T_{max}}}$ and $W_{net,\,max} = C_p[\sqrt{T_{max}} - \sqrt{T_{min}}]^2$

(ii) If isentropic efficiency of Turbine is η_T and compressor is η_c then

$$(r_p)_{opt.} = \left(\eta_T\ \eta_C \frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

Question and Solution (P K Nag)

Q13.1 **In a Stirling cycle the volume varies between 0.03 and 0.06 m^3, the maximum pressure is 0.2 MPa, and the temperature varies between 540°C and 270°C. The working fluid is air (an ideal gas). (a) Find the efficiency and the work done per cycle for the simple cycle. (b) Find the efficiency and the work done per cycle for the cycle with an ideal regenerator, and compare with the Carnot cycle having the same isothermal heat supply process and the same temperature range.**

(**Ans.** (a) 27.7%, 53.7 kJ/kg, (b) 32.2%)

Solution:

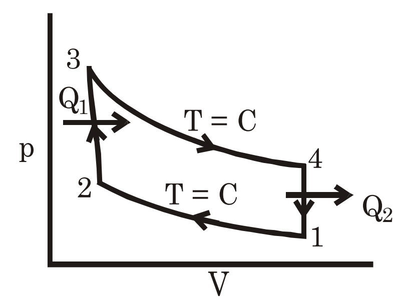
Given $V_1 = 0.06\ m^3 = V_4$

$V_2 = 0.03\ m^3 = V_3$

$p_3 = 200$ kPa

$T_1 = T_2 = 270°C = 543$ K

$T_3 = T_4 = 540°C = 813$ K



$\therefore$ Heat addition $Q_1 = Q_{2-3} = m\,c_v\ (T_3 - T_2)$

Here $m = \dfrac{p_3 V_3}{R T_3} = \dfrac{200 \times 0.03}{0.287 \times 813} = 0.025715$ kg

$\therefore\ Q_1 = 0.025715 \times 0.718\ (813 - 543)$ kJ $= 4.985$ kJ

$$W_{3-4} = \int_3^4 pdV = m\,R\,T_3\ \ln\left(\frac{V_4}{V_3}\right)$$

$pV = mRT = C$

$$W_{1-2} = \int_1^3 pdV = m\,RT_1 \ln\left(\frac{V_1}{V_2}\right)$$

$$\therefore\ p = \frac{mRT}{V}$$

$$\therefore\ \eta = \frac{m(RT_3 - RT_1)\ln\left(\dfrac{V_1}{V_2}\right)}{4.985} =$$

$$\frac{0.025715 \times 0.287\,(813-543)\ln 2}{4.985} \times 100\% = 27.7\%$$

Work done = 1.3812 kJ = 53.71 kJ/kg

For ideal regeneration

$$\eta = 1 - \frac{543}{813} = 33.21\%$$

Q13.2 **An Ericsson cycle operating with an ideal regenerator works between 1100 K and 288 K. The pressure at the beginning of isothermal compression is 1.013 bar. Determine (a) the compressor and turbine work per kg of air, and (b) the cycle efficiency.**

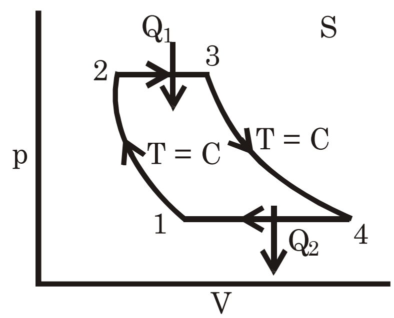
(**Ans. (a)** w_T **= 465 kJ/kg,** w_C **= 121.8 kJ/kg (b) 0.738**)

Solution: Given $T_1 = T_2 = 288$ K

$T_3 = T_4 = 1100$ K

$p_1 = 1.013$ bar = 101.3 kPa

$\therefore \quad V_1 = \dfrac{RT_1}{p_1} = 0.81595\ m^3/kg$



$$W_C = RT_1 \ln\left(\frac{V_1}{V_2}\right) \qquad W_T = R\,T_3 \ln\left(\frac{V_4}{V_3}\right)$$

$p_3 = p_2$; $p_1 = p_4$

$$\therefore\ \eta = 1 - \frac{288}{1100} = 73.82\% \qquad \therefore\ \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore\ W = \eta\, Q_1 = \eta\, C_P\,(T_1 - T_2) \qquad \therefore\ \frac{V_1}{V_2} = \frac{T_1}{T_2} \times \left(\frac{p_2}{p_1}\right) = \left(\frac{p_2}{p_1}\right)$$

$$= 0.7382 \times 1.005\,(1100 - 288)\ \text{kJ/kg} = 602.4\ \text{kJ/kg}$$

$$\therefore\ Q_{2-3} = C_P\,(T_3 - T_2) = C_v\,(T_3 - T_2) + p_2\,(V_3 - V_2)$$

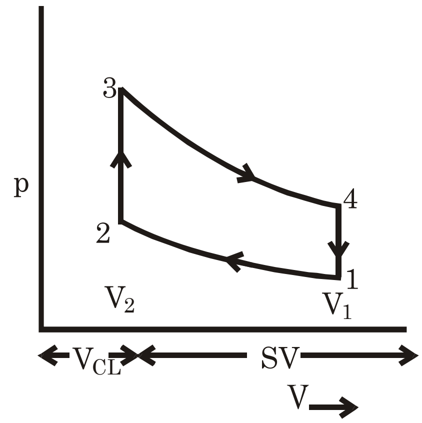
Q13.5 **An engine equipped with a cylinder having a bore of 15 cm and a stroke of 45 cm operates on an Otto cycle. If the clearance volume is 2000** cm^3**, compute the air standard efficiency.**

(**Ans.** 47.4%)

Solution:

$V_2 = 2000\ cm^3 = 0.002\ m^3$

$V_1 = V_2 + S.V.$



$$= 0.002 + \frac{\pi \times 0.15^2}{4} \times 0.45 = 0.009952\ m^3$$

$$\therefore\ r_c = \frac{V_1}{V_2} = 4.9761$$

$$\eta_{air\ std} = 1 - \frac{1}{r_c^{\gamma-1}} = 47.4\%$$

Q13.10 **Two engines are to operate on Otto and Diesel cycles with the following data: Maximum temperature 1400 K, exhaust temperature 700 K. State of air at the beginning of compression 0.1 MPa, 300 K.**
Estimate the compression ratios, the maximum pressures, efficiencies, and rate of work outputs (for 1 kg/min of air) of the respective cycles.

(**Ans**. Otto-- r_k = 5.656, p_{max} = 2.64 MPa, W = 2872 kJ/kg, η = 50%
Diesel- r_k , = 7.456, p_{max} = 1.665 MPa, W = 446.45 kJ/kg, η = 60.8%)

Solution:

$T_3 = 1400$ K

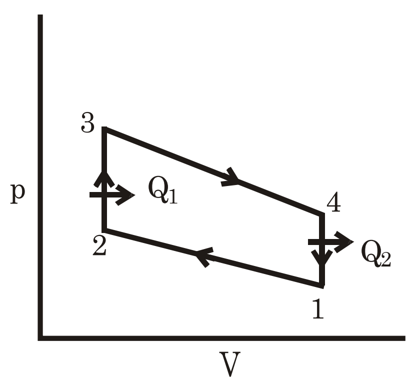
$T_4 = 700$ K

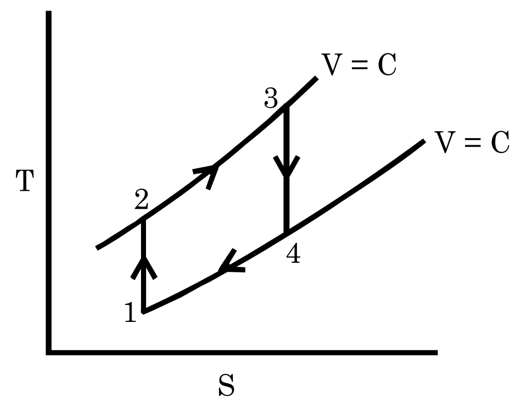
$p_1 = 100$ kPa

$T_1 = 300$ K

$$\therefore\quad v_1 = \frac{RT_1}{p_1} = 0.861\ m^3/kg$$

$$\therefore\quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$$





$$\therefore\quad \left(\frac{1400}{700}\right) = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\therefore\quad v_2 = \frac{v_1}{2^{\frac{1}{\gamma-1}}} = \frac{0.861}{2^{\frac{1}{0.9}}}$$

$= 0.1522\ m^3/kg$

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$= (5.657)^{0.4} \times 300 = 600$ K

$\therefore \quad r_c = \dfrac{v_1}{v_2} = 2^{\frac{1}{\gamma-1}} = 5.657 \qquad \dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^{\gamma} \Rightarrow P_2 = 1131.5 \text{ kPa}$

$\dfrac{p_3}{T_3} = \dfrac{p_2}{T_2} \qquad \Rightarrow p_3 = \dfrac{T_3}{T_2} \times p_2 = \dfrac{1400}{600} \times 1131.5 \text{ kPa} = 2.64 \text{ MPa}$

$\therefore \quad W = Q_1 - Q_2 = C_v (T_3 - T_2) - C_v (T_4 - T_1)$
$= 0.718 [(1400 - 600) - (700 - 300)] \text{ kJ/kg} = 287.2 \text{ kJ/kg}.$

$\therefore \quad \eta = \dfrac{Q_1 - Q_2}{Q_1} = \dfrac{287.2}{0.718\,(1400 - 600)} = 0.5 \approx 50\%$

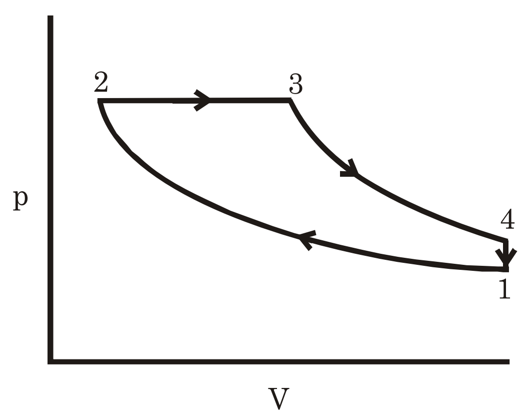
Diesel $T_3 = 1400$ K
$T_4 = 700$ K
$T_1 = 300$ K $\qquad \therefore v_1 = 0.861 \text{ m}^3/\text{kg}$
$p_1 = 100$ kPa

$\dfrac{T_3}{T_4} = \left(\dfrac{v_4}{v_3}\right)^{\gamma-1} \qquad \therefore \dfrac{1400}{700} = \left(\dfrac{v_1}{v_2}\right)^{0.4}$

$\therefore \dfrac{v_1}{v_3} = 2^{\frac{1}{0.4}} = 2^{2.5}$

$\therefore v_3 = \dfrac{v_1}{2^{3.5}} = 0.1522 \text{ m}^3/\text{kg}$

$\therefore \quad p_3 = \dfrac{RT_3}{V_3} = \dfrac{0.287 \times 1400}{0.1522} = 2639.9 \text{ kPa}$



$\therefore \quad p_2 = p_3$

$\therefore \quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{v_1}{v_2}\right)^{\gamma-1} \qquad \therefore T_2 = 764 \text{ K}$

$r_c = \dfrac{v_1}{v_2} = \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\dfrac{2639.9}{100}\right)^{\frac{1}{1.4}} = 10.36$

$p_1 = p_3 = 2.64 \text{ MPa}$

$Q_1 = Q_{2-3} = C_P (T_3 - T_2) = 1.005 (1400 - 764) \text{ kJ/kg}$
$= 638.84 \text{ kJ/kg}$

$$Q_2 = Q_{4-1} = C_v (T_4 - T_1) = 0.718 (700 - 300) = 287.2 \text{ kJ/kg}$$

$$\therefore \quad W = Q_1 - Q_2 = 351.64 \text{ kJ/kg}$$

$$\eta = \frac{W}{Q_1} = \frac{351.64}{638.84} = 55\%$$

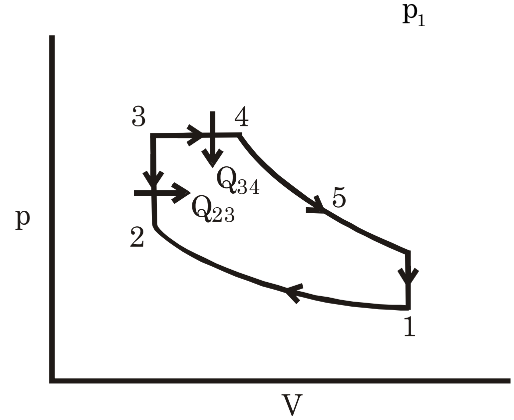
Q13.11 **An air standard limited pressure cycle has a compression ratio of 15 and compression begins at 0.1 MPa, 40°C. The maximum pressure is limited to 6 MPa and the heat added is 1.675 *MJ/kg.* Compute (a) the heat supplied at constant volume per kg of air, (b) the heat supplied at constant pressure per kg of air, (c) the work done per kg of air, (d) the cycle efficiency, (e) the temperature at the end of the constant volume heating process, (f) the cut-off ratio, and (g) the m.e.p. of the cycle.**

(**Ans.** (a) 235 *kJ/kg,* (b) 1440 *kJ/kg,* (c) 1014 *kJ/kg,* (d) 60.5%, (e) 1252 K, (f) 2.144 (g) 1.21 MPa)

Solution: $r_c = \frac{v_1}{v_2} = 15$

$p_1 = 100$ kPa

$T_1 = 40°C = 313$ K $\qquad \therefore v_1 = \frac{RT_1}{p_1} = 0.89831 \text{ m}^3/\text{kg}$



$p_3 = p_4 = 6000$ kPa

$Q_{2-4} = 1675$ kJ/kg

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (15)^{1.4-1} \Rightarrow T_2 = 924.7 \text{ K}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = 15^{1.4} \Rightarrow p_2 = 4431 \text{ kPa}$$

$$\therefore \quad \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \Rightarrow T_3 = \frac{p_3}{p_2} \times T_2 = \frac{6000}{4431} \times 924.7 = 1252 \text{ K}$$

$$\therefore \quad Q_{2-4} = C_v (T_3 - T_2) + C_P (T_4 - T_3) = 1675$$

$$\therefore \quad T_4 = T_3 + 1432.8 \text{ k} = 2684.8 \text{ K}$$

∴

∴ $v_4 = \dfrac{RT_4}{p_4} = 0.12842\ m^3/kg.$

∴ $\dfrac{T_4}{T_5} = \left(\dfrac{v_5}{v_4}\right)^{\gamma-1} = \left(\dfrac{v_1}{v_4}\right)^{\gamma-1} \Rightarrow \dfrac{T_4}{T_5} = 2.1773 \quad \therefore T_5 = 1233\ K$

(a) Heat supplied at constant volume = $C_v (T_3 - T_2) = 235$ kJ/kg

(b) Heat supplied at constant Pressure = (1675 – 235) = 1440 kJ/kg

(c) Work done = $Q_1 - Q_2 = 1675 - C_v (T_5 - T_1) = 1014.44$ kJ/kg

(d) Efficiency $\eta = \dfrac{Q_1 - Q_2}{Q_1} = \dfrac{1014.44}{1675} \times 100\% = 60.56\%$

(e) Temperature at the end of the heating (T_3) = 1252 K

(f) Cut-off ratio (ρ) = $\dfrac{v_4}{v_3} = \dfrac{0.12842}{0.05988} = 2.1444$

$$[\therefore v_3 = \frac{RT_3}{p_3} = 0.059887]$$

(g) m. e. p. $\quad \therefore p_m (V_1 - V_2) = W$

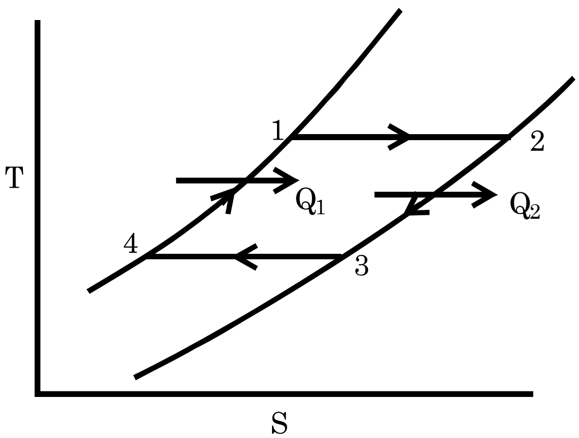
$$\therefore p_m = \frac{1014.44}{v_1 - \dfrac{v_1}{15}} = 1209.9\ kPa = 1.2099\ MPa$$

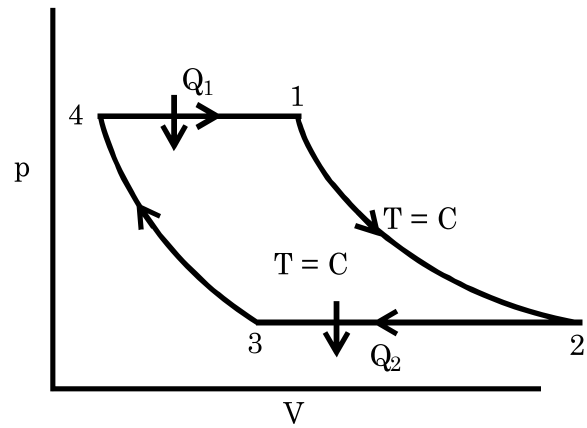
Q13.13 **Show that the air standard efficiency for a cycle comprising two constant pressure processes and two isothermal processes (all reversible) is given by**

$$\eta = \frac{(T_1 - T_2)\, ln(r_p)^{(\gamma-1)/\gamma}}{T_1\left[1 + ln(r_p)^{(\gamma-1)/\gamma} - T_2\right]}$$

Where T_1 and T_2 are the maximum and minimum temperatures of the cycle, and r_p is the pressure ratio.

Solution:





$$r_p = \frac{p_4}{p_3} = \frac{p_1}{p_2} \qquad \therefore W_{1-2} = \int pdV = RT_1\int_1^2 \frac{dV}{V} = RT_1 \ln\frac{V_2}{V_1} = RT_1 \ln r_p$$

$$Q_{1-2} = 0 + W_{1-2}$$

$$W_{3-4} = \int_3^4 pdV = RT_3 \ln\left(\frac{V_4}{V_3}\right) = -RT_3 \ln\left(\frac{V_3}{V_4}\right) = -RT_3 \ln r_p .$$

$$\therefore \qquad W_{net} = W_{1-2} + W_{3-4} = R(T_1 - T_3)\ln\left(\frac{V_2}{V_1}\right)$$

$$= R(T_1 - T_2) \ln r_p .$$

Constant pressure heat addition $= C_P(T_1 - T_4)$

$$= \frac{\gamma R}{\gamma - 2}(T_1 - T_4) \qquad T_1 = T_{max}$$

$$= \frac{\gamma R}{\gamma - 1}(T_1 - T_2) \qquad T_2 = T_{min.}$$

Total heat addition (Q_1) $= Q_{12} +$ const P_r.

$$= RT_1 \ln r_p + \frac{\gamma R}{\gamma - 1}(T_1 - T_2)$$

$$\eta = \frac{R(T_1 - T_2)\ln r_P}{R\left[(T_1 \ln r_P + \frac{\gamma}{\gamma - 1}(T_1 - T_2)\right]}$$

Multiply $\frac{\gamma - 1}{\gamma}, D^{\gamma}, N^{\gamma}$

$$= \frac{\left(\frac{\gamma - 1)}{\gamma}\right)(T_1 - T_2)\ln r_p}{(T_1 - T_2) + \frac{\gamma - 1}{\gamma}T_1 \ln r_p}$$

$$\eta = \frac{(T_1 - T_2)\ln r_p^{\left(\frac{\gamma-1}{\gamma}\right)}}{T_1\,[1 + \ln r_p^{\left(\frac{\gamma-1}{\gamma}\right)}] - T_2}$$

Q13.14 **Obtain an expression for the specific work done by an engine working on the Otto cycle in terms of the maximum and minimum**

Temperatures of the cycle, the compression ratio r_k, and constants of the working fluid (assumed to be an ideal gas).
Hence show that the compression ratio for maximum specific work output is given by

$$r_k = \left(\frac{T_{min}}{T_{max}}\right)^{1/2(1-\gamma)}$$

Solution:

$T_{min} = T_1$
$T_{max} = T_3$

$Q_1 = C_v (T_3 - T_2)$
$Q_2 = C_v (T_4 - T_1)$
$\therefore \quad W = Q_1 - Q_2$
$= C_v [(T_3 - T_2) - (T_4 - T_1)]$

Hence $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{\gamma - 1} = r_c^{\gamma - 1}$

$\therefore\ T_2 = T_1 r_c^{\gamma - 1}$

And $\dfrac{T_4}{T_3} = \left(\dfrac{v_3}{v_4}\right)^{\gamma - 1} = \left(\dfrac{v_2}{v_1}\right)^{\gamma - 1} = r_c^{-(\gamma - 1)}$ Let $r_c^{\gamma - 1}$
$= x$

$\therefore\ T_4 = T_3 \,.\, r_c^{-(r-1)} = \dfrac{T_3}{x}$

Then

$W = C_v\left[T_3 - T_1 x - \dfrac{T_3}{x} + T_1\right]$

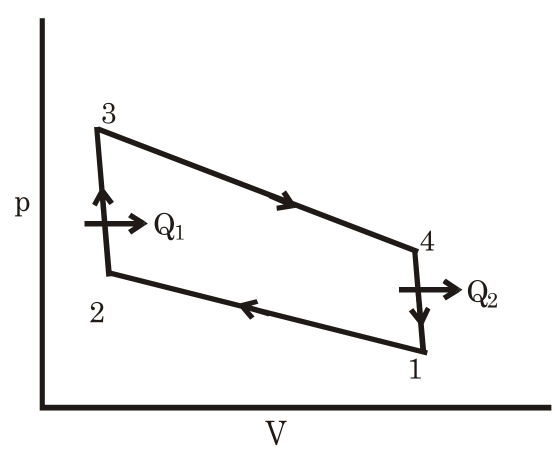
For maximum W, $\dfrac{dW}{dx} = 0$

$\therefore\ C_v\left[0 - T_1 + \dfrac{T_3}{x^2} + 0\right] = 0$

$\therefore\ x^2 = \dfrac{T_3}{T_1}$

$\therefore\ r_c^{\gamma - 1} = \sqrt{\dfrac{T_3}{T_1}} = \sqrt{\dfrac{T_{max}}{T_{min}}}$

$\therefore\ r_c = \left(\dfrac{T_{max}}{T_{min}}\right)^{\frac{1}{2(\gamma - 1)}} = \left(\dfrac{T_{min}}{T_{max}}\right)^{\frac{1}{2(1 - \gamma)}}$ Proved.



Q13.15 **A dual combustion cycle operates with a volumetric compression ratio r_k = 12, and with a cut-off ratio 1.615. The maximum pressure is given by p_{max} = 54 p_1' where p_1 is the pressure before compression. Assuming**

indices of compression and expansion of 1.35, show that the m.e.p. of the cycle

$$p_m = 10 p_1$$

Hence evaluate (a) temperatures at cardinal points with T_1 = 335 K, and (b Cycle efficiency.

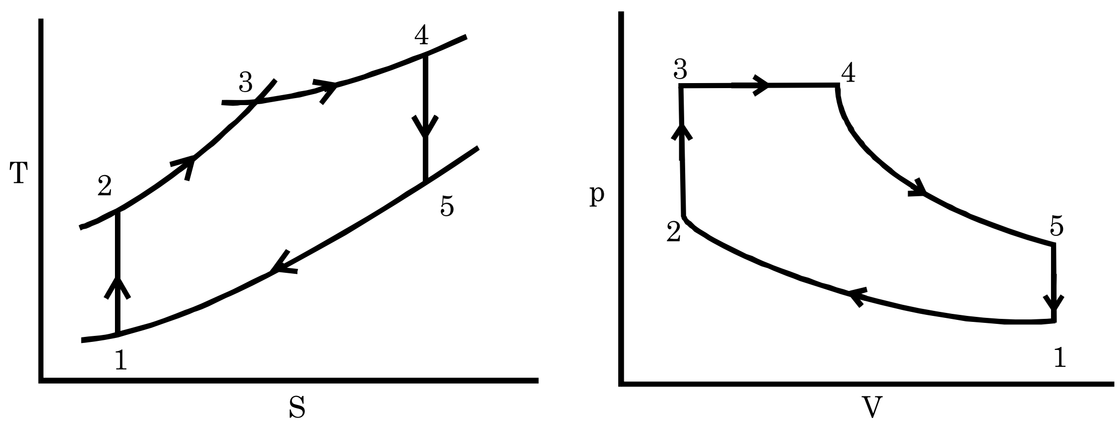
(**Ans.** (a) T_2 = 805 K, p_2 = 29.2 p_1 ′ T_3 = 1490 K, T_4 = 2410 K, T_5 = 1200 K, (b) η = 0.67)

Solution:

Here $\frac{v_1}{v_2} = r_c = 12$

$\frac{v_4}{v_3} = \rho = 1.615$ $pv^{1.35} = C$, n = 1.35

$p_{max} = p_3 = p_4 = 54\, p_1$



$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad \therefore T_2 = T_1 \times (12)^{(1.35-1)} = 2.3862\, T_1$$

$$\text{And} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{n} \quad \therefore p_2 = p_1 \times (12)^{1.35} = 28.635\, p_1$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \quad \therefore T_3 = \frac{p_3}{T_2} \times T_2 = \frac{54 p_1}{28.635 p_1} \times 2.3862\, T_1 = 4.5\, T_1$$

$$v_3 = v_2 = \left(\frac{v_1}{12}\right)$$

$$\therefore \quad v_4 = \rho\, v_3 = \frac{1.615}{12} v_1 = 0.13458\, v_1$$

$$\therefore \quad \frac{p_4 v_4}{T_4} = \frac{P_3 v_3}{T_3} \qquad p_3 = p_4$$

$$T_4 = T_3 \times \frac{v_4}{v_3} = 1.615\, T_3 = 1.615 \times 4.5\, T_1 = 7.2675\, T_1$$

$$\therefore \quad \frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{n-1} = \left(\frac{v_4}{v_1}\right)^{n-1}$$

$\therefore$ $T_5 = 3.6019\, T_1$

$\therefore$ W = [$C_v (T_3 - T_2) + C_P (T_4 - T_3) - C_v (T_5 - T_1)$ = 2.4308 T_1 kJ/kg.

$$p_m\ (v_1 - v_2) = W$$

$$\therefore \quad p_m = \frac{2.4308\ T_1}{v_1 - \frac{v_1}{12}} = \frac{2.4308\ p_1}{\frac{11}{12}\times R} = 9.25\ p_1$$

(b) $\therefore \quad \eta = \dfrac{2.4308\ T_1}{4.299\ T_1} \times 100\ \% = 56.54\%$

(a) $T_1 = 335$ K, $T_2 = 799.4$ K, $T_3 = 1507.5$ K, $T_4 = 2434.6$ K, $T_5 = 1206.6$ K.

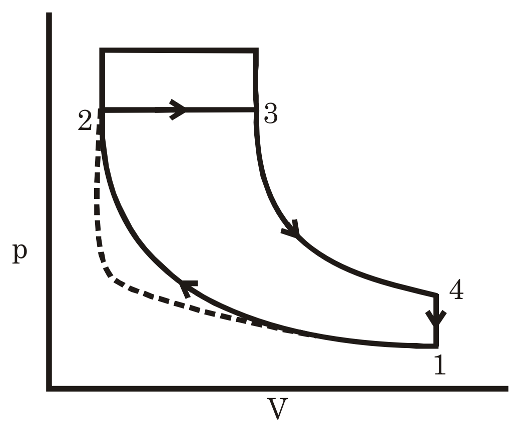
Q13.16 **Recalculate (a) the temperatures at the cardinal points, (b) the m.e.p., and (c) the cycle efficiency when the cycle of Problem 13.15 is a Diesel cycle with the same compression ratio and a cut-off ratio such as to give an expansion curve coincident with the lower part of that of the dual cycle of Problem 13.15.**

(**Ans**. (a) $T_2 = 805$ K, $T_3 = 1970$ K, $T_4 = 1142$ K
(b) $6.82\ p_1$, (c) $\eta = 0.513$)

Solution:

Given $\dfrac{v_1}{v_2} = 12 = r_c$

$\dfrac{v_3}{v_2} = \rho = 1.615$



$\therefore\ T_3 = \dfrac{v_3}{v_2} \times T_2 = 1.615 \times 799.4 = 1291$ K

Then $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{n-1}$

$\therefore\ T_2 = T_1\left(12\right)^{1.35-1}\ 1 = 799.4$ K

But $\dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^{n}$

Continue to try…..

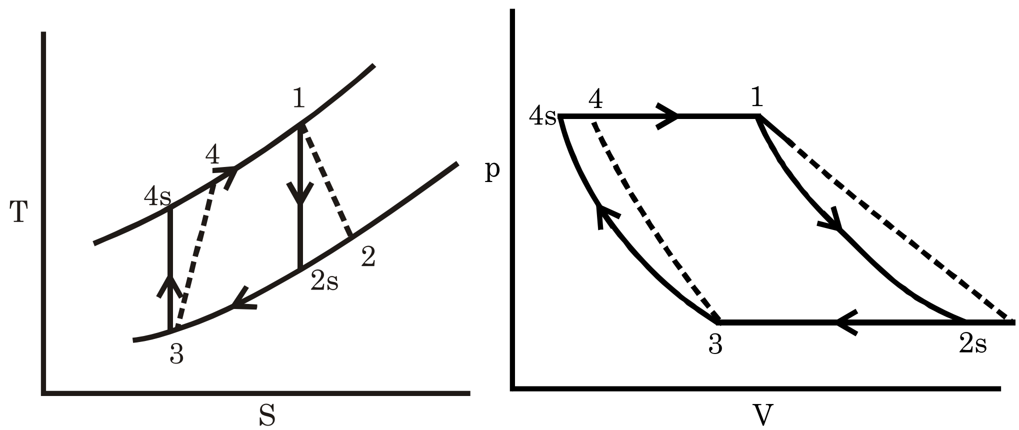
Q13.19 **In a gas turbine plant working on the Brayton cycle the air at the inlet is at 27°C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C. The turbi- ne and compressor efficiencies are each 80%. Find (a) the compressor work per kg of air, (b) the turbine work per kg of air, (c) the heat supplied per kg of air, (d) the cycle efficiency, and (e) the turbine exhaust temperature.**

(**Ans.** (a) 259.4 kJ/kg, (b) 351.68 kJ/kg, (c) 569.43 kJ/kg,
(d) 16.2%, (e) 723 K)

Solution: Maximum Temperature
$T_1 = 800°$ C $= 1073$ K
$p_3 = 100$ kPa
$T_3 = 300$ K

$r_p = 6.25$

$\frac{p_4}{p_3} = 6.25$



$\therefore \quad p_4 = 625 \text{ kPa}$ $\qquad\qquad p_2 = 100 \text{ kPa}$

$p_1 = p_4$

$\therefore \quad v_3 = \frac{RT_3}{p_3} = 0.861$ $\qquad\qquad v_3 = 0.861$

$\frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} \quad \therefore \quad \frac{v_4}{v_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{\gamma}}$ $\qquad\qquad T_3 = 300 \text{ K}$

$v_4 = v_3 \times \left(\frac{p_3}{p_4}\right)^{\frac{1}{4}}$ $\qquad\qquad p_2 = p_3$

$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1}$

$p_4 = 625 \text{ kPa}$ $\qquad \therefore T_4 = T_3 \times (3.70243)^{0.4}$ $\qquad v_{4s} = 0.23255$

$\therefore 0.8 = \frac{T_{4s} - T_3}{T_4 - T_3}$ $\qquad \therefore T_4 = 558$

$T_{4s} = 506.4 \text{ K}$ $\qquad T_{2s} = 635.6 \text{ K}$

$$\frac{T_1}{T_{2s}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = 1.68808$$

$T_4 = 558 \text{ K}$ $\qquad T_2 = 723 \text{ K}$

$$\eta = \frac{T_1 - T_2}{T_1 - T_{2s}} \Rightarrow T_1 - T_2 = 350$$

$\therefore \quad T_2 = T_1 - 350 = 723 \text{ K}$

(a) Compressor work $(W_c) = (h_4 - h_3) = C_p(T_4 - T_3) = 259.3$ kJ/kg

(b) Turbine work $\left(W_T\right)$ = $(h_1 - h_2) = C_p(T_1 - T_2) = 351.75$ kJ/kg

(c) Heat supplied $(Q_1) = C_p(T_1 - T_4) = 517.6$ kJ/kg

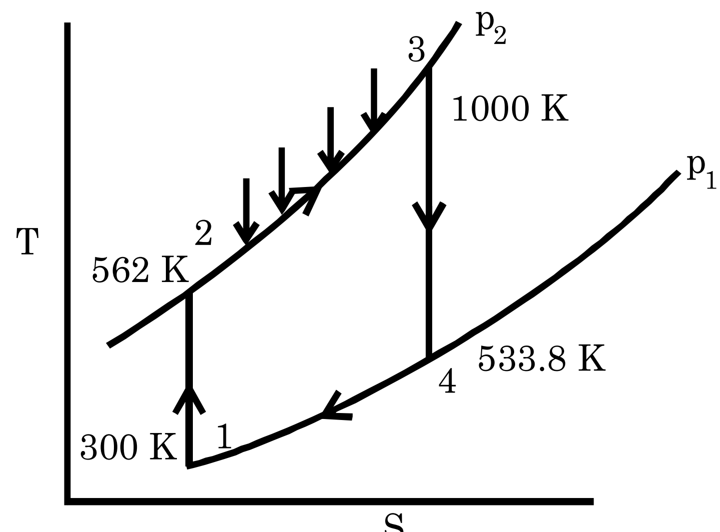
(d) Cycle efficiency $(\eta) = \dfrac{W_T - W_C}{Q_1} = 17.86\%$

(e) Turbine exhaust temperature $(T_2) = 723$ K

Q13.27 A simple gas turbine plant operating on the Brayton cycle has air inlet temperature 27°C, pressure ratio 9, and maximum cycle temperature 727°C. What will be the improvement in cycle efficiency and output if the turbine process Is divided into two stages each of pressure ratio 3, with intermediate reheating to 727°C?

(**Ans. -** 18.3%, 30.6%)

Solution:



(a)

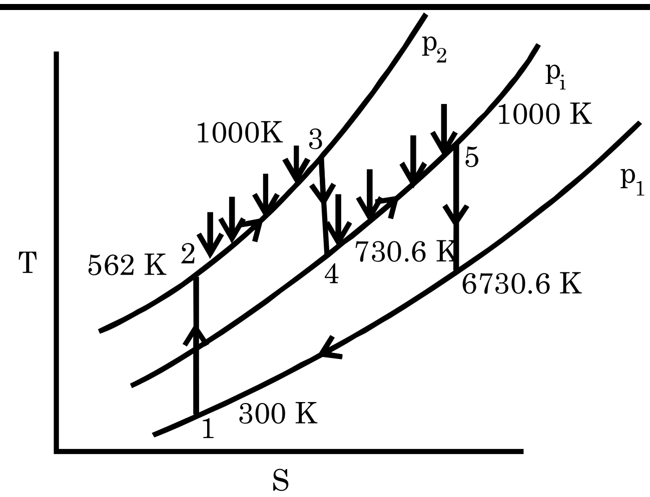
For (a) $T_1 = 300$ K

$\dfrac{p_2}{p_1} = 9$

$T_3 = 1000$ K

$\therefore \quad T_2 = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = 562$ k

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{9}\right)^{\frac{\gamma-1}{\gamma}} \qquad \therefore T_4 = \frac{T_3}{9^{\frac{\gamma-1}{\gamma}}} = 533.8 \text{ K}$$



(b)

For (b) $\frac{T_4}{T_3} = \left(\frac{p_i}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore T_4 = T_3 \times \left(\frac{1}{3}\right)^{\frac{\gamma-1}{\gamma}} = 730.6$ K

$\frac{T_6}{T_5} = \left(\frac{p_1}{p_i}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore T_6 = T_5 \times \left(\frac{1}{3}\right)^{\frac{\gamma-1}{\gamma}} = 730.6$ K

$\therefore$ For (a) $W = (h_3 - h_4) - (h_2 - h_1)$
$= C_p [T_3 - T_4) - (T_2 - T_1)] = 205.22$ kJ/kg
$Q = h_3 - h_2 = C_P (T_3 - T_2) = 440.19$ kJ/kg
$\therefore$ $\eta = 46.62$ %

For (b) $W = (h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)$
$= C_P [(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)] = 278.18$ kJ/kg

$Q = h_3 - h_2 + h_5 - h_4 = C_P [(T_3 - T_2) + (T_5 - T_4)] = 710.94$ kJ/kg

$\therefore$ $\eta = 39.13$ %

$\therefore$ Efficiency change $= \frac{39.13 - 46.62}{46.62} \times 100\% = -16.07$ %

Work output change $= \frac{278.18 - 205.22}{205.22} \times 100 = 35.6\%$

Q13.28 **Obtain an expression for the specific work output of a gas turbine unit in terms of pressure ratio, isentropic efficiencies of the compressor and turbine, and the maximum and minimum temperatures, T_3 and T_1 •**

Hence show that the pressure ratio r_p for maximum power is given by

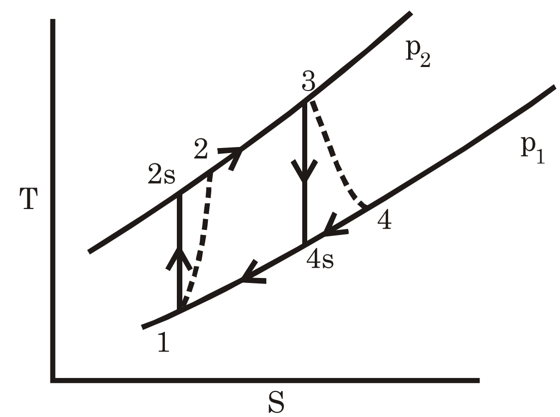
$$r_p = \left(\eta_T \eta_C \frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)}$$

If T_3 = 1073 K, T_1 = 300 K, η_C = 0.8, η_T = 0.8 and γ = 1.4 compute the optimum
Value of pressure ratio, the maximum net work output per kg of air, and corresponding cycle efficiency.

(**Ans.** 4.263, 100 kJ/kg, 17.2%)

Solution:

$T_1 = T_{min}$ $T_3 = T_{max}$



Hence

$$T_{2s} = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= T_1 \times r_P^{\frac{\gamma-1}{\gamma}}$$

Let $r_p^{\frac{\gamma-1}{\gamma}} = x$

$\therefore$ $T_{2s} = x\, T_1$

$\therefore$ If isentropic efficiency and compressor is η_c

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$\therefore$ $$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C} = T_1\left[1 + \frac{x-1}{\eta_C}\right]$$

Similarly $$T_{4s} = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{x}$$

$\therefore$ If isentropic efficiency of turbine is η_T

Then $$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4S}} \Rightarrow -T_3 + T_4 = \eta_T\ (T_{4s} - T_3)$$

$$T_4 = T_3 + \eta_T\left(\frac{T_3}{x} - T_3\right)$$

$$= T_3\left[1 + \eta_T\left(\frac{1}{x} - 1\right)\right]$$

$\therefore$ Specific work output

$$W = (h_3 - h_4) - (h_2 - h_1)$$

$$= C_P [(T_3 - T_4) - (T_2 - T_1)]$$

$$= C_P\left[\eta_T\left(T_3 - \frac{T_3}{x}\right) - \frac{xT_1 - T_1}{\eta_C}\right] \text{kJ/kg}$$

$$= C_P\left[\eta_T T_{max}\left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right) - \frac{T_{min}}{\eta_C}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)\right] \text{kJ/ kg}$$

For maximum Sp. Work $\frac{dW}{dx} = 0$

$$\therefore \quad \frac{dW}{dx} = C_P\left[\frac{\eta_T T_3}{x^2} - \frac{T_1}{\eta_C}\right] = 0$$

$$\therefore \quad x^2 = \eta_T \eta_C \frac{T_3}{T_{T_1}}$$

$$\therefore \quad x = \sqrt{\eta_T \eta_C \frac{T_{max}}{T_{min}}}$$

$$\therefore \quad r_P = \left(\eta_T \eta_C \frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}} \quad \text{Proved.}$$

$\Rightarrow$ If $T_3 = 1073$ K, $T_1 = 300$K, $\eta_1 = 0.8$, $\eta_7 = 0.8$, $\gamma = 1.4$ then

$$(r_p)_{opt.} = \left(0.8 \times 0.8 \times \frac{1073}{300}\right)^{\frac{1.4}{2(1.4-1)}} = 4.26$$

$$(r_p)_{opt}^{\frac{\gamma-1}{\gamma}} = x = 1.513$$

$$\therefore \; W_{max} = C_p\left[\eta_T \; T_3\left(1 - \frac{1}{x}\right) - \frac{T_1(x-1)}{\eta_c}\right]$$

$$= 1.005\left[0.8 \times 1073\left(1 - \frac{1}{1.513}\right) - \frac{300}{0.08}(1.513 - 1)\right] \text{kJ/kg}$$

$$= 99.18 \text{ kJ/kg}$$

Heat input $Q_1 = h_3 - h_2 = C_p (T_3 - T_2)$ $\qquad T_2 = T_1\left[1 + \frac{x-1}{\eta_c}\right]$

$= 1.005\,(1073 - 492.4)$ $\qquad = 492.4$ K

$= 583.5$ kJ/kg

$$\therefore \eta = \frac{99.18}{583.5} \times 100\% = 17\%$$

Q13.29 **A gas turbine plant draws in air at 1.013 bar, 10°C and has a pressure ratio of 5.5. The maximum temperature in the cycle is limited to 750°C. Compression is conducted in an uncooled rotary compressor having an isentropic efficiency of 82%, and expansion takes place in a turbine with an isentropic efficiency of 85%. A heat exchanger with an efficiency of 70% is fitted between the compressor outlet and combustion chamber. For an air flow of 40 kg/s, find (a) the overall cycle efficiency, (b) the turbine output, and (c) the air-fuel ratio if the calorific value of the fuel used is 45.22 MJ/kg.**

(**Ans.** (a) 30.4%, (b) 4272 kW, (c) 115)

Solution: $p_1 = 101.3$ kPa

$T_1 = 283$ K

$$\frac{p_2}{p_1} = 5.5 \text{ kPa}$$

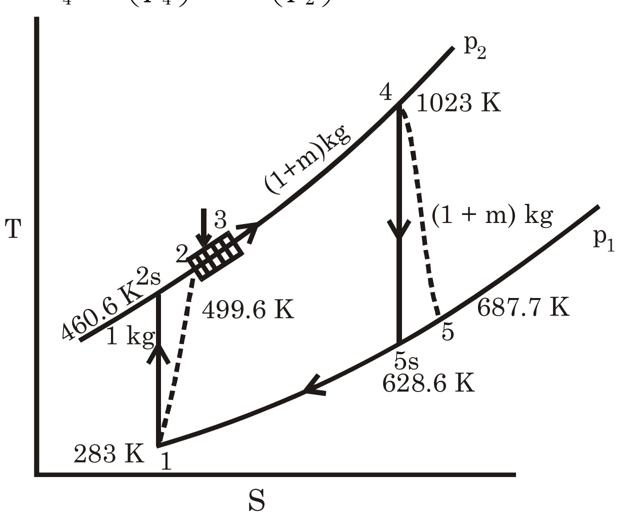
$T_4 = 750°C = 1023$ K

$$\therefore \frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2s} = 460.6 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \therefore T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

$$= 499.6\text{K}$$

$$\therefore \frac{T_{5s}}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5.5}\right)^{\frac{\gamma-1}{\gamma}}$$



$$\therefore \quad T_{5s} = T_4 \times \left(\frac{1}{5.5}\right)^{\frac{1.4-1}{1.4}} = 628.6\text{K}$$

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5s}} \therefore T_4 - T_5 = \eta_T \quad (T_4 - T_{5s}) = 335.3 \text{ K}$$

$\therefore$ $T_5 = 687.K$

Maximum possible heat from heat exchanger = $C_p (T_5 - T_2)$
$\therefore$ Actual heat from = $0.7C_p (T_5 - T_2) = 132.33$ kJ/kg of air
$\therefore$ $C_p (T_3 - T_2) = (1 + m)\ 132.33$ and

$C_pT_3 = 132.33 + 132.33\ m + C_pT_2 = 634.43 + 132.33\ m$
Heat addition $(Q_1) = C_p (T_4 - T_3) = C_pT_4 - C_pT_3$
$= 393.7 - 132.33m = m \times 45.22 \times 10^3$
$\therefore$ $m = 8.68 \times 10^{-3}$ kJ/kg of air
$\therefore$ $Q_1 = 392.6$ kJ/kg of air
$W_T = (1 + m)(h_4 - h_5) = (1 + m) C_p (T_4 - T_5)$
$= 1.00868 \times 1.005 \times (1023 - 687.7)$ kJ/kg of air 340 kJ/kg

$W_c = (h_2 - h_1) = Cp (T_2 - T_1) = 1.005 \times (499.6 - 283)$
$= 217.7$ kJ/kg of air

$\therefore$ $W_{net} = W_T - W_c = 122.32$ kJ/kg

(a) $\eta = \dfrac{122.32}{392.6} \times 100\% = 31.16\%$

(b) Turbine output = $(W_T) = 122.32$ kJ/kg of air
= 4893 kW

(c) Air fuel ratio = $\dfrac{1 \text{ kg air}}{0.00868 \text{ kg of fuel}}$ = 115.2 kg of air/kg of fuel

Q13.30 **A gas turbine for use as an automotive engine is shown in Fig. 13.43. In the first turbine, the gas expands to just a low enough pressure p_5, for the turbine to drive the compressor. The gas is then expanded through a second turbine connected to the drive wheels. Consider air as the working fluid, and assume that all processes are ideal. Determine (a) pressure p_5 (b) the net work per kg and mass flow rate, (c) temperature T_3 and cycle thermal efficiency, and (d) the $T - S$ diagram for the cycle.**

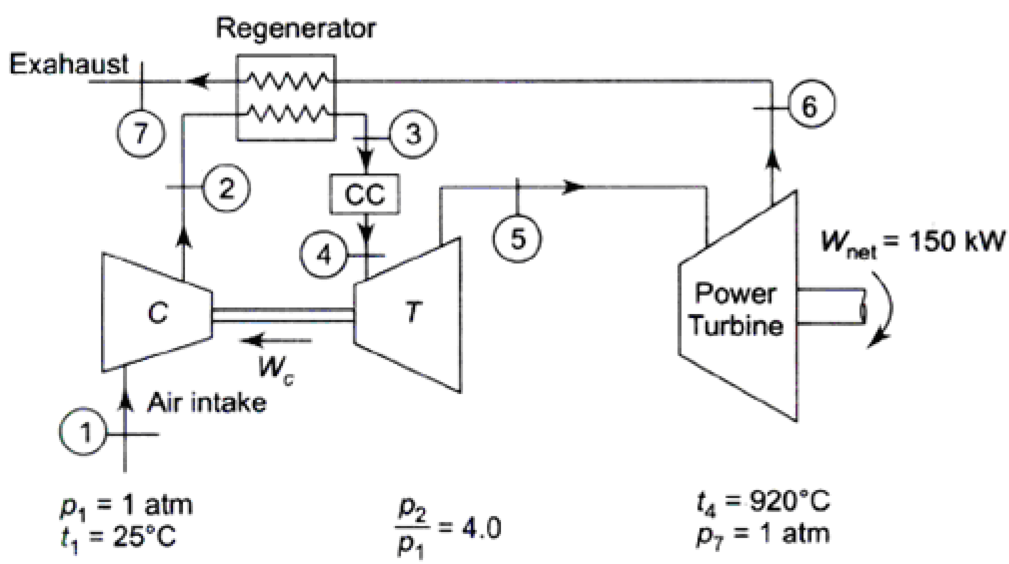


Fig.

Solution : Try please.

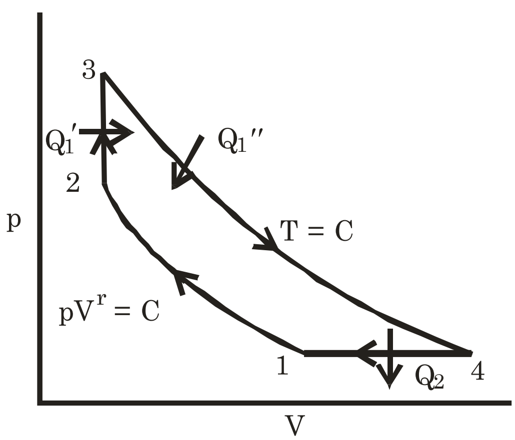
Q13.31 **Repeat Problem 13.30 assuming that the compressor has an efficiency of 80%, both the turbines have efficiencies of 85%, and the regenerator has an efficiency of 72%.**

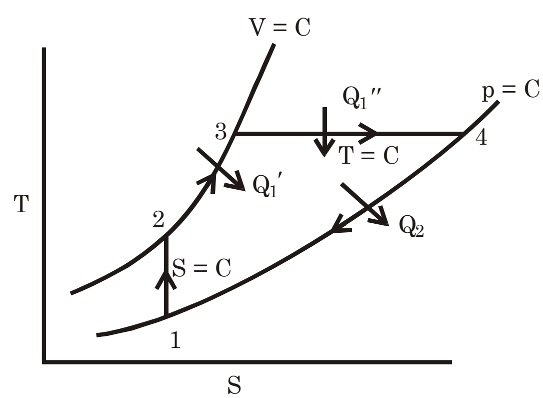
Solution: Try please.

Q13.32 **An ideal air cycle consists of isentropic compression, constant volume heat transfer, isothermal expansion to the original pressure, and constant pressure heat transfer to the original temperature. Deduce an expression for the cycle efficiency in terms of volumetric compression ratio r_k , and isothermal expansion ratio, r_k In such a cycle, the pressure and temperature at the start of compression are 1 bar and 40°C, the compression ratio is 8, and the maximum pressure is 100 bar. Determine the cycle efficiency and the m.e.p.**

(**Ans**. 51.5%, 3.45 bar)

Solution:





$\therefore$ Compression ratio, $r_c = \dfrac{V_1}{V_2}$

Expansion ratio, $r_e = \dfrac{V_4}{V_3} = \dfrac{V_4}{V_2}$

Heat addition $Q_1 = Q_1' + Q_1''$

= constant volume heat addition

(Q_1' + constant temperature heat addition Q_1'')

Heat rejection, $Q_2 = C_p\,(T_4 - T_1)$

Hence $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$

$\therefore T_2 - T_1 \,.\, r_c^{\gamma-1}$ and $v_2 = \dfrac{v_1}{r_c}$ and $p_2 = p_1\, r_c^{\gamma}$

$T_3 = T_4$

$\therefore \dfrac{p_1\, v_3}{T_3} = \dfrac{p_4\, v_4}{T_4}$ $\quad\therefore \dfrac{v_4}{v_3} = \dfrac{p_3}{p_4} = \dfrac{p_3}{p_1} = r_e$

$\therefore\ p_3 = p_1\ .\ r_e$

$\therefore\ \dfrac{p_2}{T_2} = \dfrac{p_3}{T_3}$ or $T_3 = \dfrac{p_3}{p_2} \times T_2 = \dfrac{r_e}{r_c^{\gamma}} \times T_1 \,.\, r_c^{\gamma-1} = T_1 \dfrac{r_e}{r_c} = T_4$

$$\therefore\quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p(T_4 - T_1)}{C_v(T_3 - T_2) + RT_3 \ln r_e}$$

$$= 1 - \frac{C_p\left(T_1 \,.\, \frac{r_e}{r_c} - T_1\right)}{C_v\left(T_1 \,.\, \frac{r_e}{r_c} - T_1\, r_c^{\gamma-1}\right) + R\,.\,T_1 \frac{r_e}{r_c} \ln r_e}$$

$$= 1 - \frac{\gamma\left(\frac{r_e}{r_c} - 1\right)}{\left(\frac{r_e}{r_c} - r_c^{\gamma-1}\right) + (\gamma - 1)\frac{r_e}{r_c} \ln r_e}$$

$$= 1 - \frac{\gamma(r_e - r_c)}{(r_e - r_c^{\gamma}) + (\gamma - 1)\, r_e \,l\text{n}\, r_e}$$

$$\therefore\quad \eta = 1 - \frac{\gamma[r_e - r_c]}{(r_e - r_c^{\gamma}) + (\gamma - 1)\, r_e \,l\text{n}\, r_e}$$

Given p_1 = 1 bar = 100 kPa

T_1 = 40°C = 313 K

r_c =8 and p_3 = 100 bar = 10000 kPa

$\therefore\ p_3 = p_1 \,.\, r_e \quad \therefore\ r_e = \dfrac{p_3}{p_1} = 100$

$$\therefore\ \eta = 1 - \frac{1.4\,(100 - 8)}{(100 - 8^{1.4}) + (1.4 - 1 \times \ln 100}$$

$$= 1 - \frac{128.8}{265.83}$$

$= 0.51548 = 51.548\ \%$

$\Rightarrow T_3 = T_1 \times \dfrac{r_e}{r_c} = \dfrac{313 \times 100}{8} = 3912.5$ K

$T_2 = T_1 \times r_c^{\gamma-1} = 719$ K

$\therefore$ Heat addition, $Q = C_v\,(T_3 - T_2) + R\,T_3 \ln r_e$

$= 0.718\,(3912.5 - 719) + 0.287 \times 3912.5 \times \ln 100$

$= 7464$ kJ/kg

$\therefore$ Work, $W = Q\,\eta = 3847.5$ kJ/kg

$\therefore\ p_m\,(V_4 - V_2) = W \qquad \therefore\ v_4 = 100\ v_2 \qquad v_2 = \dfrac{v_1}{r_c}$

$\therefore\ p_m\,(100 - 1)\ v_2 = W$

$\therefore\ p_m\,(99) \times \dfrac{v_1}{8} = W$

$$\therefore\ p_m = \frac{8\text{ W}}{99 \times v_1} = 346.1\text{ kPa} \qquad v_1 = \frac{RT_1}{p_1} = 0.89831\text{ kJ/kg}$$

$$= 3.461\text{ bar}$$

$$\therefore\ p_m\ (v_4 - V_3) = 40\ 58$$

$$\therefore\ p_m = \frac{4058}{v_4 - \frac{v_4}{100}} = 365\text{ bar}$$

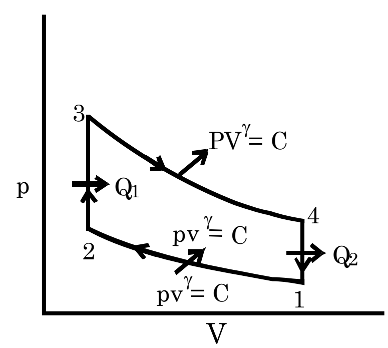
Q13.37 **Show that the mean effective pressure, p_m' for the Otto cycle is Given by**

$$p_M = \frac{\left(p_3 - p_1 r_k^{\gamma}\right)\left(1 - \frac{1}{r_k^{\gamma-1}}\right)}{(\gamma - 1)(r_k - 1)}$$

Where $p_3 = p_{max}$ **'** $p_1 = p_{min}$ **and** r_k **is the compression ratio.**

Solution:

Intake p_1, v_1, T_1



$$\therefore\ \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\therefore\ T_2 = T_1 \cdot r_c^{\gamma-1}$$

$$p_2 = p_1 \times r_c^{\gamma}$$

$$v_2 = \frac{v_1}{r_c}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$\therefore\ T3 = T_2 \times \frac{p_3}{p_2} = T_2 \times \frac{p_3}{p_1\ r_c^{\gamma}} = T_1\ \frac{r_c^{\gamma-1} \times p_3}{p_1\ r_c^{\gamma}} = \frac{T_1 \times p_3}{r_c\ p_1}$$

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\therefore\ T_4 = \frac{T_3}{r_c^{\gamma-1}} = \frac{T_1\ p_3}{r_c\ p_1 \times r_c^{\gamma-1}} = \frac{T_1\ p_3}{r_c^{\gamma}\ p_1}$$

$$W = Q_1 - Q_2$$

$$= C_v\ (T_3 - T_2) - C_v\ (T_4 - T_1)$$

$$\therefore\ p_m\ (V_1 - V_2) = W$$

$$\therefore\ p_m = \frac{C_v\ [(T_3 - T_2) - (T_4 - T_1)]}{V_1 - V_2}$$

$$= \frac{c_v \left[\dfrac{T_1\, p_3}{r_c\, p_1} - T_1\, r_c^{\gamma-1} - \dfrac{T_1\, p_3}{r_c^{\gamma}\, p_1} + T_1\right]}{v_1 - \dfrac{v_1}{r_c}}$$

$$= \frac{c_V\, T_1}{V_1\, p_1}\left[\frac{p_3 - p_1\, r_c^{\gamma} - \dfrac{p_3}{r_c^{\gamma-1}} + p_1\, r_c}{(r_c - 1)}\right]$$

$$\left[\begin{array}{l} c_V = \dfrac{R}{\gamma - 1} \\ \because p_1\, V_1 = RT_1 \end{array}\right.$$

$$= \frac{RT_1}{V_1\, p_1}\, \frac{[(p_3 - p_1\, r_c^{\gamma}) - \dfrac{p_3}{r_c^{\gamma-1}} + (p_3 - p_1\, r_c^{\gamma})]}{(\gamma - 1)\,(r_c - 1)}$$

$$= \frac{(p_3 - p_1\, r_c^{\gamma})\left(1 - \dfrac{1}{r_c^{\gamma-1}}\right)}{(\gamma - 1)(r_c - 1)} \text{ Proved}$$

Q13.38 **A gas turbine plant operates on the Bray ton cycle using an optimum pressure ratio for maximum net work output and a regenerator of 100% effectiveness. Derive expressions for net work output per kg of air and corresponding efficiency of the cycle in terms of the maximum and the minimum temperatures.**
If the maximum and minimum temperatures are 800°C and 30°C respectively, compute the optimum value of pressure ratio, the maximum net work output per kg and the corresponding cycle efficiency.

(Ans. $\left(W_{net}\right)_{\max} = C_p\left(\sqrt{T_{\max}} - \sqrt{T_{\min}}\right)^2 \; \left(\eta_{cycle}\right)_{\max} = 1 - \sqrt{\dfrac{T_{\min}}{T_{\max}}}, \left(r_p\right)_{opt} = 9.14$

$\left(W_{net}\right)_{\max} = 236.97$ kJ/kg; $\eta_{cycle} = 0.469$)

Solution:

$T_1 = T_{\min}$

$T_4 = T_{\max}$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} = x \text{ (say)}$$

$$\therefore \quad T_2 = T_1\, x$$

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{x}$$

$$\therefore \quad T_5 = \frac{T_4}{x}$$

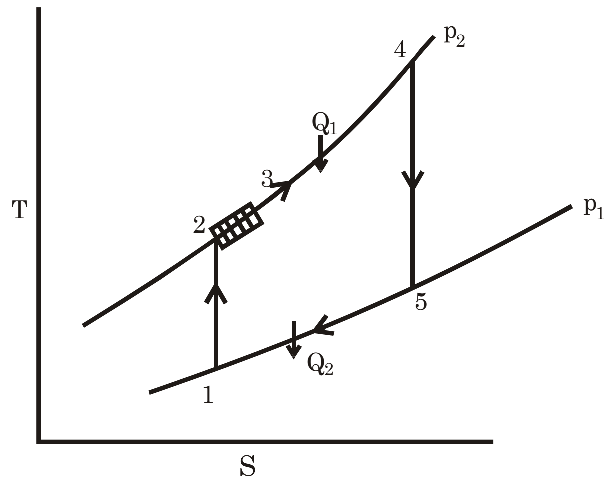
For regeneration 100% effective number

$$C_p\,(T_5 - T_2) = C_p\,(T_3 - T_2)$$

$$\therefore \quad T_3 = T_5 = \frac{T_4}{x}$$

$$W_T = h_4 - h_5 = C_p\,(T_4 - T_5)$$

$$= C_p\left(T_4 - \frac{T_4}{x}\right)$$



And $\quad W_c = h_2 - h_1 \qquad = C_pT_4\left(1 - \frac{1}{x}\right)$

$$= C_p\,(T_2 - T_1)$$
$$= C_p\,T_1\,(x - 1)$$

$$\therefore \qquad W_{net} = W_T - W_C = C_p\left[T_4\left(1 - \frac{1}{x}\right) - T_1\,(x - 1)\right]$$

For Maximum Net work done

$$\frac{\partial\, W_{net}}{\partial x} = 0 \;\therefore\; T_4 \times \frac{1}{x^2} - T_1 = 0$$

$$\therefore \qquad x^2 = \frac{T_4}{T_1} = \frac{T_{max}}{T_{min}}$$

$$\therefore \qquad x = \sqrt{\frac{T_{max}}{T_{min}}}$$

Heat addition $\therefore\; (\;r_p\,)_{opt.} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$

$$Q_1 = h_4 - h_3 = C_p\,(T_4 - T_3) = C_p\left(T_4 - \frac{T_4}{x}\right)$$

$$= C_pT_4\left(1 - \frac{1}{x}\right)$$

$$= C_pT_4\left[1 - \sqrt{\frac{T_1}{T_4}}\right]$$

$$\therefore \qquad \eta_{opt.} = \frac{W_{net}}{Q_1} = \frac{T_4\left(1 - \frac{1}{x}\right) - T_1\,(x - 1)}{T_4\left(1 - \frac{1}{x}\right)}$$

$$= 1 - \frac{T_1}{T_4} \times x = 1 - \frac{T_1}{T_4} \times \sqrt{\frac{T_4}{T_1}} = 1 - \sqrt{\frac{T_{min}}{T_{max}}}$$

$$W_{opt.} = C_p[T_4 - \sqrt{T_1T_4} - \sqrt{T_1T_4} + T_1]$$

$$= C_p[\sqrt{T_4} - \sqrt{T_1}]^2 = C_p[\sqrt{T_{max}} - \sqrt{T_{min}}]^2$$

If T_{max} = 800°C = 1073 K; T_{min} = 30°C = 303K

$$\therefore \quad r_{p,opt} = \left(\frac{1073}{303}\right)^{\frac{1.4}{2(1.4-1)}} = 9.14$$

$$\eta_{opt.} = 1 - \sqrt{\frac{T_{min}}{T_{max}}} = 46.9\%$$

$$W_{opt.} = 1.005\,(\sqrt{1073} - \sqrt{303})^2 = 236.8 \text{ kJ/kg}$$

Q13.40 **Show that for the Sterling cycle with all the processes occurring reversibly but where the heat rejected is not used for regenerative heating, the efficiency is giver: by**

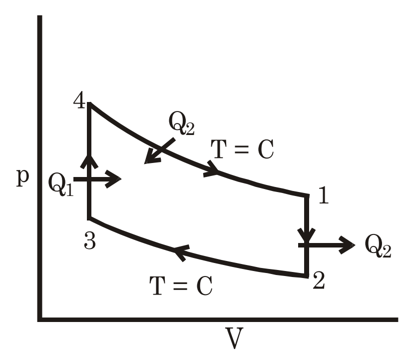
$$\eta = 1 - \frac{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\ln r}{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\frac{T_1}{T_2}\ln r}$$

Where *r* is the compression ratio and T_1/T_2 the maximum to minimum temperature ratio.

Determine the efficiency of this cycle using hydrogen *(R* = 4.307 kJ/kg K, c_p =. 14.50 *kJ/kg* K) with a pressure and temperature prior to isothermal.

Compression of 1 bar and 300 K respectively, a maximum pressure of2.55 MPa and heat supplied during the constant volume heating of 9300 *kJ/kg*. If the heat rejected during the constant volume cooling can be utilized to provide the constant volume heating, what will be the cycle efficiency? Without altering the temperature ratio, can the efficiency be further improved in the cycle?

Solution:



Minimum temperature

$(T_2) = T_{min}$

Maximum temperature

$(T_1) = T_{max}$

$\therefore$ Compression ratio

$$(r_c) = \frac{v_2}{v_3} = \frac{v_1}{v_4}$$

$\therefore$ $T_1 - T_4$ and $T_3 = T_2$

$$W_T = RT_1 \ln\frac{v_1}{v_4} = RT_1 \ln\ r_c$$

$$W_C = RT_2 \ln\left(\frac{v_2}{v_3}\right) = RT_2 \ln\ r_c$$

$\therefore$ $W_{net} = R\ln(r_c) \times [T_1 - T_2]$

Constant volume Heat addition (Q_1) = C_v ($T_1 - T_2$)

$$= \frac{R}{\gamma - 1}(T_1 - T_2)$$

Constant temperature heat addition $Q_2 = RT_2 \ln\, r_c$

$$\therefore \text{ Total heat addition } Q = Q_1 + Q_2 = R\left[T_1 \ln r_c \frac{(T_1 - T_2)}{(\gamma - 1)}\right]$$

$$\eta = \frac{W_{net}}{Q} = \frac{\ln r_c\,[T_1 - T_2]}{\left[T_1 \ln r_c - \frac{T_2 - T_1}{r - 1}\right]} = \frac{(\gamma - 1)\ln r_c\,(T_1 - T_2)}{(\gamma - 1)\,T_1 \ln r_c - (T_2 - T_1)} - 1 + 1$$

$$= 1 - \left[1 - \frac{(\gamma - 1)\ln r_c\,(T_1 - T_2)}{(\gamma - 1)\ln r_c - (T_2 - T_1)}\right]$$

$$= 1 - \frac{(\gamma - 1)\,T_1 \ln r_c - (T_2 - T_1) - (\gamma - 1)\ln r_c\,T_1 + (\gamma - 1)\,T_2 \ln r_c}{(\gamma - 1)\ln r_c - (T_2 - T_1)}$$

$$= 1 - \frac{(T_1 - T_2) + (\gamma - 1)\,T_2 \ln r_c}{(T_1 - T_2) + (\gamma - 1)\,T_1 \ln r_c}$$

$$= 1 - \frac{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\ln r_c}{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\frac{T_1}{T_2}\ln r_c} \quad \text{Proved}$$

Q13.41 **Helium is used as the working fluid in an ideal Brayton cycle. Gas enters the compressor at 27°*C* and 20 bar and is discharged at 60 bar. The gas is heated to 1000°*C* before entering the turbine. The cooler returns the hot turbine exhaust to the temperature of the compressor inlet. Determine: (a) the temperatures at the end of compression and expansion, (b) the heat supplied, the heat rejected and the net work per kg of He, and (c) the cycle efficiency and the heat rate. Take c_p = 5.1926 *kJ/kg* K.**

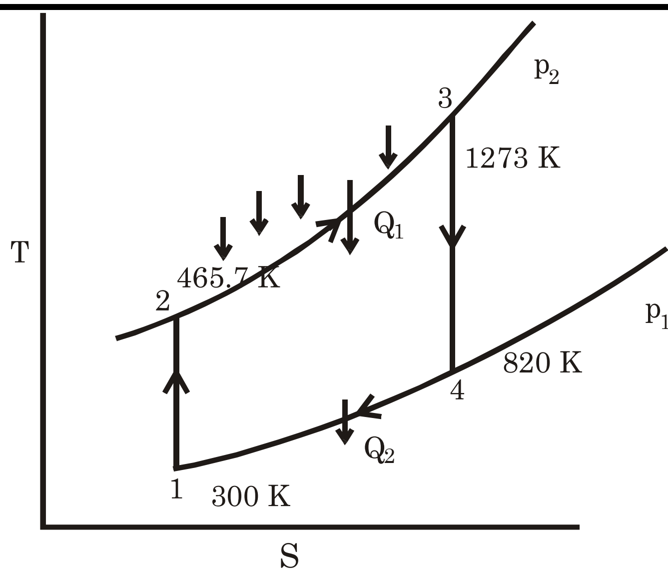
(**Ans.** (a) 4 65.5, 820.2 K, (b) 4192.5, 2701.2, 1491.3 *kJ/kg,* (c) 0.3557, 10,121kJ/kWh)

Solution:
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \frac{60}{20}$$

$C_p = 5.1926$, $R = 2.0786$

$$\therefore \quad c_v = c_p - R = 3.114$$

$$\gamma = \frac{c_p}{c_v} = \frac{5.1926}{3.114} = 1.6675 \quad \therefore \frac{\gamma - 1}{\gamma} = 0.4$$



$$\therefore \quad T_2 = T_1 \times \left(\frac{60}{20}\right)^{\frac{\gamma-1}{\gamma}} = 465.7 \text{ K}$$

$\therefore$

$$\therefore \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{20}{60}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_4 = T_3 \times \frac{1}{3^{\frac{\gamma-1}{\gamma}}} = 820 \text{ K}$$

(a) End of compressor temperature T_2 = 465.7K
End of expansion temperature T_4 = 820K

(b) Heat supplied $(Q_1) = h_3 - h_2 = C_P (T_3 - T_2)$ = 4192 kJ/kg
Heat rejected $(Q_2) = h_4 - h_1 = C_P (T_4 - T_1)$ = 2700 kJ/kg
Work, $W = Q_1 - Q_2$ = 1492 kJ/kg

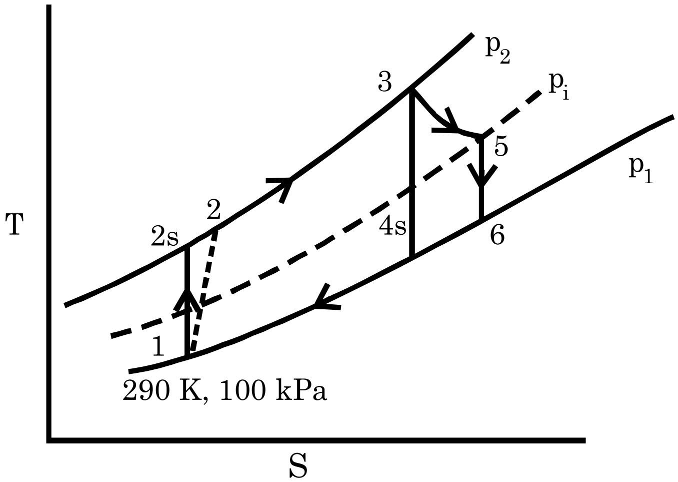
(c) $$\eta = \frac{W}{Q_1} = \frac{1492}{4192} \times 100\% = 35.6\%$$

$$\text{Heat rate} = \frac{3600}{\eta} = \frac{3600}{0.356} = 10112 \text{ kJ/kWh}$$

Q13.42 **An air standard cycle for a gas turbine jet propulsion unit, the pressure and temperature entering the compressor are 100 kPa and 290 K, respectively. The pressure ratio across the compressor is 6 to 1 and the temperature at the turbine inlet is 1400 K. On leaving the turbine the air enters the nozzle and expands to 100 kPa. Assuming that the efficiency of the compressor and turbine are both 85% and that the nozzle efficiency is 95%, determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.**

(**Ans**. 285 kPa, 760 m/s)

Solution: $\frac{p_2}{p_1} = 6 \qquad \therefore p_2 = 600 \text{ kPa}$



$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 6^{\frac{1.4-1}{1.4}}$$

$T_{2s} = 483.9$ K

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$\therefore\ T_2 - T_1 = \dfrac{T_{2s} - T_1}{\eta_c} = 228$ K

$T_2 = 518$ K

$T_3 = 1400$ K

$W_C = C_P (T_2 - T_1) = 1.005\ (518 - 290) = 229.14$ kJ/kg

$$\frac{T_{4s}}{T_3} = \left(\frac{p_i}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$\therefore\quad W_T = \dfrac{W_C}{\eta_T} = 269.9$ kJ/kg $= C_P (T_3 - T_{4s})$

$\therefore\ T_3 - T_{4s} = 268.24 \qquad \therefore \qquad T_{4s} = 1131.8$ K

$$\therefore \quad \left(\frac{1131.8}{1400}\right)^{\frac{1.4}{1.4-1}} = \frac{p_i}{p_2}$$

$$\therefore \quad p_i = p_2 \times \left(\frac{1131.8}{1400}\right)^{\frac{1.4}{1.4-1}} = 285 \text{ kPa}$$

$\Delta h = h_5 - h_6 = C_P (T_5 - T_6)$

$\dfrac{T_3 - T_5}{T_3 - T_{4s}} = \eta_T \qquad \therefore\ T_3 - T_5 = 227.97 \qquad \therefore\ T_5 = 1172$ K

$\dfrac{T_5}{T_6} = \left(\dfrac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{285}{100}\right)^{\frac{1.4-1}{1.4}} \qquad \Rightarrow T_6 = T_5 = 868.9$ K

$\therefore \quad \Delta h = C_P\ (1172 - 868.9) = 304.6$ kJ/kg

$\therefore \quad V = \sqrt{2000 \times \eta \times \Delta h} = \sqrt{2000 \times 0.95 \times 304.6}$ m/s = 760.8 m/s

Q13.43 **A stationary gas turbine power plant operates on the Brayton cycle and delivers 20 MW to an electric generator. The maximum temperature is 1200 K and the minimum temperature is 290 K. The minimum pressure is 95 kPa and the maximum pressure is 380 kPa. If the isentropic efficiencies of the turbine and compressor are 0.85 and 0.80 respectively, find (a) the mass flow rate of air to the compressor, (b) the volume flow rate of air to the compressor, (c) the fraction of the turbine work output needed to drive the compressor, (d) the cycle efficiency.**
If a regenerator of 75% effectiveness is added to the plant, what would be the changes in the cycle efficiency and the net work output?

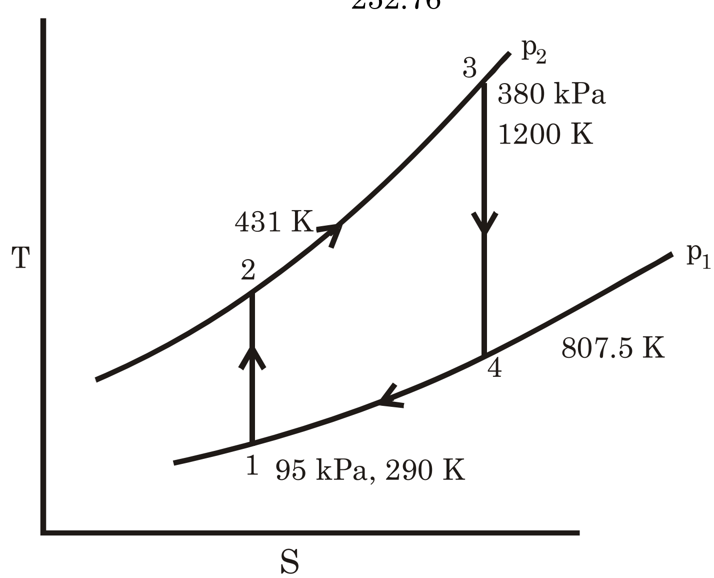
(**Ans. (**a) 126.37 kg/s, (b) 110.71 m^3/s, (c) 0.528, (d) 0.2146, $\Delta\eta$ = 0.148 ΔW_{net} = 0)

Solution:
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_2 = 431\text{K}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}; \; T_4 = 807.5 \text{ K}$$

$$\begin{aligned}\therefore \quad W_{net} &= (h_3 - h_4) - (h_2 - h_1)\\ &= C_P [(T_3 - T_4) - (T_2 - T_1)]\\ &= 252.76 \text{ kJ/kg}\end{aligned}$$

$$\therefore \text{ Mass flow rate } (\dot{m}) = \frac{20000}{252.76} = 79.13 \text{ kg/s}$$



(a) Turbine output (W_T) = $\dot{m}\, c_P$ ($T_3 - T_4$) = 31.234 MW

(b) $\eta = \dfrac{W_C}{W_T} = \dfrac{T_2 - T_1}{T_3 - T_4} = 0.3592$

(c) $(\dot{m}) = 79.13$ kg/s

(d) $v_1 = \dfrac{RT_1}{p_1} = 0.8761\ m^3/kg$ $\quad\therefore\ \dot{V} = \dot{m}v_1 = 69.33\ m^3/s$

14. Refrigeration Cycles

Some Important Notes

Heat Engine, Heat Pump

Heat engines, Refrigerators, Heat pumps:

- A heat engine may be defined as a device that operates in a thermodynamic cycle and does a certain amount of net positive work through the transfer of heat from a **high temperature body to a low temperature body.** A steam power plant is an example of a heat engine.
- A refrigerator may be defined as a device that operates in a thermodynamic cycle and transfers a certain amount of heat from a body at a **lower temperature to a body at a higher temperature** by consuming certain amount of external work. Domestic refrigerators and room air conditioners are the examples. In a refrigerator, the required output is the heat extracted from the low temperature body.
- A heat pump is similar to a refrigerator, however, here the required output is the heat rejected to the high temperature body.

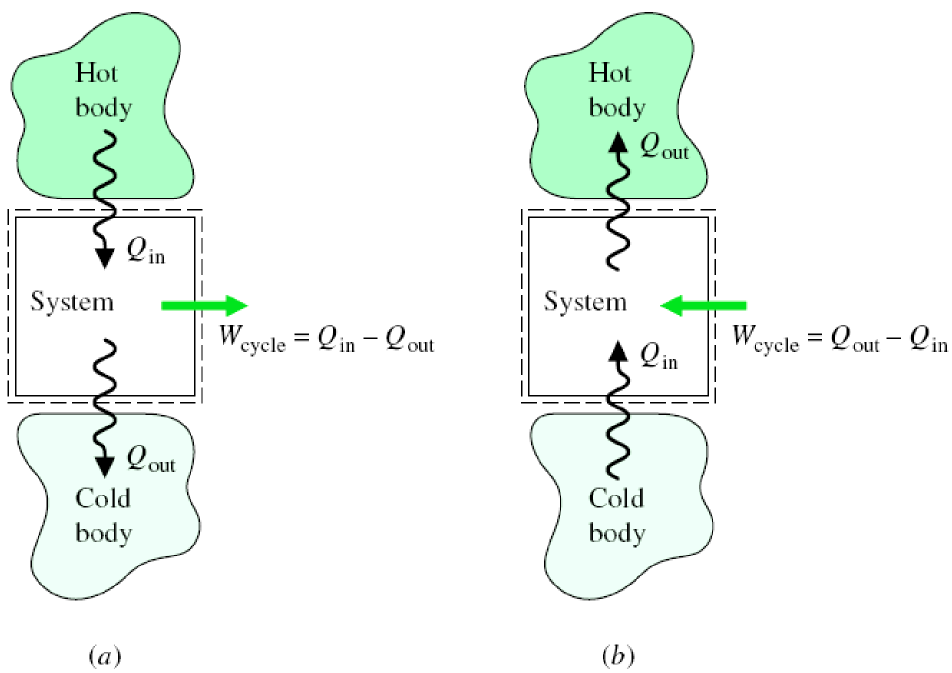


Fig. (*a*) Heat Engine (*b*) Refrigeration and heat pump cycles

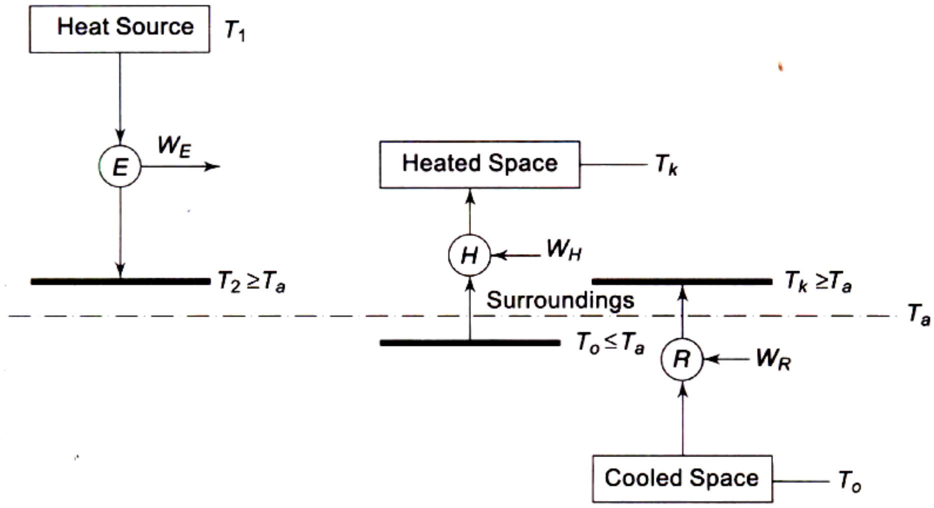


Fig. Comparison of heat engine, heat pump and refrigerating machine

$$COP_{Carnot,HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$COP_{Carnot,R} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

Where

W_{cycle} = work input to the reversible heat pump and refrigerator

Q_H = heat transferred between the system and the hot reservoir

Q_C = heat transferred between the system and cold reservoir

T_H = temperature of the hot reservoir.

T_C = temperature of the cold reservoir.

Question and Solution (P K Nag)

Q14.1 **A refrigerator using R–134*a* operates on an ideal vapour compression cycle between 0.12 and 0.7 MPa. The mass flow of refrigerant is 0.05 kg/s. Determine**
(a) The rate of heat removal from the refrigerated space
(b) The power input to the compressor
(c) The heat rejection to the environment
(d) The COP

(**Ans.** (a) 7.35 kW, (b) 1.85 kW, (c) 9.20 kW, (d) 3.97)

Solution: Try please.

Q14.2 **A Refrigerant-12 vapour compression cycle has a refrigeration load of 3 tonnes. The evaporator and condenser temperatures are – 20°C and 40°C respectively. Find**
(a) The refrigerant flow rate in kg/s
(b) The volume flow rate handled by the compressor in m³/s
(c) The work input to the compressor in kW
(d) The heat rejected in the condenser in kW
(e) The isentropic discharge temperature.

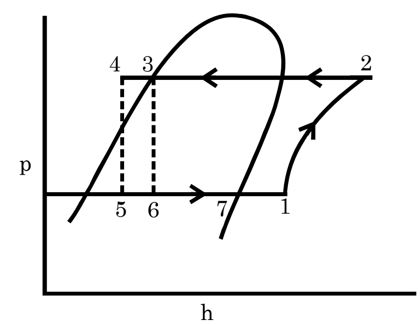
If there is 5° C of superheating of vapour before it enters the compressor, and 5° C sub cooling of liquid before it flows through the expansion valve, determine the above quantities.

Solution: As 50°C temperature difference in evaporate so evaporate temperature = – 20°C and Condenser temperature is 30°C.

$\therefore$ p_1 = 1.589 bar

p_2 = 7.450 bar

h_7 = 178.7 kJ/kg, h_3 = 64.6 kJ/kg



[Data from CP Arora]

$$h_1 = 178.7 + \frac{5}{20}(190.8 - 178.7)$$

Δh = 3.025 kJ/kg

$$s_1 = 0.7088 + \frac{5}{20}(0.7546 - 0.7088)$$

= 0.7203 kJ/kg– K

$\therefore$ $h_3 - h_4 = \Delta h = h_1 - h_7 = 3.025$

$\therefore$ $h_4 = h_3 - \Delta h = 61.6$ kJ/kg i.e. 25°C $h_g = 59.7$

30°C $h_g = 64.6 \rightarrow 0.98/v_c$

$\therefore$ Degree of sub cooling = 3.06°C

(a) Degree of super heat is discharge $= \dfrac{0.7203 - 0.6854}{0.7321 - 0.6854} \times 20 = 15°C$

$\therefore$ Discharge temperature = 15 + 30 = 45° C

$$\therefore \quad h_2 = 199.6 + \frac{15}{20}(214.3 - 199.6) = 210.63 \text{ kJ/kg}$$

$\therefore$ Compressor work (W) = $h_2 - h_1$ = 210.63 – 181.73 = 28.9 kJ/kg

Refrigerating effect (Q_0) = $h_7 - h_5 = h_7 - h4$ = (178.7 – 61.6) kJ/kg = 117.1 kJ/kg

(b) $\therefore$ $\text{COP} = \dfrac{Q_o}{W} = \dfrac{117.1}{28.9} = 4.052$

$v_1 = 0.108\ m^3/kg$ $\qquad \dot{V}_1 = \dot{m}v_1 = 0.014361\ m^3/s$

$\dfrac{\pi D^2}{4} \times L \times \dfrac{N}{60} \times n \times \eta_{vol} = \dot{V}_1$ $\qquad \dfrac{L}{D} = 1.2$

$L = 1.2\ D$

$\dfrac{\pi \times D^2}{4} \times 1.2D \times \dfrac{900}{60} \times 1 \times 0.95 = 0.014361$

$\therefore D = 0.1023\ m = 10.23\ cm$

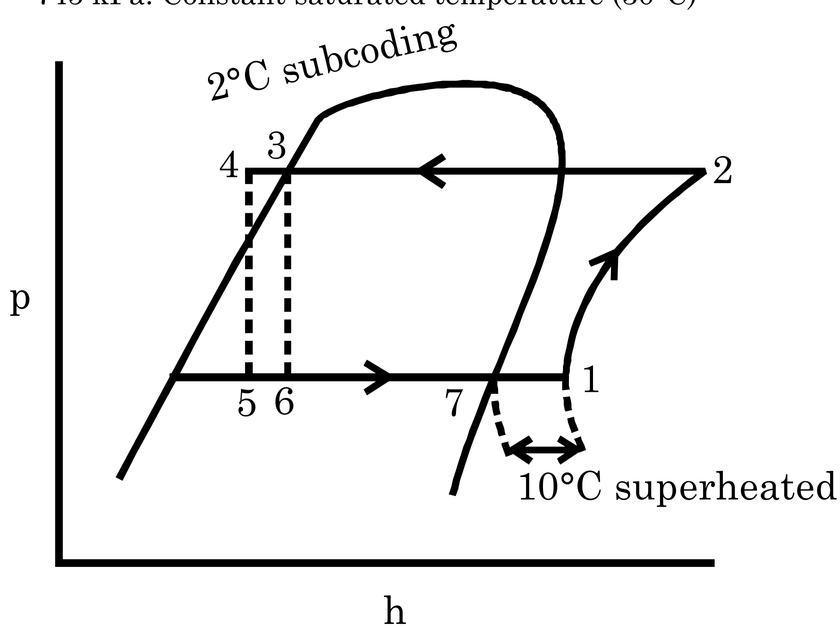
$L = 0.1227\ m = 12.27\ cm$

Q14.4 **A vapour compression refrigeration system uses R-12 and operates between pressure limits of 0.745 and 0.15 MPa. The vapour entering the compressor has a temperature of – 10°C and the liquid leaving the condenser is at 28°C. A refrigerating load of 2 kW is required. Determine the COP and the swept volume of the compressor if it has a volumetric efficiency of 76% and runs at 600 rpm.**

(**Ans.** 4.15, 243 cm^3)

Solution: $p_1 = 150$ kPa: Constant saturated temperature (– 20°C)

$p_2 = 745$ kPa: Constant saturated temperature (30°C)



$h_7 = 178.7$ kJ/kg

$h_3 = 64.6$ kJ/kg

$h_4 = h_{4\text{-}5} = 59.7 + \dfrac{3}{5}(64.6 - 59.7) = 62.64\ \text{kJ/kg} = h_5$

$h_1 = h_7 + \dfrac{10}{20}(190.8 - h_7) = 184.8\ \text{kJ/kg}$

$s_1 = 0.7088 + \dfrac{10}{20}(0.7546 - 0.7088) = 0.7317\ \text{kJ/kg-K}$

$$h_2 = 199.6 + \left(\frac{0.7317 - 0.6854}{0.7321 - 0.6854}\right)(214.3 - 199.6) = 214.2 \text{ kJ/kg}$$

$\therefore$ Compressor work (W) = $h_2 - h_1$ = 29. 374 kJ/kg

Refrigeration effect = $(h_1 - h_5)$ = (184.8 – 62.64) = 122.16 kJ/kg

$$\therefore \quad \text{COP} = \frac{122.16}{29.374} = 4.16$$

$v_1 = 0.1166 \text{m}^3/\text{kg}$

Mass flow ratio $\dot{m} \times 122.16 = 2$

$\therefore \dot{m} = 0.016372$ kg/s

$$\therefore \dot{V}_1 = \dot{m}v_1 = 1.90897 \times 10^3 \text{ m}^3/\text{s} = V_s \times 0.76 \times \frac{600}{60}$$

$\therefore V_s = 251.2 \text{ cm}^3$

Q14.6 **A R-12 vapour compression refrigeration system is operating at a condenser pressure of 9.6 bar and an evaporator pressure of 2.19 bar. Its refrigeration capacity is 15 tonnes. The values of enthalpy at the inlet and outlet of the evaporator are 64.6 and 195.7 kJ/kg. The specific volume at inlet to the reciprocating compressor is 0.082 m³/kg. The index of compression for the compressor is 1.13**

Determine:

(a) The power input in kW required for the compressor

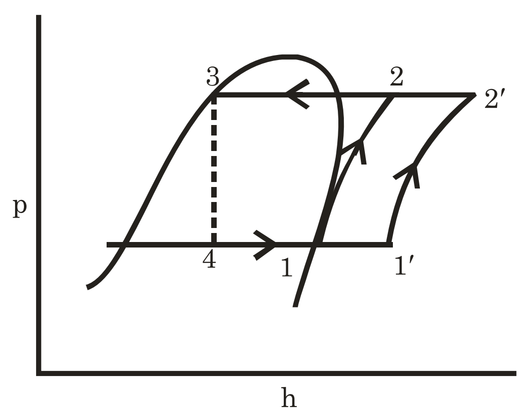
(b) The COP. Take 1 tonnes of refrigeration as equivalent to heat removal at the rate of 3.517 kW.

(**Ans.** (a) 11.57 kW, (b) 4.56)

Solution: $T_1 = -10°C$ $T_3 = 40°C$

$h_4 = 646$ kJ/kg $h_1 = 1057$ kJ/kg

$v_1 = 0.082 \text{ m}^3/\text{kg}$ $n = 1.13$



Refrigeration effect (195.7 – 64.6) kJ/kg = 131.1 kJ/kg

$\dot{m}\, Q_o = 15 \times 3.517$ $\therefore \dot{m} = 0.4024$ kg/s

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \left(\frac{2.19}{9.6}\right)^{\frac{1}{1.3}} = v_2 = 0.022173 \text{ m}^3/\text{kg}$$

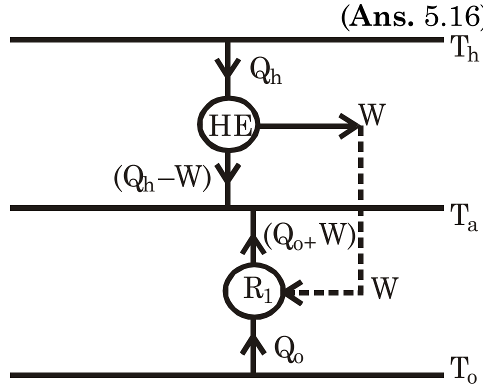
$$\therefore W_C = \frac{n}{n-1}(p_1V_1 - p_2V_2) = 28.93 \text{ kJ/kg}$$

(a) $W_{compressor} = 11.64$ KW

(b) $\quad \text{COP} = \dfrac{15 \times 3.517}{11.64} = 4532$

Q14.12 **Determine the ideal COP of an absorption refrigerating system in which the heating, cooling, and refrigeration take place at 197°C, 17°C, and –3°C respectively.**

(**Ans.** 5.16)



Solution : $\therefore \text{COP} = \dfrac{\text{Desired effort}}{\text{input}}$

$$= \frac{\text{Refregerating effect}}{\text{heat input}}$$

$$= \frac{Q_o}{Q_h}$$

$$= \frac{Q_o}{W} \times \frac{W}{Q_h}$$

$$= (\text{COP})_R \times \eta_{\text{H.E.}}$$

For ideal process

$$(\text{COP})_R = \frac{T_o}{T_a - T_o}$$

And $\quad \eta_{\text{H.E}} = \eta_{\text{Carnot}} = \left(1 - \dfrac{T_a}{T_h}\right)$

$$\therefore (\text{COP})_{\text{ideal}} = \frac{T_o}{T_a - T_o} \times \left(1 - \frac{T_a}{T_h}\right)$$

$$= \frac{T_o}{T_h} \times \frac{[T_h - T_a]}{[T_a - T_o]}$$

Given $T_o = 270$ K, $T_a = 290$ K, $T_h = 470$ K

$$\therefore (\text{COP})_{\text{ideal}} = \frac{270}{470} \times \frac{[470 - 290]}{[290 - 270]} = 5.17$$

Q14.22 **Derive an expression for the COP of an ideal gas refrigeration cycle with a regenerative heat exchanger. Express the result in terms of the minimum gas temperature during heat rejection (T_h) maximum gas temperature during heat absorption (T_1) and pressure ratio for the cycle** $(p_2 p_1)$.

$\left(\textbf{Ans.}\ COP = \dfrac{T_1}{T_h r_p^{(\gamma-1)/\gamma} - T_1}\right)$

Solution: $\therefore \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_P^{\frac{\gamma-1}{\gamma}}$

$$\therefore T_2 = T_1\, r_P^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_P^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T_5 = \frac{T_4}{r_P^{\frac{\gamma-1}{\gamma}}} = \frac{T_n}{r_P^{\frac{\gamma-1}{\gamma}}}$$

For Regeneration ideal

$C_P (T_3 - T_4) = p\ (T_1 - T_6)$

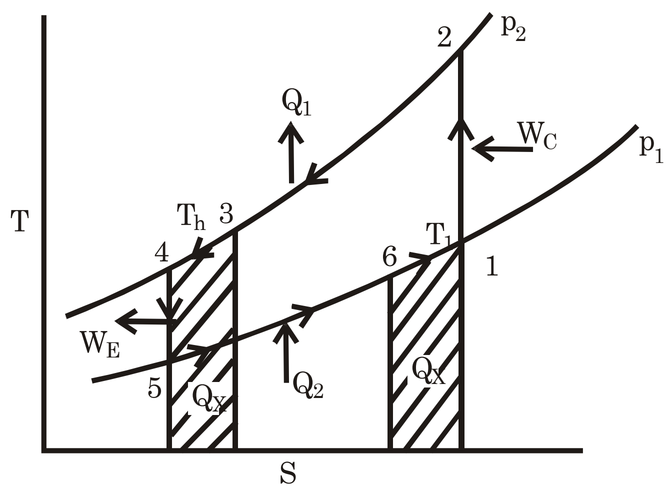
$\therefore \quad T_3 - T_h = T_1 - T_6$

$\therefore \quad$ Work input (W) $= (h_2 - h_1) - (h_4 - h_5)$

$= C_P\ [(T_2 - T_1) - (T_h - T_s)$

Heat rejection $(Q_1) = Q_2 + W = C_P\ (T_2 - T_3)$

Heat absorption $(Q_2) = C_P\ (T_6 - T_5)$



$$\therefore \quad COP = \frac{Q_2}{Q_1 - Q_2} = \frac{T_6 - T_5}{(T_2 - T_3) - (T_6 - T_5)} = \frac{1}{\frac{T_2 - T_3}{T_6 - T_5} - 1}$$

$$T_2 - T_3 = T_1\, r_P^{\frac{\gamma-1}{\gamma}} - T_1 = T_1\left(r_P^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$T_6 - T_5 = T_h - \frac{T_h}{r_P^{\frac{\gamma-1}{\gamma}}} = T_h \frac{\left(r_P^{\frac{\gamma-1}{\gamma}} - 1\right)}{r_P^{\frac{\gamma-1}{\gamma}}} = \frac{T_h}{T_1\, r_P^{\frac{\gamma-1}{\gamma}} - T_h} \quad \text{or} \quad COP = \frac{T_1}{T_h r_p^{(\gamma-1)/\gamma} - T_1}$$

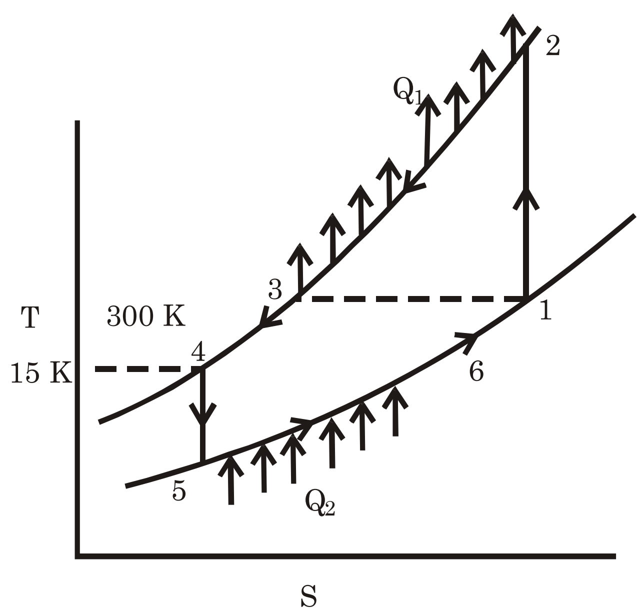
Q14.23 Large quantities of electrical power can be transmitted with relatively little loss when the transmission cable is cooled to a superconducting temperature. A regenerated gas refrigeration cycle operating with helium is used to maintain an electrical cable at 15 K. If the pressure ratio is 10 and heat is rejected directly to the atmosphere at 300 K, determine the COP and the performance ratio with respect to the Carnot cycle.

(**Ans.** 0.02, 0.38)

Solution: $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 10^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T_2 = 300 \times 10^{\frac{1.6667.1}{1.6667}} = 754 \text{ K}$$

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{10}\right)^{0.4}$$



$\therefore \quad T_5 = 5.9716$ K

Refrigerating effect (Q_2) = C_P ($T_6 - T_5$) = 9.0284 C_P

Work input (W) = C_P [($T_2 - T_1$) – ($T_4 - T_5$)] = 444. 97 C_P

$$\therefore \quad \text{COP} = \frac{0.0284\ C_P}{444.97\ C_P} = 0.0203$$

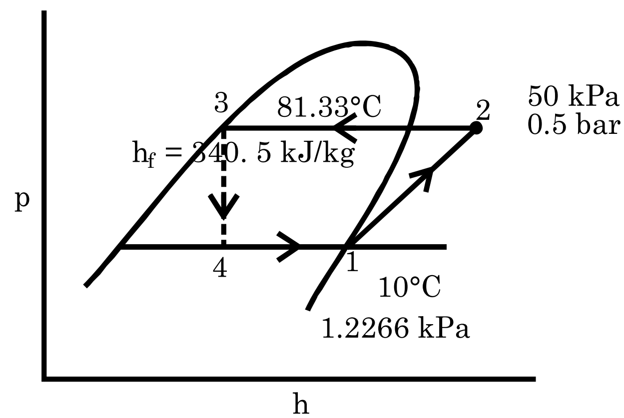
$$\text{And } (\text{COP})_{\text{carnet}} = \frac{T_6}{T_6 - T_5} = \frac{15}{300-15} = 0.05263$$

$$\frac{\text{COP}_{\text{actual}}}{\text{COP}_{\text{carnot}}} = \frac{0.0203}{0.05263} = 0.\ 3857$$

Q14.25 **A heat pump installation is proposed for a home heating unit with an output rated at 30 kW. The evaporator temperature is 10°C and the condenser pressure is 0.5 bar. Using an ideal vapour compression cycle, estimate the power required to drive the compressor if steam/water mixture is used as the working fluid, the COP and the mass flow rate of the fluid. Assume saturated vapour at compressor inlet and saturated liquid at condenser outlet.**

(**Ans.** 8.0 kW, 3.77, 0.001012 kg/s)

Solution: $h_1 = 2519.8$ kJ/kg
$s_1 = 8.9008$ kJ/kg-K
$v_1 = 106.38\ m^3/kg$



400°C 50 kPa s = 8.88642, h = 3278.9
500°C 50 kPa s = 9.1546, h = 3488.7

$$\therefore \quad h_2 = (3488.7 - 3278.9) \times \left(\frac{8.9008 - 8.8642}{9.1546 - 8.8642}\right) + 3278.9 = 3305.3 \text{ kJ/kg}$$

$$\therefore \quad \text{Compressor work (W)} = h_2 - h_1 = (3305.3 - 2519.8) = 785.5 \text{ kJ/kg}$$

$$\text{Heating (Q)} = h_2 - h_{f3} = (3305.3 - 340.5) \text{ kJ/kg} = 2964.8 \text{ kJ/kg}$$

$$\dot{m} \times Q = 30$$

$$= \dot{m} = \frac{30}{2964.8} = 0.0101187 \text{ kg/s}$$

$$\therefore \quad COP = \frac{2964.8}{785.5} = 3.77$$

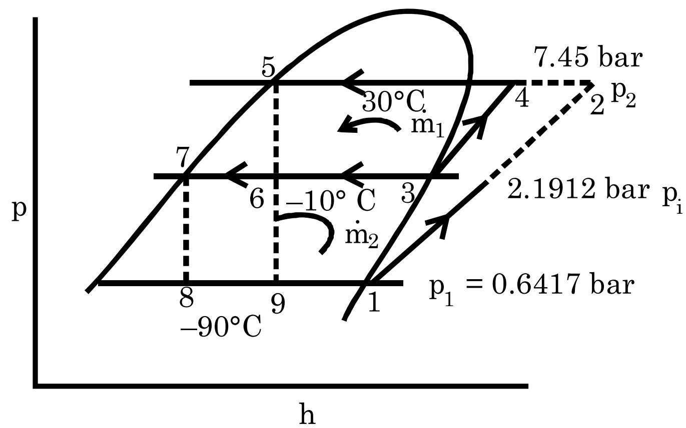
Compressor power = $\dot{m}W$ = 7.95 KW

Q14.26 A 100 tonne low temperature R-12 system is to operate on a 2-stage vapour compression refrigeration cycle with a flash chamber, with the refrigerant evaporating at – 40°C, an intermediate pressure of 2.1912 bar, and condensation at 30°C. Saturated vapour enters both the compressors and saturated liquid enters each expansion valve. Consider both stages of compression to be isentropic. Determine:

(a) The flow rate of refrigerant handled by each compressor
(b) The total power required to drive the compressor
(c) The piston displacement of each compressor, if the clearance is 2.5% for each machine
(d) The COP of the system
(e) What would have been the refrigerant flow rate, the total work of compression, the piston displacement in each compressor and the compressor and the COP, if the compression had occurred in a single stage? .

(**Ans.** (a) 2.464, 3.387 kg/s, (b) 123 kW, (c) 0.6274, 0.314 m^3/s, (d) 2.86, (e) 3.349 kg/s, 144.54 kW, 1.0236 m^3/s, 2.433)

Solution: $h_1 = 169$ kJ/kg
$h_3 = 183.2$ kJ/kg
$h_5 = 64.6$ kJ/kg $= h_6$
$h_7 = h_8 = 26.9$ kJ/kg



$S_1 = S_2 = 0.7274$ kJ/kg – K
$S_3 = S_4 = 0.7020$ kJ/kg – K
∴ From P.H chart of R12
$h_2 = 190$ kJ/kg
$h_4 = 206$ kJ/kg

$$\dot{m}_2 h_2 + \dot{m}_1 h_5 = \dot{m}_2 h_7 + \dot{m}_1 h_3$$

$$\therefore \quad \dot{m}_2(h_2 - h_7) = \dot{m}_2(h_3 - h_5)$$

$$\therefore \quad \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_2 - h_7}{h_3 - h_5} = \frac{190 - 26.9}{183.2 - 64.6} = 1.3752$$

$$\dot{m}_2\ (h_1 - h_8) = \frac{100 \times 14000}{3600}$$

(a) $\therefore\ \dot{m}_2 = 2.7367$ kg/s

$\dot{m}_1 = m_2 \times 1.3752 = 3.7635$ kg/s

(b) Power of compressor (P) $= \dot{m}_2(h_2 - h_1) + \dot{m}_1(h_4 - h_3)$
$= 14328$ kW

(d) $$\text{COP} = \frac{\text{Refrigeration efficiency}}{\text{Compressor}} = \frac{100 \times 14000}{3600 \times 143.28} = 2.7142$$

(e) For single storage
From R12 chart $h_a' = 2154$ kJ/kg, $h_g = h_s = 64.6$ kJ/kg

$$\therefore \quad \dot{m}(h_1 - h_9) = \frac{100 \times 14000}{3600} \Rightarrow \dot{m} = 3.725 \text{ kg/s}$$

Compressor power (P) $= \dot{m}\ (h_4' - h_1) = 3.725 \times 46$
$= 171.35$ kW

$$\text{COP} = \frac{\frac{100 \times 14000}{3600}}{171.35} = 2.27$$