

Fluid Mechanics and Fluid Machines

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Note

If you think there should be a change in option, don't change it by yourself send me a mail at swapan_mondal_01@yahoo.co.in I will send you complete explanation.

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Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). I would be thankful to the readers if they are brought to my attention at the following e-mail address: swapan_mondal_01@yahoo.co.in

S K Mondal

1. Properties of Fluids

Contents of this chapter

1. Definition of Fluid
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Theory at a Glance (for IES, GATE, PSU)

Definition of Fluid

A fluid is a substance which deforms continuously when subjected to external shearing forces.

Characteristics of Fluid

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.
3. It is interesting to note that a solid suffers strain when subjected to shear forces whereas a fluid suffers **Rate of Strain** i.e. it flows under similar circumstances.

Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure, velocity etc.) and fluid properties vary continuously from one point to another. Mathematical descriptions of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established.

For example density at a point is normally defined as

$$\rho = \lim_{\Delta V \rightarrow 0} \left(\frac{m}{\Delta V} \right)$$

Here ΔV is the volume of the fluid element and m is the mass

If ΔV is very large ρ is affected by the inhomogeneities in the fluid medium. Considering another extreme if ΔV is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of ΔV , before statistical fluctuations become significant. This is called continuum limit and is denoted by ΔV_c .

$$\rho = \lim_{\Delta V \rightarrow \Delta V_c} \left(\frac{m}{\Delta V} \right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is

characterised by mean free path (λ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analysed by the molecular theory.

A dimensionless parameter known as Knudsen number, $K_n = \lambda / L$, where λ is the mean free path and L is the characteristic length. It describes the degree of departure from continuum.

Usually when $K_n > 0.01$, the concept of continuum does not hold good.

Beyond this critical range of Knudsen number, the flows are known as

slip flow ($0.01 < K_n < 0.1$),

transition flow ($0.1 < K_n < 10$) and

free-molecule flow ($K_n > 10$).

However, for the flow regimes considered in this course, K_n is always less than 0.01 and it is usual to say that the fluid is a continuum.

Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good.

In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

Ideal and Real Fluids

1. Ideal Fluid

An ideal fluid is one which has

no viscosity

no surface tension

and incompressible

2. Real Fluid

An Real fluid is one which has

viscosity

surface tension

and compressible

Naturally available all fluids are real fluid.

Viscosity

Definition: Viscosity is the property of a fluid which determines its resistance to shearing stresses.

Cause of Viscosity: It is due to cohesion and molecular momentum exchange between fluid layers.

Newton's Law of Viscosity: It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

The constant of proportionality is called the co-efficient of viscosity.

When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du.

Velocity gradient =

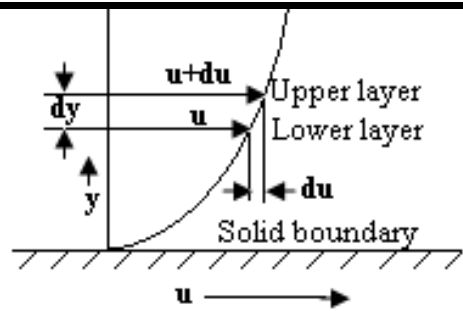
$$\frac{du}{dy}$$

According to Newton's law

$$\tau \propto \frac{du}{dy}$$

or

$$\tau = \mu \frac{du}{dy}$$



Velocity Variation near a solid boundary

Where μ = constant of proportionality and is known as co-efficient of Dynamic viscosity or only Viscosity

As $\mu = \frac{\tau}{\frac{du}{dy}}$ Thus viscosity may also be defined as the shear stress required

$$\mu = \frac{\tau}{\left[\frac{du}{dy} \right]}$$

producing unit **rate of shear strain**.

Units of Viscosity

S.I. Units: Pa.s or N.s/m²

C.G.S Unit of viscosity is Poise= dyne-sec/cm²

One Poise= 0.1 Pa.s

1/100 Poise is called centipoises.

Dynamic viscosity of water at 20°C is approx= 1 cP

Kinematic Viscosity

It is the ratio between the dynamic viscosity and density of fluid and denoted by ν . Mathematically

$$\nu = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho}$$

Units of Kinematic Viscosity

S.I units: m²/s

C.G.S units: stoke = cm²/sec

One stoke = 10⁻⁴ m²/s

Thermal diffusivity and molecular diffusivity have same dimension, therefore, by analogy, the kinematic viscosity is also referred to as the *momentum diffusivity* of the fluid, i.e. the ability of the fluid to transport momentum.

Classification of Fluids

1. Newtonian Fluids

These fluids follow Newton's viscosity equation.

For such fluids viscosity does not change with *rate of deformation*.

2. Non-Newtonian Fluids

These fluids does not follow Newton's viscosity equation.

Such fluids are relatively uncommon e.g. Printer ink, blood, mud, slurries, polymer solutions.

Non-Newtonian Fluid () $\tau \neq \mu \frac{du}{dy}$		
Purely Viscous Fluids		Visco-elastic Fluids
Time - Independent	Time - Dependent	Visco-elastic Fluids
<p>1. Pseudo plastic Fluids</p> $\tau = \mu \left(\frac{du}{dy} \right)^n; n < 1$ <p>Example: Blood, milk</p> <p>2. Dilatant Fluids</p> $\tau = \mu \left(\frac{du}{dy} \right)^n; n > 1$ <p>Example: Butter</p> <p>3. Bingham or Ideal Plastic Fluid</p> $\tau = \tau_o + \mu \left(\frac{du}{dy} \right)^n$ <p>Example: Water suspensions of clay and flash</p>	<p>1. Thixotropic Fluids</p> $\tau = \mu \left(\frac{du}{dy} \right)^n + f(t)$ <p style="text-align: right;"><i>f(t) is decreasing</i></p> <p>Example: Printer ink; crude oil</p> <p>2. Rheopectic Fluids</p> $\tau = \mu \left(\frac{du}{dy} \right)^n + f(t)$ <p style="text-align: right;"><i>f(t) is increasing</i></p> <p>Example: Rare liquid solid suspension</p>	$\tau = \mu \frac{du}{dy} + \alpha E$ <p>Example: Liquid-solid combinations in pipe flow.</p>

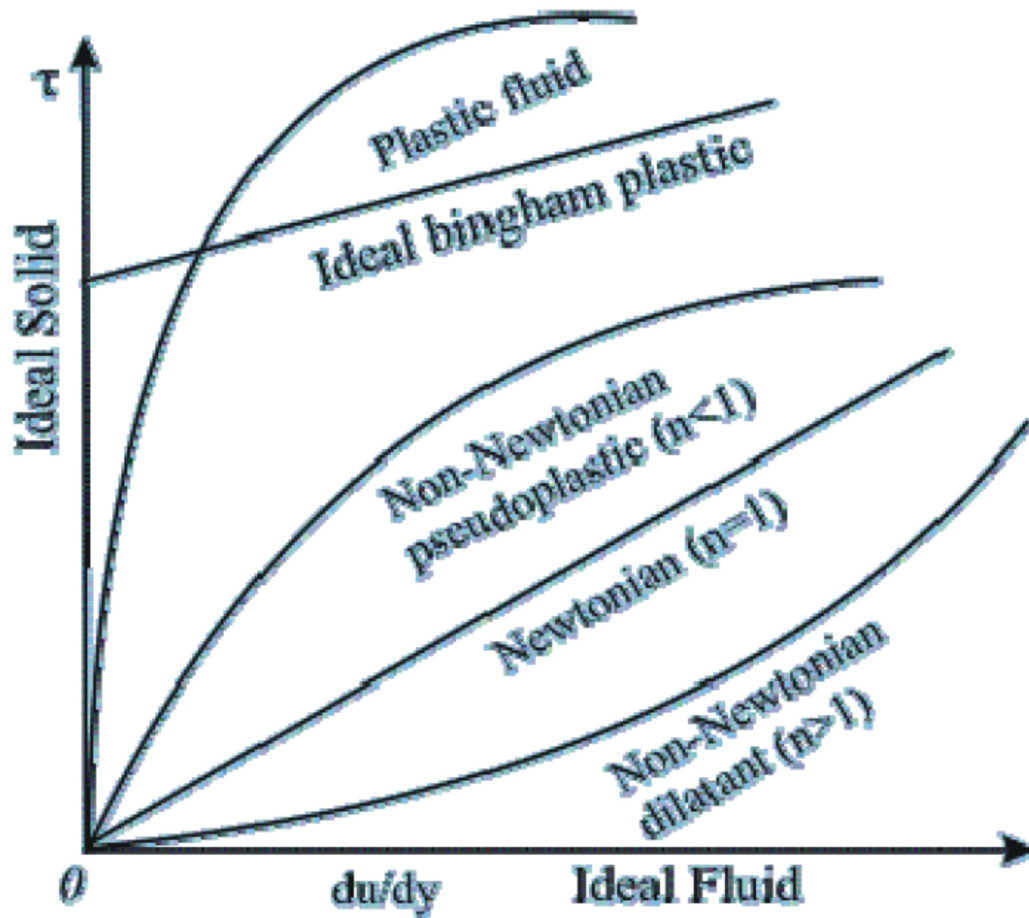


Fig. Shear stress and deformation rate relationship of different fluids

Effect of Temperature on Viscosity

With increase in temperature
 Viscosity of liquids decrease
 Viscosity of gasses increase

- Note:** 1. Temperature responses are neglected in case of Mercury.
 2. The lowest viscosity is reached at the critical temperature.

Effect of Pressure on Viscosity

Pressure has very little effect on viscosity.

But if pressure increases intermolecular gap decreases then cohesion increases so viscosity would be increase.

Surface tension

Surface tension is due to cohesion between particles at the surface.
 Capillarity action is due to both cohesion and adhesion.

Surface tension

The tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

Pressure Inside a Curved Surface

For a general curved surface with radii of curvature r_1 and r_2 at a point of interest

$$\Delta p = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

a. Pressure inside a water droplet,

$$\Delta p = \frac{4\sigma}{d}$$

b. Pressure inside a soap bubble,

$$\Delta p = \frac{8\sigma}{d}$$

c. Liquid jet.

$$\Delta p = \frac{2\sigma}{d}$$

Capillarity

A general term for phenomena observed in liquids due to inter-molecular attraction at the liquid boundary, e.g. the rise or depression of liquids in narrow tubes. We use this term for capillary action.

Capillary rise and depression phenomenon depends upon the surface tension of the liquid as well as the material of the tube.

1. General formula,

$$h = \frac{4\sigma \cos \theta}{\rho g d}$$

2. For water and glass $\theta = 0^\circ$,

$$h = \frac{4\sigma}{\rho g d}$$

3. For mercury and glass $\theta = 138^\circ$,

$$h = -\frac{4\sigma \cos 42^\circ}{\rho g d}$$

(h is negative indicates capillary depression)

Note: If adhesion is more than cohesion, the wetting tendency is more and the angle of contact is smaller.

Derive the Expression for Capillary Rise

Let us consider a glass tube of small diameter 'd' opened at both ends and is inserted vertically in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let, d = diameter of the capillary tube.

h = height of capillary rise.
 θ = angle of contact of the water

surface.
 σ = surface tension force for

unity length.
 ρ = density of liquid.

ρ
 g = acceleration due to gravity.

Under a state of equilibrium,
 Upward surface tension force (lifting force) = weight of the water column in the tube (gravity force)

or

$$\pi d \cdot \sigma \cos \theta = \frac{\pi d^2}{4} \times h \times \rho \times g$$

or

$$h = \frac{4 \sigma \cos \theta}{\rho g d}$$

If $\theta > \pi/2$, h will be negative, as in the case of mercury in a capillary
 $\theta = 138^\circ$

depression occurred.

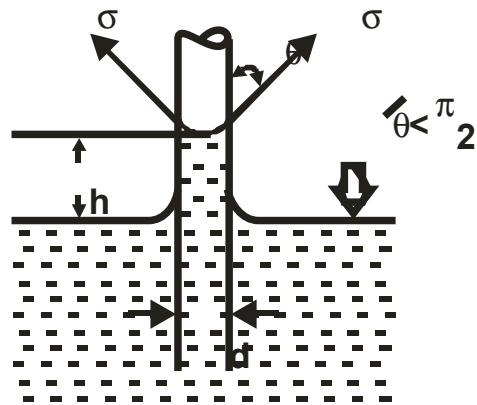


Fig. Capillary rise (As in water)

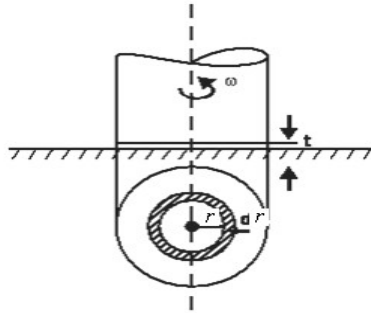
Question: A circular disc of diameter 'd' is slowly rotated in a liquid of large viscosity ' μ ' at a small distance 't' from the fixed

surface. Derive the expression for torque required to maintain the speed ' ω '.

Answer:

Radius, $R = d/2$
 Consider an elementary circular ring of radius r and thickness dr as shown.
 Area of the elements ring =

The shear stress at ring,



$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \mu \frac{r\omega}{t}$$

Shear force on the elements ring

$$dF = \tau \times \text{area of the ring} = \tau \times 2\pi r dr$$

Torque on the

$$\text{ring} = dF \times r$$

$$\therefore dT = \mu \frac{r\omega}{t} \times 2\pi r dr \times r$$

Total torque, T =

$$\begin{aligned} \therefore \int dT &= \int_0^R \frac{\mu r \omega}{t} \times 2\pi r dr \\ &= \frac{2\pi\mu\omega}{t} \int_0^R r^3 dr = \frac{\pi\mu\omega}{2t} R^4 = \frac{\pi\mu\omega}{2t} \left(\frac{d}{2}\right)^4 \end{aligned}$$

$$T = \frac{\pi\mu\omega d^4}{32t}$$

Question: A solid cone of radius R and vortex angle 2θ is to rotate at an angular velocity, ω . An oil of dynamic viscosity μ and thickness 't' fills the gap between the cone and the housing. Determine the expression for Required Torque. [IES-2000; AMIE (summer) 2002]

Answer:

Consider an elementary ring of bearing surface of radius r. at a distance h from the apex. and let $r + dr$ is the radius at

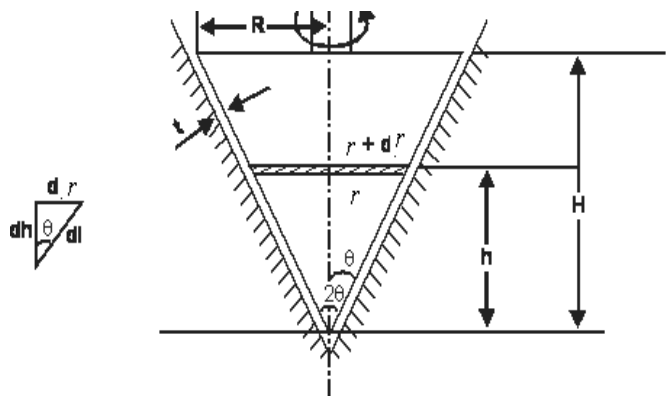
$h + dh$ distance

Bearing area =

$$2\pi r dl$$

=

$$2\pi r \cdot \frac{dr}{\sin\theta}$$



Shear stress

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \mu \frac{r\omega}{t}$$

Tangential resistance on the ring

∴

$$dF = \text{shear stress} \times \text{area of the ring}$$

$$=$$

$$\mu r \frac{\omega}{t} \times 2\pi r \frac{dr}{\sin\theta}$$

Torque due to the force dF

∴

$$dT = dF \cdot r$$

$$dT = \frac{2\pi\mu\omega}{t\sin\theta} \times r^3 dr$$

Total torque

∴

$$T = \int dT = \int_0^R \frac{2\pi\mu\omega}{t\sin\theta} \times r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta} \times \frac{R^4}{4} = \frac{\pi\mu\omega R^4}{2t\sin\theta}$$

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Viscosity

GATE-1. The SI unit of kinematic viscosity (ν) is: **[GATE-2001]**

- (a) m^2/s (b) $kg/m\cdot s$ (c) m/s^2 (d) m^3/s^2

GATE-1. Ans. (a)

GATE-2. Kinematic viscosity of air at 20°C is given to be $1.6 \times 10^{-5} m^2/s$. Its kinematic viscosity at 70°C will be vary approximately **[GATE-1999]**

- (a) $2.2 \times 10^{-5} m^2/s$ (b) $1.6 \times 10^{-5} m^2/s$ (c) $1.2 \times 10^{-5} m^2/s$ (d) $3.2 \times 10^{-5} m^2/s$

GATE-2. Ans. (a) Viscosity of gas increases with increasing temperature.

Newtonian Fluid

GATE-3. For a Newtonian fluid **[GATE-2006; 1995]**

- (a) Shear stress is proportional to shear strain
 (b) Rate of shear stress is proportional to shear strain
 (c) Shear stress is proportional to rate of shear strain
 (d) Rate of shear stress is proportional to rate of shear strain

GATE-3. Ans. (c)

Surface Tension

GATE-4. The dimension of surface tension is: **[GATE-1996]**

- (a) ML^{-1} (b) L^2T^{-1} (c) $ML^{-1}T^1$ (d) MT^{-2}

GATE-4. Ans. (d)

GATE-5. The dimensions of surface tension is: **[GATE-1995]**

- (a) N/m^2 (b) J/m (c) J/m^2
 (d) W/m

GATE-5. Ans. (c) The property of the liquid surface film to exert a tension is called the surface tension. It is the force required to maintain unit length of the film in equilibrium. In SI units surface tension is expressed in

$$N/m \left(\frac{J}{m^2} \right).$$

In metric gravitational system of units it is expressed in $kg(f)/cm$ or $kg(f)/m$.

Previous 20-Years IES Questions

Fluid

IES-1. Assertion (A): In a fluid, the rate of deformation is far more important than the total deformation itself. Reason (R): A fluid continues to deform so long as the external forces are applied. [IES-1996]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-1. Ans. (a) Both A and R correct and R is correct explanation for A

IES-2. Assertion (A): In a fluid, the rate of deformation is far more important than the total deformation itself. [IES-2009]

Reason (R): A fluid continues to deform so long as the external forces are applied.

- (a) Both A and R are individually true and R is the correct explanation of A.
- (b) Both A and R are individually true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

IES-2. Ans. (a) This question is copied from Characteristics of fluid

- 1. It has no definite shape of its own, but conforms to the shape of the containing vessel.**
- 2. Even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.**
- 3. It is interesting to note that a solid suffers strain when subjected to shear forces whereas a fluid suffers Rate of Strain i.e. it flows under similar circumstances.**

Viscosity

IES-3. Newton's law of viscosity depends upon the [IES-1998]

- (a) Stress and strain in a fluid
- (b) Shear stress, pressure and velocity
- (c) Shear stress and rate of strain
- (d) Viscosity and shear stress

IES-3. Ans. (c) Newton's law of viscosity

$$\tau = \mu \frac{du}{dy} \quad \text{where, } \tau \rightarrow \text{Shear stress}$$

$$\frac{du}{dy} \rightarrow \text{Rate of strain}$$

IES-4. What is the unit of dynamic viscosity of a fluid termed 'poise' equivalent to? [IES-2008]

- (a) dyne/cm² (b) gm s/cm (c) dyne s/cm²
 (d) gm-cm/s

IES-4. Ans. (c)

IES-5. The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 1 cm apart and moving with relative velocity of 2 m/s is: [IES-2001]

- (a) 20 N/m² (b) 196.2 N/m² (c) 29.62 N/m² (d) 40 N/m²

IES-5. Ans. (b) $du=2$ m/s; $dy=1$ cm = 0.01 m; $\mu = 9.81$ poise = 0.981 Pa.s

$$\text{Therefore } \tau = \mu \frac{du}{dy} = 0.981 \times \frac{2}{0.01} = 196.2 \text{ N/m}^2$$

IES-6. What are the dimensions of kinematic viscosity of a fluid? [IES-2007]

- (a) L^2T^{-2} (b) L^2T^{-1} (c) $ML^{-1}T^{-1}$ (d) ML^{-1}

IES-6. Ans. (b)

IES-7. An oil of specific gravity 0.9 has viscosity of 0.28 Stokes at 38°C. What will be its viscosity in Ns/m²? [IES-2005]

- (a) 0.2520 (b) 0.0311 (c) 0.0252 (d) 0.0206

IES-7. Ans. (c) Specific Gravity = 0.9 therefore Density = $0.9 \times 1000 = 900$ kg/m³
 One Stoke = 10^{-4} m²/s

$$\text{Viscosity } (\mu) = \rho \nu = 900 \times 0.28 \times 10^{-4} = 0.0252 \text{ Ns/m}^2$$

IES-8. Decrease in temperature, in general, results in [IES-1993]

- (a) An increase in viscosities of both gases and liquids
 (b) A decrease in the viscosities of both liquids and gases
 (c) An increase in the viscosity of liquids and a decrease in that of gases
 (d) A decrease in the viscosity of liquids and an increase in that of gases

IES-8. Ans. (c) The viscosity of water with increase in temperature decreases and that of air increases.

IES-9. Assertion (A): In general, viscosity in liquids increases and in gases it decreases with rise in temperature. [IES-2002]

Reason (R): Viscosity is caused by intermolecular forces of cohesion and due to transfer of molecular momentum between fluid layers; of which in liquids the former and in gases the later contribute the major part towards viscosity.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-9. Ans. (d)

Non-Newtonian Fluid

IES-10. If the Relationship between the shear stress τ and the rate of

shear strain $\frac{du}{dy}$ is expressed as $\tau = \mu \left(\frac{du}{dy} \right)^n$ then the fluid with

$$\tau = \mu \left(\frac{du}{dy} \right)^n$$

exponent $n > 1$ is known as which one of the following? [IES-2007]

- (a) Bingham Plastic (b) Dilatant Fluid
 (c) Newtonian Fluid (d) Pseudo plastic Fluid

IES-10. Ans. (b)

IES-11. Match List-I (Type of fluid) with List-II (Variation of shear stress) and select the correct answer: [IES-2001]

- | List-I | List-II |
|------------------------|---|
| A. Ideal fluid | 1. Shear stress varies linearly with the rate of strain |
| B. Newtonian fluid | 2. Shear stress does not vary linearly with the rate of strain |
| C. Non-Newtonian fluid | 3. Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain |
| D. Bingham plastic | 4. Shear stress is zero |

Codes:	A	B	C	D	A	B	C	D
(a)	3	1	2	4	(b)	4	2	1
(c)	3	2	1	4	(d)	4	1	2
								3

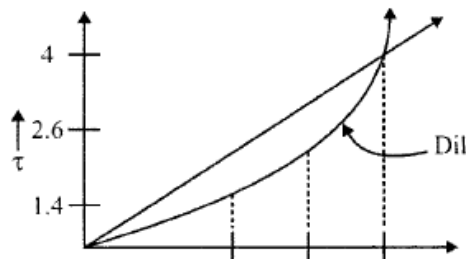
IES-11. Ans. (d)

IES-12. In an experiment, the following shear stress - time rate of shear strain values are obtained for a fluid: [IES-2008]

Time rate of shear strain (1/s):	0	2	3	4
Shear stress (kPa):	0	1.4	2.6	4

- (a) Newtonian fluid (b) Bingham plastic
 (c) Pseudo plastic (d) Dilatant

IES-12. Ans. (d)



IES-13. Match List-I (Rheological Equation) with List-II (Types of Fluids) and select the correct the answer: [IES-2003]

- | List-I | List-II |
|--|--------------------|
| A. $\tau = \mu \left(\frac{du}{dy} \right)^n$, $n=1$ | 1. Bingham plastic |

$$\tau = \mu \left(\frac{du}{dy} \right)^n$$

- B.** $\tau = \mu (du/dy)^n$, $n < 1$ **2.** Dilatant fluid
- C.** $\tau = \mu (du/dy)^n$, $n > 1$ **3.** Newtonian fluid
- D.** $\tau = \tau_0 + \mu (du/dy)^n$, $n = 1$ **4.** Pseudo-plastic fluid

Codes:	A	B	C	D		A	B	C	D	
(a)	3	2	4	1		(b)	4	1	2	3
(c)	3	4	2	1		(d)	4	2	1	3

IES-13. Ans. (c)

IES-14. Assertion (A): Blood is a Newtonian fluid. [IES-2007]
Reason (R): The rate of strain varies non-linearly with shear stress for blood.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-14. Ans. (d) A is false but R is true.

IES-15. Match List-I with List-II and select the correct answer. [IES-1995]

List-I (Properties of fluids)	List-II (Definition/ Results)
A. Ideal fluid	1. Viscosity does not change with rate of deformation
B. Newtonian fluid	2. Fluid of zero viscosity
C. μ / ρ	3. Dynamic viscosity
D. Mercury in glass	4. Capillary depression
	5. Kinematic viscosity
	6. Capillary rise

Code:	A	B	C	D		A	B	C	D	
(a)	1	2	4	6		(b)	1	2	3	4
(c)	2	1	3	6		(d)	2	1	5	4

IES-15. Ans. (d)

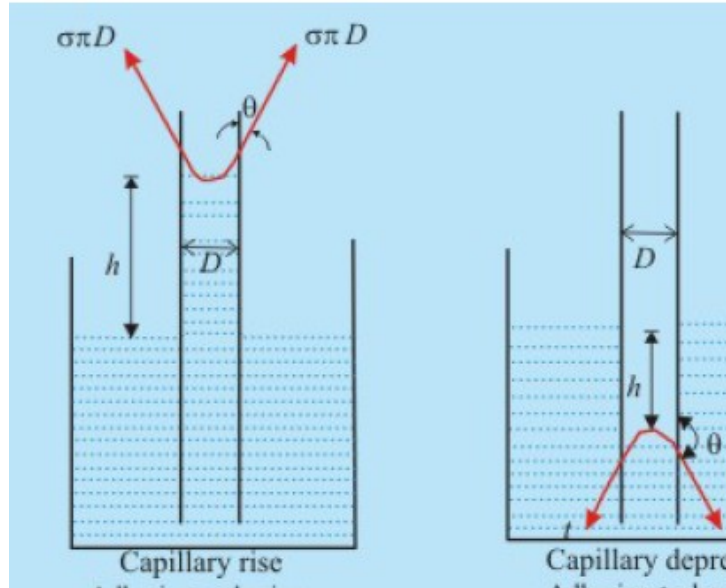
Surface Tension

IES-16. Surface tension is due to [IES-1998]

- (a) Viscous forces (b) Cohesion
 (c) Adhesion (d) The difference between adhesive and cohesive forces

IES-16. Ans. (b) Surface tension is due to cohesion between liquid particles at the surface, where as capillarity is due to both cohesion and adhesion. The property of cohesion enables a liquid to resist tensile stress, while adhesion enables it to stick to another body.

IES-17. What is the pressure difference between inside and outside of a droplet of water? [IES-2008]



- IES-22.** A capillary tube is inserted in mercury kept in an open container.
Assertion (A): The mercury level inside the tube shall rise above the level of mercury outside. [IES-2001]
Reason (R): The cohesive force between the molecules of mercury is greater than the adhesive force between mercury and glass.
- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true
- IES-22. Ans. (d)** Mercury shows capillary depression.
- IES-23.** What is the capillary rise in a narrow two-dimensional slit of width 'w'? [IES-2009]
 (a) Half of that in a capillary tube of diameter 'w'
 (b) Two-third of that in a capillary tube of diameter 'w'
 (c) One-third of that in a capillary tube of diameter 'w'
 (d) One-fourth of that in a capillary tube of diameter 'w'
- IES-23. Ans. (a)**
- IES-24.** **Assertion (A):** A narrow glass tube when immersed into mercury causes capillary depression, and when immersed into water causes capillary rise. [IES-2009]
Reason (R): Mercury is denser than water.
- (a) Both A and R are individually true and R is the correct explanation of A.
 (b) Both A and R are individually true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.
- IES-24. Ans. (b)** Causes of capillary depression: Adhesion is less than cohesion, the wetting tendency is less and the angle of contact is high.
- IES-25.** Consider the following statements related to the fluid properties:
1. Vapour pressure of water at 373 K is $101.5 \times 10^3 \text{ N/m}^2$.

2. Capillary height in cm for water in contact with glass tube and air is (tube dia)/0.268.

3. Blood is a Newtonian fluid

Which of the statements given above is/are correct? [IES-2008]

(a) 1 only (b) 1 and 3 (c) 1 and 2 (d) 2 only

IES-25. **Ans. (a)** Vapour pressure of water at 373 K means 100°C is one atmosphere = 1.01325 bar = 101.325 × 10³ N/m².

Capillary height in cm for water in contact with glass tube =

$$\frac{0.3}{d}$$

For water and glass

$$\theta = 0^\circ, h = \frac{4\sigma}{\rho g d}$$

Blood is a pseudoplastic fluid.

Where

$$\tau = \mu \left(\frac{du}{dy} \right)^n; n < 1$$

Compressibility and Bulk Modulus

IES-26. Which one of the following is the bulk modulus K of a fluid? (Symbols have the usual meaning) [IES-1997]

(a) (b) (c) (d)

$$\rho \frac{dp}{d\rho}$$

$$\frac{dp}{\rho d\rho}$$

$$\frac{\rho d\rho}{dp}$$

$$\frac{d\rho}{\rho dp}$$

IES-26. **Ans.**

(a) Bulk modulus

$$K = -\frac{dp}{\frac{dv}{v}} \quad \text{and} \quad v = \frac{1}{\rho}$$

$$\therefore K = -\frac{dp}{\frac{-d\rho/\rho^2}{1/\rho}} \quad \therefore dv = -\frac{d\rho}{\rho^2}$$

$$K = \frac{\rho d\rho}{d\rho}$$

IES-27. When the pressure on a given mass of liquid is increased from 3.0 MPa to 3.5 MPa, the density of the liquid increases from 500 kg/m³ to 501 kg/m³. What is the average value of bulk modulus of the liquid over the given pressure range? [IES-2006]

(a) 700 MPa (b) 600MPa (c) 500MPa (d) 250MPa

IES-27. **Ans.(d)**

$$\frac{500 \times (3.5 - 3.0)}{(501 - 500)} = 250 \text{ MPa}$$

Vapour Pressure

IES-28. Which Property of mercury is the main reason for use in barometers?

- (a) High Density (b) Negligible Capillary effect
(c) Very Low vapour Pressure (d) Low compressibility **[IES-2007]**

IES-28. Ans. (c)

IES-29. Consider the following properties of a fluid: **[IES-2005]**

- 1. Viscosity** **2. Surface tension**
3. Capillarity **4. Vapour pressure**

Which of the above properties can be attributed to the flow of jet of oil in an unbroken stream?

- (a) 1 only (b) 2 only (c) 1 and 3 (d) 2 and 4

IES-29. Ans. (b) Surface tension forces are important in certain classes of practical problems such as,

1. Flows in which capillary waves appear
2. Flows of small jets and thin sheets of liquid injected by a nozzle in air
3. Flow of a thin sheet of liquid over a solid surface.

Here the significant parameter for dynamic similarity is the magnitude ratio of the surface tension force to the inertia force. And we must use Weber number for similarity. Therefore the answer will be surface tension.

And you also know that Pressure inside a Liquid jet.

$$\Delta p = \frac{2\sigma}{d}$$

IES-30. Match List-I with List-II and select the correct answer using the code given below the lists: **[IES-2008]**

List-I (Variable Expression)

- A.** Dynamic Viscosity
B. Moment of momentum
C. Power
D. Volume modulus of elasticity

List-II

- 1.** $M L^2 T^{-3}$
- 2.** $M L^{-1} T^{-2}$
- 3.** $M L^{-1} T^{-1}$
- 4.** $M L^2 T^{-2}$
- 5.** $M L^2 T^{-1}$

(Dimensional)

Codes:	A	B	C	D	A	B	C	D	
(a)	1	4	2	3	(b)	3	5	1	2
(c)	1	5	2	3	(d)	3	4	1	2

IES-30. Ans. (b)

Previous 20-Years IAS Questions

Fluid

IAS-1. Which one of the following sets of conditions clearly apply to an ideal fluid? **[IAS-1994]**

- (a) Viscous and compressible (b) Non-viscous and incompressible
(c) Non-viscous and compressible (d) Viscous and incompressible

IAS-1. Ans. (b)

Viscosity

IAS-2. When a flat plate of 0.1 m^2 area is pulled at a constant velocity of 30 cm/sec parallel to another stationary plate located at a distance 0.01 cm from it and the space in between is filled with a fluid of dynamic viscosity $= 0.001\text{ Ns/m}^2$, the force required to be applied is: **[IAS-2004]**

- (a) 0.3 N (b) 3 N (c) 10 N
(d) 16 N

IAS-2. Ans. (a) Given, $\mu = 0.001\text{ Ns/m}^2$ and $du = (V - 0) = 30\text{ cm/sec} = 0.3\text{ m/s}$ and distance $(dy) = 0.01\text{ cm} = 0.0001\text{ m}$
Therefore, Shear stress $(\tau) =$

$$\tau = \mu \frac{du}{dy} = \left(0.001 \frac{\text{Ns}}{\text{m}^2}\right) \times \frac{(0.3\text{m/s})}{(0.0001\text{m})} = 3\text{N/m}^2$$

Force required $(F) = \tau \times A = 3 \times 0.1 = 0.3\text{ N}$

Newtonian Fluid

IAS-3. In a Newtonian fluid, laminar flow between two parallel plates, the ratio $\left(\frac{\tau}{\dots}\right)$ between the shear stress and rate of shear strain is given by

[IAS-1995]

- (a) $\frac{\mu}{dy^2} d^2\mu$ (b) $\mu \frac{du}{dy}$ (c) $\mu \left(\frac{du}{dy}\right)^2$ (d) $\mu \left(\frac{du}{dy}\right)^{\frac{1}{2}}$

(d) $\mu \left(\frac{du}{dy}\right)^{\frac{1}{2}}$

IAS-3. Ans. (b)

IAS-4. Consider the following statements: **[IAS-2000]**

- 1. Gases are considered incompressible when Mach number is less than 0.2**
 - 2. A Newtonian fluid is incompressible and non-viscous**
 - 3. An ideal fluid has negligible surface tension**
- Which of these statements is /are correct?**

- (a) 2 and 3 (b) 2 alone (c) 1 alone (d) 1 and 3

IAS-4. Ans. (d)

Non-Newtonian Fluid

IAS-5. The relations between shear stress (τ) and velocity gradient for ideal fluids, Newtonian fluids and non-Newtonian fluids are given below. Select the correct combination. **[IAS-2002]**

ideal fluids, Newtonian fluids and non-Newtonian fluids are given below. Select the correct combination. **[IAS-2002]**

- (a) $\tau = 0$; $\tau = \mu \left(\frac{du}{dy}\right)^2$; $\tau = \mu \left(\frac{du}{dy}\right)^3$
- (b) $\tau = 0$; $\tau = \mu \left(\frac{du}{dy}\right)$; $\tau = \mu \left(\frac{du}{dy}\right)^2$
- (c) $\tau = \mu \left(\frac{du}{dy}\right)$; $\tau = \mu \left(\frac{du}{dy}\right)^2$; $\tau = \mu \left(\frac{du}{dy}\right)^3$
- (d) $\tau = \mu \left(\frac{du}{dy}\right)$; $\tau = \mu \left(\frac{du}{dy}\right)^2$; $\tau = 0$

IAS-5. Ans. (b)

IAS-6. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are known as [IAS-1997]

- (a) Rhedopectic fluids (b) Thixotropic fluids
 (c) Pseudoplastic fluids (d) Newtonian fluids

IAS-6. Ans. (a) where $f(t)$ is increasing

$$\tau = \mu \left(\frac{du}{dy}\right)^n + f(t)$$

Surface Tension

IAS-7. At the interface of a liquid and gas at rest, the pressure is: [IAS-1999]

- (a) Higher on the concave side compared to that on the convex side
 (b) Higher on the convex side compared to that on the concave side
 (c) Equal to both sides
 (d) Equal to surface tension divided by radius of curvature on both sides.

IAS-7. Ans. (a)

Vapour Pressure

IAS-8. Match List-I (Physical properties of fluid) with List-II (Dimensions/Definitions) and select the correct answer: [IAS-2000]

List-I

- A.** Absolute viscosity
B. Kinematic viscosity
C. Newtonian fluid
D. Surface tension

List-II

- 1.** du/dy is constant
2. Newton per metre
3. Poise
4. Stress/Strain is constant
5. Stokes

Codes:	A	B	C	D		A	B	C	D	
(a)	5	3	1	2		(b)	3	5	2	4
(c)	5	3	4	2		(d)	3	5	1	2

IAS-8. Ans. (d)

Answers with Explanation (Objective)

Problem

1. A circular disc of diameter D is slowly in a liquid of a large viscosity (μ) at a small distance (h) from a fixed surface. Derive an expression of torque (T) necessary to maintain an angular velocity (ω)

1. Ans. $T =$

$$\frac{\pi \mu D^4 \omega}{32h}$$

2. A metal plate $1.25 \text{ m} \times 1.25 \text{ m} \times 6 \text{ mm}$ thick and weighting 90 N is placed midway in the 24 mm gap between the two vertical plane surfaces. The Gap is filled with an oil of specific gravity 0.85 and dynamic viscosity $3.0 \text{ N}\cdot\text{s}/\text{m}^2$. Determine the force required to lift the plate with a constant velocity of 0.15 m/s .

2. Ans. 168.08 N

3. A 400 mm diameter shaft is rotating at 200 rpm in a bearing of length 120 mm . If the thickness of oil film is 1.5 mm and the dynamic viscosity of the oil is $0.7 \text{ N}\cdot\text{s}/\text{m}^2$ determine:
 (i) Torque required overcoming friction in bearing;
 (ii) Power utilization in overcoming viscous resistance;

3. Ans. (i) 58.97 Nm (ii) 1.235 kW

4. In order to form a stream of bubbles, air is introduced through a nozzle into a tank of water at 20°C . If the process requires 3.0 mm diameter bubbles to be formed, by how much the air pressure at the nozzle must exceed that of the surrounding water? What would be the absolute pressure inside the bubble if the surrounding water is at $100.3 \text{ kN}/\text{m}^2$? ($\sigma = 0.0735 \text{ N/m}$)

σ

4. Ans. $P_{\text{abs}} = 100.398 \text{ kN}/\text{m}^2$ (Hint. Bubble of air but surface tension of water).

5. A U-tube is made up of two capillaries of diameters 1.0 mm and 1.5 mm respectively. The U tube is kept vertically and partially filled with water of surface tension $0.0075 \text{ kg}/\text{m}$ and zero contact angles. Calculate the difference in the level of the menisci caused by the capillarity.

5. Ans. 10 mm

6. If a liquid surface (density ρ) supports another fluid of density, ρ_b above the meniscus, then a balance of forces would result in capillary rise $h =$

$$\frac{4\sigma \cos\theta}{(\rho - \rho_b)gd}$$

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[the value of γ varies with height] [AMIE- WINTER

$$\frac{dT}{dz} = -\frac{g}{R} \cdot \left(\frac{\gamma-1}{\gamma} \right) \gamma$$

Answer: **-2002]**

Assumptions:

(i) Air is an Ideal fluid.

(ii) Temperature variation follows adiabatic process.

We know that temperature at any point in compressible fluid in an adiabatic process.

$$T = T_0 \left[1 - \frac{\gamma-1}{\gamma} \cdot \frac{gz}{RT_0} \right]$$

$$\therefore \frac{dT}{dz} = -\frac{g}{R} \cdot \frac{\gamma-1}{\gamma}$$

Note: If allotted marks is high for the question then more then proof from Aerostatic law also.

$$T = T_0 \left(1 - \frac{\gamma-1}{\gamma} \cdot \frac{gz}{RT_0} \right) \quad \left(\frac{\partial p}{\partial z} = -\rho g \right)$$

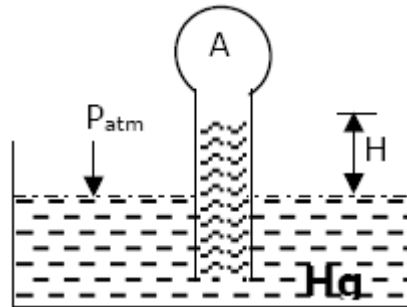
OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Absolute and Gauge Pressures

GATE-1. In given figure, if the pressure of gas in bulb A is 50 cm Hg vacuum and $P_{atm}=76$ cm Hg, then height of column H is equal to

- (a) 26 cm (b) 50 cm
 (c) 76 cm (d) 126 cm



[GATE-2000]

GATE-1. Ans. (b) If the pressure of gas in bulb A is atm. $H =$ zero.

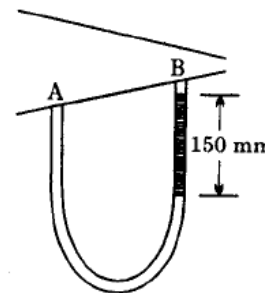
If we create a pressure of gas in bulb A is 1 cm Hg vacuum then the vacuum will lift 1 cm liquid, $H = 1$ cm.

If we create a pressure of gas in bulb A is 2 cm Hg vacuum then the vacuum will lift 2 cm liquid, $H = 2$ cm.

If we create a pressure of gas in bulb A is 50 cm Hg vacuum then the vacuum will lift 50 cm liquid, $H = 50$ cm.

Manometers

GATE-2. A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure. The density of mercury is 13600 Kg/m^3 and $g = 9.81 \text{ m/s}^2$. Which of the following is correct?



- (a) Flow Direction is A to B and $P_A - P_B = 20 \text{ KPa}$
 (b) Flow Direction is B to A and $P_A - P_B = 1.4 \text{ KPa}$
 (c) Flow Direction is A to B and $P_B - P_A = 20 \text{ KPa}$
 (d) Flow Direction is B to A and $P_B - P_A = 1.4 \text{ KPa}$

[GATE-2005]

GATE-2. Ans. (a)

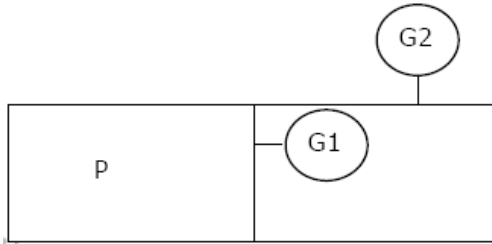
and as

$$P_B + 150 \text{ mm} - \text{Hg} = P_A \text{ Or } P_A - P_B = 0.150 \times 13600 \times 9.81 \approx 20 \text{ kPa}$$

P_A is greater than P_B therefore flow direction is A to B.

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GATE-3. The pressure gauges G_1 and G_2 installed on the system show pressures of $P_{G1} = 5.00$ bar and $P_{G2} = 1.00$ bar. The value of unknown pressure P is? (Atmospheric pressure 1.01 bars)



- (a) 1.01 bar
- (b) 2.01 bar
- (c) 5.00 bar
- (d) 7.01 bar

[GATE-2004]

GATE-3. Ans. (d) Pressure in the right cell = + Atmospheric pressure

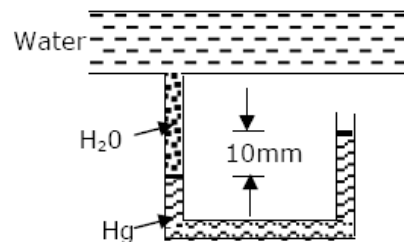
$$P_{G_2}$$

$$= 1.01 + 1.0 = 2.01 \text{ bar}$$

Therefore $P = + \text{Pressure on right cell} = 5 + 2.01 = 7.01 \text{ bar}$

$$P_{G_1}$$

GATE-4. A mercury manometer is used to measure the static pressure at a point in a water pipe as shown in Figure. The level difference of mercury in the two limbs is 10 mm. The gauge pressure at that point is



- (a) 1236 Pa
- (b) 1333 Pa
- (c) Zero
- (d) 98 Pa

[GATE-1996]

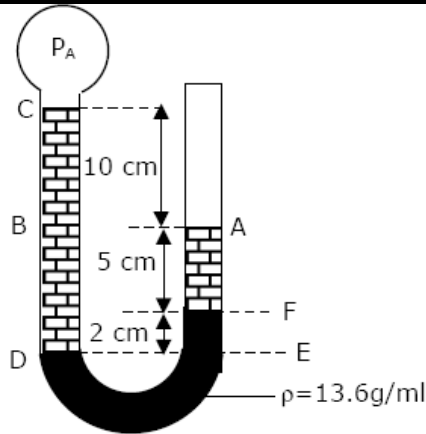
GATE-4. Ans. (a)

$$h = y \left(\frac{s_h}{s_l} - 1 \right) \text{ m of light fluid or } h = 0.010 \left(\frac{13.6}{1} - 1 \right) = 0.126 \text{ m of water column}$$

$$\text{or } P = h\rho g = 0.126 \times 1000 \times 9.81 = 1236 \text{ N/m}^2 = 1236 \text{ Pa}$$

GATE-5. Refer to Figure, the absolute pressure of gas A in the bulb is:

- (a) 771.2 mm Hg
- (b) 752.65 mm Hg
- (c) 767.35 mm Hg
- (d) 748.8 mm Hg



[GATE-1997]

GATE-5. Ans. (a) Use 'hs' formula;

$$H_A + 170 \times 1 - 20 \times 13.6 - 50 \times 1 = h_{am.} (760 \times 13.6) \text{ [All mm of water]}$$

$$\text{Or } H_A = 10488 / 13.6 \text{ mm of Hg} = 771.2 \text{ mm of Hg (Abs.)}$$

Mechanical Gauges

GATE-6. A siphon draws water from a reservoir and discharges it out at atmospheric pressure. Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is: [GATE-2006]

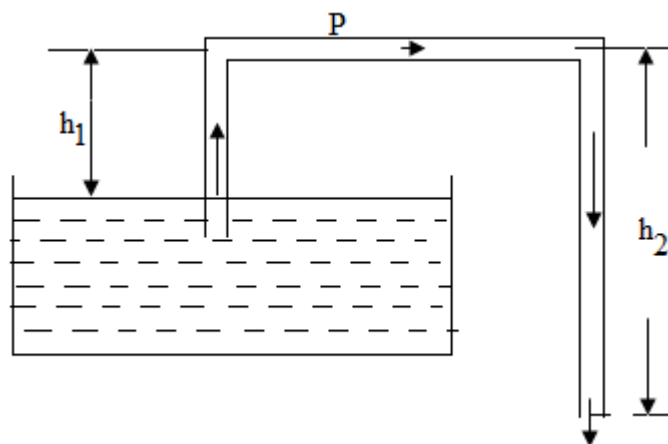
- (a) $\sqrt{2gh_1}$
- (b) $\sqrt{2gh_2}$
- (c) $\sqrt{2g(h_2 - h_1)}$
- (d) $\sqrt{2g(h_2 + h_1)}$

GATE-6. Ans. (c) By energy conservation, velocity at point Q =

$$\sqrt{2g(h_2 - h_1)}$$

As there is a continuous and uniform flow, so velocity of liquid at point Q and P is same.
 $V_p =$

$$\sqrt{2g(h_2 - h_1)}$$



Previous 20-Years IES Questions

Pressure of a Fluid

IES-1. A beaker of water is falling freely under the influence of gravity. Point B is on the surface and point C is vertically below B near the bottom of the beaker. If P_B is the pressure at point B and P_C the pressure at point C, then which one of the following is correct? [IES-2006]

- (a) $P_B = P_C$ (b) $P_B < P_C$ (c) $P_B > P_C$ (d) Insufficient data

IES-1. Ans. (a) For free falling body relative acceleration due to gravity is zero
 $P = \rho gh$ if $g=0$ then $p=0$ (but it is only hydrostatic pr.) these will be
 $\therefore \rho$

atmospheric pressure through out the liquid.

IES-2. Assertion (A): If a cube is placed in a liquid with two of its surfaces parallel to the free surface of the liquid, then the pressures on the two surfaces which are parallel to the free surface, are the same. [IES-2000]

Reason (R): Pascal's law states that when a fluid is at rest, the pressure at any plane is the same in all directions.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-2. Ans. (d)

IES-3. In an open U tube containing mercury, kerosene of specific gravity 0.8 is poured into one of its limbs so that the length of column of kerosene is about 40 cm. The level of mercury column in that limb is lowered approximately by how much? [IES-2008]

- (a) 2.4 cm (b) 1.2 cm (c) 3.6 cm (d) 0.6 cm

IES-3. Ans. (b) $0.8 \times 40 = 13.6 \times (2h) \Rightarrow h = 1.2 \text{ cm}$

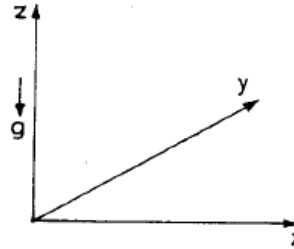
Hydrostatic Law and Aerostatic Law

IES-4. Hydrostatic law of pressure is given as [IES 2002; IAS-2000]

- | | | | |
|--|-------------------------------------|-------------------------------------|---|
| (a) | (b) | (c) | (d) |
| $\frac{\partial p}{\partial z} = \rho g$ | $\frac{\partial p}{\partial z} = 0$ | $\frac{\partial p}{\partial z} = z$ | $\frac{\partial p}{\partial z} = \text{const.}$ |

IES-4. Ans. (a)

IES-5. If z is vertically upwards, ρ is the density and g gravitational acceleration (see figure) then the



$\frac{\partial p}{\partial z}$

pressure in a fluid at rest due to gravity is given by

- (a) $\rho g z^2 / 2$ (b) $-\rho g$
 (c) $-\rho g z$ (d) $-\rho g / z$

[IES-1995; 1996]

IES-5. Ans. (b) Pressure at any point at depth z due to gravitational acceleration is, $p = \rho g h$. Since z is vertically upwards, $\frac{\partial p}{\partial z} = -\rho g$

$\frac{\partial p}{\partial z}$

Absolute and Gauge Pressures

IES-6. The standard atmospheric pressure is 762 mm of Hg. At a specific location, the barometer reads 700 mm of Hg. At this place, what does at absolute pressure of 380 mm of Hg correspond to? **[IES-2006]**

- (a) 320 mm of Hg vacuum (b) 382 of Hg vacuum
 (c) 62 mm of Hg vacuum (d) 62 mm of Hg gauge

IES-6. Ans. (a)

Manometers

IES-7. The pressure difference of two very light gasses in two rigid vessels is being measured by a vertical U-tube water filled manometer. The reading is found to be 10 cm. what is the pressure difference? **[IES-2007]**

- (a) 9.81 kPa (b) 0.0981 bar (c) 98.1 Pa
 (d) 981 N/m²

IES-7. Ans. (d) $p = h \rho g = 0.1 \times 1000 \times 9.81 \text{ N/m}^2 = 981 \text{ N/m}^2$

$\Delta \Delta \times \times \times \times$

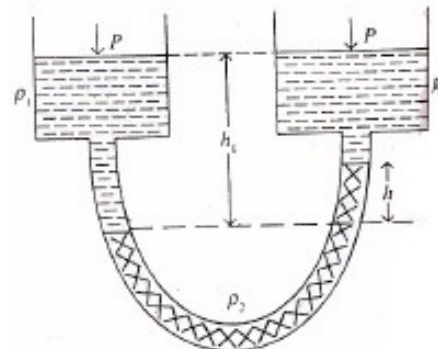
IES-8. The balancing column shown in the diagram contains 3 liquids of different densities ρ_1 , ρ_2 , and ρ_3 . The

$\rho_1 \quad \rho_2 \quad \rho_3$

liquid level of one limb is h_1 below the top level and there is a difference of h relative to that in the other limb.

What will be the expression for h ?

- (a) (b)



[IES-2004]

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(c)

$$\frac{\rho_1 - \rho_3}{\rho_2 - \rho_3} h_1$$

(d)

$$\frac{\rho_1 - \rho_2}{\rho_2 - \rho_3} h_1$$

IES-8. Ans. (c)

$$h_1 \rho_1 = h \rho_2 + (h_1 - h) \rho_3$$

IES-9. A mercury-water manometer has a gauge difference of 500 mm (difference in elevation of menisci). What will be the difference in pressure? [IES-2004]

(a) 0.5 m

(b) 6.3 m

(c) 6.8 m

(d) 7.3 m

IES-9. Ans. (b)

$$h = y \left(\frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.5 \left(\frac{13.6}{1} - 1 \right) = 6.3 \text{ m of water.}$$

IES-10. To measure the pressure head of the fluid of specific gravity S flowing through a pipeline, a simple micro-manometer containing a fluid of specific gravity S₁ is connected to it. The readings are as indicated as the diagram. The pressure head in the pipeline is:

(a) $h_1 S_1 - h S - \Delta h (S_1 - S)$

Δ

(b) $h_1 S_1 - h S + \Delta h (S_1 - S)$

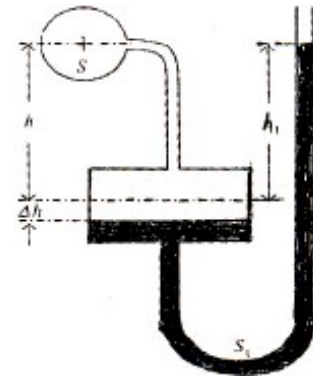
Δ

(c) $h S - h_1 S_1 - \Delta h (S_1 - S)$

Δ

(d) $h S - h_1 S_1 + \Delta h (S_1 - S)$

Δ



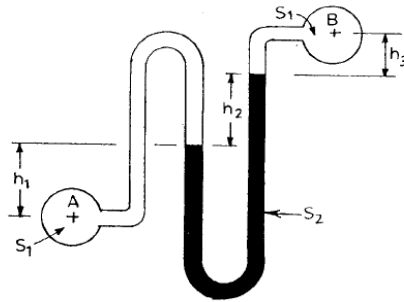
[IES-2003]

IES-10. Ans. (a) Use 'hs' rules;

The pressure head in the pipeline (H_p)

$$H_p + h S + \Delta h S - \Delta h S_1 - h_1 S_1 = 0 \text{ or } H_p = h_1 S_1 - h S - \Delta h (S_1 - S)$$

IES-11. Two pipe lines at different pressures, P_A and P_B , each carrying the same liquid of specific gravity S_1 , are connected to a U-tube with a liquid of specific gravity S_2 resulting in the level differences h_1 , h_2 and h_3 as shown in the figure. The difference in pressure head between points A and B in terms of head of water is:



[IES-1993]

- (a) $h_1 S_2 + h_2 S_1 + h_3 S_1$ (b) $h_1 S_1 + h_2 S_2 - h_3 S_1$
 (c) $h_1 S_1 - h_2 S_2 - h_3 S_1$ (d) $h_1 S_1 + h_2 S_2 + h_3 S_1$

IES-11. Ans. (d) Using hS formula: P_A and P_B (in terms of head of water)

$$P_A - h_1 S_1 - h_2 S_2 - h_3 S_1 = P_B \text{ or } P_A - P_B = h_1 S_1 + h_2 S_2 + h_3 S_1$$

IES-12. Pressure drop of flowing through a pipe (density 1000 kg/m^3) between two points is measured by using a vertical U-tube manometer. Manometer uses a liquid with density 2000 kg/m^3 . The difference in height of manometric liquid in the two limbs of the manometer is observed to be 10 cm. The pressure drop between the two points is:

[IES-2002]

- (a) 98.1 N/m^2 (b) 981 N/m^2 (c) 1962 N/m^2
 (d) 19620 N/m^2

IES-12. Ans. (b)

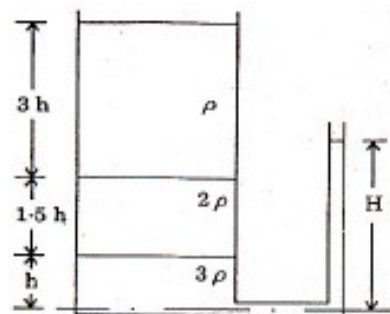
$$h = y \left(\frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.1 \left(\frac{2}{1} - 1 \right) = 0.1 \text{ m of light fluid}$$

The pressure drop between the two points is $= h \rho g = 0.1 \times 9.81 \times 1000 = 981 \text{ N/m}^2$

IES-13. Three immiscible liquids of specific densities ρ , 2ρ and 3ρ are kept in a jar.

The height of the liquids in the jar and at the piezometer fitted to the bottom of the jar is as shown in the given figure. The ratio H/h is :

- (a) 4 (b) 3.5
 (c) 3 (d) 2.5



[IES-2001]

IES-13. Ans. (c) Use 'hs' formula

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$$3h \times \rho + 1.5h \times 2\rho + h \times 3\rho - H \times 3\rho = 0 \text{ Or } H/h = 3$$

IES-14. Differential pressure head measured by mercury oil differential manometer (specific gravity of oil is 0.9) equivalent to a 600 mm difference of mercury levels will nearly be: [IES-2001]

- (a) 7.62 m of oil (b) 76.2 m of oil
 (c) 7.34 m of oil (d) 8.47 m of oil

IES-14. Ans. (d)

$$h = y \left(\frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.600 \left(\frac{13.6}{0.9} - 1 \right) = 8.47 \text{ m of oil}$$

IES-15. How is the difference of pressure head, "h" measured by a mercury-oil differential manometer expressed? [IES-2008]

- (a) $h = x \left[1 - \frac{S_g}{S_o} \right]$ (b) $h = x [S_g - S_o]$
 (c) $h = x [S_o - S_g]$ (d) $h = x \left[\frac{S_g}{S_o} - 1 \right]$

Where x = manometer reading; S_g and S_o are the specific gravities of mercury and oil, respectively.

IES-15. Ans. (d) Measurement of h using U tube manometer

Case 1. When specific gravity of manometric liquid is more than specific gravity of liquid flowing

$$h = y \left(\frac{S_g}{S_o} - 1 \right)$$

In m of liquid flowing through pipe (i.e. m of light liquid)

Case 2. When specific gravity of manometric fluid is less than the specific gravity of liquid flowing.

$$h = y \left(1 - \frac{S_g}{S_o} \right)$$

In m of liquid flowing through pipe (i.e. m of heavy liquid)

IES-16. The differential manometer connected to a Pitot static tube used for measuring fluid velocity gives [IES-1993]

- (a) Static pressure (b) Total pressure
 (c) Dynamic pressure (d) Difference between total pressure and dynamic pressure

IES-16. Ans. (c) Fig. 6 shows a Pitot static tube used for measuring fluid velocity in a pipe and connected through points A and B to a differential manometer.

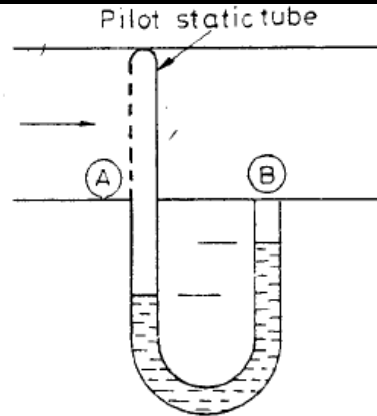


Fig. 6

Point A measures velocity head

$$\frac{V^2}{2g}$$

+ static pressure.

Whereas point B senses static pressure.

In actual practice point B is within the tube and not separate on the pipe. Thus manometer reads only dynamic pressure ()

$$\frac{V^2}{2g}$$

IES-17. Assertion (A): U-tube manometer connected to a venturimeter fitted in a pipeline can measure the velocity through the pipe.

[IES-1996]

Reason (R): U-tube manometer directly measures dynamic and static heads.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-17. Ans. (a)

IES-18. Pressure drop of water flowing through a pipe (density 1000 kg/m³) between two points is measured by using a vertical U-tube manometer. Manometer uses a liquid with density 2000 kg / m³. The difference in height of manometric liquid in the two limbs of the manometer is observed to be 10 cm. The pressure drop between the two points is:

[IES-2002]

- (a) 98.1 N/m²
- (b) 981 N/m²
- (c) 1962 N/m²
- (d) 19620 N/m²

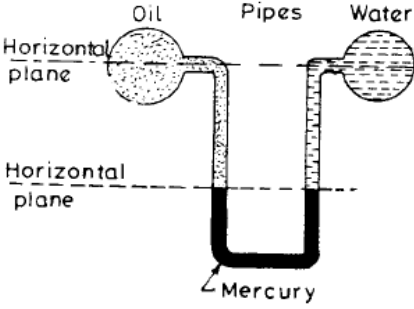
IES-18. Ans.

(b)
$$\frac{P_1 - P_2}{\rho g} = h \left(\frac{S_m}{S} - 1 \right)$$

$$(P_1 - P_2) = 1000 \times 0.1(2 - 1) = 981 \text{ N / m}^2$$

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IES-19. The manometer shown in the given figure connects two pipes, carrying oil and water respectively. From the figure one



- (a) Can conclude that the pressures in the pipes are equal.
- (h) Can conclude that the pressure in the oil pipe is higher.
- (c) Can conclude that the pressure in the water pipe is higher.
- (d) Cannot compare the pressure in the two pipes for want of sufficient data.

[IES-1996]

IES-19. Ans. (b) Oil has density lower than that of water. Thus static head of oil of same height will be lower. Since mercury is at same horizontal plane in both limbs, the lower static head of oil can balance higher static head of water when oil pressure in pipe is higher.

IES-20. In order to increase sensitivity of U-tube manometer, one leg is usually inclined by an angle θ . What is the sensitivity of inclined

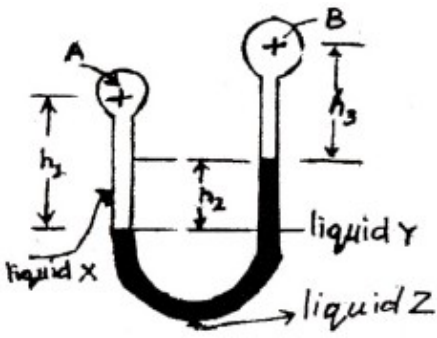
tube compared to sensitivity of U-tube?

[IES-2009]

- (a) $\sin \theta$
- (b) $\frac{1}{\sin \theta}$
- (c) $\frac{1}{\cos \theta}$
- (d) $\tan \theta$

IES-20. Ans. (b)

IES-21. A differential manometer is used to measure the difference in pressure at points A and B in terms of specific weight of water, W . The specific gravities of the liquids X, Y and Z are respectively s_1 , s_2 and s_3 . The correct difference is given by :



$$\left(\frac{P_A}{W} - \frac{P_B}{W} \right)$$

[IES-

1997]

- (a) $h_3 s_2 - h_1 s_1 + h_2 s_3$
- (b) $h_1 s_1 + h_2 s_3 - h_3 s_2$
- (c) $h_3 s_1 - h_1 s_2 + h_2 s_3$
- (d) $h_1 s_1 + h_2 s_2 - h_3 s_3$

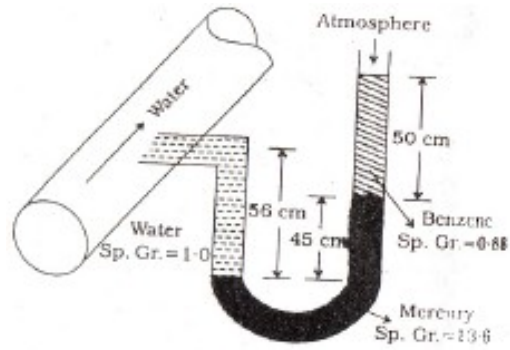
IES-21. Ans (a) Use 'hs' formula

$$\frac{P_A}{W} + h_1 s_1 - h_2 s_3 - h_3 s_2 = \frac{P_B}{W} \text{ Or } \frac{P_A}{W} - \frac{P_B}{W} = h_3 s_2 - h_1 s_1 + h_2 s_3$$

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IES-22. A U-tube manometer is connected to a pipeline conveying water as shown in the Figure. The pressure head of water in the pipeline is
 (a) 7.12 m (b) 6.56 m
 (c) 6.0 m (d) 5.12 m

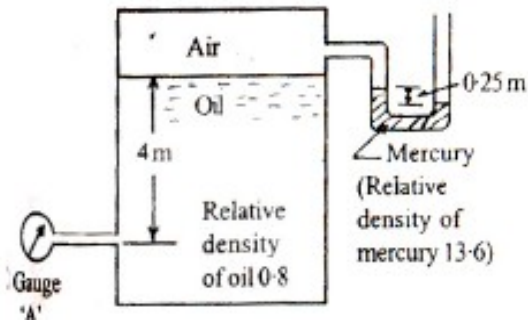


[IES-2000]

IES-22. Ans. (c) Use 'hs' formula;

$$H + 0.56 \times 1 - 0.45 \times 13.6 - 0.5 \times 0.88 = 0$$

IES-23. The reading of gauge 'A' shown in the given figure is:
 (a) -31.392 kPa
 (b) -1.962 kPa
 (c) 31.392 kPa
 (d) +19.62 kPa



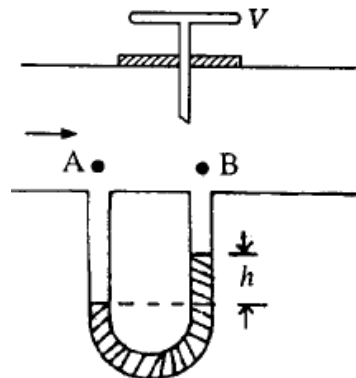
[IES-1999]

IES-23. Ans. (b) Use 'hs' formula;

$$H_A - 4 \times 0.8 + 0.25 \times 13.6 = 0 \text{ Or } H_A = -0.2 \text{ m of water column}$$

$$= -0.2 \times 9.81 \times 1000 \text{ N/m}^2 = -1.962 \text{ kPa}$$

IES-24. A mercury manometer is fitted to a pipe. It is mounted on the delivery line of a centrifugal pump. One limb of the manometer is connected to the upstream side of the pipe at 'A' and the other limb at 'B', just below the valve 'V' as shown in the figure. The manometer reading 'h' varies with different valve positions.
Assertion (A): With gradual closure of the valve, the magnitude of 'h' will go on increasing and even a situation may arise when mercury will be sucked in by the water flowing around 'B'.
Reason (R): With the gradual closure of the valve, the pressure at 'A' will go on increasing.



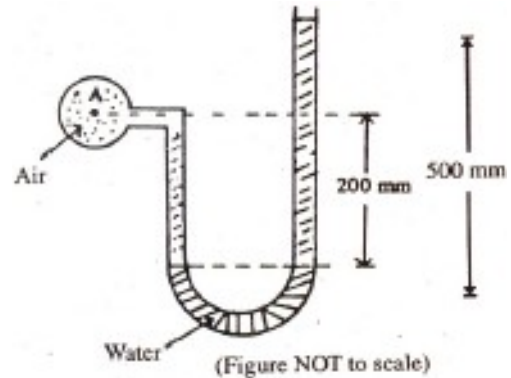
[IES-1998]

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- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-24. Ans. (a) With gradual closure the valve, the valve will be restricted the flow of liquid. Then pressure at A will be increased.

IES-25. In the figure shown below air is contained in the pipe and water is the manometer liquid. The pressure at 'A' is approximately:



- (a) 10.14 m of water absolute
- (b) 0.2 m of water
- (c) 0.2 m of water vacuum
- (d) 4901 pa

[IES-1998]

IES-25. Ans. (d) Use 'hs' formula;

$$H_{air} + 0.2 \times S_{air} (1.3/1000) - 0.5 \times 1 = 0 \text{ or } H_{air} = 0.49974 \text{ m of water column (Gauge)}$$

$$= 0.49974 \times 9.81 \times 1000 \text{ Pa} = 4902 \text{ Pa (gauge)}$$

It is not 10.14 m of absolute because atmospheric pressure = 10.33 m of water column if we add 0.49974 m gauge with atmosphere it will give us 10.83 m of absolute pressure. Without any calculation we are able to give the same answer as elevation of point A is lower than right limb then pressure at point A will be more than atmospheric (10.33m of water column).

IES-26. A manometer is made of a tube of uniform bore of 0.5 cm² cross-sectional area, with one limb vertical and the other limb inclined at 30° to the horizontal. Both of its limbs are open to atmosphere and, initially, it is partly filled with a manometer liquid of specific gravity 1.25. If then an additional volume of 7.5 cm³ of water is poured in the inclined tube, what is the rise of the meniscus in the vertical tube?

[IES-2006]

- (a) 4 cm
- (b) 7.5 cm
- (c) 12 cm
- (d) 15 cm

IES-26. Ans. (a) Let 'x' cm will be rise of the meniscus in the vertical tube. So for this 'x' cm rise quantity of 1.25 s.g. liquid will come from inclined limb. So we have to lower our reference line = $x \sin 30^\circ = x/2$. Then Pressure balance gives us

$$\left(x + \frac{x}{2}\right) \times 1250 \times 9.81 = \left(\frac{7.5}{0.5}\right) \sin 30^\circ \times 1000 \times 9.81 \quad \text{or } x = 4$$

IES-27. The lower portion of a U-tube of uniform bore, having both limbs vertical and open to atmosphere, is initially filled with a liquid of

specific gravity $3S$. A lighter liquid of specific gravity S is then poured into one of the limbs such that the length of column of lighter liquid is X . What is the resulting movement of the meniscus of the heavier liquid in the other limb? [IES-2008]

- (a) X (b) $X/2$ (c) $X/3$
 (d) $X/6$

IES-27. Ans. (d) $(s) \times (x) = (3s) \times (y)$

$$\therefore y = \frac{x}{3}$$

Resulting movement of meniscus =

$$\frac{x}{6}$$

Piezometer

IES-28. A vertical clean glass tube of uniform bore is used as a piezometer to measure the pressure of liquid at a point. The liquid has a specific weight of 15 kN/m^3 and a surface tension of 0.06 N/m in contact with air. If for the liquid, the angle of contact with glass is zero and the capillary rise in the tube is not to exceed 2 mm , what is the required minimum diameter of the tube? [IES-2006]

- (a) 6 mm (b) 8 mm (c) 10 mm (d) 12 mm

IES-28. Ans. (b)

$$h = \frac{4\sigma \cos \theta}{\rho g d} \leq 0.002 \quad \text{or} \quad d \geq \frac{4 \times 0.06 \times \cos 0^\circ}{15000 \times 0.002} = 8 \text{ mm}$$

IES-29. When can a piezometer be not used for pressure measurement in pipes?

- (a) The pressure difference is low [IES-2005]
 (b) The velocity is high
 (c) The fluid in the pipe is a gas
 (d) The fluid in the pipe is highly viscous

IES-29. Ans. (c)

Mechanical Gauges

IES-30. In a pipe-flow, pressure is to be measured at a particular cross-section using the most appropriate instrument. Match List-I (Expected pressure range) with List-II (Appropriate measuring device) and select the correct answer: [IES-2002]

- | List-I | List-II |
|---|----------------------------------|
| A. Steady flow with small positive and negative pressure | 1. Bourdon pressure gauge |
| B. Steady flow with small positive and negative pressure | 2. Pressure transducer |
| | 3. Simple piezometer |

- C. Steady flow with high gauge pressure 4. U-tube manometer
D.

Unsteady flow with fluctuating pressure

Codes:	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	1	4	3	2
(c)	3	4	1	2	(d)	1	2	3	4

IES-30. Ans. (c)

Previous 20-Years IAS Questions

Pressure of a Fluid

- IAS-1. The standard sea level atmospheric pressure is equivalent to**
 (a) 10.2 m of fresh water of $\rho = 998 \text{ kg/m}^3$ **[IAS-2000]**
 (b) 10.1 m of salt water of $\rho = 1025 \text{ kg/m}^3$
 (c) 12.5 m of kerosene of $\rho = 800 \text{ kg/m}^3$
 (d) 6.4 m of carbon tetrachloride of $\rho = 1590 \text{ kg/m}^3$

IAS-1. Ans. (b) ρgh must be equal to 1.01325 bar = 101325 N/m²
 For (a)

$$998 \times 9.81 \times 10.2 = 99862 \text{ N/m}^2$$

(b) $1025 \times 9.81 \times 10.1 = 101558 \text{ N/m}^2$

(c) $800 \times 9.81 \times 12.5 = 98100 \text{ N/m}^2$

(d) $1590 \times 9.81 \times 6.4 = 99826 \text{ N/m}^2$

Hydrostatic Law and Aerostatic Law

- IAS-2. Hydrostatic law of pressure is given as** **[IES 2002; IAS-2000]**
 (a) (b) (c) (d)

$$\frac{\partial p}{\partial z} = \rho g$$

$$\frac{\partial p}{\partial z} = 0$$

$$\frac{\partial p}{\partial z} = z$$

$$\frac{\partial p}{\partial z} = \text{const.}$$

IAS-2. Ans. (a)

- IAS-3. Match List-I (Laws) with List-II (Phenomena) and select the correct answer using the codes given below the lists: [IAS-1996]**

List-I

A. Hydrostatic law

List-II

1. Pressure at a point is equal in all directions in a fluid at rest

- | | |
|---------------------------|--|
| B. Newton's law | 2. Shear stress is directly proportional to velocity gradient in fluid flow |
| C. Pascal's law | 3. Rate of change of pressure in a vertical direction is proportional to specific weight of fluid |
| D. Bernoulli's law | |

Codes:	A	B	C	D	A	B	C	D	
(a)	2	3	-	1	(b)	3	2	1	-
(c)	2	-	1	3	(d)	2	1	-	3

IAS-3. Ans. (b)

Absolute and Gauge Pressures

IAS-4. The reading of the pressure gauge fitted on a vessel is 25 bar. The atmospheric pressure is 1.03 bar and the value of g is 9.81m/s^2 . The absolute pressure in the vessel is: [IAS-1994]
 (a) 23.97 bar (b) 25.00 bar (c) 26.03 bar (d) 34.84 bar

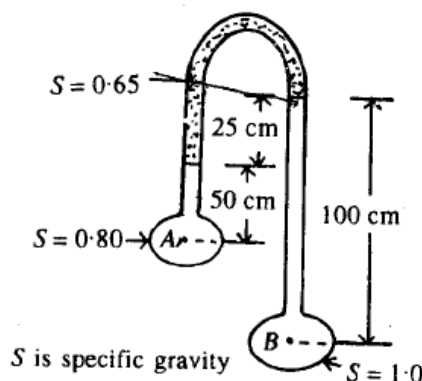
IAS-4. Ans. (c) Absolute pressure = Atmospheric pressure + Gauge Pressure
 = $25 + 1.03 = 26.03$ bar

IAS-5. The barometric pressure at the base of a mountain is 750mm Hg and at the top 600 mm Hg. If the average air density is 1kg/m^3 , the height of the mountain is approximately [IAS-1998]
 (a) 2000m (b) 3000 m (c) 4000 m (d) 5000 m

IAS-5. Ans. (a) Pressure difference = $(750 - 600) = 150$ mmHg = H m of air column
 = height of mountain. Therefore
 $0.150 \times (13.6 \times 10^3) \times g = H \times 1 \times g$ or $H = 2040\text{m} \approx 2000\text{m}$

Manometers

IAS-6. The pressure difference between point B and A (as shown in the above figure) in centimeters of water is:
 (a) -44 (b) 44
 (c) -76 (d) 76



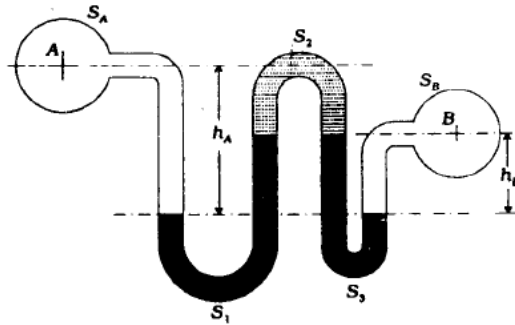
[IAS-2002]

IAS-6. Ans. (b) Use 'hs' formula

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$$h_A - 50 \times 0.8 - 25 \times 0.65 + 100 \times 1 = h_B \text{ or } h_B - h_A = 43.75 \text{ cm of water column}$$

IAS-7. A double U-tube manometer is connected to two liquid lines A and B. Relevant heights and specific gravities of the fluids are shown in the given figure. The pressure difference, in head of water, between fluids at A and B is



- (a) $S_A h_A + S_1 h_B - S_3 h_B + S_B h_B$
- (b) $S_A h_A - S_1 h_B - S_2 (h_A - h_B) + S_3 h_B - S_B h_B$
- (c) $S_A h_A + S_1 h_B + S_2 (h_A - h_B) - S_3 h_B + S_B h_B$
- (d)

$$h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$$

[IAS-2001]

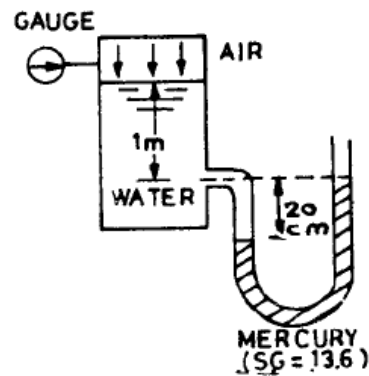
IAS-7. Ans. (d) Use 'hs' formula

$$H_A + h_A S_A - (h_A - h_B) S_1 + (h_A - h_B) S_3 - h_B S_B = H_B$$

or, $H_B - H_A = h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$

IAS-8. The pressure gauge reading in meter of water column shown in the given figure will be

- (a) 3.20 m
- (b) 2.72 m
- (c) 2.52 m
- (d) 1.52 m



[IAS-1995]

IAS-8. Ans. (d) Use 'hs' formula;

$$H_G + 1 \times 1 + 0.2 \times 1 - 0.2 \times 13.6 = 0 \text{ or } H_G = 1.52 \text{ m of water column}$$

Mechanical Gauges

IAS-9. Match List-I with List-II and select the correct answer using the codes given below the lists: **[IAS-1999]**

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List-I (Device)

- A.** Barometer
- B.** Hydrometer
- C.** U-tube manometer
- D.** Bourdon gauge

Codes:	A	B	C	D
(a)	2	3	1	4
(c)	3	2	4	1

List-II (Use)

- 1.** Gauge pressure
- 2.** Local atmospheric pressure
- 3.** Relative density
- 4.** Pressure differential

	A	B	C	D
(b)	3	2	1	4
(d)	2	3	4	1

IAS-9. Ans. (d)

3. Hydrostatic Forces on Surfaces

Contents of this chapter

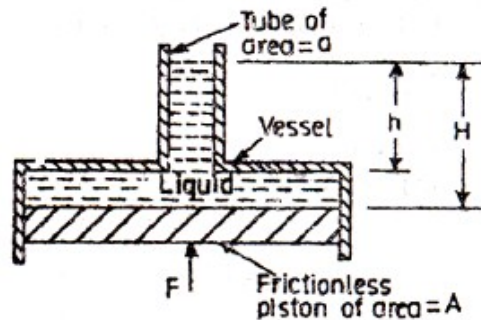
1. Hydrostatic forces on plane surface
 2. Hydrostatic forces on plane inclined surface
 3. Centre of pressure
 4. Hydrostatic forces on curved surface
 5. Resultant force on a sluice gate
 6. Lock gate
-

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. The force F needed to support the liquid of density d and the vessel on top is:

- (a) $gd[ha - (H - h) A]$
- (b) $gdHA$
- (c) $GdHa$
- (d) $gd(H - h)A$



[GATE-1995]

GATE-1. Ans. (b)

GATE-2. A water container is kept on a weighing balance. Water from a tap is falling vertically into the container with a volume flow rate of Q ; the velocity of the water when it hits the water surface is U . At a particular instant of time the total mass of the container and water is m . The force registered by the weighing balance at this instant of time is:

- (a) $mg + \frac{QU}{\rho}$
- (b) $mg + 2 \frac{QU}{\rho}$
- (c) $mg + \frac{QU^2/2}{\rho}$
- (d) $\frac{QU^2/2}{\rho}$

[GATE-2003]

GATE-2. Ans. (a) Volume flow rate = Q
 Mass of water strike = $\frac{QU}{\rho}$

Velocity of the water when it hit the water surface = U
 Force on weighing balance due to water strike

$$= \text{Initial momentum} - \text{final momentum}$$

$$= \frac{QU}{\rho} - 0 = \frac{QU}{\rho}$$

(Since final velocity is perpendicular to initial velocity)

Now total force on weighing balance = $mg + \frac{QU}{\rho}$

Previous 20-Years IES Questions

IES-1. A tank has in its side a very small horizontal cylinder fitted with a frictionless piston. The head of liquid above the piston is h and the piston area a , the liquid having a specific weight γ . What is

the force that must be exerted on the piston to hold it in position against the hydrostatic pressure? **[IES-2009]**

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(a) $2 \gamma ha$ (b) γha (c) _____ (d) $\frac{2 \gamma ha}{3}$ $\gamma \frac{ha}{2}$ _____

IES-1. Ans. (b) Pressure of the liquid above the piston = γh
 Force exerted on the piston to hold it in position = γha

IES-2. Which one of the following statements is correct? [IES-2005]
The pressure centre is:

- (a) The cycloid of the pressure prism
- (b) A point on the line of action of the resultant force
- (c) At the centroid of the submerged area
- (d) Always above the centroid of the area

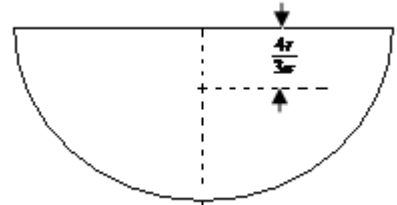
IES-2. Ans. (b)

IES-3. A semi-circular plane area of diameter 1 m, is subjected to a uniform gas pressure of 420 kN/m². What is the moment of thrust (approximately) on the area about its straight edge? [IES-2006]

- (a) 35 kNm (b) 41 kNm (c) 55 kNm (d) 82 kNm

IES-3. Ans. (a) Force (P) = p.A =

$$420 \times \frac{\pi \cdot 1^2}{4 \times 2}$$



Moment (M) =

$$P \times \bar{h}$$

=

$$420 \times \frac{\pi \times 1^2}{4 \times 2} \times \frac{4 \times (1/2)}{3 \times \pi} = 35 \text{ kNm}$$

IES-4. A horizontal oil tank is in the shape of a cylinder with hemispherical ends. If it is exactly half full, what is the ratio of magnitude of the vertical component of resultant hydraulic thrust on one hemispherical end to that of the horizontal component? [IES-2006]

- (a) $\frac{2}{\pi}$ (b) $\frac{1}{2}$ (c) $\frac{4}{3\pi}$ (d) $\frac{3}{4\pi}$

IES-4. Ans. (b) =

$$P_H = \rho g A \bar{x} = \rho g \left(\frac{\pi \cdot r^2}{4 \times 2} \right) \frac{4r}{3\pi} = \frac{2}{3} \rho g r^3$$

$$P_V = \rho g \nabla = \rho g \cdot \frac{1}{4} \left(\frac{4}{3} \pi r^3 \right) \therefore \frac{P_V}{P_H} = \frac{\pi}{2}$$

IES-5. A rectangular water tank, full to the brim, has its length, breadth and height in the ratio of 2: 1: 2. The ratio of hydrostatic forces at the bottom to that at any larger vertical surface is: [IES-1996]

- (a) 1/2 (b) 1 (c) 2 (d) 4

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IES-5. Ans. (b) Hydrostatic force at bottom = (length = 2x;

$$\rho g A \bar{z} = \rho g (2x \times 1x) \times 2x$$

breadth = 1x; height = 2x) =

$$4\rho g x^3$$

Hydrostatic force at larger vertical surface = $\rho g (2x \times 2x) \times \frac{2x}{2} = 4\rho g x^3$

∴ Ratio of above two forces = 1

IES-6. A circular plate 1.5 m diameter is submerged in water with its greatest and least depths below the surface being 2 m and 0.75 m respectively. What is the total pressure (approximately) on one face of the plate?

[IES-2007, IAS-2004]

- (a) 12kN (b) 16kN (c) 24kN (d) None of the above

IES-6. Ans. (c)

$$P = \rho g A \bar{x} = \rho g \left(\frac{\pi \times 1.5^2}{4} \right) \times \left(\frac{0.75 + 2}{2} \right) = 24 \text{ kN}$$

IES-7. A tank with four equal vertical faces of width t and depth h is

filled up with a liquid. If the force on any vertical side is equal to the force at the bottom, then the value of h/t will be: [IAS-2000, IES-1998]

- (a) 2 (b) $\sqrt{2}$ (c) 1 (d) 1/2

IES-7. Ans. (a) or

$$P_{\text{bottom}} = P_{\text{side}} \quad h \rho g \cdot t \cdot t = \rho g t h \cdot (h/2) \text{ or } \frac{h}{t} = 2$$

IES-8. The vertical component of the hydrostatic force on a submerged curved surface is the [IAS-1998, 1995, IES-1993, 2003]

- (a) Mass of liquid vertically above it
 (b) Weight of the liquid vertically above it
 (c) Force on a vertical projection of the surface
 (d) Product of pressure at the centroid and the surface area

IES-8. Ans. (b)

IES-9. What is the vertical component of pressure force on submerged curved surface equal to? [IES-2008]

- (a) Its horizontal component
 (b) The force on a vertical projection of the curved surface
 (c) The product of the pressure at centroid and surface area
 (d) The gravity force of liquid vertically above the curved surface up to the free surface

IES-9. Ans. (d) The vertical component of the hydrostatic force on a submerged curved surface is the weight of the liquid vertically above it.

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[IAS-1998, 1995, IES-1993, 2003]

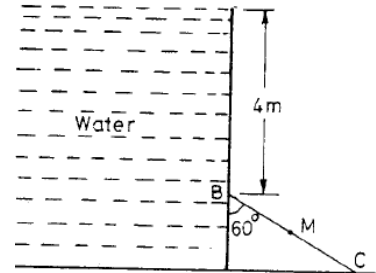
IES-10. Resultant pressure of the liquid in case of an immersed body acts through which one of the following? [IES-2007]

- (a) Centre of gravity (b) Centre of pressure
(c) Metacentre (d) Centre of buoyancy

IES-10. Ans. (b)

IES-11. In the situation shown in the given figure, the length BC is 3 m and M is the mid-point of BC. The hydrostatic force on BC measured per unit width (width being perpendicular to the plane of the paper) with 'g' being the acceleration due to gravity, will be

- (a) 16500 g N/m passing through M
(b) 16500 g N/m passing through a point between M and C
(c) 14250 g N/m passing through M
(d) 14250 g N/m passing through a point between M and C



[IES-1993]

IES-11. Ans. (d) The hydrostatic force on BC = . Vertical component (F_v)

$$\sqrt{F_v^2 + F_H^2}$$

= weight over area BC (for unit width)

$$= \left(\frac{4 + 5.5}{2} \right) \times \frac{3\sqrt{3}}{2} \times 10^3 g$$

$$= 12.34 \times 10^3 g \text{ N/m}$$

F_H = Projected area of BC, i.e. BD \times depth upon centre of BD $\times 10^3 g$
 = $1.5 \times 4.75 \times 10^3 g$ (for unit width)
 = $7.125 \times 10^3 g \text{ N/m}$

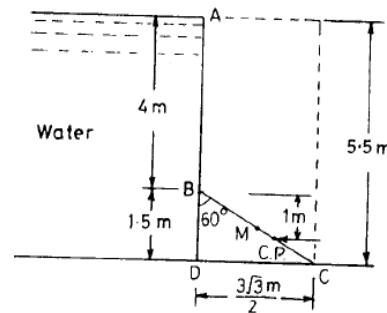
Resultant =

$$10^3 g \sqrt{12.34^2 + 7.125^2} = 14250 g \text{ N/m}$$

This resultant acts at centre of pressure, i.e., at $\frac{2}{3}$ of BD or between M

$\frac{2}{3}$

and C.



IES-12. A circular annular plate bounded by two concentric circles of diameter 1.2m and 0.8 m is immersed in water with its plane making an angle of 45° with the horizontal. The centre of the circles is 1.625m below the free surface. What will be the total pressure force on the face of the plate? [IES-2004]

- (a) 7.07 kN (b) 10.00 kN (c) 14.14 kN (d) 18.00kN

IES-12. Ans. (b)

$$\rho g A \bar{x} = 1000 \times 9.81 \times \frac{\pi}{4} (1.2^2 - 0.8^2) \times 1.625 \approx 10 \text{ kN}$$

IES-13. A plate of rectangular shape having the dimensions of 0.4m x 0.6m is immersed in water with its longer side vertical. The total hydrostatic thrust on one side of the plate is estimated as 18.3 kN. All other conditions remaining the same, the plate is turned through 90° such that its longer side remains vertical. What would be the total force on one of the plate? [IES-2004]

- (a) 9.15 kN (b) 18.3 kN (c) 36.6 kN (d) 12.2 kN

IES-13. Ans. (b)

IES-14. Consider the following statements about hydrostatic force on a submerged surface: [IES-2003]

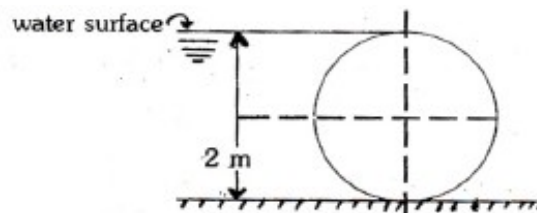
1. It remains the same even when the surface is turned.
2. It acts vertically even when the surface is turned.

Which of these is/are correct?

- (a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) Neither 1 nor 2

IES-14. Ans. (a)

IES-15. A cylindrical gate is holding water on one side as shown in the given figure. The resultant vertical component of force of water per meter width of gate will be:



[IES-1997]

- (a) Zero (b) 7700.8 N/m (c) 15401.7 N/m (d) 30803.4 N/m

IES-15. Ans. (c) Vertical component of force = Weight of the liquid for half cylinder portion

$$= \text{Volume} \times \rho g = \left(\frac{\pi D^2 L}{4} \right) \times \rho g = \left(\frac{\pi 2^2 \times 1}{4} \right) \times 1000 \times 9.81 = 15409.5 \text{ N/m}$$

IES-16. The vertical component of force on a curved surface submerged in a static liquid is equal to the [IES-1993]

- (a) Mass of the liquid above the curved surface
- (b) Weight of the liquid above the curved surface
- (c) Product of pressure at C.G. multiplied by the area of the curved surface
- (d) Product of pressure at C.G. multiplied by the projected area of the curved surface

IES-16. Ans. (b) The correct choice is (b) since the vertical component of force on a curved surface submerged in a static liquid is the weight of the liquid above the curved surface.

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IES-17. The depth of centre of pressure for a rectangular lamina immersed vertically in water up to height 'h' is given by: [IES-2003]

- (a) $h/2$ (b) $h/4$ (c) $2h/3$
 (d) $3h/2$

IES-17. Ans. (c)

IES-18. The point of application of a horizontal force on a curved surface submerged in liquid is: [IES-2003]

- (a) $\frac{I_G}{Ah} - \bar{h}$ (b) $\frac{I_G + Ah^2}{Ah}$ (c) $\frac{Ah}{I_G} + \bar{h}$ (d) $\frac{I_G}{h} + Ah$

$\frac{I_G}{h} + Ah$

Where A = area of the immersed surface
 = depth of centre of surface immersed

\bar{h}

I_G = Moment of inertia about centre of gravity

IES-18. Ans. (b)

IES-19. What is the vertical distance of the centre of pressure below the centre of the plane area? [IES-2009]

- (a) $\frac{I_G}{A.h}$ (b) $\frac{I_G \cdot \sin\theta}{A.h}$ (c) $\frac{I_G \cdot \sin^2\theta}{A.h}$ (d) $\frac{I_G \cdot \sin^2\theta}{A.h^2}$

IES-19. Ans. (c)

IES-20. What is the depth of centre of pressure of a vertical immersed surface from free surface of liquid equal to? [IES-2008]

- (a) $\frac{I_G}{Ah} + \bar{h}$ (b) $\frac{I_G A}{h} + \bar{h}$ (c) $\frac{I_G \bar{h}}{A} + \bar{h}$ (d) $\frac{Ah}{I_G} + \bar{h}$

(Symbols have their usual meaning)

IES-20. Ans. (a) Without knowing the formula also you can give answer option (b), (c) and (d) are not dimensionally homogeneous.

IES-21. Assertion (A): The centre of pressure for a vertical surface submerged in a liquid lies above the centroid (centre of gravity) of the vertical surface. [IES-2008]

Reason (R): Pressure from the free surface of the liquid for a vertical surface submerged in a liquid is independent of the density of the liquid.

- (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is NOT the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

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IES-21. Ans. (d) Centre of pressure lies always below the centre of gravity of vertical surface . Therefore the distance of centre of pressure

$$\bar{h} = \bar{x} + \frac{I_{CG}}{A\bar{x}}$$

from free surface is independent of density of liquid. So, (d) is the answer.

IES-22. Assertion (A): A circular plate is immersed in a liquid with its periphery touching the free surface and the plane makes an angle θ with the free surface with different values of θ , the position of centre of pressure will be different. [IES-2004]

Reason (R): Since the center of pressure is dependent on second moment of area, with different values of θ , second moment of area for the circular plate will change.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-22. Ans. (c)

$$\frac{I_G \cdot \sin \theta}{A \cdot \bar{h}}$$

IES-23. A rectangular plate 0.75 m × 2.4 m is immersed in a liquid of relative density 0.85 with its 0.75 m side horizontal and just at the water surface. If the plane of plate makes an angle of 60° with the horizontal, what is the approximate pressure force on one side of the plate?

[IES-2008]

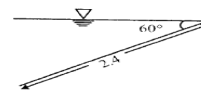
- (a) 7.80 kN
- (b) 15.60 kN
- (c) 18.00 kN
- (d) 24.00 kN

IES-23. Ans. (b) Pressure force on one side of plate

$$= wA\bar{x}$$

$$= (0.85 \times 9.81) \times (0.75 \times 2.4) \times \left(\frac{2.4 \sin 60}{2} \right)$$

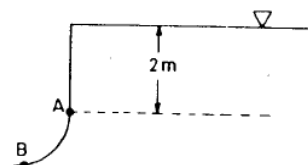
$$= 15.60 \text{ kN}$$



Pressure force on one side of plate
 $= wA\bar{x}$
 $= (0.85 \times 9.81) \times (0.75 \times 2.4)$
 $\times \left(\frac{2.4 \sin 60}{2} \right)$
 $= 15.60 \text{ kN}$

IES-24. The hydrostatic force on the curved surface AB shown in given figure acts.

- (a) Vertically downwards
- (b) Vertically upwards
- (c) Downwards, but at an angle with the vertical plane.
- (d) Upwards, but at an angle with the vertical plane



[IES-1994]

IES-24. Ans. (d) Total hydrostatic force on the curved surface is upwards, but at an angle with the vertical plane.

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IES-25. A dam is having a curved surface as shown in the figure. The height of the water retained by the dam is 20m; density of water is 1000kg/m^3 . Assuming g as 9.81 m/s^2 , the horizontal force acting on the dam per unit length is: [IES-2002]

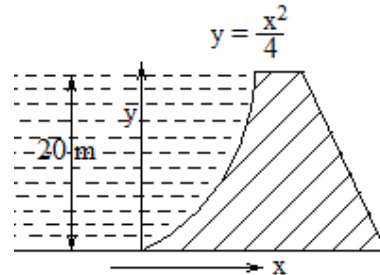
- (a) $1.962 \times 10^2\text{ N}$ (b) $2 \times 10^5\text{ N}$
 (c) $1.962 \times 10^6\text{ N}$ (d) $3.924 \times 10^6\text{ N}$

IES-25. Ans. (c)

$$P_H = \rho g A \bar{x}$$

$$= 1000 \times 9.81 \times (20 \times 1) \times (20/2)$$

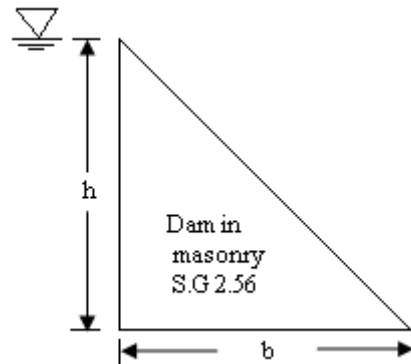
$$= 1.962 \times 10^6\text{ N}$$



IES-26. A triangular dam of height h and base width b is filled to its top with water as shown in the given figure. The condition of stability

- (a) $b = h$ (b) $b = 2.6h$
 (c) $b = \sqrt{3h}$ (d) $b = 0.425h$

$$\sqrt{3h}$$



[IES-1999]

IES-26. Ans. (d) Taking moment about topple point

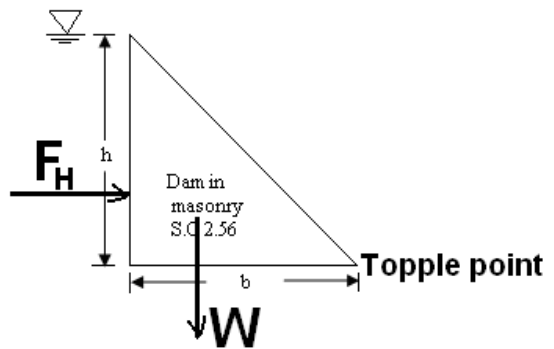
$$F_H \times \frac{h}{3} = W \times \frac{2b}{3}$$

$$\left(\rho_w g h \times 1 \times \frac{h}{2} \right) \times \frac{h}{3}$$

$$= \rho_w \times 2.56 \times g \times \left(\frac{bh}{2} \times 1 \right) \times \frac{2b}{3}$$

$$\text{or } b = 0.442h$$

Nearest answer (d)



IES-27. What acceleration would cause the free surface of a liquid contained in an open tank moving in a horizontal track to dip by 45° ? [IES-2008]

- (a) $g/2$ (b) $2g$ (c) g (d) $(3/2)g$

IES-27. Ans. (c)

IES-28. A vertical sludge gate, 2.5 m wide and weighting 500 kg is held in position due to horizontal force of water on one side and associated friction force. When the water level drops down to 2 m above the bottom of the gate, the gate just starts sliding

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down. The coefficient of friction between the gate and the supporting structure is: [IES-1999]

- (a) 0.20 (b) 0.10 (c) 0.05 (d) 0.02

IES-28. Ans. (b) or or

$$\mu P = W \quad \mu \rho g A \bar{x} = mg \quad \mu = \frac{m}{\rho A \bar{x}} = \frac{500}{1000 \times (2 \times 2.5) \times (2/2)} = 0.1$$

Previous 20-Years IAS Questions

IAS-1. A circular plate 1.5 m diameter is submerged in water with its greatest and least depths below the surface being 2 m and 0.75 m respectively. What is the total pressure (approximately) on one face of the plate?

[IES-2007, IAS-2004]

- (a) 12kN (b) 16kN (c) 24kN (d) None of the above

IAS-1. Ans. (c)

$$P = \rho g A \bar{x} = \rho g \left(\frac{\pi \times 1.5^2}{4} \right) \times \left(\frac{0.75 + 2}{2} \right) = 24 \text{ kN}$$

IAS-2. A tank with four equal vertical faces of width l and depth h is

filled up with a liquid. If the force on any vertical side is equal to the force at the bottom, then the value of h/l will be: [IAS-2000,

l

IES-1998]

- (a) 2 (b) $\sqrt{2}$ (c) 1 (d) 1/2

IAS-2. Ans. (a) or

$$P_{\text{bottom}} = P_{\text{side}} \quad h \rho g . l . l = \rho g t h . (h/2) \text{ or } \frac{h}{t} = 2$$

IAS-3. The vertical component of the hydrostatic force on a submerged curved surface is the [IAS-1998, 1995, IES-1993, 2003]

- (a) Mass of liquid vertically above it
 (b) Weight of the liquid vertically above it
 (c) Force on a vertical projection of the surface
 (d) Product of pressure at the centroid and the surface area

IAS-3. Ans. (b)

IAS-4. What is the vertical component of pressure force on submerged curved surface equal to? [IAS-1998, 1995, IES-1993, 2003]

- (a) Its horizontal component
 (b) The force on a vertical projection of the curved surface
 (c) The product of the pressure at centroid and surface area
 (d) The gravity force of liquid vertically above the curved surface up to the free surface

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IAS-4. Ans. (d) The vertical component of the hydrostatic force on a submerged curved surface is the weight of the liquid vertically above it.

IAS-5. Consider the following statements regarding a plane area submerged in a liquid: **[IAS-1995]**

1. The total force is the product of specific weight of the liquid, the area and the depth of its centroid.
2. The total force is the product of the area and the pressure at its centroid.

Of these statements:

- | | |
|----------------------------|------------------------------|
| (a) 1 alone is correct | (b) 2 alone is correct |
| (c) Both 1 and 2 are false | (d) Both 1 and 2 are correct |

IAS-5. Ans. (d)

IAS-6. A vertical dock gate 2 meter wide remains in position due to horizontal force of water on one side. The gate weights 800 Kg and just starts sliding down when the depth of water upto the bottom of the gate decreases to 4 meters. Then the coefficient of friction between dock gate and dock wall will be: **[IAS-1995]**

- | | | |
|----------|---------|----------|
| (a) 0.5 | (b) 0.2 | (c) 0.05 |
| (d) 0.02 | | |

IAS-6. Ans. (c)

or

or

$$\mu P = W \quad \mu \rho g(4 \times 2) \cdot (4/2) = 800 \times g \quad \mu = 0.05$$

IAS-7. A circular disc of radius 'r' is submerged vertically in a static fluid up to a depth 'h' from the free surface. If $h > r$, then the position of centre of pressure will: **[IAS-1994]**

- | | |
|-----------------------------------|------------------------------------|
| (a) Be directly proportional to h | (b) Be inversely proportional of h |
| (c) Be directly proportional to r | (d) Not be a function of h or r |

IAS-7. Ans. (a)

IAS-8. Assertion (A): The total hydrostatic force on a thin plate submerged in a liquid, remains same, no matter how its surface is turned. **[IAS-2001]**

Reason (R): The total hydrostatic force on the immersed surface remains the same as long as the depth of centroid from the free surface remains unaltered.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-8. Ans. (d) If \bar{x} changes it also change. $P = \rho g A \bar{x}$

$$\bar{x}$$

IAS-9. A rectangular tank of base 3 m × 3 m contains oil of specific gravity 0.8 upto a height of 8 m. When it is accelerated at 2.45 m/s² vertically upwards, the force on the base of the tank will be: **[IAS-1999]**

- | | | | |
|-------------|-------------|-------------|-------------|
| (a) 29400 N | (b) 38240 N | (c) 78400 N | (d) 49050 N |
|-------------|-------------|-------------|-------------|

IAS-9. Ans. (c)

4. Buoyancy and Flotation

Contents of this chapter

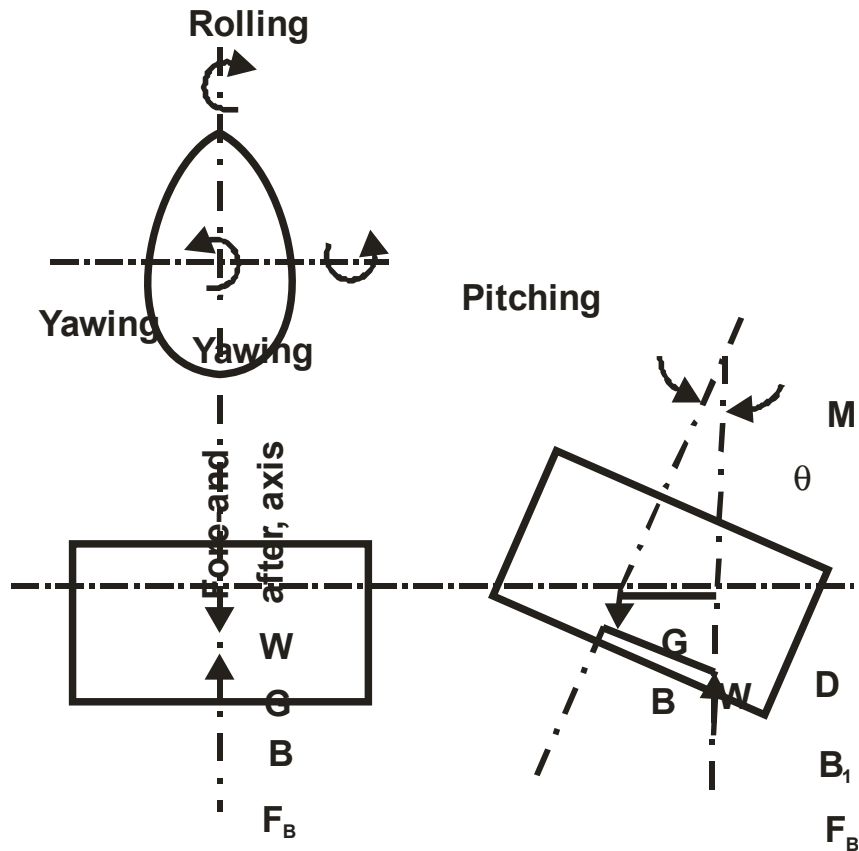
1. Buoyancy
2. Equilibrium of floating bodies
3. Equilibrium of floating bodies
4. Metacentric height
5. Time period of rolling of a floating body

Question: *Show that, for small angle of tilt, the time period of oscillation of a ship floating in stable equilibrium in water is given by*

$$T = \frac{2\pi k}{\sqrt{GM \cdot g}}$$

[AMIE (WINTER)-2001]

Answer: Consider a floating body in a liquid is given a small angular displacement θ at an instant of time t . The body starts oscillating about its metacenter (M).



Let, $W =$ Weight of floating body.

θ = Angle (in rad) through which the body is depressed.

θ

= Angular acceleration of the body in rad/

α

s^2

T = Time of rolling (i.e. one complete oscillation) in seconds.

k = Radius of gyration about G.

I = Moment of inertia of the body about its center of gravity

G.

=

$$\frac{W}{g} \cdot k^2$$

GM = Metacentric height of the body.

When the force which has caused angular displacement is removed the body is acted by an external restoring moment due to its weight and buoyancy forces; acting parallel to each other GD apart. Since the weight and buoyancy force must be equal and distance GD equals GM sin θ . The external moment equals W . GM sin θ and acts so as to

θ

θ

oppose the tilt θ .

θ

$$- W.GM \sin \theta =$$

$$\therefore \theta \quad I \frac{d^2 \theta}{dt^2}$$

$$\text{or, } W.GM \sin \theta =$$

$$\theta \quad \frac{W}{g} k^2 \frac{d^2 \theta}{dt^2}$$

$$\text{or, } \quad = 0$$

$$\frac{d^2 \theta}{dt^2} + \frac{g}{k^2} GM \sin \theta$$

For θ is very small sin θ

θ

$$\theta \approx \theta$$

$$= 0$$

$$\therefore \frac{d^2 \theta}{dt^2} + \frac{g.GM}{k^2} \cdot \theta$$

The solution of this second order linear differential

eqⁿ

$$\theta = C_1 \sin \left(\sqrt{\frac{g.GM}{k^2}} \cdot t \right) + C_2 \cos \left(\sqrt{\frac{g.GM}{k^2}} \cdot t \right)$$

Boundary conditions (applying)

(i) $\theta = 0$ at $t = 0$

θ

$$\therefore 0 = C_1 \sin \theta + C_2 \cos \theta$$

$$\therefore C_2 = 0$$

(ii) When

$$t = \frac{T}{2}, \theta = 0$$

$$\therefore 0 = C_1 \sin \left(\sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} \right)$$

As $C_1 \neq 0$

$$\therefore \sin \left(\sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} \right) = 0$$

$$\text{or } \sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} = \pi$$

$$\text{or } T = 2\pi \times \sqrt{\frac{k^2}{g \cdot GM}}$$

$$\text{or } \boxed{T = \frac{2\pi k}{\sqrt{GM \cdot g}}} \text{ required expression.}$$

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. Bodies in flotation to be in stable equilibrium, the necessary and sufficient condition is that the centre of gravity is located below the..... **[GATE-1994]**

- | | |
|-----------------------|------------------------|
| (a) Metacentre | (b) Centre of pressure |
| (c) Centre of gravity | (d) Centre of buoyancy |

GATE-1. Ans. (a)

Previous 20-Years IES Questions

IES-1. What is buoyant force? **[IES-2008]**

- (a) Lateral force acting on a submerged body
- (b) Resultant force acting on a submerged body
- (c) Resultant force acting on a submerged body
- (d) Resultant hydrostatic force on a body due to fluid surrounding it

IES-1. Ans. (d) When a body is either wholly or partially immersed in a fluid, a lift is generated due to the net vertical component of hydrostatic pressure forces experienced by the body. This lift is called the buoyant force and the phenomenon is called buoyancy.

IES-2. Assertion (A): The buoyant force for a floating body passes through the centroid of the displaced volume. **[IES-2005]**

Reason (R): The force of buoyancy is a vertical force & equal to the weight of fluid displaced.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-2. Ans. (a) When a solid body is either wholly or partially immersed in a fluid, the hydrostatic lift due to net vertical component of the hydrostatic pressure forces experienced by the body is called the buoyant force. The buoyant force on a submerged or floating body is equal to the weight of liquid displaced by the body and acts vertically upward through the centroid of displaced volume known as centre of buoyancy.

The x coordinate of the center of the buoyancy is obtained as

$$x_b = \frac{1}{V} \iiint_V x dV$$

Which is the centroid of the displaced volume. It is due to the buoyant force is equals to the weight of liquid displaced by the submerged body of volume and the force of buoyancy is a vertical force.

IES-3. Which one of the following is the condition for stable equilibrium for a floating body? **[IES-2005]**

- (a) The metacenter coincides with the centre of gravity
- (b) The metacenter is below the center of gravity
- (c) The metacenter is above the center of gravity
- (d) The centre of buoyancy is below the center of gravity

IES-3. Ans. (c)

IES-4. Resultant pressure of the liquid in case of an immersed body acts through which one of the following? [IES-2007]

- (a) Centre of gravity
- (b) Centre of pressure
- (c) Metacenter
- (d) Centre of buoyancy

IES-4. Ans. (b) For submerged body it acts through centre of buoyancy.

IES-5. Assertion (A): If a boat, built with sheet metal on wooden frame, has an average density which is greater than that of water, then the boat can float in water with its hollow face upward but will sink once it overturns. [IES-1999]

Reason (R): Buoyant force always acts in the upward direction.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-5. Ans. (b) Both A and R are true, but R does not give sufficient explanation for phenomenon at A. Location of Metacentre and centre of buoyancy decide about floating of a body.

IES-6. Which of the following statement is true? [IES-1992]

- (a) For an ideal fluid $\mu = 0$, $\rho = \text{constant}$, $K = 0$
- (b) A floating body is in stable, unstable or neutral equilibrium according to as the metacentric height zero, positive or negative.
- (c) The exact analysis of viscous flow problems can be made by Euler's equation
- (d) The most economical diameter of a pipe is the one for which the annual fixed cost and annual power cost (to overcome friction) are minimum.

IES-6. Ans. (d) Here (a) is wrong: For an ideal fluid $\mu = 0$, $\rho = \text{constant}$, $K = \text{const.}$

$$\mu = 0$$

(b) is wrong: A floating body is in stable, unstable or neutral equilibrium according to as the metacentric height positive or negative or zero respectively. (c) is wrong: The exact analysis of viscous flow problems can be made by Navier stroke equations. Euler's equation is valid for non-viscous fluid.

IES-7. A hydrometer weighs 0.03 N and has a stem at the upper end which is cylindrical and 3 mm in diameter. It will float deeper in oil of specific gravity 0.75, than in alcohol of specific gravity 0.8 by how much amount? [IES-2007]

- (a) 10.7 mm
- (b) 43.3 mm
- (c) 33 mm
- (d) 36 mm

IES-7. Ans. (d) and Now

$$V_{oil} = \frac{W}{\rho_{oil}g} \quad V_{al} = \frac{W}{\rho_{al}g} \quad V_{oil} - V_{al} = \frac{W}{g} \left(\frac{1}{\rho_{oil}} - \frac{1}{\rho_{al}} \right) = \frac{\pi \cdot (0.003)^2 h}{4}$$

IES-8. A wooden rectangular block of length l is made to float in water

with its axis vertical. The centre of gravity of the floating body is $0.15l$ above the centre of buoyancy. What is the specific gravity

of the wooden block?

(a) 0.6

(b) 0.65

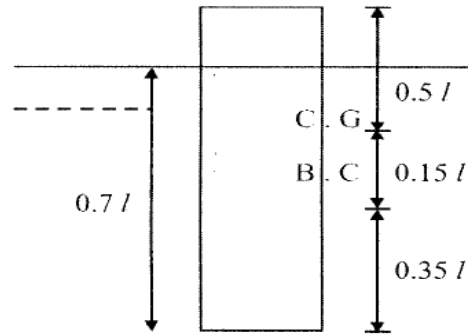
(c) 0.7

(d) 0.75

IES-8. Ans. (c)

$$0.7l A \rho_w g = l A \rho_b g$$

$$\therefore \rho_b = 0.7 \rho_w \Rightarrow \frac{\rho_b}{\rho_w} = 0.7$$



$$0.7l A \rho_w g = l A \rho_b g$$

$$\therefore \rho_b = 0.7 \rho_w$$

IES-9. A body weighs 30 N and 15 N when weighed under submerged conditions in liquids of relative densities 0.8 and 1.2 respectively. What is the volume of the body? **[IES-2009]**

(a) 12.50

(b) 3.82

(c) 18.70

(d) 75.50

IES-9. Ans. (d)

$$W - V\rho_1g = 30 \quad \dots(i)$$

$$W - V\rho_2g = 15 \quad \dots(ii)$$

$$\Rightarrow \frac{W - V(800)(9.8)}{W - V(1200)(9.8)} = 2$$

$$\Rightarrow W - V(800)(9.8) = 2W - 2V(1200)(9.8)$$

$$\Rightarrow W = 15680V$$

Putting the value of W in Equation (i)

$$15680V - V(800)(9.8) = 30$$

$$\Rightarrow V = 3.82 \times 10^{-3} \text{ m}^3 = 3.82 \text{ liters}$$

IES-10. If B is the centre of buoyancy, G is the centre of gravity and M is the Metacentre of a floating body, the body will be in stable equilibrium if

(a) $MG = 0$

(b) M is below G

(c) $BG=0$

(d) M is above G

[IES-1994; 2007]

IES-10. Ans. (d)

IES-11. For floating bodies, how is the metacentric radius defined? [IES-2009]

- (a) The distance between centre of gravity and the metacentre.
- (b) Second moment of area of plane of flotation about centroidal axis perpendicular to plane of rotation/immersed volume.
- (c) The distance between centre of gravity and the centre of buoyancy.
- (d) Moment of inertia of the body about its axis of rotation/immersed volume.

IES-11. Ans. (a) Metacentric Radius or Metacentric Height is the distance between Centre of Gravity and the Metacentre.

IES-12. The metacentric height of a passenger ship is kept lower than that of a naval or a cargo ship because [IES-2007]

- (a) Apparent weight will increase
- (b) Otherwise it will be in neutral equilibrium
- (c) It will decrease the frequency of rolling
- (d) Otherwise it will sink and be totally immersed

IES-12. Ans. (c)

IES-13. Consider the following statements: [IES-1996]

The metacentric height of a floating body depends

- 1. Directly on the shape of its water-line area.**
- 2. On the volume of liquid displaced by the body.**
- 3. On the distance between the metacentre and the centre of gravity.**
- 4. On the second moment of water-line area.**

Of these statements correct are:

- (a) 1 and 2
- (b) 2 and 3
- (c) 3 and 4
- (d) 1 and 4

IES-13. Ans. (b) The metacentric height of a floating body depends on (2) and (3), i.e. volume of liquid displaced by the body and on the distance between the metacentre and the centre of gravity.

IES-14. A cylindrical vessel having its height equal to its diameter is filled with liquid and moved horizontally at acceleration equal to acceleration due to gravity. The ratio of the liquid left in the vessel to the liquid at static equilibrium condition is: [IES-2001]

- (a) 0.2
- (b) 0.4
- (c) 0.5
- (d) 0.75

IES-14. Ans. (c)

IES-15. How is the metacentric height, GM expressed? [IES-2008]

- (a) $GM = BG - (I/V)$
- (b) $GM = (V/I) - BG$
- (c) $GM = (I/V) - BG$
- (d) $GM = BG - (V/I)$

Where I = Moment of inertia of the plan of the floating body at the water surface

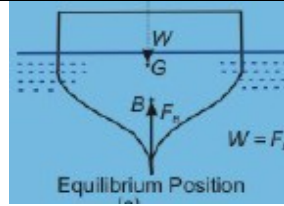
V = Volume of the body submerged in water

BG = Distance between the centre of gravity (G) and the centre of buoyancy (B).

IES-15. Ans. (c)

$$GM = BM - BG \text{ and } BM =$$

$$\frac{I}{V}$$



IES-16. Stability of a floating body can be improved by which of the following?

1. Making its width large
2. Making the draft small
3. Keeping the centre of mass low
4. Reducing its density

[IES-2008]

Select the correct answer using the code given below:

- | | |
|-------------------|---------------------|
| (a) 1, 2, 3 and 4 | (b) 1, 2 and 3 only |
| (c) 1 and 2 only | (d) 3 and 4 only |

IES-16. Ans. (b) Stability of a floating body can be improved by making width large which will increase.

I and will thus increase the metacentric height and keeping the centre of mass low and making the draft small.

IES-17. The distance from the centre of buoyancy to the meta-centre is given by I/V_d where V_d is the volume of fluid displaced. [IES-2008] What does I represent?

- (a) Moment of inertia of a horizontal section of the body taken at the surface of the fluid
- (b) Moment of inertia about its vertical centroidal axis
- (c) Polar moment of inertia
- (d) Moment of inertia about its horizontal centroidal axis

IES-17. Ans. (a) $BM =$ distance between centre of buoyancy to metacentre is given by $\frac{I}{V_d}$, where V_d is volume of fluid displaced. I represents the

$$\frac{I}{V_d}$$

Moment of Inertia of a horizontal section of a body taken at the surface of the fluid.

IES-18. A 25 cm long prismatic homogeneous solid floats in water with its axis vertical and 10 cm projecting above water surface. If the same solid floats in some oil with axis vertical and 5 cm projecting above the liquid surface, what is the specific gravity of the oil? [IES-2006]

- | | | | |
|----------|----------|----------|----------|
| (a) 0.60 | (b) 0.70 | (c) 0.75 | (d) 0.80 |
|----------|----------|----------|----------|

IES-18. Ans. (c)

$$\text{Let Area is } A \text{ cm}^2 \therefore A \times 25 \times s_s \times g = A \times 15 \times s_w \times g \text{ or } s_s = \frac{15}{25}$$

$$A \times 25 \times s_s \times g = A \times 20 \times s_{oil} \times g \text{ or } s_{oil} = \frac{25}{20} s_s = \frac{25}{20} \times \frac{15}{25} = 0.75$$

IES-19. Consider the following statements: [IES-1997, 1998] Filling up a part of the empty hold of a ship with ballasts will

1. Reduce the metacentric height.
2. Lower the position of the centre of gravity.

3. Elevate the position of centre of gravity.

4. Elevate the position of centre of buoyancy.

Of these statements

(a) 1, 3 and 4 are correct

(b) 1 and 2 are correct

(c) 3 and 4 are correct

(d) 2 and 4 are correct

IES-19. Ans. (d) Making bottom heavy lowers the c.g. of ship. It also increases displaced volume of water and thus the centre of displaced water, i.e. centre of buoyancy is elevated.

IES-20. Assertion (A): A circular plate is immersed in a liquid with its periphery touching the free surface and the plane makes an angle θ with the free surface with different values of θ , the

θ

θ

position of centre of pressure will be different.

[IES-2004]

Reason (R): Since the centre of pressure is dependent on second moment of area, with different values of θ , second moment of

θ

area for the circular plate will change.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IES-20. Ans. (c)

IES-21. An open rectangular box of base 2m × 2m contains a liquid of specific gravity 0.80 up to a height of 2.5m. If the box is imparted a vertically upward acceleration of 4.9 m/s², what will the pressure on the base of the tank? **[IES-2004]**

(a) 9.81 kPa

(b) 19.62 kPa

(c) 36.80 kPa

(d) 29.40 kPa

IES-21. Ans. (d)

$$p = h\rho(g + a)$$

IES-22. Assertion (A): For a vertically immersed surface, the depth of the centre of pressure is independent of the density of the liquid.

Reason (R): Centre of pressure lies above the centre of area of the immersed surface. **[IES-2003]**

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IES-22. Ans. (c)

IES-23. Match List-I with List-II and select the correct answer:[IES-1995, 2002]

List-I (Stability)

List-II (Conditions)

A. Stable equilibrium of a floating body

1. Centre of buoyancy below the centre of gravity

B. Stable equilibrium of a submerged body

2. Metacentre above the centre of gravity

- IES-27. Stability of a freely floating object is assured if its centre of**
 (a) Buoyancy lies below its centre of gravity [IES-1999]
 (b) Gravity coincides with its centre of buoyancy
 (c) Gravity lies below its metacenter
 (d) Buoyancy lies below its metacenter

IES-27. Ans. (c)

- IES-28. Match List-I with List-II regarding a body partly submerged in a liquid and select answer using the codes given below: [IES-1999]**

List-I

- A.** Centre of pressure
B. Centre of gravity
C. Centre of buoyancy
D. Metacenter

List-II

- 1.** Points of application of the weight of displace liquid
2. Point about which the body starts oscillating when tilted by a small angle
3. Point of application of hydrostatic pressure force
4. Point of application of the weight of the body

Codes:	A	B	C	D	A	B	C	D
(a)	4	3	1	2	(b)	4	3	2
(c)	3	4	1	2	(d)	3	4	2

IES-28. Ans. (c)

- IES-29. A cylindrical piece of cork weighting 'W' floats with its axis in horizontal position in a liquid of relative density 4. By anchoring the bottom, the cork piece is made to float at neutral equilibrium position with its axis vertical. The vertically downward force exerted by anchoring would be: [IES-1998]**
 (a) 0.5 W (b) W (c) 2W
 (d) 3 W

IES-29. Ans. (d) Due to own weight of cylinder, it will float upto $1/4^{\text{th}}$ of its height in liquid of relative density of 4. To make it float in neutral equilibrium, centre of gravity and centre of buoyancy must coincide, i.e. cylinder upto full height must get immersed.
 For free floating: Weight (W) = Buoyancy force (i.e. weight of liquid equal to $1/4^{\text{th}}$ volume cork)
 The vertically downward force exerted by anchoring would be weight of liquid equal to $3/4^{\text{th}}$ volume cork = 3W.

- IES-30. If a piece of metal having a specific gravity of 13.6 is placed in mercury of specific gravity 13.6, then [IES-1999]**
 (a) The metal piece will sink to the bottom
 (b) The metal piece simply float over the mercury with no immersion
 (c) The metal piece will be immersed in mercury by half
 (d) The whole of the metal piece will be immersed with its top surface just at mercury level.

IES-30. Ans. (d)

- IES-31. A bucket of water hangs with a spring balance. if an iron piece is suspended into water from another support without touching the sides of the bucket, the spring balance will show [IES-1999]**
 (a) An increased reading
 (b) A decreased reading
 (c) No change in reading

(d) Increased or decreased reading depending on the depth of immersion

IES-31. Ans. (c)

IES-32. A large metacentric height in a vessel [IES-1997]

- (a) Improves stability and makes periodic time to oscillation longer
- (b) Impairs stability and makes periodic time of oscillation shorter
- (e) Has no effect on stability or the periodic time of oscillation
- (d) Improves stability and makes the periodic time of oscillation shorter

IES-32. Ans. (d) Large metacentric height improves stability and decreases periodic time of oscillation.

IES-33. The least radius of gyration of a ship is 9 m and the metacentric height is 750 mm. The time period of oscillation of the ship is: [IES-1999]

- (a) 42.41 s
- (b) 75.4 s
- (c) 20.85 s
- (d) 85 s

IES-33. Ans. (c) = = 20.85 s

$$T = 2\pi \sqrt{\frac{k^2}{GM \cdot g}} = 2\pi \sqrt{\frac{9^2}{0.750 \times 9.81}}$$

IES-34. What are the forces that influence the problem of fluid static? [IES-2009]

- (a) Gravity and viscous forces
- (b) Gravity and pressure forces
- (c) Viscous and surface tension forces
- (d) Gravity and surface tension forces

IES-34. Ans. (b) Gravity and pressure forces influence the problem of Fluid statics.

Previous 20-Years IAS Questions

IAS-1. A metallic piece weighs 80 N air and 60 N in water. The relative density of the metallic piece is about [IAS-2002]

- (a) 8
- (b) 6
- (c) 4
- (d) 2

IAS-1. Ans. (c)

IAS-2. Assertion (A): A body with wide rectangular cross section provides a highly stable shape in floatation. [IAS-1999]

Reason (R) The center of buoyancy shifts towards the tipped end considerably to provide a righting couple.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-2. Ans. (a)

IAS-3. Match List-I (Nature of equilibrium of floating body) with List-II (Conditions for equilibrium) and select the correct answer using the codes given below the Lists: [IAS-2002]

List-I (Nature of equilibrium of floating body)	List-II (Conditions for equilibrium)
A. Unstable equilibrium B. Neutral equilibrium	1. $MG = 0$ 2. M is above G

C. Stable equilibrium

3. M is below G

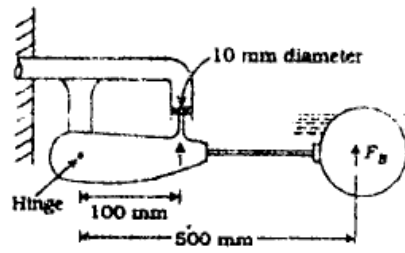
4. $BG = 0$

(Where M,G and B are metacenter, centre of gravity and centre of buoyancy respectively)

Codes:	A	B	C	A	B	C
(a)	1	3	2	(b)	3	1
(c)	1	3	4	(d)	4	2
					2	3

IAS-3. Ans. (b)

IAS-4. A float valve of the 'ball-clock' type is required to close an opening of a supply pipe feeding a cistern as shown in the given figure.



The buoyant force F_B required to be exerted by the float to keep the valve closed against a pressure of 0.28 N/mm is:

- (a) 4.4 N (b) 5.6N
 (c) 7.5 N (d) 9.2 N

[IAS-2000]

IAS-4. Ans. (a)

$$\text{Pressure force on valve } (F_V) = \text{Pressure} \times \text{area} = 0.28 \times \frac{\pi \times 10^2}{4} N = 22 N$$

$$\text{Taking moment about hinge, } F_V \times 100 = F_B \times 500 \text{ or } F_B = 4.4 N$$

IAS-5. **Assertion (A): A body with rectangular cross section provides a highly stable shape in floatation.** [IAS-1999]

Reason (R): The centre of buoyancy shifts towards the tipped end considerably to provide a righting couple.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IAS-5. Ans. (a)

IAS-6. A weight of 10 tonne is moved over a distance of 6m across the deck of a vessel of 1000 tonne floating in water. This makes a pendulum of length 2.5m swing through a distance of 12.5cm horizontally. The metacentric height of the vessel is: [IAS-1997]

- (a) 0.8m (b) 1.0m (c) 1.2m (d) 1.4m

IAS-6. Ans. (c)

IAS-7. The fraction of the volume of a solid piece of metal of relative density 8.25 floating above the surface of a container of mercury of relative density 13.6 is: [IAS-1997]

- (a) 1.648 (b) 0.607 (c) 0.393 (d) 0.352

IAS-7. Ans. (c)

IAS-8. If a cylindrical wooden pole, 20 cm in diameter, and 1m in height is placed in a pool of water in a vertical position (the gravity of wood is 0.6), then it will: [IAS-1994]

- (a) Float in stable equilibrium (b) Float in unstable equilibrium
(c) Float in neutral equilibrium (d) Start moving horizontally

IAS-8. Ans. (b)

IAS-9. An open tank contains water to depth of 2m and oil over it to a depth of 1m. If the specific gravity of oil is 0.8, then the pressure intensity at the interface of the two fluid layers will be: [IAS-1994]

- (a) 7848 N/m² (b) 8720 N/m² (c) 9747 N/m²
(d) 9750 N/m²

IAS-9. Ans. (a)

**IAS-10. Consider the following statements [IAS-1994]
For a body totally immersed in a fluid.**

- I. The weight acts through the centre of gravity of the body.
II. The up thrust acts through the centroid of the body.**

Of these statements:

- (a) Both I and II are true (b) I is true but II is false
(c) I is false but II is true (d) Neither I nor II is true

IAS-10. Ans. (b)

IAS-11. Consider the following statements regarding stability of floating bodies: [IAS-1997]

- 1. If oscillation is small, the position of Metacentre of a floating body will not alter whatever be the axis of rotation**
- 2. For a floating vessel containing liquid cargo, the stability is reduced due to movements of gravity and centre of buoyancy.**
- 3. In warships and racing boats, the metacentric height will have to be small to reduce rolling**

Of these statements:

- (a) 1, 2 and 3 are correct (b) 1 and 2 are correct
(c) 2 alone is correct (d) 3 alone is correct

IAS-11. Ans. (c)

IAS-12. Assertion (A): To reduce the rolling motion of a ship, the metacentric height should be low. [IAS-1995]

Reason (R): Decrease in metacentric height increases the righting couple.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IAS-12. Ans. (c) A is true but R is false

Since high metacentric height will result in faster restoring action, rolling will be more. Thus to reduce rolling metacentric height should be low. However reason (R) is reverse of true statement.

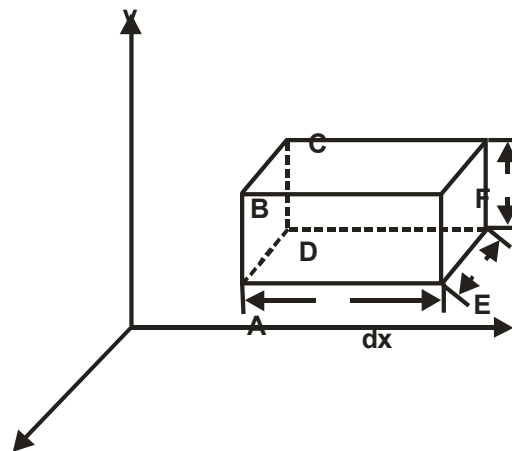
5. Fluid Kinematics

Contents of this chapter

1. Velocity
2. Acceleration
3. Tangential and Normal Acceleration
4. Types of Flow
5. Stream Line
6. Path Line
7. Streak Line
8. Continuity Equation
9. Circulation and Vorticity
10. Velocity Potential Function
11. Stream Function
12. Flow Net
13. Acceleration in Fluid Vessel

Question: Derive three dimensional general continuity equations in differential form and extend it to 3-D in compressible flow (Cartesian Coordinates).

Answer: Consider a fluid element (control volume) - Parallelepiped with sides dx , dy and dz as shown in Fig. Let ρ = mass density of the fluid at a particular instant t . u, v, w = components of velocity of the flow entering the three faces x, y and z respectively, of the parallelepiped.



Rate of mass of fluid entering the face ABCD (i.e. fluid influx)

$$= \rho u \, dy \, dz$$

$$\text{Area ABCD} = dy \, dz$$

and Rate of mass of fluid leaving the face EFGH (i.e. fluid efflux)

$$= \rho u \, dy \, dz + \frac{\partial}{\partial x} (\rho \, dy \, dz) \, dx$$

The gain in mass per unit time due to flow in the x-direction is given by the difference between the fluid influx and fluid efflux.

Mass accumulated per unit time due to flow in x-direction

$$\begin{aligned} \therefore & \\ &= \rho u \, dy \, dz - \rho u \, dy \, dz - \frac{\partial}{\partial x}(\rho u \, dy \, dz) \, dx = -\frac{\partial}{\partial x}(\rho u) \, dx \, dy \, dz \end{aligned}$$

Similarly, the gain in fluid mass per unit time in the parallelepiped due to flow in Y and z-directions.

$$= \text{(in Y-direction)}$$

$$-\frac{\partial}{\partial y}(\rho v) \, dx \, dy \, dz$$

$$= \text{(in z-direction)}$$

$$-\frac{\partial}{\partial z}(\rho \omega) \, dx \, dy \, dz$$

The total (or net) gain in fluid mass per unit time for fluid flow along 3 co-

$$\therefore \text{ordinate axes} = \text{Rate of change of mass of the parallelepiped (c. v.)}$$

$$\therefore -\frac{\partial}{\partial x}(\rho u) \, dx \, dy \, dz - \frac{\partial}{\partial y}(\rho v) \, dx \, dy \, dz - \frac{\partial}{\partial z}(\rho \omega) \, dx \, dy \, dz = \frac{\partial}{\partial t}(\rho \, dx \, dy \, dz)$$

or General continuity in 3-D

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho \omega) + \frac{\partial \rho}{\partial t} = 0} \quad \text{eq. 5.1}$$

For incompressible fluid

$$\rho = \text{const} \therefore \frac{\partial \rho}{\partial t} = 0$$

For 3-D incompressible flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0$$

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Acceleration

GATE-1. In a two-dimensional velocity field with velocities u and v along the x and y directions respectively, the convective acceleration along the x -direction is given by: [GATE-2006]

- (a) u (b) u

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y}$$

(c) $u + v$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

(d) v

$$\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y}$$

GATE-1. Ans. (a)

GATE-2. For a fluid flow through a divergent pipe of length L having inlet and outlet radii of R_1 and R_2 respectively and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is: **[GATE-2004]**

(a)

$$\frac{2Q(R_1 - R_2)}{\pi LR_2^3}$$

(b)

$$\frac{2Q^2(R_1 - R_2)}{\pi LR_2^3}$$

(c)

$$\frac{2Q^2(R_1 - R_2)}{\pi^2 LR_2^5}$$

(d)

$$\frac{2Q^2(R_2 - R_1)}{\pi^2 LR_2^5}$$

GATE-2. Ans (c) At a distance x from the inlet radius (R_x) =

$$\left(R_1 + \frac{R_2 - R_1}{L} x \right)$$

Area

$$\therefore A_x = \pi R_x^2$$

$$u = \frac{Q}{A_x}$$

$$\therefore u = \frac{Q}{\pi \left(R_1 + \frac{R_2 - R_1}{L} x \right)^2}$$

Total acceleration $a_x = u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$ for constant flow rate i.e. steady flow

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial t} = 0$$

$$a_x = u \frac{\partial u}{\partial x}$$

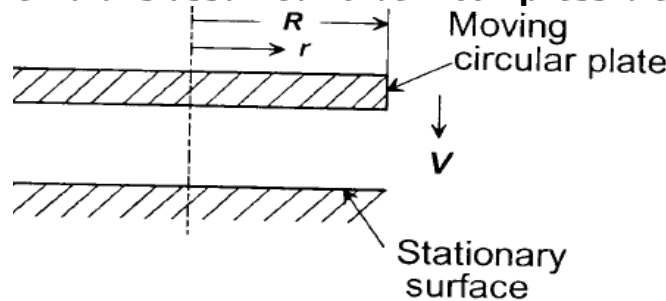
$$\therefore a_x = \frac{Q}{\pi \left(R_1 + \frac{R_2 - R_1}{L} x \right)^2} \times \frac{-2Q \frac{R_2 - R_1}{L}}{\pi \left(R_1 + \frac{R_2 - R_1}{L} x \right)^3}$$

at $x = L$ it gives

$$\frac{2Q^2(R_1 - R_2)}{\pi^2 LR_2^5}$$

Statement for Linked Answers & Questions Q3 and Q4:

The gap between a moving circular plate and a stationary surface is being continuously reduced. as the circular plate comes down at a uniform speed V towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



GATE-3. The radial velocity v_r at any radius r , when the gap width is h , is: [GATE-2008]

- (a) $v_r = \frac{Vr}{2h}$ (b) $v_r = \frac{Vr}{h}$ (c) $v_r = \frac{2Vr}{h}$ (d) $v_r = \frac{Vr}{h}$

GATE-3. Ans. (a) At a distance r take a small strip of dr .
 volume of liquid will pass through $2\pi r$ of length within one second.

$\pi r^2 V$ $2\pi r$

$$\therefore v_r = \frac{\pi r^2 V}{2\pi r h} = \frac{rV}{2h}$$

GATE-4. The radial component of the fluid acceleration at $r = R$ is: [GATE-2008]

- (a) $\frac{3V^2 R}{4h^2}$ (b) $\frac{V^2 R}{4h^2}$ (c) $\frac{V^2 R}{2h^2}$ (d) $\frac{V^2 h}{4R^2}$

GATE-4. Ans. (c) Acceleration (a_r) =

$$\frac{\partial(v_r)}{\partial t} = \frac{\partial}{\partial t} \left(\frac{RV}{2h} \right) = \frac{RV}{2} \cdot \frac{\partial}{\partial t} \left(\frac{1}{h} \right) = -\frac{RV}{2h^2} \cdot \frac{\partial h}{\partial t}$$

(given) Therefore (a_r) =

$$\frac{\partial h}{\partial t} = -V \qquad \frac{V^2 R}{2h^2}$$

Tangential and Normal Acceleration

GATE-5. For a fluid element in a two dimensional flow field (x-y plane), if it will undergo [GATE-1994]

- (a) Translation only (b) Translation and rotation
 (c) Translation and deformation (d) Deformation only

GATE-5. Ans. (b)

Types of Flow

GATE-6. You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two-dimensional velocity fields in the xy -plane, are made available. [GATE-2009]

P. $u = 2y, v = -3x$ Q. $u = 3xy, v = 0$ R. $u = -2x, v = 2y$

Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational?

- (a) P and R (b) Q (c) Q and R (d) R

GATE-6. Ans. (d)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \text{ continuity}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \text{ irrotational}$$

Stream Line

GATE-7. A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamlines is: [GATE-2006]

- (a) $x^2y = \text{constant}$ (b) $xy^2 = \text{constant}$
 (c) $xy = \text{constant}$ (d) not possible to determine
 or integrating both side or

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{x^2t} = \frac{dy}{-2xyt} \qquad \int \frac{dx}{x} = -\frac{1}{2} \int \frac{dy}{y}$$

$$\ln(x^2y) = 0$$

GATE-7. Ans. (a)

GATE-8. In adiabatic flow with friction, the stagnation temperature along a streamline [GATE-1995]

- (a) Increases (b) Decreases (c) Remains constant
 (d) None

GATE-8. Ans. (c)

Streak Line

GATE-9. Streamlines, path lines and streak lines are virtually identical for [GATE-1994]

- (a) Uniform flow (b) Flow of ideal fluids
 (c) Steady flow (d) Non uniform flow

GATE-9. Ans. (c)

Continuity Equation

GATE-10. The velocity components in the x and y directions of a two dimensional potential flow are u and v, respectively. Then $\frac{\partial u}{\partial x}$ is

$$\frac{\partial u}{\partial x}$$

equal to:

[GATE-2005]

- | | | |
|--------------------------------------|-------------------------------------|-------------------------------------|
| (a) $\frac{\partial v}{\partial x}$ | (b) $\frac{\partial v}{\partial x}$ | (c) $\frac{\partial v}{\partial y}$ |
| (d) $-\frac{\partial v}{\partial y}$ | | |

GATE-10. Ans. (d) From continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

GATE-11. The velocity components in the x and y directions are given by:

$$u = \lambda xy^3 - x^2y \quad \text{and} \quad v = xy^2 - \frac{3}{4}y^4$$

The value of λ for a possible flow field involving an incompressible fluid is: [GATE-1995]

- | | | |
|----------|----------|---------|
| (a) -3/4 | (b) -4/3 | (c) 4/3 |
| (d) 3 | | |

GATE-11. Ans. (d) Just use continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

GATE-12. The continuity equation in the form $\Delta \vec{V} = 0$ always represents

$$\Delta \vec{V} = 0$$

an incompressible flow regardless of whether the flow is steady or unsteady. [GATE-1994]

GATE-12. Ans. True General continuity equation, if

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0 \quad \rho = \text{const} \quad \Delta \vec{V} = 0$$

GATE-13. If \vec{V} is velocity vector of fluid, then $\Delta \vec{V} = 0$ is strictly true for

$$\vec{V} \quad \Delta \vec{V} = 0$$

which of the following? [IAS-2007; GATE-2008]

- (a) Steady and incompressible flow
- (b) Steady and irrotational flow
- (c) Inviscid flow irrespective of steadiness

(d) Incompressible flow irrespective of steadiness

GATE-13. Ans. (d)

$$\nabla \cdot \vec{V} = 0 \quad \text{Or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Circulation and Vorticity

GATE-14. Circulation is defined as line integral of tangential component of velocity about a [GATE-1994]

- (a) Closed contour (path) in a fluid flow
- (b) Open contour (path) in a fluid flow
- (c) Closed or open contour (path) in a fluid flow
- (d) None

GATE-14. Ans. (a)

Velocity Potential Function

GATE-15. Existence of velocity potential implies that [GATE-1994]

- (a) Fluid is in continuum
- (b) Fluid is irrotational
- (c) Fluid is ideal
- (d) Fluid is compressible

GATE-15. Ans. (b)

Stream Function

GATE-16. The 2-D flow with, velocity $= (x + 2y + 2)\mathbf{i} + (4 - y)\mathbf{j}$ is:

[GATE-2001]

- (a) Compressible and irrotational
- (b) Compressible and not irrotational
- (c) Incompressible and irrotational
- (d) Incompressible and not irrotational

GATE-16. Ans. (d) Continuity equation satisfied but

$$\omega_z \neq 0$$

Flow Net

GATE-17. In a flow field, the streamlines and equipotential lines [GATE-1994]

- (a) Are Parallel
- (b) Are orthogonal everywhere in the flow field
- (c) Cut at any angle
- (d) Cut orthogonally except at the stagnation points

GATE-17. Ans. (d)

Previous 20-Years IES Questions

Acceleration

IES-1. The convective acceleration of fluid in the x-direction is given by:

(a)

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial \omega}{\partial z}$$

(b)

$$\frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial \omega}{\partial t}$$

[IES-2001]

(c)

$$u \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial y} + u \frac{\partial \omega}{\partial z}$$

(d)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z}$$

IES-1. Ans. (d)

IES-2. For a steady two-dimensional flow, the scalar components of the velocity field are $V_x = -2x$, $V_y = 2y$, $V_z = 0$. What are the components of acceleration? [IES-2006]

(a) $a_x = 0$, $a_y = 0$

(b) $a_x = 4x$, $a_y = 0$

(c) $a_x = 0$, $a_y = 4y$

(d) $a_x = 4x$, $a_y = 4y$

IES-2. Ans (d) $a_x = u$

Given $u = V_x = -2x$; $v = V_y = 2y$ and $w =$

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$V_z = 0$$

$$a_y = u$$

$$\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

IES-3. A steady, incompressible flow is given by $u = 2x^2 + y^2$ and $v = -4xy$. What is the convective acceleration along x-direction at point (1, 2)?

(a) $a_x = 6$ unit

(b) $a_x = 24$ unit

[IES-2008]

(c) $a_x = -8$ unit

(d) $a_x = -24$ unit

IES-3. Ans. (c) Convective acceleration along x direction at point (1, 2)

$$\begin{aligned} a_x &= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (2x^2 + y^2)(4x) + (-4xy)(2y) \\ &= (2+4)(4) + (-4 \cdot 1 \cdot 2)(2 \cdot 2) = 24 - 32 = -8 \text{ unit} \end{aligned}$$

IES-4. The area of a 2m long tapered duct decreases as $A = (0.5 - 0.2x)$ where 'x' is the distance in meters. At a given instant a discharge of $0.5 \text{ m}^3/\text{s}$ is flowing in the duct and is found to increase at a rate of $0.2 \text{ m}^3/\text{s}^2$. The local acceleration (in m/s^2) at $x = 0$ will be: [IES-2007]

(a) 1.4
0.667

(b) 1.0

(c) 0.4

(d)

IES-4. Ans.(c) $u =$ $=$ local acceleration at $x =$

$$\therefore \frac{Q}{A_x} = \frac{Q}{(0.5 - 0.2x)} \quad \frac{\partial u}{\partial t} = \frac{1}{(0.5 - 0.2x)} \times \frac{\partial Q}{\partial t}$$

0

=0.4

$$\frac{\partial u}{\partial t} = \frac{1}{(0.5)} \times 0.2$$

IES-5. Assertion (A): The local acceleration is zero in a steady motion. Reason (R): The convective component arises due to the fact that a fluid element experiences different velocities at different locations.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false **[IES-2009]**
- (d) A is false but R is true

IES-5. Ans. (b) • A flow is said to be steady when conditions do not change with time at any point.

- In a converging steady flow, there is only convective acceleration.
- Local acceleration is zero in steady flow.

IES-6. The components of velocity in a two dimensional frictionless incompressible flow are $u = t^2 + 3y$ and $v = 3t + 3x$. What is the approximate resultant total acceleration at the point (3, 2) and $t = 2$?

[IES-2004]

- (a) 5 (b) 49 (c) 59 (d) 54

IES-6. Ans. (c) $a_x = u$ and $a_y = v$

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \qquad \qquad \qquad \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

or $a_x = (t^2 + 3y).(0) + (3t + 3x).(3) + 2t$ and $a_y = (t^2+3y).(3)+(3t+3x).(0)+3$

at $x = 3, y = 2$ and $t = 2$
 $a = \qquad \qquad \qquad = 59.08$

$$\sqrt{a_x^2 + a_y^2} \quad \sqrt{49^2 + 33^2}$$

IES-7. Match List-I (Pipe flow) with List-II (Type of acceleration) and select the correct answer: **[IES-1999]**

List-I

List-II

- | | |
|--|--|
| <p>A. Flow at constant rate passing through a bend</p> <p>B. Flow at constant rate passing through a straight uniform diameter pipe</p> <p>C. Gradually changing flow through a bend</p> <p>D. Gradually changing flow through a straight pipe</p> | <p>1. Zero acceleration</p> <p>2. Local and convective acceleration</p> <p>3. Convective acceleration</p> <p>4. Local acceleration</p> |
|--|--|

Codes:	A	B	C	D		A	B	C	D
(a)	3	1	2	4	(b)	3	1	4	2
(c)	1	3	2	4	(d)	1	3	4	2

IES-7. Ans. (a)

Types of Flow

IES-8. Match List-I (Flows Over or Inside the Systems) with List-II (Type of Flow) and select the correct answer: [IES-2003]

List-I	List-II
A. Flow over a sphere	1. Two dimensional flow
B. Flow over a long circular cylinder	2. One dimensional flow
C. Flow in a pipe bend	3. Axisymmetric flow
D. Fully developed flow in a pipe at constant flow rate	4. Three dimensional flow
Codes:	A B C D
(a) 3 1 2 4	(b) 1 4 3 2
(c) 3 1 4 2	(d) 1 4 2 3

IES-8. Ans. (c)

IES-9. Match List-I with List-II and select the correct answer using the code given below the lists: [IES-2007]

List-I (Condition)	List-II (Regulating Fact)
A. Existence of stream function	1. Irrotationality of flow
B. Existence of velocity potential	2. Continuity of flow
C. Absence of temporal Variations	3. Uniform flow
D. Constant velocity vector	4. Steady flow
Codes:	A B C D
(a) 4 3 2 1	(b) 2 1 4 3
(c) 4 1 2 3	(d) 2 3 4 1

IES-9. Ans. (b)

IES-10. Irrotational flow occurs when: [IES-1997]

- (a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.
- (b) Streamlines are curved.
- (c) There is no net rotation of the fluid element about its mass center.
- (d) Fluid element does not undergo any change in size or shape.

IES-10. Ans. (c) If the fluid particles do not rotate about their mass centres while moving in the direction of motion, the flow is called as an irrotational flow.

IES-11. Which one of the following statements is correct? [IES-2004]
Irrotational flow is characterized as the one in which

- (a) The fluid flows along a straight line
- (b) The fluid does not rotate as it moves along
- (c) The net rotation of fluid particles about their mass centres remains zero.
- (d) The streamlines of flow are curved and closely spaced

IES-11. Ans. (c)

IES-12. In a two-dimensional flow in x-y plane, if _____, then the fluid

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

element will undergo [IES-1996]

- (a) Translation only
- (b) Translation and rotation
- (c) Translation and deformation
- (d) Rotation and deformation.

IES-12. Ans. (a) In a two-dimensional flow in x-y plane, if i.e. irrotational,

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

then the fluid element will undergo translation and deformation.

Remember: Any fluid element can undergo translation, rotation and RATE OF deformation. As i.e. irrotational flow, rotation is not there but

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

not translation and RATE OF deformation must be possible.

IES-13. Two flows are specified as [IES-2008]

(A) $u = y, v = - (3/2) x$ (B) $u = xy^2, v = x^2y$

Which one of the following can be concluded?

- (a) Both flows are rotational
- (b) Both flows are irrotational
- (c) Flow A is rotational while flow B is irrotational
- (d) Flow A is irrotational while flow B is rotational

IES-13. Ans. (c)

$$(A) \omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \left(\frac{\partial}{\partial x} \left(-\frac{3}{2}x \right) - \frac{\partial}{\partial y} (y) \right)$$

$$= \frac{1}{2} \left(-\frac{3}{2} - 1 \right) = \frac{1}{2} \times \left(-\frac{5}{2} \right) = -\frac{5}{4} \neq 0$$

∴ Flow A is rotational

$$(B) \omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \left(\frac{\partial}{\partial x} (x^2y) - \frac{\partial}{\partial y} (xy^2) \right) = \frac{1}{2} (2xy - 2xy) = 0$$

∴ Flow (b) is irrotational

IES-14. Match List-I with List-II and select the correct answer: [IES-2002]

List-I (Example)

List-II (Types of flow)

- | | |
|---|---|
| <p>A. Flow in a straight long pipe with varying flow rate</p> <p>B. Flow of gas through the nozzle of a jet engine</p> <p>C. Flow of water through the hose of a fire fighting pump</p> <p>D. Flow in a river during tidal bore</p> | <p>1. Uniform, steady</p> <p>2. Non-uniform, steady</p> <p>3. Uniform, unsteady</p> <p>4. Non-uniform, unsteady</p> |
|---|---|

Codes:	A	B	C	D		A	B	C	D
(a)	1	4	3	2	(b)	3	2	1	4
(c)	1	2	3	4	(d)	3	4	1	2

IES-14. Ans. (b)

Stream Line

IES-15. A streamline is a line: [IES-2003]

- (a) Which is along the path of the particle
- (b) Which is always parallel to the main direction of flow
- (c) Along which there is no flow
- (d) On which tangent drawn at any point given the direction of velocity

IES-15. Ans. (d)

IES-16. Assertion (A): Stream lines are drawn in the flow field such that at a given instant of time there perpendicular to the direction of flow at every point in the flow field. [IES-2002]

Reason (R): Equation for a stream line in a two dimensional flow is given by $V_x dy - V_y dx = 0$.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-16. Ans. (d) A streamline in a fluid flow is a line tangent to which at any point is in the direction of velocity at that point at that instant.

IES-17. Assertion (A): Streamlines can cross one another if the fluid has higher velocity. [IES-2003]

Reason (R): At sufficiently high velocity, the Reynolds number is high and at sufficiently high Reynolds numbers, the structure of the flow is turbulent type.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-17. Ans. (d)

IES-18. The streamlines and the lines of constant velocity potential in an inviscid irrotational flow field form. [IES-1994]

- (a) Parallel grid lines placed in accordance with their magnitude.
- (b) Intersecting grid-net with arbitrary orientation.
- (c) An orthogonal grid system
- (d) None of the above.

IES-18. Ans. (c) Flow net:

Streamline $\psi = \text{const.}$

Velocity potential line $\phi = \text{const.}$

- The streamlines and velocity potential lines form an orthogonal net work in a fluid flow.
- Observation of a flow net enables us to estimate the velocity variation.
- Streamline and velocity potential lines must constitute orthogonal net work except at the stagnation points.

IES-19. A velocity field is given by $u = 3xy$ and $v = \frac{3}{2}(x^2 - y^2)$. What is the

relevant equation of a streamline?

[IES-2008]

(a) (b) (c) (d)

$$\frac{dx}{dy} = \frac{(x^2 - y^2)}{xy}$$

$$\frac{3xy}{(x^2 - y^2)}$$

$$\frac{2xy}{(x^2 - y^2)}$$

$$\frac{dx}{dy} = \frac{(x^2 - y^2)}{2xy}$$

IES-19. Ans. (c)

$$u = 3xy \quad v = \frac{3}{2}(x^2 - y^2)$$

Equation of streamline is given by

$$vdx = udy$$

$$\Rightarrow \frac{dx}{dy} = \frac{u}{v}$$

$$\Rightarrow \frac{dx}{dy} = \frac{3xy}{\frac{3}{2}(x^2 - y^2)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{2xy}{(x^2 - y^2)}$$

Path Line

IES-20. Consider the following statements regarding a path line in fluid flow:

- 1. A path line is a line traced by a single particle over a time interval.**
- 2. A path line shows the positions of the same particle at successive time instants. [IES-2006]**
- 3. A path line shows the instantaneous positions of a number of a particle, passing through a common point, at some previous time instants.**

Which of the statements given above are correctly?

- (a) Only 1 and 3 (b) only 1 and 2
(c) Only 2 and 3 (d) 1, 2 and 3

IES-20. Ans. (b) 3 is wrong because it defines Streak line.

- (i) A path line is the trace made by a single particle over a period of time. i.e. It is the path followed by a fluid particle in motion.

$$\text{Equation } x = \int u dt; y =$$

$$\int v dt; z = \int w dt$$

- (ii) Streak line indicates instantaneous position of particles of fluid passing through a point.

Streak Line

IES-21. Which one of the following is the correct statement? [IES-2007] Streamline, path line and streak line are identical when the

- (a) Flow is steady
- (b) Flow is uniform
- (c) Flow velocities do not change steadily with time

(d) Flow is neither steady nor uniform

IES-21 Ans (a)

Continuity Equation

IES-22. The differential form of continuity equation for two-dimensional flow of fluid may be written in the following form in

$$\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0$$

which u and v are velocities in the x and y-direction and ρ is the density. This is valid for [IES-1995]

- (a) Compressible, steady flow (b) Compressible, unsteady flow
 (c) Incompressible, unsteady flow (d) Incompressible, steady flow

IES-22. Ans. (d) The equation is valid for incompressible steady

$$\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0$$

flow.

IES-23. The general form of expression for the continuity equation in a Cartesian coordinate system for incompressible or compressible flow is given by: [IES-1996]

(a)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(b)

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

(c)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

(d)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 1$$

IES-23. Ans. (c) general form valid for

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

incompressible or compressible flow, steady and unsteady flow.

valid for incompressible or compressible flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

but steady flow.

valid for incompressible flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

IES-24. Which of the following equations are forms of continuity equations?

(ρ is the velocity and $\nabla \cdot \mathbf{V}$ is volume)

[IES-1993]

$\rho \nabla \cdot \mathbf{V}$

$\nabla \cdot \mathbf{V}$

1. $A_1 V_1 = A_2 V_2$

2. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

3. $\int_s \rho V \cdot dA + \frac{\partial}{\partial t} \int_V \rho dV = 0$

4. $\frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0$

Select the correct answer using the codes given below:

Codes:

(a) 1, 2, 3 and 4

(b) 1 and 2

(c) 3 and 4

(d) 2, 3 and 4

IES-24. Ans. (b) const. In case of compressible fluid.

$$\rho AV = \text{const.}$$

AV = const. In case of incompressible fluid.

Differential form of continuity equation in Cartesian co-ordinates system.

, Vector form , for incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla \cdot V = 0$$

General form

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

Vector form .(

$$\nabla \cdot (\rho \vec{V}) + \frac{\partial \rho}{\partial t} = 0$$

General form valid for

Viscous or Inviscid; steady or unsteady; uniform or non-uniform; compressible or incompressible.

Integral form: .dA+

$$\int_s \rho V \cdot dA + \frac{\partial}{\partial t} \int_V \rho dV = 0$$

Differential form of continuity equation in cylindrical co-ordinate system

, for incompressible flow.

$$\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

IES-25. Consider the following equations:

[IES-2009]

1. $A_1 v_1 = A_2 v_2$

2. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

3. $\oint_s v \cdot dA + \frac{\partial}{\partial t} \left(\int_V \rho dV \right) = 0$

4. $\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{\partial v_z}{\partial z} = 0$

Which of the above equations are forms of continuity equations? (Where u, v are velocities and V is volume)

(a) 1 only

(b) 1 and 2

(c) 2 and 3

(d) 3 and 4

IES-25. Ans. (b) General continuity in 3-D:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

For incompressible fluid

$$\rho = \text{const} \therefore \frac{\partial \rho}{\partial t} = 0$$

For 3-D incompressible flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For 2-D incompressible flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

IES-26. In a two-dimensional incompressible steady flow, the velocity component $u = Ae^x$ is obtained. What is the other component v of velocity? [IES-2006]

- (a) $v = Ae^{xy}$ (b) $v = Ae^{xy}$ (c) $v = -Ae^x y + f(x)$ (d) $v = -Ae^x y + f(y)$

IES-26. Ans. (c) From continuity eq. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -Ae^x$$

$$v = -Ae^x y + f(x)$$

IES-27. For steady incompressible flow, if the u -component of velocity is $u = Ae^x$, then what is the v -component of velocity? [IES-2008]

- (a) Ae^y (b) Ae^{xy} (c) $-Ae^x y$ (d) $-Ae^x$

IES-27. Ans. (c) Using continuity equation

$$\begin{aligned} \Rightarrow \frac{du}{dx} + \frac{dv}{dy} &= 0 & \Rightarrow \frac{d(Ae^x)}{dx} + \frac{dv}{dy} &= 0 \\ \Rightarrow Ae^x + \frac{dv}{dy} &= 0 & \Rightarrow v &= -Ae^x y \end{aligned}$$

IES-28. Which one of the following stream functions is a possible irrotational flow field? [IES-2003]

- (a) $\psi = x^3 y$ (b) $\psi = 2xy$ (c) $\psi = Ax^2 y^2$ (d) $\psi = Ax + By^2$

IES-28. Ans. (b) Use continuity equation

IES-29. Which one of the following stream functions is a possible irrotational flow field ? [IES-2007]

- (a) (b) (c) (d)

$$\psi = y^2 - x^2$$

$$\psi = A \sin(xy)$$

$$\psi = Ax^2y^2$$

$$\psi = Ax + By^2$$

IES-29 Ans. (a) Satisfy Laplace Equation.

IES-30. The continuity equation for a steady flow states that [IES-1994]

- (a) Velocity field is continuous at all points in flow field
 (b) The velocity is tangential to the streamlines.
 (c) The stream function exists for steady flows.
 (d) The net efflux rate of mass through the control surfaces is zero

IES-30. Ans. (c) It is a possible case of fluid flow therefore the stream function exists for steady flows.

Circulation and Vorticity

IES-31. Which of the following relations must hold for an irrotational two-dimensional flow in the x-y plane? [IAS-2003, 2004, IES-1995]

- (a) $\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 0$ (b) $\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$
 (c) $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$ (d) $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

IES-31. Ans. (d) i.e.

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

IES-32. Irrotational flow occurs when: [IES-1998]

- (a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.
 (b) Streamlines are curved.
 (c) There is no net rotation of the fluid element about its mass center. [IES-1998]
 (d) Fluid element does not undergo any change in size or shape.

IES-32. Ans. (c) If the fluid particles do not rotate about their mass centre and while moving in the direction of motion. The flow is called as an irrotational flow.

IES-33. The curl of a given velocity field indicates the rate of [IES-

$$\left(\nabla \times \vec{V} \right)$$

1996]

- (a) Increase or decrease of flow at a point. (b) Twisting of the lines of flow.
 (c) Deformation (d) Translation.

IES-33. Ans. (c) The curl of a given velocity field $(\nabla \times \vec{V})$ indicates the rate of

angular deformation.

IES-34. If the governing equation for a flow field is given by $\vec{V}^2 \phi = 0$ and the velocity is given by $\vec{V} = \nabla \phi$, then **[IES-1993]**

- (a) $\nabla \times \vec{V} = 0$ (b) $\nabla \times \vec{V} = 1$ (c) $\nabla^2 \times \vec{V} = 1$ (d) $(\vec{V} \cdot \nabla) \times \vec{V} = \frac{\partial \vec{V}}{\partial t}$

IES-34. Ans. (a)

Velocity Potential Function

IES-35. The relation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ for an irrotational flow is known as which

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

one of the following?

- (a) Navier - Stokes equation (b) Laplace equation
(c) Reynolds equation (d) Euler's equation

[IES-2007]

IES-35. Ans. (b)

IES-36. Consider the following statements: **[IES-1994]**

For a two-dimensional potential flow

- 1. Laplace equation for stream function must be satisfied.**
- 2. Laplace equation for velocity potential must be satisfied.**
- 3. Streamlines and equipotential lines are mutually perpendicular.**
- 4. Stream function and potential function are not interchangeable.**

- (a) 1 and 4 are correct (b) 2 and 4 are correct
(c) 1, 2 and 3 are correct (d) 2, 3 and 4 are correct

IES-36. Ans. (c)

IES-37. Which of the following functions represent the velocity potential in a two-dimensional flow of an ideal fluid? **[IES-2004, 1994]**

- 1. $2x + 3y$ 2. $4x^2 - 3y^2$ 3. $\cos(x - y)$ 4. $\tan^{-1}(x/y)$**

Select the correct answer using the codes given below:

- (a) 1 and 3 (b) 1 and 4 (c) 2 and 3 (d) 2 and 4

IES-37. (a) Checking $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ for all the above.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Stream Function

IES-38. If for a flow, a stream function ψ exists and satisfies the Laplace equation, then which one of the following is the correct statement?

- (a) The continuity equation is satisfied and the flow is irrotational. **[IES-2005]**
- (b) The continuity equation is satisfied and the flow is rotational.
- (c) The flow is irrotational but does not satisfy the continuity equation.
- (d) The flow is rotational.

IES-38. Ans (a) if a stream function ψ exists means a possible case of flow, if it satisfies the Laplace equation then flow is irrotational.

IES-39. In a two-dimensional flow, the velocity components in x and y directions in terms of stream function ψ are: **[IES-1995]**

(ψ)

- | | |
|--|---|
| (a) $u = \partial\psi / \partial x, v = \partial\psi / \partial y$ | (b) $u = \partial\psi / \partial y, v = \partial\psi / \partial x$ |
| (c) $u = (-) \partial\psi / \partial y, v = \partial\psi / \partial x$ | (d) $u = \partial\psi / \partial x, v = -\partial\psi / \partial y$ |

IES-39. Ans. (c) The stream function ψ is defined as a scalar function of space

(ψ)

and time, such that its partial derivative with respect to any direction given the velocity component at right angles (in the counter clockwise direction) to this direction. Hence,

$$\frac{\partial\psi}{\partial y} = -U \quad \text{and} \quad \frac{\partial\psi}{\partial x} = V.$$

IES-40. Of the possible irrotational flow functions given below, the incorrect relation is (where ψ = stream function and ϕ = velocity

ϕ

potential).

[IES-1995]

- | | |
|---|--|
| (a) $\psi = xy$ | (b) $\psi = A(x^2 - y^2)$ |
| (c) $\phi = ur \cos\theta + \frac{u}{r} \cos\theta$ | (d) $\phi = \left(r - \frac{2}{r} \right) \sin\theta$ |

IES-40. Ans. (d) Equation at (d) is not irrotational. If the stream function satisfy the Laplace equation the flow is irrotational, otherwise rotational.

If ψ (Laplace equation)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

Then,

$$\omega_z = 0$$

A.

$$\psi = xy$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2(xy)}{\partial x^2} + \frac{\partial^2(xy)}{\partial y^2} = 0 + 0 = 0$$

Satisfy the Laplace equation therefore flow is irrotational.

B.

$$\psi = A(x^2 - y^2)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2[A(x^2 - y^2)]}{\partial x^2} + \frac{\partial^2[A(x^2 - y^2)]}{\partial y^2} = 2A - 2A = 0 + 0 = 0$$

Satisfy Laplace equation, therefore flow is rotational.

C.

$$\nabla_\phi^2 = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2[A(x^2 - y^2)]}{\partial x^2} + \frac{\partial^2[A(x^2 - y^2)]}{\partial y^2} = 2A - 2A = 0 + 0 = 0$$

$$V_r = \frac{\partial \phi}{\partial r} \quad \text{and} \quad V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

$$2w_z = \frac{1}{r} \frac{\partial}{\partial r} (V_\theta) - \frac{1}{r^2} \frac{\partial}{\partial \theta} (V_r)$$

$$2w_z = \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial \phi}{\partial \theta} \right] - \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[\frac{\partial \phi}{\partial r} \right]$$

$$\phi = ur \cos \theta + \frac{u}{r} \cos \theta$$

$$V_r = -\frac{\partial \phi}{\partial r} = -u \cos \theta + \frac{u}{r^2} \cos \theta$$

$$V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{1}{r} \left[-ur \sin \theta - \frac{u}{r} \sin \theta \right]$$

$$V_\theta = u \sin \theta + \frac{u}{r^2} \sin \theta$$

$$\begin{aligned} 2w_z &= \frac{1}{r} \frac{\partial}{\partial r} \left[u \sin \theta + \frac{u}{r^2} \sin \theta \right] - \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[u \cos \theta + \frac{u}{r^2} \cos \theta \right] \\ &= \frac{1}{r} \left(-\frac{2u}{r^3} \sin \theta \right) - \frac{1}{r^2} \left[-u \sin \theta + \frac{u}{r^2} \sin \theta \right] \\ &= -\frac{2u \sin \theta}{r^4} - \frac{u \sin \theta}{r^2} + \frac{u}{r^4} \sin \theta \end{aligned}$$

IES-44. Ans. (c)

$$\text{Since, } u = x - 4y; \quad \therefore \frac{\partial u}{\partial x} = 1$$

$$\text{and, } v = -y - 4x; \quad \therefore \frac{\partial v}{\partial y} = -1$$

$$\therefore \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 1 - 1 = 0$$

Hence, the flow satisfies the continuity equation

$$\text{Also, } \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -4 - (-4) = 0$$

Hence, the flow is irrotational.

IES-45. The stream function $\psi = x^3 - y^3$ is observed for a two dimensional flow field. What is the magnitude of the velocity at point (1, -1)?

[IES-2004; IES-1998]

(a) 4.24
2.83

(b) 2.83

(c) 0

(d) -

IES-45. Ans. (a)

and

 $= -3$

$$u = \frac{\partial \psi}{\partial y} = -3y^2 = -3$$

$$= 4.24$$

$$v = -\frac{\partial \psi}{\partial x} = -3x^2 \quad \therefore$$

$$\sqrt{(-3)^2 + (-3)^2}$$

IES-46. The stream function in a flow field is given by $\psi = 2xy$. In the same flow field, what is the velocity at a point (2, 1)? [IES-2008]

(a) 4 unit

(b) 5.4 unit

(c) 1.73 unit

(d) 4.47 unit

IES-46. Ans. (d)

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y$$

$$u(2, 1) = -4 \quad \text{and} \quad v(2, 1) = 2$$

$$\therefore \text{Velocity at point } (2, 1) = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = 4.47 \text{ unit}$$

IES-47. For irrotational and incompressible flow, the velocity potential and stream function are given by, respectively. Which one

 ϕ and ψ

of the following sets is correct?

[IES-2006]

- | | |
|--|---|
| (a)
$\nabla^2\phi = 0, \nabla^2\psi = 0$ | (b)
$\nabla^2\phi \neq 0, \nabla^2\psi = 0$ |
| (c)
$\nabla^2\phi = 0, \nabla^2\psi \neq 0$ | (d)
$\nabla^2\phi \neq 0, \nabla^2\psi \neq 0$ |

IES-47. Ans. (a)

IES-48. Which one of the following statements is true to two-dimensional flow of ideal fluids? [IES-1996]

- (a) Potential function exists if stream function exists.
- (b) Stream function may or may not exist.
- (c) Both potential function and stream function must exist for every flow.
- (d) Stream function will exist but potential function may or may not exist.

IES-48. Ans. (d) For a possible case of fluid flow Stream function will exist, but potential function will exist only for irrotational flow. In this case flow may be rotational or irrotational.

IES-49. The realisation of velocity potential in fluid flow indicates that the

- (a) Flow must be irrotational [IES-1993]
- (b) Circulation around any closed curve must have a finite value
- (c) Flow is rotational and satisfies the continuity equation
- (d) Vorticity must be non-zero

IES-49. Ans. (a) The realisation of velocity potential in fluid flow indicates that the flow must be irrotational.

Flow Net

IES-50. For an irrotational flow, the velocity potential lines and the streamlines are always. [IES-1997]

- (a) Parallel to each other
- (b) Coplanar
- (c) Orthogonal to each other
- (d) Inclined to the horizontal.

IES-50. Ans. (c) Slope of velocity potential =

$$\left(\frac{dy}{dx}\right)_1 = -\frac{v}{u}$$

Slope of stream line

$$\left(\frac{dy}{dx}\right)_2 = \frac{v}{u}$$

$$\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -\frac{v}{u} \times \frac{v}{u} = -1$$

Hence, they are orthogonal to each other.

Acceleration in Fluid Vessel

IES-51. A cylindrical vessel having its height equal to its diameter is filled with liquid and moved horizontally at acceleration equal to acceleration due to gravity. The ratio of the liquid left in the vessel to the liquid at static equilibrium condition is: [IES-2001]

- (a) 0.2
- (b) 0.4
- (c) 0.5
- (d) 0.75

IES-51. Ans. (c)

Previous 20-Years IAS Questions

Tangential and Normal Acceleration

IAS-1. Which one of the following statements is correct? [IAS-2004]

A steady flow of diverging straight stream lines

- (a) Is a uniform flow with local acceleration
- (b) Has convective normal acceleration
- (c) Has convective tangential acceleration
- (d) Has both convective normal and tangential accelerations

IAS-1. Ans. (c)

Stream Line

IAS-2. In a two-dimensional flow, where u is the x -component and v is the y -component of velocity, the equation of streamline is given by [IAS-1998]

- (a) $u dx - v dy = 0$
- (b) $v dx - u dy = 0$
- (c) $u v dx + dy = 0$
- (d) $u dx + v dy = 0$

IAS-2. Ans. (b) or

$$\frac{dx}{u} = \frac{dy}{v} \qquad v dx - u dy = 0$$

Streak Line

IAS-3. Consider the following statements: [IAS-2001]

1. Streak line indicates instantaneous position of particles of fluid passing through a point.
2. Streamlines are paths traced by a fluid particle with constant velocity.
3. Fluid particles cannot cross streamlines irrespective of the type of flow.
4. Streamlines converge as the fluid is accelerated, and diverge when retarded.

Which of these statements are correct?

- (a) 1 and 4
- (b) 1, 3 and 4
- (c) 1, 2 and 4
- (d) 2 and 3

IAS-3. Ans. (b) 2 is wrong.

Continuity Equation

IAS-4. Which one of the following is the continuity equation in differential form? (The symbols have usual meanings) [IAS-2004; IAS-2003]

- (a) $\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = \text{const.}$
- (b) $\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$

(c)

$$\frac{A}{dA} + \frac{V}{dV} + \frac{\rho}{d\rho} = \text{const.}$$

(d) $AdA + VdV + \frac{d\rho}{\rho} = 0$

IAS-4. Ans. (b) $= 0$

$$\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho}$$

Integrating, we get $\log A + \log V + \log P = \log C$

\therefore

or, $\log(AV\rho) = \log C$

$$AV\rho = C$$

$\therefore \rho$

which is the continuity equation

IAS-5. Which one of the following equations represents the continuity equation for steady compressible fluid flow? [IAS-2000]

(a)

$$\Delta.\rho V + \frac{\partial \rho}{\partial t} = 0$$

(b)

$$\Delta.\rho V + \frac{\partial \rho}{\partial t} = 0$$

(c)

$$\Delta.V = 0$$

(d)

$$\Delta.\rho V = 0$$

IAS-5. Ans. (d) General continuity equation

$$\nabla.\rho V + \frac{\partial \rho}{\partial t} = 0$$

For steady flow $\frac{\partial \rho}{\partial t} = 0$ and for compressible fluid the equation $\nabla.\rho V = 0$

$$\frac{\partial \rho}{\partial t} = 0,$$

$$\nabla.\rho V$$

For steady, incompressible flow $\rho = \text{const.}$ So the equation $\nabla.V = 0$

$$\frac{\partial \rho}{\partial t} = 0$$

$$\nabla.V = 0$$

IAS-6. The continuity equation for 3-dimentional flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

applicable to:

[IAS-1999; IAS-1998, 1999]

(a) Steady flow

(b) Uniform flow

(c) Ideal fluid flow

(d) Ideal as well as viscous flow

IAS-6. Ans. (c)

IAS-7. In a steady, incompressible, two dimensional flow, one velocity component in the x-direction is given by $u = cx^2/y^2$. The velocity component in the y-direction will be: [IAS-1997]

(a) $V = -c(x + y)$

(b) $v = -cx/y$

(c) $v = -xy$

(d) $v = -cy/x$

IAS-7. Ans. (b)

IAS-8. The components of velocity u and v along x - and y -direction in a 2-D flow problem of an incompressible fluid are: [IAS-1994]

1. $u = x^2 \cos y; v = -2x \sin y$ 2. $u = x + 2; v = 1 - y$

3. $u = xy; v = x^3 - y^2 t / 2$ 4. $u = \ln x + y; v = xy - y/x$

Those which would satisfy the continuity equation would include

- (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 3 and 4 (d) 1 and 2

IAS-8. Ans. (a) Checking $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for all cases.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

IAS-9. If \vec{V} is velocity vector of fluid, then $\Delta \vec{V} = 0$ is strictly true for

$$\vec{V}$$

$$\Delta \vec{V} = 0$$

which of the following?

[IAS-2007; GATE-2008]

- (a) Steady and incompressible flow
- (b) Steady and irrotational flow
- (c) Inviscid flow irrespective of steadiness
- (d) Incompressible flow irrespective of steadiness

IAS-9. Ans. (d)

$$\nabla \cdot \vec{V} = 0 \quad \text{Or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Circulation and Vorticity

IAS-10. Which one of the following is the expression of the rotational component for a two-dimensional fluid element in x - y plane?

- (a) $\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ (b) $\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$ [IAS-2004; IAS-2003]

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

(c)

(d)

$$\omega_z = \frac{1}{2} \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

IAS-10. Ans. (a)

IAS-11. Which of the following relations must hold for an irrotational two-dimensional flow in the x - y plane? [IAS-2003, 2004, IES-1995]

(a)

(b)

$$\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

(c)

(d)

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

IAS-11. Ans. (d) i.e.

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

- IAS-12. A liquid mass readjusts itself and undergoes a rigid body type of motion when it is subjected to a** **[IAS-1998]**
 (a) Constant angular velocity (b) Constant angular acceleration
 (c) Linearly varying velocity (d) Linearly varying acceleration

IAS-12. Ans. (a)

Velocity Potential Function

- IAS-13. The velocity potential function in a two dimensional flow fluid is given by $\phi = x^2 - y^2$. The magnitude of velocity at the point (1, 1) is:**

$$\phi =$$

[IAS-2002]

- (a) 2 (b) 4 (c) $2\sqrt{2}$ (d) $4\sqrt{2}$

IAS-13. Ans. (c) $u =$

$$-\frac{\partial \phi}{\partial x} = -2x, \quad v = -\frac{\partial \phi}{\partial y} = +2y$$

$V =$ unit

$$\sqrt{u^2 + v^2} = \sqrt{(2x)^2 + (2y)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

Stream Function

- IAS-14. For a stream function to exist, which of the following conditions should hold?** **[IAS-1997]**

1. The flow should always be irrotational.
 2. Equation of continuity should be satisfied.
 3. The fluid should be incompressible.
 4. Equation of continuity and momentum should be satisfied.
- Select the correct answer using the codes given below:

Codes:

- (a) 1, 2, 3 and 4 (b) 1, 3 and 4 (c) 2 and 3 (d) 2 alone

IAS-14. Ans. (d)

- IAS-15. Consider the following statements:** **[IAS-2002]**

1. For stream function to exist, the flow should be irrotational.
2. Potential functions are possible even though continuity is not satisfied.
3. Streamlines diverge where the flow is accelerated.
4. Bernoulli's equation will be satisfied for flow across a cross-section.

Which of the above statements is/are correct?

- (a) 1, 2, 3 and 4 (b) 1, 3 and 4 (c) 3 and 4 (d) 2 only

- IAS-15. Ans. (c)** 1. Stream function is exist for possible case of fluid flow i.e. if continuity is satisfied but flow may be rotational or irrotational, 1 is wrong.
 2. Potential function will exist for possible and irrotational flow so both continuity and irrotational must be satisfied, 2 is wrong.

Flow Net

IAS-16. Consider the following statements for a two dimensional potential flow:

1. Laplace equation for stream function must be satisfied. [IAS-2002]
2. Laplace equation for velocity potential must be satisfied.
3. Streamlines and equipotential lines are mutually perpendicular.
4. Streamlines can intersect each other in very high speed flows.

Which of the above statements are correct?

- (a) 1 and 4 (b) 2 and 4 (c) 1, 2 and 3 (d) 2, 3 and 4

IAS-16. Ans. (c) Streamlines never intersect each other.



Fluid Dynamics

Contents of this chapter

1. Bernoulli's Equation
 2. Euler's Equation
 3. Venturimeter
 4. Orifice Meter
 5. Pitot Tube
 6. Free Liquid Jet
 7. Impulse Momentum Equation
 8. Forced Vortex
 9. Free Vortex
-

Question: Derive from the first principles the Euler's equation of motion for steady flow along a streamline. Obtain Bernoulli's equation by its integration. State the assumptions made. [IES – 1997] [Marks-10]

Answer: Consider steady flow of an ideal fluid along the stream tube. Separate out a small element of fluid of cross-sectional area dA and length dS from stream tube as a free body from the moving fluid.

Fig (below) shows such a small element LM of fluid of cross section area dA and length dS .

Let, p = Pressure on the element at L.
 $p + dp$ = Pressure on the element at M and
 V = velocity of fluid along stream line.

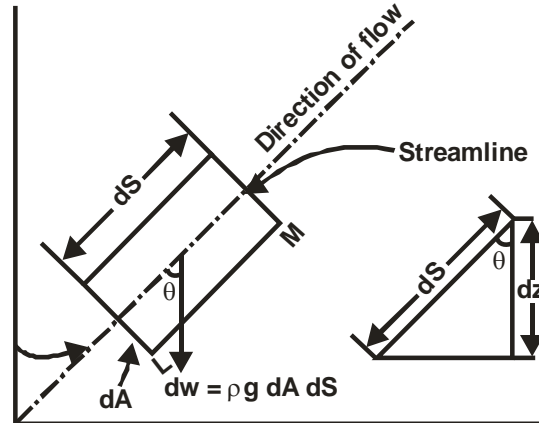


Fig: Force on a fluid element

The external forces tending to accelerate the fluid element in the direction of stream line are as follows.

1. Net pressure force in the direction of flow is
 $p \cdot dA - (p + dp) dA = - dp dA$
2. Component of weight of the fluid element in the direction of flow is
 $= -\rho g dA dS \cos \theta$
 $= -\rho g dA dS \left(\frac{dZ}{dS} \right) \quad \left[\because \cos \theta = \frac{dZ}{dS} \right]$
 $= -\rho g dA dS dZ$

Mass of the fluid element = $\rho dA dS$

The acceleration of a fluid element

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \cdot \frac{dv}{ds} \quad (v \text{ along the direction of streamline})$$

\therefore According to Newton second law of motion

Force = mass \times acceleration

$$-dp dA - \rho g dA dz = \rho dA dS \cdot v \frac{dv}{dS}$$

$$\text{or } -dp dA - \rho g dA dz = \rho dA \cdot (v \cdot dv)$$

Dividing both side by ρdA

$$\frac{-dp}{\rho} - g dz = v \cdot dv$$

or, $\frac{dp}{\rho} + v \cdot dv + g dz = 0$ Euler's equation of motion for steady flow along a stream line.

Question: *Derive Bernoulli's Equation*

Answer: Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int \frac{dp}{\rho} + \int g dZ + \int v dv = \text{const.}$$

∴ For incompressible flow ($\rho = \text{const.}$)

$$\frac{p}{\rho} + gZ + \frac{v^2}{2} = \text{const}$$

$$\text{or } \frac{p}{\rho g} + Z + \frac{v^2}{2g} = \text{const.}$$

$$\text{or } \boxed{\frac{p}{w} + \frac{v^2}{2g} + z = \text{const.}} \text{ where } w = \text{unit weight (specific weight)}$$

For compressible flow $\left(\frac{p}{\rho^\gamma} = c\right)$ **undergoing adiabatic flow.**

$$p = c \cdot \rho^\gamma$$

$$\text{or } dp = c \cdot \gamma \cdot \rho^{\gamma-1} d\rho \quad \text{and} \quad \int \frac{c \cdot \gamma \cdot \rho^{\gamma-1} d\rho}{\rho g} + \frac{1}{g} \int v dv + \int dz = \text{const.}$$

$$\text{or } \frac{\gamma \cdot c}{g} \int \rho^{\gamma-2} d\rho + \frac{v^2}{2g} + z = \text{const.}$$

$$\text{or } \frac{\gamma}{g} \cdot \frac{p}{\rho^\gamma} \cdot \frac{\rho^{\gamma-1}}{\gamma-1} + \frac{v^2}{2g} + z = \text{const.}$$

$$\text{or } \boxed{\frac{\gamma}{\gamma-1} \cdot \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{const.}} \text{ For compressible flow undergoing adiabatic process.}$$

For compressible flow $\left(\frac{p}{\rho} = c\right)$ **undergoing isothermal process**

$$\rho = \frac{p}{c}$$

$$\therefore c \int \frac{dp}{\rho} + g \int dz + \int v dv = \text{const}$$

$$\text{or } \frac{p}{\rho} \cdot \ln p + gz + \frac{v^2}{2} = \text{const.}$$

$$\text{or } \boxed{\frac{p \ln p}{\rho g} + \frac{v^2}{2g} + z = \text{const.}} \text{ For compressible flow undergoing isothermal process.}$$

Question: *Define Bernoulli's theorem and explain what corrections are to be made in the equation for ideal fluid, if the fluid is a real fluid.*

[AMIE (Win) 2002; AMIE MAY 1974]

Answer: **Statement of Bernoulli's Theorem:** "It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant".

The total energy consists of

(i) Pressure energy = $\frac{p}{\rho g}$

(ii) Kinetic energy = $\frac{v^2}{2g}$, and

(iii) Datum or potential energy = z

Thus Mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Correction:

(i) Bernoulli's equation has restriction of frictionless flow. For real fluid this is accommodated by introducing a loss of energy term (h_L) i.e.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

(ii) Restriction of irrotational flow is waived in most of the cases.

Question: *Velocity distribution in a pipe is given by $\frac{u}{u_{\max}} = 1 - \left(\frac{r}{R}\right)^n$*

Where, u_{\max} = Maximum velocity at the centre of pipe.

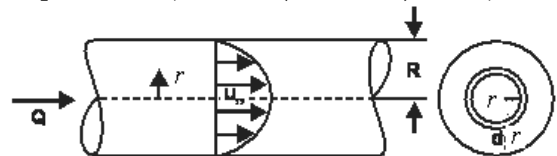
u = velocity at a distance r

R = radius of the pipe.

Obtain an expression for mean velocity in terms of u_{\max} and n .

[IES-1997; AMIE (summer)-1998, 2001]

Answer: Consider an elementary strip at a distance r from the center and thickness dr .
 \therefore Area, $dA = 2\pi r dr$



As velocity is u at that point so discharge then the elementary ring

$$dQ = dA \times u = 2\pi r dr \cdot u$$

$$\therefore dQ = 2\pi r u_{\max} \left(1 - \frac{r^n}{R^n}\right) dr$$

\therefore Total flow Q is

$$\begin{aligned} Q &= \int dQ = \int_0^R 2\pi u_{\max} r \left(1 - \frac{r^n}{R^n}\right) dr = 2\pi u_{\max} \int_0^R \left(r - \frac{r^{n+1}}{R^n}\right) dr \\ &= 2\pi u_{\max} \left[\frac{R^2}{2} - \frac{R^{n+2}}{(n+2)R^n} \right] = 2\pi u_{\max} R^2 \left(\frac{1}{2} - \frac{1}{n+2} \right) \\ &= \pi u_{\max} R^2 \left(\frac{n}{n+2} \right) \end{aligned}$$

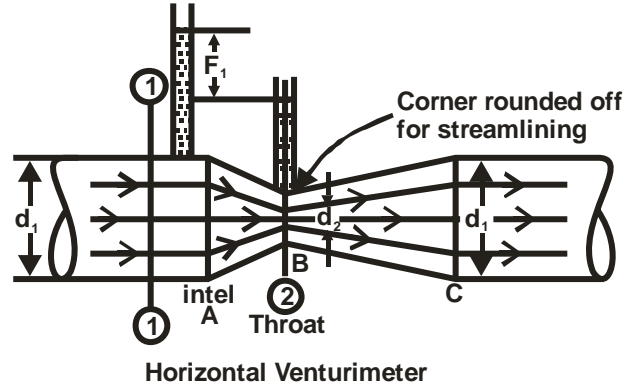
If mean velocity is \bar{U} then flow $Q = \pi R^2 \bar{U}$

$$\therefore \pi R^2 \bar{U} = \pi u_{\max} R^2 \frac{n}{(n+2)}$$

$$\therefore \bar{U} = u_{\max} \times \frac{n}{(n+2)}$$

Question: *Derive an expression for rate of flow through Horizontal Venturimeter. What changes have to be made for vertical & inclined Venturimeter?* [IES-2003]

Answer: A venturimeter consists of the following three parts
 (i) A short converging part (AB)
 (ii) Throat, B, and
 (iii) Diverging part, BC
 Fig (below) shows a venturimeter fitted in horizontal pipe through which an incompressible fluid is flowing.



Let, d_1 = diameter at inlet

$$A_1 = \text{Area at inlet} \left(\frac{\pi d_1^2}{4} \right)$$

p_1 = Pressure at inlet

V_1 = Velocity at inlet

and, d_2, A_2, p_2 and V_2 are the corresponding values at throat.

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\text{or } \frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\text{or } h = \frac{V_2^2 - V_1^2}{2g} \quad [\text{where } h = \text{difference of pressure head from manometer}]$$

$$\text{or } V_2^2 - V_1^2 = 2gh$$

Applying continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\text{or } V_1 = \frac{A_2}{A_1} V_2$$

$$\therefore V_2^2 - V_1^2 = 2gh$$

$$\text{or } V_2^2 - \left(\frac{A_2}{A_1} V_2 \right)^2 = 2gh$$

$$\text{or } V_2^2 \left\{ A_1^2 - A_2^2 \right\} = A_1^2 \times 2gh$$

$$\text{or } V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

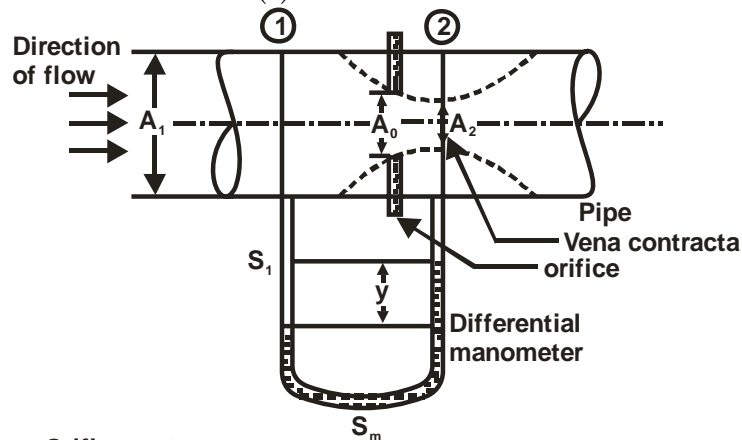
$$\therefore \text{Discharge (Q)} = A_2 V_2 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

If co-efficient of discharge is C_d the actual discharge

$$Q_{\text{actual}} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

Question: Derive an expression for discharge through orifice meter.

Answer: It consists of a flat circular plate having a circular sharp edged hole (called orifice) and a differential manometer is connected between section (1) and vena contracta section (2)



: Orifice meter :

Let, A_1 = Area of pipe

A_2 = Area of vena-contracta

A_0 = Orifice Area

p_1 = Pressure at section (1)

p_2 = Pressure at vena contracta, section (2)

V_1 = Velocity at section (1)

V_2 = Velocity at section (2)

$$C_c = \text{Co-efficient of contraction} \left(\frac{A_2}{A_0} \right)$$

$$C_d = \text{Co-efficient of discharge} \left[C_d = C_c \times \frac{\sqrt{1 - \left(\frac{A_0}{A_1} \right)^2}}{\sqrt{1 - \left(\frac{A_0}{A_1} \right)^2} \cdot C_c^2} \right]$$

Applying Bernoulli's equation between section (1) and (2) we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \left(\frac{p_1}{\rho g} + Z_1 \right) - \left(\frac{p_2}{\rho g} + Z_2 \right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\text{or } h = \frac{V_2^2 - V_1^2}{2g} \left[\text{where } h \text{ is the differential head (h) } = y \left\{ \frac{S_m}{S_l} - 1 \right\} \right]$$

Using continuity equation $A_1 V_1 = A_2 V_2$ and $A_2 = A_0 \cdot C_c$

$$V_1 = \frac{A_2}{A_1} \cdot V_2 = \frac{A_0 C_c}{A_1} \cdot V_2$$

$$\therefore V_2^2 - \frac{A_0^2 C_c^2}{A_1^2} V_2^2 = 2gh$$

$$\text{or } V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} \cdot C_c^2}}$$

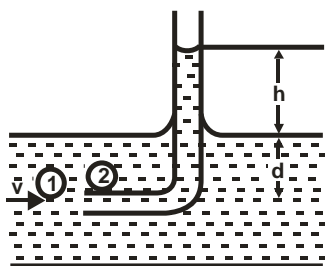
$$\therefore \text{Discharge (Q)} = A_2 V_2 = A_0 C_c \cdot V_2$$

$$= \frac{A_0 \cdot C_c \cdot \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} \cdot C_c^2}} = A_0 \cdot \frac{C_d}{\sqrt{1 - \frac{A_0^2}{A_1^2}}} \cdot \sqrt{2gh} = \frac{C_d \cdot A_1 \cdot A_0 \cdot \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

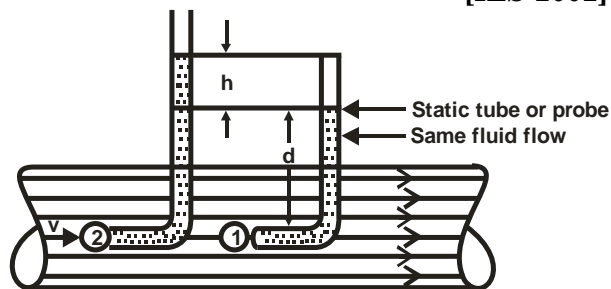
$$\therefore Q = \frac{C_d A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

Question: Describe the working principal of a pitot-static tube with the help of neat sketch and explain how it can be used to measure the flow rate.

[IES-2002]



A. Pitot-tube in a open channel



B. Pitot-tube in Pipe flow

The 'stagnation pressure' at a point in a fluid flow is the total pressure which would result if the fluid were brought to rest isentropically. In actual practice, a stagnation point is created by bringing the fluid to rest at the desired point and the pressure at the point corresponds to the stagnation pressure as shown is above fig. A & B, 'h'. A simple device to measure the stagnation pr. is a tube with a hole in the front inserted in the flow such that the velocity is normal to the plane of the hole.

Stagnation Pressure = static Pressure + dynamic Pressure

$$\therefore p_0 = p + \frac{\rho}{2} v^2$$

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Here $z_1 = z_2$ and $v_2 = 0$ and $\frac{p_1}{\rho g} = d$ and $\frac{p_2}{\rho g} = (d+h)$

$$\therefore d + \frac{v_1^2}{2g} = d+h$$

$$\text{or } v_1 = \sqrt{2gh}$$

Here v_1 is theoretical velocity $\therefore v_{\text{actual}} = c_v \cdot c_1 = c_v \sqrt{2gh}$

Where c_v = co-efficient of velocity depends on tube.

\therefore If velocity is known then

$$\text{Discharge (Q)} = A \cdot v_{\text{actual}} = c_v A \sqrt{2gh}$$

Question: *What is vortex flow? What is the difference between Free vortex flow and forced vortex flow?*

Answer: **Vortex flow:** A flow in which the whole fluid mass rotates about an axis. In vortex flow streamlines are curved.

Forced vortex flow: Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency.

Here angular velocity, $\omega = \frac{v}{r} = \text{constant}$

Example: Rotation of water through the runner of a turbine.

Free vortex flow: Free vortex flow is one in which the fluid mass rotates without any external impressed contact force.

The whole fluid mass rotates either due to fluid pr. itself or the gravity or due to rotation previously imparted.

Here Moment of Momentum = const.

i.e. $V \times r = \text{constant}$

Example: A whirlpool in a river.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

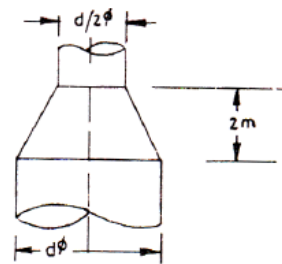
Previous 20-Years GATE Questions

Bernoulli's Equation

GATE-1. Bernoulli's equation can be applied between any two points on a streamline for a rotational flow field. [GATE-1994]

GATE-1. Ans. True

GATE-2. Water flows through a vertical contraction from a pipe of diameter d to another of diameter $d/2$. The flow velocity at the inlet to the contraction is 2m/s and pressure 200 kN/m^2 if the height of the contraction measures 2m , the pressure at the exit of the contraction will be very nearly



- (a) 168 kN/m^2
- (b) 192 kN/m^2
- (c) 150 kN/m^2
- (d) 174 kN/m^2

[GATE-1999]

GATE-2. Ans. (c)

GATE-3. Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm . The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa . The fluid has a vapour pressure of 50 kPa and a specific weight of 5 kN/m^3 . Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is:

- (a) 0.05
- (b) 0.16
- (c) 0.27
- (d) 0.38

[GATE -2009]

GATE-3. Ans. (b) $\rho g = 5000\text{N/m}^3$

$$A_1 V_1 = A_2 V_2 \quad \dots\dots(1)$$

$$\text{or } V_2 = \left(\frac{20}{10}\right)^2 \times V_1 = 4V_1$$

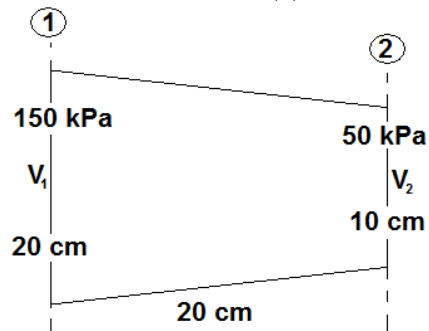
$$\frac{150 \times 10^3}{5000} + \frac{V_1^2}{2g} = \frac{50 \times 10^3}{5000} + \frac{V_2^2}{2g} \quad \dots\dots(2)$$

$$\text{or } 30 + \frac{V_1^2}{2g} = 10 + \frac{V_2^2}{2g} \quad 16 \frac{V_1^2}{g}$$

$$\text{or } 20 = 15 \frac{V_1^2}{2g}$$

$$\text{or } V_1 = \sqrt{\frac{20 \times 2 \times g}{15}}$$

$$\text{Then } Q = A_1 V_1 = 0.16$$



Euler's Equation

GATE-4. Navier Stoke's equation represents the conservation of [GATE-2000]
 (a) Energy (b) Mass (c) Pressure (d) Momentum

GATE-4. Ans. (d)

Venturimeter

GATE-5. In a venturimeter, the angle of the diverging section is more than that of converging section. [GATE-1994]

- (a) True (b) False (c) Insufficient data (d) Can't say

GATE-5. Ans. (b) The angle of diverging section is kept small to reduce the possibility of flow separation. Due to this the angle of converging section is more as compared to its diverging section.

GATE-6. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is:

[GATE-2005]

- (a) 0.2 m/s (b) 1.0 m/s (c) 1.4 m/s (d) 2.0 m/s

GATE-6. Ans. (d) We know, $A_1 V_1 = A_2 V_2$

$$\Rightarrow V_2 = \frac{D_1^2}{D_2^2} V_1 = \frac{16}{4} V_1$$

$$\therefore V_2 = 4V_1$$

Applying Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

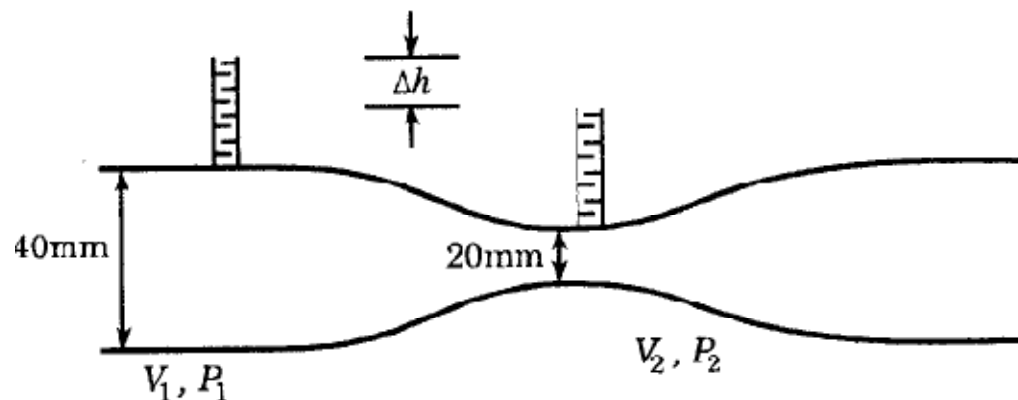
$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow \frac{15V_1^2}{2} = \frac{30 \times 10^3}{1000}$$

$$\Rightarrow V_1^2 = 4$$

$$\Rightarrow V_1 = 2.0 \text{ m/s}$$

So velocity of flow is 2.0m/sec.



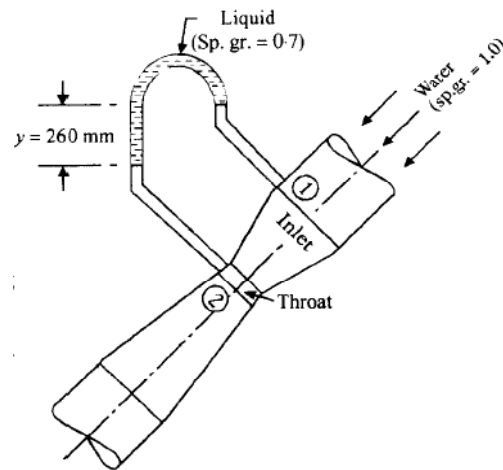
GATE-7. Air flows through a venture and into atmosphere. Air density is ρ ; atmospheric pressure P_a ; throat diameter is D_t ; exit diameter is D and

At point P: Spring force = pressure force due air

$$-kx = \frac{\pi D_s^2}{4} \times \frac{\rho U^4}{2} \left[1 - \frac{D^4}{D_t^4} \right]$$

$$\Rightarrow x = \frac{\pi D_s^2 \rho U^2}{8k} \left[1 - \frac{D^4}{D_t^4} \right]$$

GATE-8. Determine the rate of flow of water through a pipe 300 mm diameter placed in an inclined position where a Venturimeter is inserted having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0.7 in an inverted V-tube which gives a reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of the pipe.



- (a) 0.0222 m³/s (b) 0.4564 m³/s (c) 1.m³/s [GATE-1985]
 (d) m³/s

GATE-8. Ans. (a)

Free Liquid Jet

GATE-9. Two balls of mass m and 2 m are projected with identical velocities from the same point making angles 30° and 60° with the vertical axis, respectively. The heights attained by the balls will be identical.

- (a) True (b) False (c) None [GATE-1994]
 (d) Can't say

GATE-9. Ans. (b)

Forced Vortex

GATE-10. Which combination of the following statements about steady incompressible forced vortex flow is correct? [GATE-2007]

P: Shear stress is zero at all points in the flow.

Q: Vorticity is zero at all points in the flow

R: Velocity is directly proportional to the radius from the centre of the vortex.

S: Total mechanical energy per unit mass is constant in the entire flow field.

- (a) P and Q (b) R and S (c) P and R (d) P and S

GATE-10. Ans. (b)

GATE-11. A closed cylinder having a radius R and height H is filled with oil of density ρ. If the cylinder is rotated about its axis at an angular velocity of ω, then thrust at the bottom of the cylinder is: [GATE-2004]

- (a) πR² ρgH (b) πR² $\frac{\rho\omega^2 R^2}{4}$

(c) $\pi R^2(\rho\omega^2 R^2 + \rho gH)$

(d) $\pi R^2 \left(\frac{\rho\omega^2 R^2}{4} + \rho gH \right)$

GATE-11. Ans. (d) We know that

$$\frac{\partial P}{\partial r} = \frac{\rho v^2}{r} = \frac{\rho \cdot \omega^2 r}{r} = \rho \omega^2 r.$$

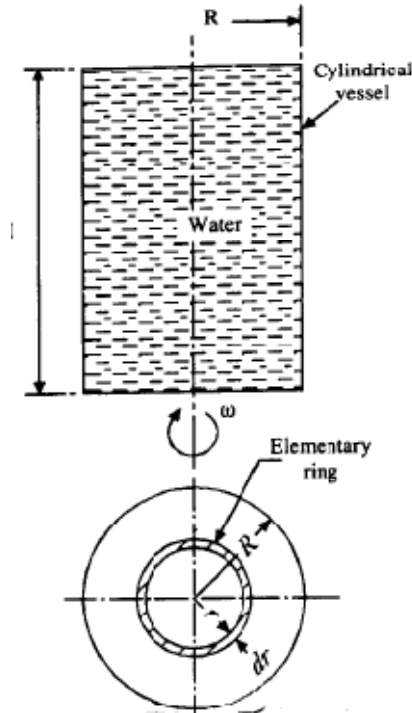
[∵ $v = \omega \times r$]

$$\therefore \int_0^p \partial p = \int_0^r \rho \omega^2 r dr \quad [p = \frac{\rho}{2} \omega^2 r^2]$$

Area of circular ring = $2\pi r dr$
 Force on elementary ring =
 Intensity of pressure \times Area of ring
 $= \frac{\rho}{2} \omega^2 r^2 2\pi r dr$

∴ Total force on the top of the cylinder
 $= \int_0^R \frac{\rho}{2} \omega^2 r^2 2\pi r dr = \frac{\rho}{2} \omega^2 2\pi \int_0^R r^3 dr$
 $= \frac{\rho}{2} \cdot \omega^2 2\pi \frac{R^4}{4} = \frac{\rho}{4} \omega^2 \times \pi R^4$

Thrust at the bottom of the cylinder
 = Weight of water in cylinder +
 Total force on the top of cylinder
 $= \rho g \times \pi R^2 \times H + \frac{\rho}{4} \omega^2 \times \pi R^4$
 $= \pi R^2 \left[\frac{\rho \omega^2 R^2}{4} + \rho g h \right]$



Previous 20-Years IES Questions

Bernoulli's Equation

IES-1. Bernoulli's equation represents the [IES-1994]

- (a) Forces at any point in the flow field and is obtained by integrating the momentum equation for viscous flows.
- (b) Energies at any point in the flow field and is obtained by integrating the Euler equations.
- (c) Momentum at any point in the flow field and is obtained by integrating the equation of continuity.
- (d) Moment of momentum and is obtained by integrating the energy equation.

IES-1. Ans. (b)

IES-2. When is Bernoulli's equation applicable between any two points in a flow fields? [IES-2009]

- (a) The flow is steady, incompressible and rotational
- (b) The flow is steady, compressible and irrotational
- (c) The flow is unsteady, incompressible and irrotational
- (d) The flow is steady, incompressible and irrotational

IES-2. Ans. (d) The assumptions made for Bernoulli's equation.

- (i) The liquid is ideal (viscosity, surface tension is zero and incompressible)
- (ii) The flow is steady and continuous
- (iii) The flow is along the streamline (it is one-dimensional)
- (iv) The velocity is uniform over the section and is equal to the mean velocity.
- (v) The only forces acting on the fluid are the gravity force and the pressure force

The assumptions NOT made for Bernoulli's equation

- (i) The flow is uniform
- (ii) The flow is irrotational

IES-3. Assertion (A): Two table tennis balls hang parallelly maintaining a small gap between them. If air is blown into the gap between the balls, the balls will move apart. [IES-1994]

Reason (R): Bernoulli's theorem is applicable in this case.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-3. Ans. (c)

IES-4. Which of the following assumptions are made for deriving Bernoulli's equation? [IES-2002]

1. Flow is steady and incompressible
2. Flow is unsteady and compressible
3. Effect of friction is neglected and flow is along a stream line.
4. Effect of friction is taken into consideration and flow is along a stream line.

Select the correct answer using the codes given below:

- (a) 1 and 3
- (b) 2 and 3
- (c) 1 and 4
- (d) 2 and 4

IES-4. Ans. (a)

IES-5. The expression $(p + \rho gz + \rho v^2/2)$ commonly used to express Bernoulli's equation, has units of [IES-1995]

- (a) Total energy per unit mass
- (b) Total energy per unit weight
- (c) Total energy per unit volume
- (d) Total energy per unit cross - sectional area of flow

IES-5. Ans. (c) The expression $p + \rho gz + \rho v^2/2$, has units of $\frac{N}{m^2}$ or $\frac{Nm}{m^3}$ $\left(\frac{\text{energy}}{\text{volume}} \right)$

IES-6. The expression: [IES-2003]

$$\frac{\partial \phi}{\partial t} + \int \frac{\partial p}{\rho} + \frac{1}{2} |\Delta \phi|^2 + gz = \text{constant}$$

represents :

- (a) Steady flow energy equation
- (b) Unsteady irrotational Bernoulli's equation
- (c) Steady rotational Bernoulli's equation
- (d) Unsteady rotational Bernoulli's equation

IES-6. Ans. (b)
$$\frac{\partial \phi}{\partial t} + \int \frac{\partial \rho}{\rho} + \frac{1}{2} (\nabla \phi)^2 + gz = \text{const.}$$

\downarrow
Unsteady

\downarrow
irrotational

IES-7. Which one of the following statements is correct? While using boundary layer equations, Bernoulli's equation [IES-2006]

- (a) Can be used anywhere
- (b) Can be used only outside the boundary layer
- (c) Can be used only inside the boundary layer
- (d) Cannot be used either inside or outside the boundary layer

IES-7. Ans. (b)

IES-8. Assertion (A): Bernoulli's equation is an energy equation. [IES-1997]
Reason (R): Starting from Euler's equation, one can arrive at Bernoulli's equation.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-8. Ans. (b) Starting from Euler's equation, one can arrive at Bernoulli's equation. And we know that Euler equation is a momentum equation and integrating Euler equation we can arrive at Bernoulli's equation.

IES-9. Assertion (A): After the fluid has re-established its flow pattern downstream of an orifice plate, it will return to same pressure that it had upstream of the orifice plate. [IES-2003]

Reason (R): Bernoulli's equation when applied between two points having the same elevation and same velocity gives the same pressure at these points.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-9. Ans. (d) There is a loss of energy due to eddy formation and turbulence. This is the reason for that pressure is less than that it had upstream of the orifice plate.

Euler's Equation

IES-10. Consider the following assumptions: [IES-1998]

1. The fluid is compressible
2. The fluid is inviscid.
3. The fluid is incompressible and homogeneous.
4. The fluid is viscous and compressible.

The Euler's equation of motion requires assumptions indicated in :

- (a) 1 and 2
- (b) 2 and 3
- (c) 1 and 4
- (d) 3 and 4

IES-10. Ans. (b)

IES-11. The Euler's equation of motion is a statement of [IES-2005]

- (a) Energy balance
- (b) Conservation of momentum for an inviscid fluid

- (c) Conservation of momentum for an incompressible flow
- (d) Conservation of momentum for real fluid

IES-11. Ans. (b)

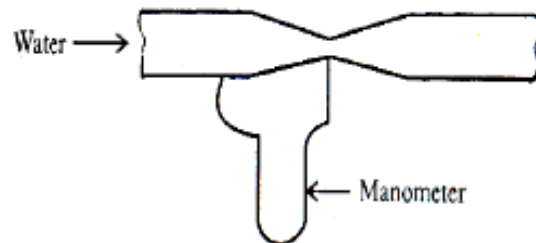
IES-12. The Euler equations of motion for the flow of an ideal fluid is derived considering the principle of conservation of [IES-1994]

- (a) Mass and the fluid as incompressible and inviscid.
- (b) Momentum and the fluid as incompressible and viscous.
- (c) Momentum and the fluid as incompressible and inviscid.
- (d) Energy and the fluid as incompressible and inviscid.

IES-12. Ans. (c) For inviscid flows, the steady form of the momentum equation is the Euler equation. For an inviscid incompressible fluid flowing through a duct, the steady flow energy equation reduces to Bernoulli equation.

Venturimeter

IES-13. A horizontal pipe of cross-sectional area 5 cm^2 is connected to a venturimeter of throat area 3 cm^2 as shown in the below figure. The manometer reading is equivalent to 5 cm of water. The discharge in cm^3/s is nearly:



- (a) 0.45
- (b) 5.5
- (c) 21.0
- (d) 370

IES-13. Ans. (d) Just use the Venturimeter formula: $Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$ [IES-1998]

Here $C_d = 1.0$; $A_1 = 5 \text{ cm}^2$; $A_2 = 3 \text{ cm}^2$; $g = 981 \text{ cm/s}^2$ and $h = 5 \text{ cm}$

$$Q_{act} = \frac{3 \times 5}{\sqrt{5^2 - 3^2}} \times \sqrt{2 \times 981 \times 5} = 371.42 \text{ cm}^3 / \text{s}$$

IES-14. An orifice meter with $C_d = 0.61$ is substituted y Venturimeter with $C_d = 0.98$ in a pipeline carrying crude oil, having the same throat diameter as that of the orifice. For the same flow rate, the ratio of the pressure drops for the Venturimeter and the orifice meter is: [IES-2003]

- (a) $0.61 / 0.98$
- (b) $(0.61)^2 / (0.98)^2$
- (c) $0.98 / 0.61$
- (d) $(0.98)^2 / (0.61)^2$

IES-14. Ans. (b)

IES-15. A Venturimeter in an oil (sp. gr. 0.8) pipe is connected to a differential manometer in which the gauge liquid is mercury (sp.gr.13.6). For a flow rate of $0.16 \text{ m}^3/\text{s}$, the manometer registers a gauge differential of 20 cm . The oil-mercury manometer being unavailable, an air-oil differential manometer is connected to the same venturimeter. Neglecting variation of discharge coefficient for the venturimeter, what is the new gauge differential for a flow rate of $0.08 \text{ m}^3/\text{s}$? [IES-2006]

- (a) 64 cm
- (b) 68 cm
- (c) 80 cm
- (d) 85 cm

IES-15. Ans. (c)

Orifice Meter

IES-16. An orifice meter, having an orifice of diameter d is fitted in a pipe of diameter D . For this orifice meter, what is the coefficient of discharge C_d ? [IES-2007]

- (a) A function of Reynolds number only
- (b) A function of d/D only
- (c) A function of d/D and Reynolds number
- (d) Independent of d/D and Reynolds number

IES-16. Ans.(b) $C_d = C_c \times \frac{\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2}}{1 - C_c^2 \times \left(\frac{A_0}{A_1}\right)^2}$ or, $C_A = f\left(\frac{A_0}{A_1}\right) = F\left(\frac{d}{D}\right)$

IES-17. A tank containing water has two orifices of the same size at depths of 40 cm and 90 cm below the free surface of water. The ratio of discharges through these orifices is: [IES-2000]

- (a) 1 : 1
- (b) 2 : 3
- (c) 4 : 9
- (d) 16 : 81

IES-17. Ans. (b)

IES-18. How is the velocity coefficient C_v , the discharge coefficient C_d , and the contraction coefficient C_c of an orifice related? [IES-2006]

- (a) $C_v = C_c C_d$
- (b) $C_c = C_v C_d$
- (c) $C_d = C_c C_v$
- (d) $C_c C_v C_d = 1$

IES-18. Ans. (c)

Pitot Tube

IES-19. The velocity of a water stream is being measured by a L-shaped Pilot-tube and the reading is 20 cm. Then what is the approximate value of velocity? [IES-2007]

- (a) 19.6 m/s
- (b) 2.0 m/s
- (c) 9.8 m/s
- (d) 20 cm/s

IES-19. Ans. (b) $\frac{V^2}{2g} = h$ or, $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.2} = 1.981 \text{ m/s}$

IES-20. A Prandtl Pilot tube was used to measure the velocity of a fluid of specific gravity S_1 . The differential manometer, with a fluid of specific gravity S_2 , connected to the Pitot tube recorded a level difference as h . The velocity V is given by the expression. [IES-1995]

- (a) $\sqrt{2gh(S_1/S_2 - 1)}$
- (b) $\sqrt{2gh(S_2/S_1 - 1)}$
- (c) $\sqrt{2gh(S_1 - S_2)}$
- (d) $\sqrt{2gh(S_2 - S_1)}$

IES-20. Ans. (b) $\frac{P_1}{\rho g} + y + \frac{hS_2}{S_1} = \frac{P_2}{\rho g} + (h + y)$

$$\therefore \frac{P_2 - P_1}{\rho g} = h \left(\frac{S_2}{S_1} - 1 \right) = \frac{V_1^2}{2g}$$

$$\therefore V_1 = \sqrt{2gh \left(\frac{S_2}{S_1} - 1 \right)}$$

IES-21. A Pitot-static tube ($C = 1$) is used to measure air flow. With water in the differential manometer and a gauge difference of 75 mm, what is the value of air speed if $\rho = 1.16 \text{ kg/m}^3$? [IES-2004]

- (a) 1.21 m/s (b) 16.2 m/s (c) 35.6 m/s (d) 71.2 m/s

IES-21. Ans. (c) $\frac{V^2}{2g} = h$ or $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 64.58} = 35.6 \text{ m/s}$

$$h = y \left(\frac{S_h}{S_l} - 1 \right) = 0.075 \left(\frac{1000}{1.16} - 1 \right) = 64.58 \text{ m of air colume}$$

IES-22. What is the difference in pressure head, measured by a mercury-oil differential manometer for a 20 cm difference of mercury level?

(Sp. gravity of oil = 0.8) [IES-2009]

- (a) 2.72 m of oil (b) 2.52 m of oil (c) 3.40 m of oil (d) 2.00 m of oil

IES-22. Ans. (c) Difference in pressure head in m of oil (i.e. light liquid) =

$$= \left(\frac{S_h}{S_l} - 1 \right) \times y = \left(\frac{13.6}{0.8} - 1 \right) \times 0.2 = 3.2 \text{ m}$$

IES-23. Match List-I (Measuring Devices) with List-II (Measured Parameter) and select the correct answer using the codes given below: [IES-2004]

List-I					List-II				
A. Pitot tube					1. Flow static pressure				
B. Micro-manometer					2. Rate of flow (indirect)				
C. Pipe band meter					3. Differential pressure				
D. Wall pressure tap					4. Flow stagnation pressure				
Codes:	A	B	C	D		A	B	C	D
(a)	1	3	2	4	(b)	4	3	2	1
(c)	1	2	3	4	(d)	4	2	3	1

IES-23. Ans. (b)

IES-24. The instrument preferred in the measurement of highly fluctuating velocities in air flow is: [IES-2003]

- (a) Pitot-static tube (b) Propeller type anemometer
 (c) Three cup anemometer (d) Hot wire anemometer

IES-24. Ans. (d)

IES-25. If a calibration chart is prepared for a hot-wire anemometer for measuring the mean velocities, the highest level of accuracy can be: [IES-1996]

- (a) Equal to accuracy of a Pitot tube
 (b) Equal to accuracy of a Rotameter
 (c) Equal to accuracy of a venturimeter
 (d) More than that of all the three instruments mentioned above

IES-25. Ans. (d) Hot wire anemometer is more accurate than Pitot tube, rotanmeter, or venturi meter.

IES-26. Which one of the following is measured by a Rotameter? [IES-2006]

- (a) Velocity of fluids (b) Discharge of fluids
 (c) Viscosity of fluids (d) Rotational speed of solid shafts

IES-26. Ans. (b)

IES-27. In a rotameter as the flow rate increase, the float [IES-1992]

- (a) Rotates at higher speed (b) Rotates at lower speed
 (c) Rises in the tube (d) Drops in the tube

IES-27. Ans. (c)

IES-28. A Pitot static tube is used to measure the velocity of water using a differential gauge which contains a manometric fluid of relative density 1.4. The deflection of the gauge fluid when water flows at a velocity of 1.2 m/s will be (the coefficient of the tube may be assumed to be 1) [IES-2000]

- (a) 183.5 mm (b) 52.4 mm (c) 5.24 mm (d) 73.4 mm

IES-28. Ans. (a) Use $V = \sqrt{2gh}$ where, $h = y \left(\frac{s_h}{s_l} - 1 \right)$

Given $y = ?$ and $V = 1.2$ m/s, $\frac{s_h}{s_l} = 1.4$

IES-29. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1993]

List-I (Discharge measuring device)	List-II (Characteristic feature)																											
A. Rotameter B. Venturimeter C. Orifice meter D. Flow nozzle	1. Vena contracta 2. End contraction 3. Tapering tube 4. Convergent – divergent 5. Bell mouth entry																											
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">Codes:</td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(a)</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>(c)</td> <td>5</td> <td>4</td> <td>2</td> <td>1</td> </tr> </table>	Codes:	A	B	C	D	(a)	1	2	3	4	(c)	5	4	2	1	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(b)</td> <td>3</td> <td>4</td> <td>1</td> </tr> <tr> <td>(d)</td> <td>3</td> <td>5</td> <td>1</td> </tr> </table>	A	B	C	D	(b)	3	4	1	(d)	3	5	1
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(d)	3	5	1																									

IES-29. Ans. (b)

IES-30. A glass tube with a 90° bend is open at both the ends. It is inserted into a flowing stream of oil, $S = 0.90$, so that one opening is directed upstream and the other is directed upward. Oil inside the tube is 50 mm higher than the surface of flowing oil. The velocity measured by the tube is, nearly, [IES-2001]

- (a) 0.89 m/s (b) 0.99 m/s (c) 1.40 m/s (d) 1.90 m/s

IES-30. Ans. (b)

IES-31. The speed of the air emerging from the blades of a running table fan is intended to be measured as a function of time. The point of measurement is very close to the blade and the time period of the speed fluctuation is four times the time taken by the blade to complete one revolution. The appropriate method of measurement would involve the use of [IES-1993]

- (a) A Pitot tube (b) A hot wire anemometer
 (c) High speed photography (d) A Schlieren system

IES-31. Ans. (b) A Pitot tube is used for measuring speed in closed duct or pipe. Hot wire anemometer is used for measuring fluctuation of speed. High speed photography may be useful to measure blade speed but not of air.

Free Liquid Jet

IES-32. A constant-head water tank has, on one of its vertical sides two identical small orifices issuing two horizontal jets in the same vertical

plane. The vertical distance between the centres of orifices is 1.5 m and the jet trajectories intersect at a point 0.5 m below the lower orifice. What is the approximate height of water level in the tank above the point of intersection of trajectories? [IES-2004]

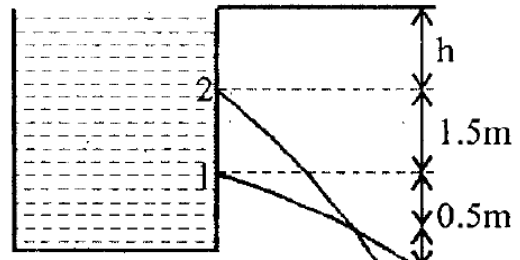
- (a) 1.0 m (b) 2.5 m (c) 0.5 m (d) 2.0 m

IES-32. Ans. (b) $C_{v1} = \sqrt{\frac{x^2}{4HY}}$

$$\therefore \frac{x^2}{4 \times 0.5 \times (h + 1.5)} = \frac{x^2}{4 \times 2 \times h}$$

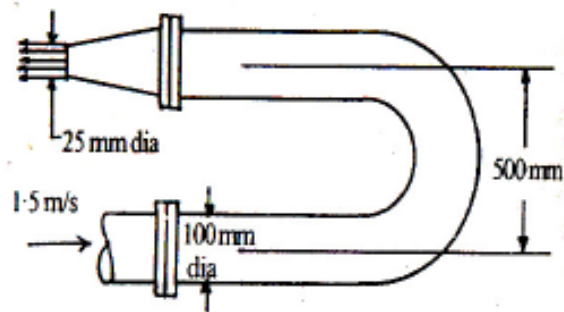
$$\therefore h = 0.5 \text{ m}$$

$$\text{Total height} = 0.5 + 1.5 + 0.5 = 2.5 \text{ m}$$



IES-33. The elbow nozzle assembly shown in the given figure is in a horizontal plane. The velocity of jet issuing from the nozzle is:

- (a) 4 m/s (b) 16 m/s
(c) 24 m/s (d) 30 m/s



[IES-1999]

IES-33. Ans. (c)

Impulse Momentum Equation

IES-34. Which one of the following conditions will linearize the Navier-Stokes equations to make it amenable for analytical solutions? [IES-2007]

- (a) Low Reynolds number ($Re \ll 1$) (b) High Reynolds number ($Re \gg 1$)
(c) Low Mach number ($M \ll 1$) (d) High Mach number ($M \gg 1$)

IES-34. Ans. (c)

Forced Vortex

IES-35. Which one of the statements is correct for a forced vortex? [IES-2009]

- (a) Turns in an opposite direction to a free vortex
(b) Always occurs in conjunction with a free vortex
(c) Has the linear velocity directly proportional to the radius
(d) Has the linear velocity inversely proportional to the radius

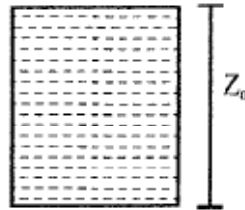
IES-35. Ans. (c) Forced vortex flow: Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency. Where $(v) = \omega \times r$ As ω is constant linear velocity (v) is directly proportional to the radius (r).

IES-36. A right circular cylinder, open at the top is filled with liquid of relative density 1.2. It is rotated about its vertical axis at such a speed that half liquid spills out. The pressure at the centre of the bottom will be:

- (a) Zero
(b) One-fourth of the value when the cylinder was full
(c) Half of the value when the cylinder was full
(d) Not determinable from the given data

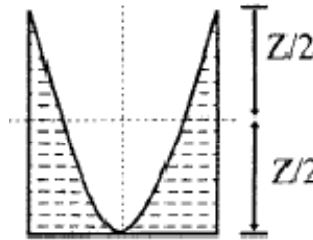
[IES-1998]

IES-36. Ans. (a) When cylinder is rotated such that half of the liquid spills out. Then liquid left in cylinder at height $\frac{z}{2}$,



initial condition

and liquid will rise at the wall of the cylinder by the same amount as it falls at the centre from its original level at rest.



Final condition

IES-37. Assertion (A) : A cylinder, partly filled with a liquid is rotated about its vertical axis. The rise of liquid level at the ends is equal to the fall of liquid level at the axis. [IES-1999]

Reason (R) : Rotation creates forced vortex motion.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-37. Ans. (b)

IAS-38. For a real fluid moving with uniform velocity, the pressure [IES-1993]

- (a) Depends upon depth and orientation
- (b) Is independent of depth but depends upon orientation
- (c) Is independent of orientation but depends upon depth
- (d) Is independent of both depth and orientation

IAS-38. Ans. (d) In case of a real fluid moving with uniform velocity, the velocity head and pressure head are dependent on each other and their total sum remains constant. The pressure is thus independent of both depth and orientation, but in case of fluids under static condition, the pressure would depend on depth.

IES-39. A right circular cylinder is filled with a liquid upto its top level. It is rotated about its vertical axis at such a speed that half the liquid spills out then the pressure at the point of intersection of the axis and bottom surface is: [IES-2001]

- (a) Same as before rotation
- (b) Half of the value before rotation
- (c) Quarter of the value before rotation
- (d) Equal to the atmospheric pressure

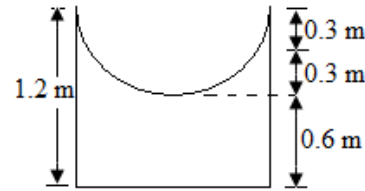
IES-39 Ans. (d)

IES-40. An open circular cylinder 1.2 m high is filled with a liquid to its top. The liquid is given a rigid body rotation about the axis of the cylinder and the pressure at the centre line at the bottom surface is found to be 0.6 m of liquid. What is the ratio of Volume of liquid spilled out of the cylinder to the original volume? [IES-2007]

- (a) 1/4
- (b) 3/8
- (c) 1/2
- (d) 3/4

IES-40. Ans. (a) $\frac{\text{Volume of paraboloid}}{\text{Total volume}}$

$$= \frac{(1/2) \times A \times 0.6}{A \times 1.2} = \frac{1}{4}$$



Free Vortex

IES-41. Both free vortex and forced vortex can be expressed mathematically as functions of tangential velocity V at the corresponding radius r . Free vortex and forced vortex are definable through V and r as [IES-1993]

Free vortex	Forced vortex
(a) $V = r \times \text{const.}$	$Vr = \text{const.}$
(b) $V \times r = \text{const.}$	$V^2 = r \times \text{const.}$
(c) $V \times r = \text{const.}$	$V = r \times \text{const.}$
(d) $V^2 \times r = \text{const.}$	$V = r \times \text{const.}$

IES-41. Ans. (c) Free vortex can be expressed mathematically as $V \times r = \text{constant}$ and the forced vortex as $V = r \times \text{constant}$.

IES-42. An incompressible fluid flows radially outward from a line source in a steady manner. How does the velocity in any radial direction vary? [IES-2008]

- (a) r (b) r^2 (c) $1/r^2$ (d) $1/r$

IES-42. Ans. (d) For an incompressible fluid flow radially outward from a line source in a steady manner. Angular momentum is conserved.

$$\Rightarrow mvr = \text{constant} \quad \Rightarrow vr = \text{constant} \quad \Rightarrow v = \frac{\text{constant}}{r}$$

$$\therefore v \propto \frac{1}{r}$$

IES-43. In a cylindrical vortex motion about a vertical axis, r_1 and r_2 are the radial distances of two points on the horizontal plane ($r_2 > r_1$). If for a given tangential fluid velocity at r_1 , the pressure difference between the points in free vortex is one-half of that when the vortex is a forced one, then what is the value of the ratio (r_2/r_1)? [IES-2007]

- (a) $\sqrt{3/2}$ (b) $\sqrt{2}$ (c) $3/2$ (d) $\sqrt{3}$

IES-43. Ans. (b) For free vortex, $\omega r_1 = \text{const.}(k)$

For forced vortex, $V_1 = \text{const.}(k) = \frac{c}{r_1}$ Or $c = \omega r_1^2$

$$(\Delta P)_{\text{forced}} = \frac{\rho \omega^2}{2} [r_2^2 - r_1^2], \quad (\Delta P)_{\text{free}} = \frac{\rho c^2}{2} \left[\frac{1}{r_1^2} - \frac{1}{r_2^2} \right] \quad \because c = \omega r_1^2$$

$$2 (\Delta P)_{\text{free}} = (\Delta P)_{\text{forced}} \quad \text{Or} \quad \frac{r_2}{r_1} = \sqrt{2}$$

IES-44. An inviscid, irrotational flow field of free vortex motion has a circulation constant Ω . The tangential velocity at any point in the flow field is given by Ω/r , where, r is the radial distance from the centre. At the centre, there is a mathematical singularity which can be physically

substituted by a forced vortex. At the interface of the free and forced vortex motion ($r = r_c$), the angular velocity ω is given by: [IES-1997]

- (a) $\Omega / (r_c)^2$ (b) Ω / r_c (c) Ωr_c (d) Ωr_c^2

IES-44. Ans. (a) Free vortex,

$$V_r = \text{constant} = \Omega \text{ (given)}$$

$$V = \frac{\Omega}{r}$$

Forced vortex,

$$V = r\omega$$

$$\omega = \frac{V}{r}$$

$$r = r_c$$

$$V = \frac{\Omega}{r_c} \quad \text{(Free vortex)}$$

$$\omega = \frac{V}{r_c} \quad \text{(Forced vortex)}$$

$$\omega = \frac{\left(\frac{\Omega}{r_c}\right)}{r_c} \Rightarrow \omega = \frac{\Omega}{r_c^2}$$

Previous 20-Years IAS Questions

Bernoulli's Equation

IAS-1. The Bernoulli's equation refers to conservation of [IAS-2003]

- (a) Mass (b) Linear momentum
(c) Angular momentum (d) Energy

IAS-1. Ans. (d)

IAS-2. Bernoulli's equation $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$, is applicable to for

- (a) Steady, frictionless and incompressible flow along a streamline [IAS-1999]
(b) Uniform and frictionless flow along a streamline when p is a function of p
(c) Steady and frictionless flow along a streamline when p is a function of p
(d) Steady, uniform and incompressible flow along a streamline

IAS-2. Ans. (a)

IAS-3. Bernoulli's theorem $\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{constant}$ is valid [IAS-1996]

- (a) Along different streamlines in rotational flow
(b) Along different streamlines in irrotational flow
(c) Only in the case of flow of gas
(d) Only in the case of flow of liquid

IAS-3. Ans. (a)

Venturimeter

IAS-4. Fluid flow rate Q, can be measured easily with the help of a venturi tube, in which the difference of two pressures, ΔP , measured at an

upstream point and at the smallest cross-section and at the smallest cross-section of the tube, is used. If a relation $\Delta P \propto Q^n$ exists, then n is equal to: [IAS-2001]

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

IAS-4. Ans. (d) $Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \therefore Q^2 \propto \Delta h$ or $Q^2 \propto \Delta \rho$

IAS-5. Two venturimeters of different area ratios are connected at different locations of a pipeline to measure discharge. Similar manometers are used across the two venturimeters to register the head differences. The first venturimeter of area ratio 2 registers a head difference 'h', while the second venturimeter registers '5h'. The area ratio for the second venturimeter is: [IAS-1999]

- (a) 3 (b) 4 (c) 5 (d) 6

IAS-5. Ans. (b) $Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{C_d A_1 A_2' \sqrt{2g5h}}{\sqrt{A_1^2 - A_2'^2}}$ $A_1 = 2A_2$ and $A_2 = (A_1/2)$

That gives $\frac{A_1}{A_2} = 4$

Orifice Meter

IAS-6. If a fluid jet discharging from a 50 mm diameter orifice has a 40 mm diameter at its vena contracta, then its coefficient of contraction will be: [IAS-1996]

- (a) 0.32 (b) 0.64 (c) 0.96 (d) 1.64

IAS-6. Ans. (b)

IAS-7. What is the percentage error in the estimation of the discharge due to an error of 2% in the measurement of the reading of a differential manometer connected to an orifice meter? [IAS-2004]

- (a) 4 (b) 3 (c) 2 (d) 1

IAS-7. Ans. (d) $Q = \frac{C_d A_2 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = const. \times \sqrt{h}$

or, $\ln Q = \ln(const.) + \frac{1}{2} \ln h$

or, $\frac{dQ}{Q} = 2 \frac{dh}{h} = \frac{1}{2} \times 2 = 1\%$

Pitot Tube

IAS-8. A simple Pitot tube can be used to measure which of the following quantities? [IAS-1994]

1. Static head 2. Datum head 3. Dynamic head
4. Friction head 5. Total head

Select the correct answer using the codes given below

Codes:

- (a) 1, 2 and 4 (b) 1,3 and 5 (c) 2, 3 and 4 (d) 2, 3 and 5

IAS-8. Ans. (b)

IAS-9. An instrument which offers no obstruction to the flow, offers no additional loss and is suitable for flow rate measurement is: [IAS-1997]
 (a) Venturimeter (b) Rotameter (c) Magnetic flow meter (d) Bend meter

IAS-9. Ans. (c)

IAS-10. The following instruments are used in the measurement of discharge through a pipe: [IAS-1996]

1. Orifice meter 2. Flow nozzle 3. Venturimeter

Decreasing order of use:

- (a) 1, 3, 2 (b) 1, 2, 3 (c) 3, 2, 1 (d) 2, 3, 1

IAS-10. Ans. (c)

IAS-11. Match List-I with List-II and select the correct answer: [IAS-2000]

List-I

- A. Orifice meter
 B. Broad crested weir
 C. Pitot tube

List-II

1. Measurement of flow in a channel
 2. Measurement of velocity in a pipe/ channel
 3. Measurement of flow in a pipe of any inclination
 4. Measurement of upward flow in a vertical pipe

D. Rotameter

Codes: A B C D A B C D

(a) 3 1 4 2 (b) 1 3 2 4

(c) 3 1 2 4 (d) 1 3 4 2

IAS-11. Ans. (c)

IAS-12. Assertion (A): In a rotameter the fluid flows from the bottom of the conical rotameter tube with divergence in the upward direction and the position of the metering float indicated the discharge. [IAS-1996]

Reason (R): Rotameter float indicates the discharge in terms of its rotation.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IAS-12. Ans. (c)

Free Liquid Jet

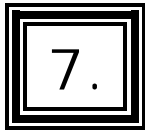
IAS-13. A liquid jet issues from a nozzle inclined at an angle of 60° to the horizontal and is directed upwards. If the velocity of the jet at the nozzle is 18m/s, what shall approximately be the maximum vertical distance attained by the jet from the point of exit of the nozzle? [IAS-2004]

- (a) 4.2 m (b) 12.4 m (c) 14.3m (d) 16.5m

IAS-13. Ans. (b) $H = u \sin \theta \times t - \frac{1}{2} g t^2$

$$\frac{dH}{dt} = u \sin \theta - gt \quad \text{or } t = \frac{u \sin \theta}{g}$$

$$\therefore H_{\max} = u \sin \theta \times \frac{u \sin \theta}{g} - \frac{1}{2} g \times \left(\frac{u^2 \sin^2 \theta}{2g} \right) = \frac{18^2 \sin^2 60}{2 \times 9.8} = 12.4m$$



7. Dimensional & Model Analysis

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-

Question: *Discuss the importance of Dimensional Analysis.* [IES-2003]

- Answer:**
1. Dimensional Analysis help in determining a systematic arrangement of the variable in the physical relationship, combining dimensional variable to form meaningful non- dimensional parameters.
 2. It is especially useful in presenting experimental results in a concise form.
 3. Dimensional Analysis provides partial solutions to the problems that are too complex to be dealt with mathematically.
 4. Design curves can be developed from experimental data.

Question: *Explain clearly Buckingham's π -theorem method and Rayleigh's method of dimensional Analysis.* [IES-2003]

Answer: **Buckingham's π - theorem:** statement "If there are n variable (dependent and independent) in a dimensionally homogeneous equation and if these

variable contain m fundamental dimensions, then the variables are arranged into (n-m) dimensionless terms.”

These dimensionless terms are called π -terms.

Mathematically, if any variables x_1 , depends on independent variables, x_2, x_3, \dots, x_n , the functional eqⁿ may be written as

$$x_1 = f(x_2, x_3, \dots, x_n)$$

or $F(x_1, x_2, \dots, x_n) = 0$

It is a dimensionally homogeneous eqⁿ and contains ‘n’ variables. If there are m fundamental dimensions, then According to Buckingham’s π -theorem it can be written in (n-m) numbers of π -terms (dimensionless groups)

$$\therefore F(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0$$

Each dimensionless π -term is formed by combining m variables out of the total n variable with one of the remaining (n-m) variables. i.e. each π -term contain (m+1) variables.

These m variables which appear repeatedly in each of π -term are consequently called repeating variables and are chosen from among the variable such that they together involve all the fundamental dimensions and they themselves do not form a dimensionless parameter.

Let x_2, x_3 and x_4 are repeating variables then

$$\pi_1 = x_2^{a_1} x_3^{b_1} x_4^{c_1} \cdot x_1$$

$$\pi_2 = x_2^{a_2} x_3^{b_2} x_4^{c_2} \cdot x_5$$

$$\pi_{n-m} = x_2^{a_{n-m}} x_3^{b_{n-m}} x_4^{c_{n-m}} \cdot x_n$$

Where $a, b, c; a_2, b_2, c_2$ etc are constant and can be determined by considering dimensional homogeneity.

Rayleigh’s Method: This method gives a special form of relationship among the dimensional group. In this method a functional relationship of some variables is expressed in the form of an exponential equation which must be dimensionally homogeneous.

Thus, if x is a dependent variable which depends $x_1, x_2, x_3, \dots, x_n$ The functional equation can be written as:

$$x = f(x_1, x_2, x_3, \dots, x_n)$$

or $x = c x_1^a x_2^b x_3^c \dots x_n^n$

Where c is a non-dimensional const and a, b, c, ..., n, are the arbitrary powers. The value of a, b, c, ..., n, are obtained by comparing the power of the fundamental dimensions on both sides.

Question: *What is meant by similitude? Discuss.* **[AMIE-(Winter)-2002]**

Answer: Similitude is the relationship between model and a prototype. Following three similarities must be ensured between the model and the prototype.

1. Geometric similarity
2. Kinematic similarity, and
3. Dynamic similarity.

Question: *What is meant by Geometric, kinematic and dynamic similarity?*

Are these similarities truly attainable? If not why ?

[AMIE- summer-99]

Answer: **Geometric similarity:** For geometric similarity the ratios of corresponding length in the model and in the prototype must be same and the include angles between two corresponding sides must be the same.

Kinematic similarity: kinematic similarity is the similarity of motion. It demands that the direction of velocity and acceleration at corresponding points in the two flows should be the same.

For kinematic similarity we must have; $\frac{(v_1)_m}{(v_1)_P} = \frac{(v_2)_m}{(v_2)_P} = v_v$, velocity ratio

and $\frac{(a_1)_m}{(a_1)_P} = \frac{(a_2)_m}{(a_2)_P} = a_v$, acceleration ratio.

Dynamic similarity: Dynamic similarity is the similarity of forces at the corresponding points in the flows.

Then for dynamic similarity, we must have

$$\frac{(F_{inertia})_m}{(F_{inertia})_P} = \frac{(F_{viscous})_m}{(F_{viscous})_P} = \frac{(F_{gravity})_m}{(F_{gravity})_P} = F_\gamma, \text{ Force ratio}$$

Note: Geometric, kinematic and dynamic similarity are mutually independent. Existence of one does not imply the existence of another similarity.

No these similarity are not truly attainable.

Because:

- (a) The geometric similarity is complete when surface roughness profiles are also in the scale ratio. Since it is not possible to prepare a 1/20 scale model to 20 times better surface finish, so complete geometric similarity can not be achieved.
- (b) The kinematic similarity is more difficult because the flow patterns around small objects tend to be quantitatively different from those around large objects.
- (c) Complete dynamic similarity is practically impossible. Because Reynold's number and Froude number cannot be equated simultaneously.

Question:

- (i) Define (IES-03)
- (ii) Significance (IES-02)
- (iii) Area of application (IES-01)
of Reynolds number; Froude number; Mach number; Weber number; Euler number.

Answer:

Sl. No.	Dimension-less no.	Aspects			
		Symbol	Group of variable	Significance	Field of application
1.	Reynolds number	R_e	$\frac{\rho vL}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	Laminar viscous flow in confined passages (where viscous effects are significant).

2.	Froude's number	F_r	$\frac{v}{\sqrt{Lg}}$	$\sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}}$	Free surface flows (where gravity effects are important)
3.	Euler's Number	E_u	$\frac{v}{\sqrt{P/\rho}}$	$\sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$	Conduit flow (where pressure variation are significant)
4.	Weber's Number	W_e	$\frac{\rho v^2 L}{\sigma}$	$\frac{\text{Inertia force}}{\text{Surface tension force}}$	Small surface waves, capillary and sheet flow (where surface tension is important)
5.	Mach's number	M	$\frac{v}{\sqrt{k/\rho}}$	$\sqrt{\frac{\text{Inertia force}}{\text{elastic force}}}$	High speed flow (where compressibility effects are important)

Forces

- (i) **Inertia force** = $\rho A v^2 = \rho L^2 v^2$
- (ii) **Viscous force** = $\mu \frac{du}{dv} A = \mu \frac{v}{L} A = \mu \frac{v}{L} L^2 = \mu v L$
- (iii) **Gravity force** = $\rho \times \text{volume} \times g = \rho L^3 g$
- (iv) **Pressure force** = $PA = PL^2$
- (v) **Surface tension force** = σL
- (vi) **Elastic force** = $k A = kL^2$

(a) Reynolds Model Law

- (i) Motion of air planes,
- (ii) Flow of incompressible fluid in closed pipes,
- (iii) Motion of submarines completely under water, and
- (iv) Flow around structures and other bodies immersed completely under moving fluids.

(b) Froude Model Law

- (i) Free surface flows such as flow over spillways, sluices etc.
- (ii) Flow of jet from an orifice or nozzle.
- (iii) Where waves are likely to be formed on the surface
- (iv) Where fluids of different mass densities flow over one another.

$$V_r = T_r = \sqrt{L_r} \quad \text{And} \quad Q_r = L_r^{2.5}; \quad F_r = L_r^3$$

(c) Weber Model Law

Weber model law is applied in the following flow situations:

- (i) Flow over weirs involving very low heads;
- (ii) Very thin sheet of liquid flowing over a surface;
- (iii) Capillary waves in channels;
- (iv) Capillary rise in narrow passages;
- (v) Capillary movement of water in soil.

(d) Mach Model Law

The similitude based on Mach model law finds application in the following:

- (i) Aerodynamic testing;
- (ii) Phenomena involving velocities exceeding the speed of sound;

- (iii) Hydraulic model testing for the cases of unsteady flow, especially water hammer problems.
- (iv) Under-water testing of torpedoes.

(e) Euler Model Law

- (i) Enclosed fluid system where the turbulence is fully developed so that viscous forces are negligible and also the forces of gravity and surface tensions are entirely absent;
- (ii) Where the phenomenon of cavitation occurs.

Question: *Considering Froude number as the criterion of dynamic similarity for a certain flow situation, work out the scale factor of velocity, time, discharge, acceleration, force, work, and power in terms of scale factor for length.* [IES-2003]

Answer: Let, V_m = velocity of fluid in model.
 L_m = Length (or linear dimension) of the model.
 g_m = acceleration due to gravity (at a place where model is tested, and V_p, L_p and g_p are the corresponding value of prototype.

According to Froude model law

$$(F_r)_m = (F_r)_p$$

$$\text{or } \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

As site of model and prototype is same the $g_m = g_p$

$$\therefore \frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\text{or } \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$\therefore \text{(i) Velocity ratio, } V_r = (L_r)^{1/2}$$

$$\text{(ii) Time scale ratio, } T_r = \frac{T_p}{T_m} = \frac{L_p / V_p}{L_m / V_m} = L_r \cdot (L_r)^{-1/2} = (L_r)^{-1/2} \quad [\because V_r = L_r^{1/2}]$$

(iii) Discharge scale ratio, Q_r

$$\text{Discharge (Q)} = AV = L^2 \cdot \frac{L}{T} = \frac{L^3}{T}$$

$$\therefore Q_r = \frac{Q_p}{Q_m} = \frac{L_p^3 / T_p}{L_m^3 / T_m} = \left(\frac{L_p}{L_m} \right)^3 \times \frac{1}{\left(\frac{T_p}{T_m} \right)} = \frac{L_r^3}{L_r^{1/2}} = L_r^{2.5}$$

(iv) Acceleration ratio (a_r)

$$\text{Acceleration, } a = \frac{V}{t}$$

$$\therefore a_r = \frac{a_p}{a_m} = \frac{V_p / T_p}{V_m / T_m} = \frac{V_p}{V_m} \times \frac{1}{\left(\frac{T_p}{T_m} \right)} = L_r^{1/2} \times \frac{1}{L_r^{1/2}} = 1$$

(v) For scale ratio (F_r)

$$\text{Force, } F = \text{mass} \times \text{acceleration} = \rho L^3 \times \frac{V}{T}$$

$$\therefore F_r = \frac{F_p}{F_m} = \frac{\rho_p L_p^3 V_p / T_p}{\rho_m L_m^3 V_m / T_m} = (L_r)^3 \times L_r^{1/2} \times \frac{1}{L_r^{1/2}} = (L_r)^3$$

(vi) Work done scale ratio or energy scale ratio (E_r)

$$\text{Energy, } E = \frac{1}{2} m V^2 = \frac{1}{2} \rho L^3 V^2$$

$$\therefore E_r = \frac{E_p}{E_m} = \frac{\frac{1}{2} \rho_p L_p^3 V_p^2}{\frac{1}{2} \rho_m L_m^3 V_m^2} = (L_r)^3 \times (L_r^{1/2})^2 = L_r^4$$

(vii) Power scale ratio (P_r)

$$\text{Power} = \frac{1}{2} \frac{m V^2}{T} = \frac{1}{2} \rho \frac{L^3 V^2}{T}$$

$$\begin{aligned} \therefore \text{Power Ratio, } (P_r) &= \frac{P_p}{P_m} = \frac{\frac{1}{2} \rho_p L_p^3 V_p^2 / T_p}{\frac{1}{2} \rho_m L_m^3 V_m^2 / T_m} \\ &= \left(\frac{L_p}{L_m}\right)^3 \left(\frac{V_p}{V_m}\right)^2 \times \frac{1}{\left(\frac{T_p}{T_m}\right)} = L_r^3 (L_r^{1/2})^2 \times \frac{1}{(L_r)^{1/2}} = L_r^{3.5} \end{aligned}$$

Question: Obtain an expression for the length scale of a model, which has to satisfy both Froude's model law and Reynold's model law.

[AMIE (winter) 2000]

Answer: (i) Applying Reynold's model law

$$\left(\frac{\rho V L}{\mu}\right)_p = \left(\frac{\rho V L}{\mu}\right)_m$$

$$\text{or } \frac{V_p}{V_m} = \left(\frac{\rho_m \times \mu_p}{\rho_p \times \mu_m}\right) \times \left(\frac{L_m}{L_p}\right)$$

$$\text{or } \frac{V_p}{V_m} = C_1 \times \frac{L_m}{L_p} \quad \dots(i)$$

C_1 is unity for the same fluid used in the two flows, but it can be another constant in accordance with the properties of the two fluids.

Applying Froude's model law:

$$\left(\frac{V}{\sqrt{Lg}}\right)_p = \left(\frac{V}{\sqrt{Lg}}\right)_m$$

$$\text{or } \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} \times \sqrt{\frac{g_p}{g_m}} = \sqrt{\frac{L_p}{L_m}} \quad \dots(ii) \quad [\because g_p = g_m \text{ at same site}]$$

Conditions (i) and (ii) are entirely different showing that the Reynolds number and the Froude's number cannot be equated simultaneously. i.e. complete dynamic similarity is practically impossible.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Buckingham's π -method/theorem

GATE-1. If the number of fundamental dimensions equals 'm', then the repeating variables shall be equal to: [IES-1999, IES-1998, GATE-2002]

- (a) m and none of the repeating variables shall represent the dependent variable.
 (b) m + 1 and one of the repeating variables shall represent the dependent variable
 (c) m + 1 and none of the repeating variables shall represent the dependent variable.
 (d) m and one of the repeating variables shall represent the dependent variable.

GATE-1. Ans. (c)

Reynolds Number (Re)

GATE-2. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)i$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is: [GATE-2007]

- (a) $\frac{L}{u_0}$ (b) $\frac{L}{3u_0} \ln 4$ (c) $\frac{L}{4u_0}$ (d) $\frac{L}{2.5u_0}$

GATE-2. Ans. (b) Velocity, $V = \frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = u_0 \left(1 + \frac{3x}{L}\right) \Rightarrow \frac{dx}{\left(1 + \frac{3x}{L}\right)} = u_0 dt$$

Integrating both side, we get

$$u_0 \int_0^t dt = \int_0^L \frac{dx}{\left(1 + \frac{3x}{L}\right)}$$

$$u_0 t = \frac{L}{3} \left[\ln \left(1 + \frac{3x}{L}\right) \right]_0^L$$

$$u_0 t = \frac{L}{3} \ln 4$$

$$t = \frac{L}{3u_0} \ln 4$$

GATE-3. The Reynolds number for flow of a certain fluid in a circular tube is specified as 2500. What will be the Reynolds number when the tube diameter is increased by 20% and the fluid velocity is decreased by 40% keeping fluid the same? [GATE-1997]

- (a) 1200 (b) 1800 (c) 3600 (d) 200

GATE-3. Ans. (b) $R_e = \frac{\rho VD}{\mu}$

$$R_{e2} = \frac{\rho VD}{\mu} = \frac{\rho(0.6V)(1.2D)}{\mu} = 0.6 \times 1.2 \times 2500 = 1800$$

Froude Number (*Fr*)

GATE-4. The square root of the ratio of inertia force to gravity force is called
[GATE-1994, IAS-2003]

- (a) Reynolds number (b) Froude number
(c) Mach number (d) Euler number

GATE-4. Ans. (b)

Mach Number (*M*)

GATE-5. An aeroplane is cruising at a speed of 800 kmph at altitude, where the air temperature is 0° C. The flight Mach number at this speed is nearly
[GATE-1999]

- (a) 1.5 (b) 0.254 (c) 0.67 (d) 2.04

GATE-5. Ans. (c)

GATE-6. In flow through a pipe, the transition from laminar to turbulent flow does not depend on
[GATE-1996]

- (a) Velocity of the fluid (b) Density of the fluid
(c) Diameter of the pipe (d) Length of the pipe

GATE-6. Ans. (d) $R_e = \frac{\rho VD}{\mu}$

GATE-7. List-I

- (A) Fourier number
(B) Weber number
(C) Grashoff number
(D) Schmidt number

List-II

1. Surface tension
2. Forced convection
3. Natural convection
4. Radiation
5. Transient heat conduction
6. Mass diffusion

[GATE-1996]

Codes:	A	B	C	D	A	B	C	D	
(a)	1	2	6	4	(b)	4	5	2	1
(c)	5	1	3	6	(d)	4	2	3	1

GATE-7. Ans. (c)

Previous 20-Years IES Questions

Dimensions

IES-1. The dimensionless group formed by wavelength λ , density of fluid ρ , acceleration due to gravity g and surface tension σ , is: [IES-2000]

- (a) $\sigma / \lambda^2 g \rho$ (b) $\sigma / \lambda g^2 \rho$ (c) $\sigma g / \lambda^2 \rho$ (d) $\rho / \lambda g \sigma$

IES-1. Ans. (a)

IES-2. Match List-I (Fluid parameters) with List-II (Basic dimensions) and select the correct answer: [IES-2002]

List-I	List-II
A. Dynamic viscosity	1. M / t^2
B. Chezy's roughness coefficient	2. $M / L t^2$
C. Bulk modulus of elasticity	3. $M / L t$
D. Surface tension (σ)	4. $\sqrt{L/t}$
Codes:	
(a)	(b)
(c)	(d)

	A	B	C	D
(a)	3	2	4	1
(c)	3	4	2	1

IES-2. Ans. (c)

IES-3. In M-L-T system. What is the dimension of specific speed for a rotodynamic pump? [IES-2006]

- (a) $L^{\frac{-3}{4}} T^{\frac{3}{2}}$ (b) $M^{\frac{1}{2}} L^{\frac{1}{4}} T^{\frac{-5}{2}}$ (c) $L^{\frac{3}{4}} T^{\frac{-3}{2}}$ (d) $L^{\frac{3}{4}} T^{\frac{3}{2}}$

IES-3. Ans. (c)

IES-4. A dimensionless group formed with the variables ρ (density), ω (angular velocity), μ (dynamic viscosity) and D (characteristic diameter) is: [IES-1995]

- (a) $\rho\omega\mu / D^2$ (b) $\rho\omega D^2 / \mu$ (c) $\rho\omega\mu D^2$ (d) $\rho\omega\mu D$

IES-4. Ans. (b) Let $\phi = \rho^a D^b \mu^c \omega$

$$M^0 L^0 T^0 = [ML^{-3}]^a [L]^b [ML^{-1}T^{-1}]^c [T^{-1}]$$

$$a + c = 0 \quad (1)$$

$$-3a + b - c = 0 \quad (2)$$

$$-c - 1 = 0 \quad (3)$$

Hence, $a = 1$, $b = 2$, and $c = -1$

$$\therefore \phi = \frac{\rho\omega D^2}{\mu}$$

Alternate solution: check the dimensions individually.

IES-5. Which of the following is not a dimensionless group? [IES-1992]

- (a) $\frac{\Delta p}{\rho N^2 D^2}$ (b) $\frac{gH}{N^2 D^2}$ (c) $\frac{\rho\omega D^2}{\mu}$ (d) $\frac{\Delta p}{\rho V^3}$

IES-5. Ans. (d) $\frac{\Delta p}{\rho V^3} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}][LT^{-1}]^3} = L^{-1}T$, hence dimensionless.

IES-6. What is the correct dimensionless group formed with the variable ρ -density, N -rotational speed, d -diameter and π coefficient of viscosity?

(a) $\frac{\rho N d^2}{\pi}$ (b) $\frac{\rho N d}{\pi}$ [IES-2009]

(c) $\frac{Nd}{\rho\pi}$ (d) $\frac{Nd^2}{\rho\pi}$

IES-6. Ans. (a)

IES-7. Match List-I (Fluid parameters) with List-II (Basic dimensions) and select the correct answer: [IES-2002]

List-I

List-II

A. Dynamic viscosity	1. M/t^2
B. Chezy's roughness coefficient	2. M/Lt^2
C. Bulk modulus of elasticity	3. M/Lt
D. Surface tension (σ)	4. $\sqrt{L/t}$
Codes:	A B C D
(a)	3 2 4 1
(c)	3 4 2 1
(b)	1 4 2 3
(d)	1 2 4 3

IES-7. Ans. (c)

Rayleigh's Method

IES-8. Given power 'P' of a pump, the head 'H' and the discharge 'Q' and the specific weight 'w' of the liquid, dimensional analysis would lead to the result that 'P' is proportional to: [IES-1998]

- (a) $H^{1/2} Q^2 w$ (b) $H^{1/2} Q w$ (c) $H Q^{1/2} w$ (d) $HQ w$

IES-8. Ans. (d)

IES-9. Volumetric flow rate Q, acceleration due to gravity g and head H form a dimensionless group, which is given by: [IES-2002]

- (a) $\sqrt{\frac{gH^5}{Q}}$ (b) $\frac{Q}{\sqrt{gH^5}}$ (c) $\frac{Q}{\sqrt{gH^3}}$ (d) $\frac{Q}{\sqrt{g^2H}}$

IES-9. Ans. (b)

Buckingham's π-method/theorem

IES-10. If the number of fundamental dimensions equals 'm', then the repeating variables shall be equal to: [IES-1999, IES-1998, GATE-2002]

- (a) m and none of the repeating variables shall represent the dependent variable.
 (b) m + 1 and one of the repeating variables shall represent the dependent variable
 (c) m + 1 and none of the repeating variables shall represent the dependent variable.
 (d) m and one of the repeating variables shall represent the dependent variable.

IES-10. Ans. (c)

IES-11. The time period of a simple pendulum depends on its effective length I and the local acceleration due to gravity g. What is the number of dimensionless parameter involved? [IES-2009]

- (a) Two (b) One (c) Three (d) Zero

IES-11. Ans. (b) $m = 3$ (time period, length and acceleration due to gravity); $n = 2$ (length and time). Then the number of dimensionless parameter = $m - n$.

IES-12. In a fluid machine, the relevant parameters are volume flow rate, density, viscosity, bulk modulus, pressure difference, power consumption, rotational speed and characteristic dimension. Using the Buckingham pi (π) theorem, what would be the number of independent non-dimensional groups? [IES-1993, 2007]

- (a) 3 (b) 4 (c) 5 (d) None of the above

IES-12. Ans. (c) No of variable = 8

No of independent dimension (m) = 3

∴ No of π term = $n - m = 8 - 3 = 5$

IES-13. The variable controlling the motion of a floating vessel through water are the drag force F , the speed v , the length l , the density ρ , dynamic viscosity μ of water and gravitational constant g . If the non-dimensional groups are Reynolds number (Re), Weber number (We), Prandtl number (Pr) and Froude number (Fr), the expression for F is given by: [IES-1997]

- | | |
|--|--|
| (a) $\frac{F}{\rho v^2 l^2} = f(Re)$ | (b) $\frac{F}{\rho v^2 l^2} = f(Re, Pr)$ |
| (c) $\frac{F}{\rho v^2 l^2} = f(Re, We)$ | (d) $\frac{F}{\rho v^2 l^2} = f(Re, Fr)$ |

IES-13. Ans. (d) To solve this problem we have to use Buckingham's π -Theory.

IES-14. Consider the following statements: [IES-2003]

1. Dimensional analysis is used to determine the number of variables involved in a certain phenomenon
2. The group of repeating variables in dimensional analysis should include all the fundamental units.
3. Buckingham's π theorem stipulates the number of dimensionless groups for a given phenomenon.
4. The coefficient in Chezy's equation has no dimension.

Which of these are correct?

- (a) 1, 2, 3 and 4 (b) 2, 3 and 4 (c) 1 and 4 (d) 2 and 3

IES-14. Ans. (d) 1 and 4 are wrong, coefficient in Chezy's equation has dimension $[L^{1/2}T^{-1}]$

Reynolds Number (Re)

IES-15. Match List-I with List-II and select the correct: [IES-1996]

- | List-I | List-II |
|--------------------|---|
| A. Reynolds Number | 1. Film coefficient, pipe diameter, thermal conductivity |
| B. Prandtl Number | 2. Flow velocity, acoustic velocity |
| C. Nusselt Number | 3. Heat capacity, dynamic viscosity, thermal conductivity |
| D. Mach Number | 4. Flow velocity, pipe diameter, kinematic viscosity |

Code:	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	4	3	1	2
(c)	2	3	1	4	(d)	2	1	3	4

IES-15. Ans. (b) As. $Re = \frac{\rho V l}{\mu}$ $Pr = \frac{\mu C_p}{k}$ $N_u = \frac{h l}{k}$ $M = \frac{V}{V_a}$

Euler Number (Eu)

IES-16. Euler number is defined as the ratio of inertia force to: [IES-1997]

- | | |
|--------------------|-------------------|
| (a) Viscous force | (b) Elastic force |
| (c) Pressure force | (d) Gravity force |

IES-16. Ans. (c) Euler number

$$Eu = \left(\frac{\text{Inertia force}}{\text{Pressure force}} \right)^{1/2} = \frac{V}{\sqrt{P/\rho}}$$

Weber number

$$W = \left(\frac{\text{Inertia force}}{\text{Surface force}} \right)^{1/2} = \frac{V}{\sqrt{\sigma/\rho L}}$$

Mach number

$$M = \left(\frac{\text{Inertia force}}{\text{Elastic force}} \right)^{1/2} = \frac{V}{\sqrt{K/\rho}}$$

Mach Number (*M*)

IES-17. Match List-I (Dimensionless number) with List-II (Nature of forces involved) and select the correct answer using the code given below the lists: [IES-2008]

List-I

- A. Euler number
- B. Weber number
- C. Mach number
- D. Froude number

List-II

- 1. Surface tension
- 2. Gravity
- 3. Pressure
- 4. Elastic

Code:	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	4	1	2	3
(d)	4	2	1	3

IES-17. Ans. (a)

- | | | |
|----|-----------------|---|
| 1. | Reynolds number | $\frac{\text{Inertia force}}{\text{Viscous force}}$ |
| 2. | Froude's number | $\frac{\text{Inertia force}}{\text{Gravity force}}$ |
| 3. | Euler's number | $\frac{\text{Inertia force}}{\text{Pressure force}}$ |
| 4. | Weber's number | $\frac{\text{Inertia force}}{\text{Surface tension}}$ |
| 5. | Mach's number | $\frac{\text{Inertia force}}{\text{Elastic force}}$ |

IES-18. Match List-I (Dimensionless numbers) with List-II (Definition as the ratio of) and select the correct answer: [IES-2001]

List-I

- A. Reynolds number
- B. Froude number
- C. Weber number
- D. Mach number

List-II

- 1. Inertia force and elastic force
- 2. Inertia force and surface tension force
- 3. Inertia force and gravity force
- 4. Inertia force and viscous force

Codes:	A	B	C	D	A	B	C	D	
(a)	1	2	3	4	(b)	4	3	2	1
(c)	1	3	2	4	(d)	4	2	3	1

IES-18. Ans. (b)

IES-19. Which one of the dimensionless numbers identifies the compressibility effect of a fluid? [IES-2005]

- (a) Euler number (b) Froude number
(c) Mach number (d) Weber number

IES-19. Ans. (c)

IES-20. It is observed in a flow problem that total pressure, inertia and gravity forces are important. Then, similarly requires that [IES-2006]

- (a) Reynolds and Weber numbers be equal
(b) Mach and Froude numbers be equal
(c) Euler and Froude numbers be equal
(d) Reynolds and Mach numbers be equal

IES-20. Ans. (c)

IES-21. Match List-I (Flow/Wave) with List-II (Dimensionless number) and select the correct answer: [IES-2003]

List-I					List-II						
A. Capillary waves in channel					1. Reynolds number						
B. Testing of aerofoil					2. Froude number						
C. Flow around bridge piers					3. Weber number						
D. Turbulent flow through pipes					4. Euler number						
					5. Mach number						
Codes:	A	B	C	D							
(a)	5	4	3	2	(b)	3	5	4	1		
(c)	5	4	2	1	(d)	3	5	2	1		

IES-21. Ans. (d)

IES-22. Match List-I (Predominant force) with List-II (Dimensionless numbers) and select the correct answer [IES-1996]

List-I					List-II						
A. Compressibility force					1. Euler number						
B. Gravity force					2. Prandtl number						
C. Surface tension force					3. Mach number						
D. Viscous force					4. Reynolds number						
					5. Weber number						
Codes:	A	B	C	D							
(a)	1	2	3	4	(b)	3	2	5	4		
(c)	3	1	4	5	(d)	2	3	5	1		

IES-22. Ans. (b) When compressibility force is predominant, mach number is used; when gravity force predominates, Froude number is adopted. Similarly for surface tension force and viscous force, Weber number and Reynolds number are considered.

IES-23. Match List-I (Forces) with List-II (Dimensionless groups) and select the correct answer. [IES-1994]

List-I				List-II			
A. Viscous force				1. Reynolds number			
B. Elastic force				2. Froude number			
C. Surface tension				3. Waber number			

<p>D. Gravity</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">Codes:</td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(a)</td> <td>1</td> <td>4</td> <td>2</td> <td>3</td> </tr> <tr> <td>(c)</td> <td>3</td> <td>4</td> <td>1</td> <td>2</td> </tr> </table>	Codes:	A	B	C	D	(a)	1	4	2	3	(c)	3	4	1	2	<p>4. Mach number</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"></td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(b)</td> <td>1</td> <td>2</td> <td>4</td> <td>3</td> </tr> <tr> <td>(d)</td> <td>1</td> <td>4</td> <td>3</td> <td>2</td> </tr> </table>		A	B	C	D	(b)	1	2	4	3	(d)	1	4	3	2
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(d)	1	4	3	2																											

IES-23. Ans. (d)

IES-24. List-I gives 4 dimensionless numbers and List-II gives the types of forces which are one of the constituents describing the numbers. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1993]

<p>List-I</p> <p>A. Euler number B. Froude number C. Mach number D. Webber number</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">Codes:</td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(a)</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>(c)</td> <td>2</td> <td>1</td> <td>3</td> <td>4</td> </tr> </table>	Codes:	A	B	C	D	(a)	2	3	4	5	(c)	2	1	3	4	<p>List-II</p> <p>1. Pressure force 2. Gravity force 3. Viscous force 4. Surface tension 5. Elastic force</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"></td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(b)</td> <td>3</td> <td>2</td> <td>4</td> <td>5</td> </tr> <tr> <td>(d)</td> <td>1</td> <td>2</td> <td>5</td> <td>4</td> </tr> </table>		A	B	C	D	(b)	3	2	4	5	(d)	1	2	5	4
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IES-24. Ans. (d) Euler number is concerned with pressure force and this choice is available for A in code (d) only. If one is confident, then there is no need to look for items B, C and D. However a cross checks will show that Froude number is concerned with gravity force, Mach number with elastic force, and Weber number with surface tension. Hence the answer is (d) only.

IES-25. Match List-I (Type of Model) with List-II (Transference Ratio for Velocity) and select the correct answer: [IES-2004]

<p>List-I</p> <p>A. Reynolds model B. Froude model C. Weber model D. Mach model</p> <p>(Where symbols g, μ, ρ, σ and k have their usual meanings and subscript r refers to the ratio)</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">Codes:</td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(a)</td> <td>3</td> <td>1</td> <td>2</td> <td>4</td> </tr> <tr> <td>(c)</td> <td>2</td> <td>1</td> <td>3</td> <td>4</td> </tr> </table>	Codes:	A	B	C	D	(a)	3	1	2	4	(c)	2	1	3	4	<p>List-II</p> <p>1. $\sqrt{K_r / \rho_r}$ 2. $\sqrt{\sigma_r / (\rho_r l_r)}$ 3. $\mu_r / (\rho_r l_r)$ 4. $\sqrt{g_r l_r}$</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"></td> <td style="width: 10%;">A</td> <td style="width: 10%;">B</td> <td style="width: 10%;">C</td> <td style="width: 10%;">D</td> </tr> <tr> <td>(b)</td> <td>3</td> <td>4</td> <td>2</td> <td>1</td> </tr> <tr> <td>(d)</td> <td>2</td> <td>4</td> <td>3</td> <td>1</td> </tr> </table>		A	B	C	D	(b)	3	4	2	1	(d)	2	4	3	1
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(d)	2	4	3	1																											

IES-25. Ans. (b)

Model (or Similarity) Laws

IES-26. Consider the following statements: [IES-2005]

1. For achieving dynamic similarity in model studies on ships, Froude numbers are equated.

2. Reynolds number should be equated for studies on aerofoil for dynamic similarity.
3. In model studies on a spillway, the ratio of width to height is equated for kinematic similarity.

What of the statements given above are correct?

- (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-26. Ans. (d) Mach number should be equated for studies on aerofoil for dynamic similarity.

IES-27. Kinematic similarity between model and prototype is the similarity of **[IES-1996]**

- (a) Shape (b) Discharge (c) Stream line pattern (d) Forces

IES-27. Ans. (c) Kinematic similarity between a model and its prototype is said to exist if the flow patterns are in geometric i.e. velocity, acceleration etc are similar. Remember discharge is not related with kinematic similarity.

Reynolds Model Law

IES-28. Assertion (A): Reynolds number must be same for the model and prototype immersed in subsonic flows. **[IES-2003]**

Reason (R): Equality of Reynolds number for the model and prototype satisfies the dynamic similarity criteria.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-28. Ans. (b)

IES-29. A model test is to be conducted in a water tunnel using a 1: 20 model of a submarine, which is to travel at a speed of 12 km/h deep under sea surface. The water temperature in the tunnel is maintained, so that is kinematic viscosity is half that of sea water. At what speed is the model test to be conducted to produce useful data for the prototype?

[IES-2002]

- (a) 12 km/h (b) 240 km/h (c) 24 km/h (d) 120 km/h

IES-29. Ans. (d) Apply Reynolds Model law.

IES-30. A sphere is moving in water with a velocity of 1.6 m/s. Another sphere of twice the diameter is placed in a wind tunnel and tested with air which is 750 times less dense and 60 times less viscous than water. The velocity of air that will give dynamically similar conditions is:

[IES-1999]

- (a) 5 m/s (b) 10 m/s (c) 20 m/s (d) 40 m/s

IES-30. Ans. (b)

IES-31. The model of a propeller, 3 m in diameter, cruising at 10 m/s in air, is tested in a wind tunnel on a 1: 10 scale model. If a thrust of 50 N is measured on the model at 5 m/s wind speed, then the thrust on the prototype will be: **[IES-1995]**

- (a) 20,000 N (b) 2,000 N (c) 500 N (d) 200 N

IES-31. Ans. (a) Force ratio $= \frac{\rho_m}{\rho_p} \times \frac{L_m^2}{L_p^2} \times \frac{V_m^2}{V_p^2}$; $\frac{F_m}{F_p} = 1 \times \left(\frac{1}{10}\right)^2 \times \left(\frac{5}{10}\right)^2$
 or $\frac{50}{F_p} = \frac{1}{100} \times \frac{1}{4}$; $F_p = 50 \times 400 = 20000 \text{ N}$

Froude Model Law

IES-32. A 1.0 m log model of a ship is towed at a speed of 81 cm/s in a towing tank. To what speed of the ship, 64 m long does this correspond to?

[IES-2004]

- (a) 7.20 m/s (b) 6.48 m/s (c) 5.76 m/s (d) 3.60 m/s

IES-32. Ans. (b) Apply Froude Model law $(F_r)_m = (F_r)_p$ or $\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$

or $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \Rightarrow \frac{0.81}{V_p} = \sqrt{\frac{1}{64}}$

IES-33. A ship model 1/60 scale with negligible friction is tested in a towing tank at a speed of 0.6 m/s. If a force of 0.5 kg is required to tow the model, the propulsive force required to tow the prototype ship will be:

[IES-1999]

- (a) 5 MN (b) 3 MN (c) 1 MN (d) 0.5 MN

IES-33. Ans. (c)

IES-34. A1:256 scale model of a reservoir is drained in 4 minutes by opening the sluice gate. The time required to empty the prototype will be: [IES-1999]

- (a) 128 min (b) 64 min (c) 32 min (d) 25.4 min

IES-34. Ans. (b)

IES-35. A ship whose full length is 100 m is to travel at 10 m/s. For dynamic similarity, with what velocity should a 1: 25 model of the ship be towed?

[IES-2004]

- (a) 2 m/s (b) 10 m/s (c) 25 m/s (d) 250 m/s

IES-35. Ans. (a) For ship Froude Model law is used.

$\therefore \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$ or $V_m = V_p \times \sqrt{\frac{L_m}{L_p}} = 10 \times \sqrt{\frac{1}{25}} = 2 \text{ m/s}$

IES-36. A $\frac{1}{25}$ model of a ship is to be tested for estimating the wave drag. If the speed of the ship is 1 m/s, then the speed at which the model must be tested is:

[IES-1992, IAS-2002]

- (a) 0.04 m/s (b) 0.2 m/s (c) 5.0 m/s (d) 25.0 m/s

IES-36. Ans. (b) Apply Froude Model law $(F_r)_m = (F_r)_p$ or $\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$

or $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$ or $V_m = \frac{1}{5} = 0.2 \text{ m/s}$.

IES-37. In a flow condition where both viscous and gravity forces dominate and both the Froude number and the Reynolds number are the same in model and prototype; and the ratio of kinematic viscosity of model to that of the prototype is 0.0894. What is the model scale? [IES-2004]

- (a) 1: 3.3 (b) 3.3: 1 (c) 5: 1 (d) 1:5

IES-37. Ans. (c) $(R_e)_{\text{model}} = (R_e)_{\text{prototype}}$ gives $\frac{V_m}{V_p} \times \frac{L_m}{L_p} \times \frac{\nu_p}{\nu_m} = 1$ ----(i)

and $(F_r)_{\text{model}} = (F_r)_{\text{prototype}}$ gives $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$ ---- (ii)

(i) and (ii) gives $\left(\frac{L_m}{L_p}\right)^{3/2} = \frac{\nu_p}{\nu_m} = 0.0894$ or $\frac{L_m}{L_p} = 0.2$

$$L_m : L_p = 5 : 1$$

IES-38. A 1:20 model of a spillway dissipates 0.25 hp. The corresponding prototype horsepower dissipated will be: [IES-1998]

- (a) 0.25 (b) 5.00 (c) 447.20 (d) 8944.30

IES-38. Ans. (d) $P_r = L_r^{3.5} = 20^{3.5}$ Therefore $P_p = 0.25 \times 20^{3.5} = 8944$ hp

IES-39. A ship with hull length of 100 m is to run with a speed of 10 m/s. For dynamic similarity, the velocity for a 1: 25 model of the ship in a towing tank should be: [IES-2001]

- (a) 2 m/s (b) 10 m/s (c) 20 m/s (d) 25 m/s

IES-39. Ans. (a) Use $V_r = \sqrt{L_r}$

IES-40. A ship's model, with scale 1: 100, has a wave resistance of 10 N at its design speed. What is the corresponding prototype wave resistance in kN? [IES-2007]

- (a) 100 (b) 1000 (c) 10000
(d) Cannot be determined because of insufficient data

IES-40. Ans.(c) We know that $F_r = L_r^3$

$$\text{or, } \frac{F_p}{F_m} = \left(\frac{L_p}{L_m}\right)^3 \text{ or } F_p = F_m \times \left(\frac{L_p}{L_m}\right)^3 = 10 \times (100)^3 \text{ N} = 10000 \text{ kN}$$

IES-41. A model test is to be conducted for an under water structure which each likely to be exposed for an under water structure, which is likely to be exposed to strong water currents. The significant forces are known to be dependent on structure geometry, fluid velocity, fluid density and viscosity, fluid depth and acceleration due to gravity. Choose from the codes given below, which of the following numbers must match for the model with that of the prototype: [IES-2002]

1. Mach number 2. Weber number
3. Froude number 4. Reynolds number.

- (a) 3 alone (b) 1, 2, 3 and 4 (c) 1 and 2 (d) 3 and 4

IES-41. Ans. (d)

Types of Models (Undistorted models, distorted models)

IES-42. Consider the following statements: [IES-2003]

1. Complete similarity between model and prototype envisages geometric and dynamic similarities only.
2. Distorted models are necessary where geometric similarity is not possible due to practical reasons.
3. In testing of model of a ship, the surface tension forces are generally neglected.
4. The scale effect takes care of the effect of dissimilarity between model and prototype.

Which of these statements are correct?

- (a) 1 and 3 (b) 1, 2 and 4 (c) 2 and 3 (d) 2 and 4

IES-42. Ans. (c) 1 is wrong. Complete similarity between model and prototype envisages geometric, kinematic and dynamic similarities only.

4 is also wrong. The scale effect takes care of the effect of dissimilarity (size difference) between model and prototype.

Previous 20-Years IAS Questions

Similitude

IAS-1. The drag force D on a certain object in a certain flow is a function of the coefficient of viscosity μ , the flow speed v and the body dimension L (for geometrically similar objects); then D is proportional to: [IAS-2001]

- (a) $L\mu V$ (b) $\frac{\mu^2 V^2}{L^2}$ (c) $\mu^2 v^2 L^2$ (d) $\frac{\mu L}{V}$

IAS-1. Ans. (a)

IAS-2. For a 1: m scale model of a hydraulic turbine, the specific speed of the model N_{sm} is related to the prototype specific speed N_{sp} as [IAS-1997]

- (a) $N_{sm} = N_{sp}/m$ (b) $N_{sm} = mN_{sp}$ (c) $N_{sm} = (N_{sp})^{1/m}$ (d) $N_{sm} = N_{sp}$

IAS-2. Ans. (d)

Froude Number (Fr)

IAS-3. The square root of the ratio of inertia force to gravity force is called [GATE-1994, IAS-2003]

- (a) Reynolds number (b) Froude number
(c) Mach number (d) Euler number

IAS-3. Ans. (b)

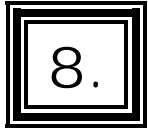
Froude Model Law

IAS-4. A $\frac{1}{25}$ model of a ship is to be tested for estimating the wave drag. If the speed of the ship is 1 m/s, then the speed at which the model must be tested is: [IES-1992, IAS-2002]

- (a) 0.04 m/s (b) 0.2 m/s (c) 5.0 m/s (d) 25.0 m/s

IAS-4. Ans. (b) Apply Froude Model law $(F_r)_m = (F_r)_p$ or $\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$

$$\text{or } \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{25}} = \frac{1}{5} \text{ or } V_m = \frac{1}{5} = 0.2 \text{ m/s.}$$



8. Boundary Layer Theory

Contents of this chapter

1. Boundary Layer Definitions and Characteristics
 2. Boundary Layer Thickness (δ)
 3. Displacement Thickness (δ^*)
 4. Momentum Thickness (θ)
 5. Energy thickness (δ_e)
 6. Momentum Equation for Boundary Layer by Von-karman
 7. Laminar Boundary Layer
 8. Turbulent Boundary Layer
 9. Total Drag Due to Laminar and Turbulent Layers
 10. Boundary Layer Separation and its Control
 11. Thermal Boundary Layer
-

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

Momentum Equation for Boundary Layer by Von-karman

GATE-1. For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed respectively, as [GATE-2004]

$$\frac{U}{U_a} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3; \quad \delta = \frac{4.64x}{\sqrt{\text{Re}_x}}$$

If the free stream velocity is 2 m/s, and air has kinematic viscosity of $1.5 \times 10^{-5} \text{m}^2/\text{s}$ and density of 1.23kg/m^3 , then wall shear stress at $x = 1 \text{m}$, is

- (a) $2.36 \times 10^2 \text{N/m}^2$ (b) $43.6 \times 10^{-3} \text{N/m}^2$
 (c) $4.36 \times 10^{-3} \text{N/m}^2$ (d) $2.18 \times 10^{-3} \text{N/m}^2$

GATE-1. Ans. (c) Given: $\rho = 1.23 \text{kg/m}^3$, $\nu = \frac{\mu}{\rho} = 1.5 \times 10^{-5} \text{m}^2/\text{s}$, $u = 2 \text{m/s}$, $x = L = 1 \text{m}$.

$$\text{Re}_x = \frac{\rho u L}{\mu} = \frac{2 \times 1}{\left(\frac{\mu}{\rho} \right)} = \frac{2 \times 1}{1.5 \times 10^{-5}} = 1.34 \times 10^5$$

$$\text{Now, shear stress, } \tau_0 = \mu \left(\frac{du}{dy} \right)_{y=0}$$

$$\text{Where, } \frac{u}{U} = \frac{3}{2} \frac{y}{\delta} - \frac{y^3}{2\delta^3} \text{ or } \frac{du}{dy} = U \left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3} \right]$$

$$\text{Hence, } \left(\frac{du}{dy} \right)_{y=0} = \frac{3U}{2\delta}$$

$$\text{Given: } \delta = \frac{4.64x}{\sqrt{\text{Re}_x}} = \frac{4.64 \times x}{\sqrt{\frac{\rho U x}{\mu}}}$$

$$\text{Putting } x = 1, (\delta)_{x=1} = \frac{4.64}{\sqrt{\frac{2 \times 1}{1.5 \times 10^{-5}}}} = 0.0127$$

$$\therefore \tau_0 = \mu \cdot \frac{du}{dy} = \frac{3}{2} \frac{\mu U}{\delta} = \frac{3}{2} \times \frac{(1.5 \times 10^{-5} \times 1.23) \times 2}{0.0127} = 4.355 \times 10^{-3} \text{N/m}^2$$

Laminar Boundary Layer

GATE-2. The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]

- (a) X (b) $X^{\frac{1}{2}}$ (c) $X^{\frac{1}{5}}$ (d) $X^{\frac{4}{5}}$

GATE-2. Ans. (b) $\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re}_x}}$ or $\delta \propto \frac{5x}{\sqrt{\rho \nu x}}$ or $\delta \propto \sqrt{x}$

Total Drag Due to Laminar and Turbulent Layers

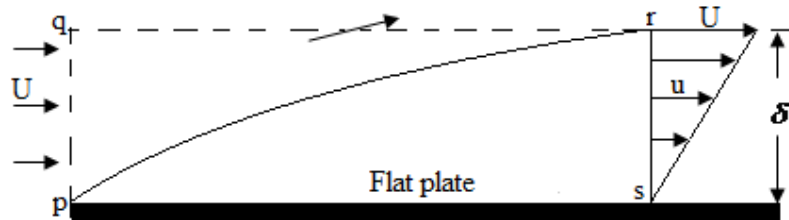
GATE-3. Consider an incompressible laminar boundary layer flow over a flat plate of length L , aligned with the direction of an oncoming uniform free stream. If F the ratio of the drag force on the front half of the plate to the drag force on the rear half, then [GATE-2007]

- (a) $F < 1/2$ (b) $F = 1/2$ (c) $F = 1$ (d) $F > 1$

GATE-3. Ans. (d) $F_D = \text{some Const} \times \int_0^L x^{-1/2} dx$. Therefore ratio = $\frac{\sqrt{L/2} - 0}{L - \sqrt{L/2}} = \frac{1}{\sqrt{2} - 1} > 1$

Statement for Linked Answer and Questions Q4–Q5:

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at $U = 10$ m/s. The thickness of the boundary layer at section r - s is 10 mm, the breadth of the plate is 1 m (into the paper) and the density of the gas $\rho = 1.0$ kg/m³. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, $u = U(y/\delta)$, at the section r - s , where y is the height from plate.



GATE-4. The mass flow rate (in kg/s) across the section $q - r$ is: [GATE-2006]
 (a) Zero (b) 0.05 (c) 0.10 (d) 0.15

GATE-4. Ans. (b) Mass entering from side $q - p =$ Mass leaving from side $q - r +$ Mass leaving the side $r - s$.

GATE-5. The integrated drag force (in N) on the plate, between $p - s$, is: [GATE-2006]
 (a) 0.67 (b) 0.33 (c) 0.17 (d) Zero

GATE-5. Ans. (c) By momentum equation, we can find drag force.

Boundary Layer Separation and Its Control

GATE-6. Flow separation is caused by: [IAS-1996; IES-1994, 1997; 2000; GATE-2002]

- (a) Reduction of pressure to local vapour pressure
 (b) A negative pressure gradient
 (c) A positive pressure gradient
 (d) Thinning of boundary layer thickness to zero.

GATE-6. Ans. (c) i.e. an adverse pressure gradient.

When the pressure goes increasing $\left(\frac{\partial P}{\partial x} > 0\right)$ in the direction of flow, the pressure force acts against the direction of direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickenings the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as “Boundary Layer Separation”.

GATE-7. The necessary and sufficient condition which brings about separation of boundary layer is $\frac{dp}{dx} > 0$ [GATE-1994]

GATE-7. Ans. False because Separation takes place where $\frac{dp}{dx} > 0$ and $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

Thermal Boundary Layer

GATE-8. The temperature distribution within thermal boundary layer over a heated isothermal flat plate is given by $\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$, where

T_w and T_∞ are the temperature of plate and free stream respectively, and y is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness δ_t is given by

[GATE-2007]

- (a) 1.33 (b) 1.50 (c) 2.0 (d) 4.64

GATE-8. Ans. (b) $Nu = \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$

where, $T^* = \frac{T - T_w}{T_\infty - T_w}$ and $y^* = \frac{y}{\delta}$

Where T_w is surface temperature and T_∞ is free-stream temperature.

$$\Rightarrow Nu = \frac{3}{2} - \frac{3}{2} \left(\frac{y}{\delta_t}\right) \Big|_{y=0} = \frac{3}{2} - \frac{3}{2} y^* \Big|_{y^*=0} = 1.5$$

GATE-9. Consider a laminar boundary layer over a heated flat plate. The free stream velocity is U_∞ . At some distance x from the leading edge the velocity boundary layer thickness is δ_v . If the Prandtl number is greater than 1, then [GATE-2003]

- (a) $\delta_v > \delta_T$ (b) $\delta_T > \delta_v$ (c) δ_v

GATE-9. Ans. (a) Prandtl number = $\frac{\text{Molecular diffusivity of mom}}{\text{Molecular diffusivity of heat}}$

From question, since Prandtl number > 1

\therefore Velocity boundary thickness (δ_v) > 1 thermal boundary thickness.

GATE-10. A fluid flowing over a flat plate has the following properties:

Dynamic viscosity: 25×10^{-6} kg/ms

[GATE-1992]

Specific heat: 2.0 kJ/kgK

Thermal conductivity: 0.05 W/mk

The hydrodynamic boundary layer thickness is measured to be 0.5 mm.

The thickness of thermal boundary layer would be:

- (a) 0.1 mm (b) 0.5 mm (c) 1.0 mm (d) None of the above

GATE-10. Ans. (b) $\frac{\delta}{\delta_{th}} = (Pr)^{1/3}$ if $Pr = \frac{\mu C_p}{k} = \frac{25 \times 10^{-6} \text{ kg/ms} \times 2000 \text{ J/kgK}}{0.05 \text{ kg/mK}} = 1$. So, $\delta = \delta_{th}$

GATE-11. For flow of fluid over a heated plate, the following fluid properties are known: viscosity = 0.001 Pa.s ; specific heat at constant pressure = 1 kJ/kg.K; thermal conductivity = 1 W/m.K. [GATE-2008]

The hydrodynamic boundary layer thickness at a specified location on the plate is 1 mm. The thermal boundary layer thickness at the same location is:

- (a) 0.001 mm (b) 0.01 mm (c) 1 mm (d) 1000 mm

GATE-11. Ans. (c) We know that $\frac{\delta}{\delta_h} = (P_r)^{1/3}$.

$$\text{Here Prandtl number } (P_r) = \frac{\mu c_p}{k} = \frac{0.001 \times 1000}{1} = 1. \text{ So, } \delta = \delta_h.$$

GATE-12. For air near atmosphere conditions flowing over a flat plate, the laminar thermal boundary layer is thicker than the hydrodynamic boundary layer. [GATE-1994]

GATE-12. Ans. False

Previous 20-Years IES Questions

Boundary Layer Definitions and Characteristics

IES-1. Boundary layer is defined as [IES-1998]

- (a) A thin layer at the surface where gradients of both velocity and temperature are small
 (b) A thin layer at the surface where velocity and velocity gradients are large
 (c) A thick layer at the surface where velocity and temperature gradients are large
 (d) A thin layer at the surface where gradients of both velocity and temperature are large

IES-1. Ans. (b)

- The **boundary layer** of a flowing fluid is **the thin layer close to the wall**
- In a flow field, **viscous stresses are very prominent within this layer.**
- Although the layer is thin, it is very important to know the details of flow within it.
- The **main-flow velocity** within this layer **tends to zero** while approaching the wall (**no-slip condition**).
- Also the gradient of this velocity component in a direction normal to the surface is large as compared to the gradient in the stream wise direction.

IES-2. In the boundary layer, the flow is: [IES-2006]

- (a) Viscous and rotational (b) Inviscid and irrotational
 (c) Inviscid and rotational (d) Viscous and irrotational

IES-2. Ans. (a)

IES-3. Assertion (A): In the boundary layer concept, the shear stress at the outer edge of the layer is considered to be zero. [IES-2008]

Reason (R): Local velocity is almost equal to velocity in potential flow.

- (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is NOT the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IES-3. Ans. (a)

IES-4. In the region of the boundary layer nearest to the wall where velocity is not equal to zero, the viscous forces are: [IES-1993, 1995]

- (a) Of the same order of magnitude as the inertial forces
- (b) More than inertial forces
- (c) Less than inertial forces
- (d) Negligible

IES-4. Ans. (c) Reynold's number = Inertia force / Viscous force
and it is more than one therefore the viscous forces are less than inertial forces.

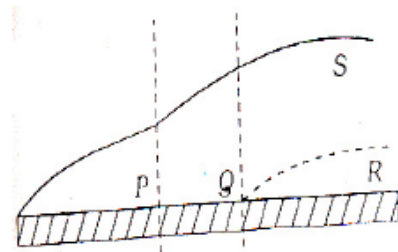
IES-5. The critical value of Reynolds number for transition from laminar to turbulent boundary layer in external flows is taken as: [IES-2002]

- (a) 2300
- (b) 4000
- (c) 5×10^5
- (d) 3×10^6

IES-5. Ans. (c)

IES-6. The development of boundary layer zones labeled P, Q, R and S over a flat plate is shown in the given figure.

Based on this figure, match List-I (Boundary layer zones) with List-II (Type of boundary layer) and select the correct answer:



[IES-2000]

List-I

- A. P
- B. Q
- C. R
- D. S

Codes: A B C D

- (a) 3 1 2 4
- (c) 4 2 1 3

List-II

- 1. Transitional
- 2. Laminar viscous sub-layer
- 3. Laminar
- 4. Turbulent

A B C D

- (b) 3 2 1 4
- (d) 4 1 2 3

IES-6. Ans. (a)

IES-7. The transition Reynolds number for flow over a flat plate is 5×10^5 . What is the distance from the leading edge at which transition will occur for flow of water with a uniform velocity of 1 m/s? [For water, the kinematic viscosity, $\nu = 0.858 \times 10^{-6} \text{ m}^2/\text{s}$] [IES-1994]

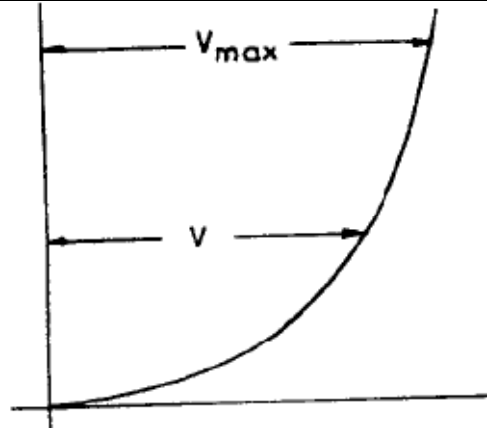
- (a) 1 m
- (b) 0.43 m
- (c) 43 m
- (d) 103 m

IES-7. Ans. (b) $R_N = 5 \times 10^5, R_N = \frac{Vx}{\nu}, \text{ or } x = \frac{R_N \times \nu}{V} = \frac{5 \times 10^5 \times 0.858 \times 10^{-6}}{1} = 0.43 \text{ m}$

IES-8. The 'velocity defect law' is so named because it governs a [IES-1993]

- (a) Reverse flow region near a wall
- (b) Slip-stream flow at low pressure
- (c) Flow with a logarithmic velocity profile a little away from the wall
- (d) Re-circulating flow near a wall

IES-8. Ans. (c) Figure shows the logarithmic velocity profile a little away from the wall. Velocity difference ($V_{\max} - V$) is known as velocity defect. So velocity defect law occurs due to occurrence of flow with a logarithmic velocity profile a little away from the wall.



IES-9. Which one of the following velocity distributions of u/u_α satisfies the boundary conditions for laminar flow on a flat plate? (Here u_α is the free stream velocity, u is velocity at any normal distance y from the flat plate, $\eta = y/\delta$ and δ is boundary layer thickness) [IES-1996]

- (a) $\eta - \eta^2$ (b) $1.5\eta - 0.5\eta^3$ (c) $3\eta - \eta^2$ (d) $\cos(\pi\eta/2)$

IES-9. Ans. (b) The relation $\frac{u}{u_\alpha} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta}\right)^3$ satisfies boundary condition for laminar flow on a flat plate.

IES-10. The predominant forces acting on an element of fluid in the boundary layer over a flat plate placed in a uniform stream include [IES-1996]

- (a) Inertia and pressure forces (b) Viscous and pressure forces
(c) Viscous and body forces (d) Viscous and inertia forces

IES-10. Ans. (d) That so why we are using Reynold's number for analysis.

Boundary Layer Thickness (δ)

IES-11. The hydrodynamic boundary layer thickness is defined as the distance from the surface where the [IES-1999]

- (a) Velocity equals the local external velocity
(b) Velocity equals the approach velocity
(c) Momentum equals 99% of the momentum of the free stream
(d) Velocity equals 99% of the local external velocity

IES-11. Ans. (d)

IES-12. Assertion (A): The thickness of boundary layer is an ever increasing one as its distance from the leading edge of the plate increases.

Reason (R): In practice, 99% of the depth of the boundary layer is attained within a short distance of the leading edge. [IES-1999]

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

IES-12. Ans. (c) Why A is true? For laminar boundary layer:

$$\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re } x}} \quad \text{or} \quad \delta \alpha \frac{5x}{\sqrt{\rho v x}} \quad \text{or} \quad \delta \alpha \sqrt{x}$$

For turbulent boundary layer:

$$\frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}} \text{ or } \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5} \text{ or } \delta \propto x^{4/5}$$

Therefore for both the cases if x increases δ will increase.

Why R is false? There is no term in the boundary layer like “depth of the boundary layer”. Till today we never heard about depth of the boundary layer.

IES-13. For the velocity profile $u / u_\infty = \square$, the momentum thickness of a laminar boundary layer on a flat plate at a distance of 1 m from leading edge for air (kinematic viscosity = 2×10^{-5} m²/s) flowing at a free stream velocity of 2 m/s is given by: [IES-2001]

- (a) 3.16 mm (b) 2.1 mm (c) 3.16 m (d) 2.1 m

IES-13. Ans. (b) Thickness of Boundary layer, $\delta = \frac{5x}{\sqrt{\text{Re}_x}} = \frac{5x}{\sqrt{Ux/\nu}} = \frac{5 \times 1}{\sqrt{2 \times 2 / 2 \times 10^{-5}}} =$

0.01118 m and for such velocity distribution $\theta = \frac{\delta}{6} = 1.863 \text{ mm}$ nearest ans. (b)

IES-14. A flat plate, 2m × 0.4m is set parallel to a uniform stream of air (density 1.2kg/m³ and viscosity 16 centistokes) with its shorter edges along the flow. The air velocity is 30 km/h. What is the approximate estimated thickness of boundary layer at the downstream end of the plate? [IES-2004]

- (a) 1.96 mm (b) 4.38 mm (c) 13.12 mm (d) 9.51 mm

IES-14. Ans. (b) Thickness of Boundary layer,

$$\delta = \frac{5x}{\sqrt{\text{Re}_x}} = \frac{5L}{\sqrt{\frac{UL}{\nu}}} = \frac{5 \times 0.4}{\sqrt{\frac{30 \times (5/18) \times 0.4}{16 \times 10^{-6}}}} = 4.38 \text{ mm}$$

IES-15. A laminar boundary layer occurs over a flat plate at zero incidence to the flow. The thickness of boundary layer at a section 2 m from the leading edge is 2 mm. The thickness of boundary layer at a section 4 m from the leading edge will be: [IES-1995]

- (a) $2 \times (2)^2$ mm (b) $2 \times (2)^{1/2}$ mm (c) $2 \times (2)^{4/5}$ mm (d) $2 \times (2)^{1/5}$ mm

IES-15. Ans. (b) Thickness of boundary layer at 4 mm from leading edge = $2 \times (4/2)^{1/2} = 2 \times 2^{1/2}$

Displacement Thickness (δ^*)

IES-16. If the velocity distribution in a turbulent boundary layer is given by

$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$ then the ratio of displacement thickness to nominal layer

thickness will be:

- (a) 1.0 (b) 0.6 (c) 0.3 (d) 0.1

[IAS-1998; IES-2006]

IES-16. Ans. (d) Displacement thickness (δ^*) = $\delta \int_0^1 (1 - z^{1/9}) dz = 0.1\delta$

IES-17. For linear distribution of velocity in the boundary layer on a flat plate, what is the ratio of displacement thickness (δ^*) to the boundary layer thickness (δ)? [IES-2005]

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

IES-17. Ans. (c) Remember it.

Momentum Thickness (θ)

IES-18. If U_∞ = free stream velocity, u = velocity at y and δ = boundary layer thickness, then in a boundary layer flow, the momentum thickness θ is given by: [IES-1997; IAS-2004]

- (a) $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy$ (b) $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$
 (c) $\theta = \int_0^\delta \frac{u^2}{U_\infty^2} \left(1 - \frac{u}{U_\infty}\right) dy$ (d) $\theta = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

IES-18. Ans. (a)

IES-19. Given that [IES-1997]

- δ = boundary layer thickness, δ^* = displacement thickness
 δ_e = energy thickness θ = momentum thickness

The shape factor H of a boundary layer is given by

- (a) $H = \frac{\delta_e}{\delta}$ (b) $H = \frac{\delta^*}{\theta}$ (c) $H = \frac{\delta}{\theta}$ (d) $H = \frac{\delta}{\delta^*}$

IES-19. Ans. (b) Shape factor = $\frac{\text{Displacement thickness}}{\text{Momentum thickness}}$

IES-20. The velocity distribution in the boundary layer is given as $u / u_s = y / \delta$, where u is the velocity at a distance y from the boundary, u_s is the free stream velocity and δ is the boundary layer thickness at a certain distance from the leading edge of a plate. The ratio of displacement to momentum thicknesses is: [IES-2001; 2004]

- (a) 5 (b) 4 (c) 3 (d) 2

IES-20. Ans. (c) Remember it.

IES-21. The velocity profile in a laminar boundary layer is given by $u/U = y/\delta$. The ratio of momentum thickness to displacement thickness for the boundary is given by which one of the following? [IES-2008]

- (a) 2 : 3 (b) 1 : 2 (c) 1 : 6 (d) 1 : 3

IES-21. Ans. (d) Velocity profile of laminar boundary layer is given

$$\frac{u}{U} = \frac{y}{\delta}$$

Displacement thickness:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \frac{\delta}{2}$$

Momentum thickness:

$$\theta = \int_0^{\delta} \left(1 - \frac{u}{U}\right) \frac{u}{U} dy = \int_0^{\delta} \left(\frac{y}{\delta}\right) \left(1 - \frac{y}{\delta}\right) dy = \frac{\delta}{6}$$

$$\therefore \frac{\theta}{\delta^*} = \frac{1}{3}$$

Energy Thickness (δ_e)

IES-22. The energy thickness for a laminar boundary layer flow depends on local and free stream velocities within and outside the boundary layer δ . The expression for the energy thickness is given by (symbols have the usual meaning). [IES-1994]

(a) $\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) dy$	(b) $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u}{U_{\infty}}\right) dy$
(c) $\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right)^2 dy$	(d) $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u^2}{U_{\infty}^2}\right) dy$

IES-22. Ans. (d)

IES-23. Which one of the following is the correct relationship between the boundary layer thickness δ , displacement thickness δ^* and the momentum thickness θ ? [IAS-2004; IES-1999]

(a) $\delta > \delta^* > \theta$ (b) $\delta^* > \theta > \delta$ (c) $\theta > \delta > \delta^*$ (d) $\theta > \delta^* > \delta$

IES-23. Ans. (a) $\delta > \delta^* > \theta > \delta^{**}$

IES-24. List-I give the different items related to a boundary layer while List-II gives the mathematical expressions. Match List-I with List-II and select the correct answer using the codes given below the lists: (symbols have their usual meaning). [IES-1995]

List-I	List-II
A. Boundary layer thickness	1. $y = \delta, u = 0.99 U_{\infty}$
B. Displacement thickness	2. $\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) dy$
C. Momentum thickness	3. $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u}{U_{\infty}}\right) dy$
D. Energy thickness	4. $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u^2}{U_{\infty}^2}\right) dy$
Code:	A B C D
(a)	1 2 3 4 (b) 2 1 4 3
(c)	2 1 3 4 (d) 1 2 4 3

IES-24. Ans. (a)

Momentum Equation for Boundary Layer by Von-karman

IES-25. According to Blasius law, the local skin friction coefficient in the boundary-layer over a flat plate is given by: [IES-2001, 1994]

(a) $0.332/\sqrt{R_e}$ (b) $0.664/\sqrt{R_e}$ (c) $0.647/\sqrt{R_e}$ (d) $1.328/\sqrt{R_e}$

IES-25. Ans. (b)

IES-26. Match List-I (Variables in Laminar Boundary layer Flow over a Flat Plate Set Parallel to the Stream) with List-II (Related Expression with usual notations) and select the correct answer using the codes given below: [IES-2004; IAS-1999]

List-I					List-II				
A. Boundary layer thickness					1. $1.729 / \sqrt{Ux / \nu}$				
B. Average skin-friction coefficient					2. $0.332 \rho U^2 / \sqrt{Ux / \nu}$				
C. Shear stress at boundary					3. $5\sqrt{\nu x / U}$				
D. Displacement thickness					4. $0.664 \sqrt{\nu / Ux}$				
					5. $1.328 / \sqrt{UL / \nu}$				
Codes:	A	B	C	D	A	B	C	D	
(a)	3	5	4	2	(b)	2	4	1	3
(c)	3	5	2	1	(d)	5	4	1	2

IES-26. Ans. (c)

IES-27. The equation of the velocity distribution over a plate is given by $u = 2y - y^2$ where u is the velocity in m/s at a point y meter from the plate measured perpendicularly. Assuming $\mu = 8.60$ poise, the shear stress at a point 15 cm from the boundary is: [IES-2002]

- (a) 1.72 N/m² (b) 1.46 N/m² (c) 14.62 N/m² (d) 17.20 N/m²

IES-27. Ans. (b)

Laminar Boundary Layer

IES-28. The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]

- (a) X (b) $X^{\frac{1}{2}}$ (c) $X^{\frac{1}{5}}$ (d) $X^{\frac{4}{5}}$

IES-28. Ans. (b) $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ or $\delta \propto \frac{5x}{\sqrt{\frac{\rho \nu x}{\mu}}}$ or $\delta \propto \sqrt{x}$

IES-29. For laminar flow over a flat plate, the thickness of the boundary layer at a distance from the leading edge is found to be 5 mm. The thickness of the boundary layer at a downstream section, which is at twice the distance of the previous section from the leading edge will be: [IES-1994]

- (a) 10 mm (b) $5\sqrt{2}$ mm (c) $5\sqrt{2}$ mm (d) 2.5 mm

IES-29. Ans. (b) Thickness of boundary layer for laminar flow over a flat plate is proportional to square root of ratio of distances from the leading edge. Thus new thickness = $5 \times \sqrt{2}$ mm.

IES-30. The laminar boundary layer thickness, δ at any point x for flow over a flat plate is given by: $\delta / x =$ [IES-2002]

- (a) $\frac{0.664}{\sqrt{Re_x}}$ (b) $\frac{1.328}{\sqrt{Re_x}}$ (c) $\frac{1.75}{\sqrt{Re_x}}$ (d) $\frac{5.0}{\sqrt{Re_x}}$

IES-30. Ans. (d)

Turbulent Boundary Layer

IES-31. The velocity profile for turbulent layer over a flat plate is: [IES-2003]

(a) $\frac{u}{U} = \sin\left(\frac{\pi}{2} - \frac{y}{\delta}\right)$

(b) $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

(c) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(d) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

IES-31. Ans. (b)

IES-32. The velocity distribution in a turbulent boundary layer is given by $u/U = (y/\delta)^{1/7}$. [IES-2008]

What is the displacement thickness δ^* ?

- (a) δ (b) $\delta/7$ (c) $(7/8)\delta$ (d) $\delta/8$

IES-32. Ans. (d) $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy = \left(y - \frac{7}{8} \frac{y^{8/7}}{\delta^{1/7}}\right) \Big|_0^\delta = \delta - \frac{7}{8}\delta = \frac{\delta}{8}$

IES-33. The thickness of turbulent boundary layer at a distance x from the leading edge over a flat plate varies as

[IAS-2003; 2004; 2007; IES-1996, 1997; 2000]

- (a) $x^{4/5}$ (b) $x^{1/2}$ (c) $x^{1/5}$ (d) $x^{3/5}$

IES-33. Ans. (a) $\frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}} \text{ or } \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5} \text{ or } \delta \propto x^{4/5}$

IES-34. For turbulent boundary layer low, the thickness of laminar sublayer ' δ ' is given by: [IES-1999]

- (a) v / u^* (b) $5 v / u^*$ (c) $5.75 \log v / u^*$ (d) $2300 v / u^*$

IES-34. Ans. (b)

IES-35. Assertion (A): The 'dimples' on a golf ball are intentionally provided. Reason (R): A turbulent boundary layer, since it has more momentum than a laminar boundary layer, can better resist an adverse pressure gradient. [IES-2009]

- (a) Both A and R are individually true and R is the correct explanation of A.
 (b) Both A and R are individually true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

IES-35. Ans. (a)

Boundary Layer Separation and Its Control

IES-36. The boundary layer flow separates from the surface if [IES-1995, 2002]

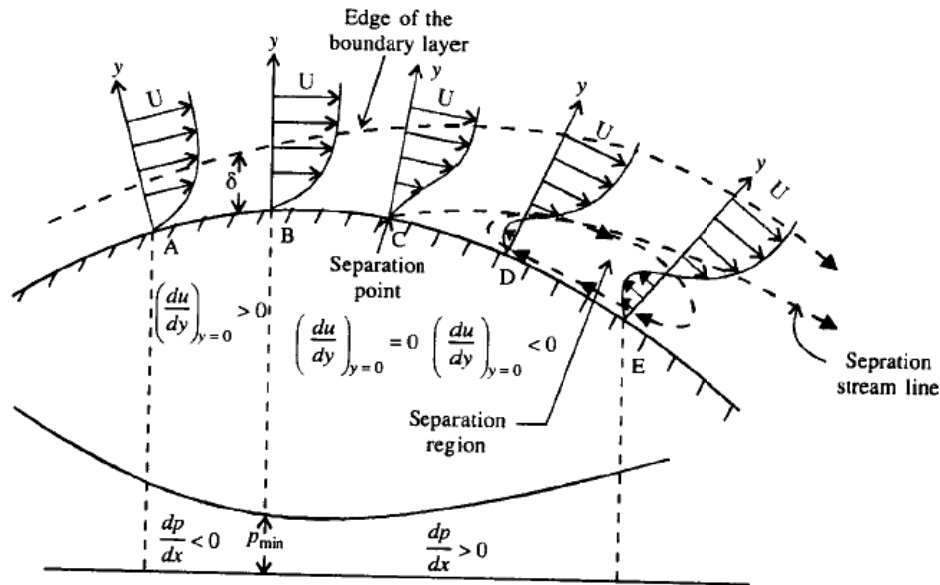
- (a) $du/dy = 0$ and $dp/dx = 0$
 (b) $du/dy = 0$ and $dp/dx > 0$
 (c) $du/dy = 0$ and $dp/dx < 0$
 (d) The boundary layer thickness is zero

IES-36. Ans. (b)

IES-37. In a boundary layer developed along the flow, the pressure decreases in the downstream direction. The boundary layer thickness would:

- (a) Tend to decrease (b) Remain constant [IES-1998]
 (c) Increase rapidly (d) Increase gradually

IES-37. Ans. (d) Consider point A to B where pressure decreases in the downstream direction but boundary layer thickness increases.



IES-38. Flow separation is caused by: [IAS-1996; IES-1994, 1997;2000; GATE-2002]

- (a) Reduction of pressure to local vapour pressure
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Thinning of boundary layer thickness to zero.

IES-38. Ans. (c) i.e. an adverse pressure gradient.

When the pressure goes increasing $\left(\frac{\partial P}{\partial x} > 0\right)$ in the direction of flow, the pressure force acts against the direction of direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickenings the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as “Boundary Layer Separation”.

IES-39. At the point of boundary layer separation [IES-1996]

- (a) Shear stress is maximum
- (b) Shear stress is zero
- (c) Velocity is negative
- (d) Density variation is maximum.

IES-39. Ans. (b) At the verge of separation $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is zero

\therefore Shear stress, $\tau = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ is also zero.

IES-40. Laminar sub-layer may develop during flow over a flat-plate. It exists in [IES-1992]

- (a) Laminar zone
- (b) Transition zone
- (c) Turbulent zone
- (d) Laminar and transition zone

IES-40. Ans. (c)

IES-41. Consider the following statements regarding laminar sublayer of boundary layer flow: [IES-2004]

1. The laminar sublayer exists only in a region that occurs before the formation of laminar boundary layer
2. The laminar sublayer is a region next to the wall where the viscous force is predominant while the rest of the flow is turbulent
3. The laminar sublayer occurs only in turbulent flow past a smooth plate

Which of the statements given above is/are correct?

- (a) 1, 2 and 3 (b) 1 and 2 (c) Only 2 (d) 1 and 3

IES-41. Ans. (c)

IES-42. Consider the following statements pertaining to boundary layer:

1. Boundary layer is a thin layer adjacent to the boundary where maximum viscous energy dissipation takes place.
2. Boundary layer thickness is a thickness by which the ideal flow is shifted.
3. Separation of boundary layer is caused by presence of adverse pressure gradient.

Which of these statements are correct?

[IES-2003]

- (a) 1, 2 and 3 (b) 1 and 2 (c) 1 and 3 (d) 2 and 3

IES-42. Ans. (c) 2 is wrong it defines displacement thickness.

Boundary layer thickness: The thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity reaches 99 percent of the velocity of the free stream. It is denoted by the symbol δ

Displacement thickness: $\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$

It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.

or,

It is an additional "wall thickness" that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation

IES-43. Drag on cylinders and spheres decreases when the Reynolds number is in the region of 2×10^5 since [IES-1993]

- (a) Flow separation occurs due to transition to turbulence
- (b) Flow separation is delayed due to onset of turbulence
- (c) Flow separation is advanced due to transition to turbulence
- (d) Flow reattachment occurs

IES-43. Ans. (d) In the region of 2×10^5 (Reynolds number), the boundary layer on the cylinders and sphere begins to become unstable and thus boundary layer is said to reattach and the separation point moves back along the cylinder. Due to flow reattachment, a pressure recovery takes place over the back side and thus the drag force decreases.

IES-44. During the flow over a circular cylinder, the drag coefficient drops significantly at a critical Reynolds number of 2×10^5 . This is due to [IES-1996]

- (a) Excessive momentum loss in the boundary layer
- (b) Separation point travelling upstream
- (c) Reduction in skin-friction drag
- (d) The delay in separation due to transition to turbulence

IES-44. Ans. (d) The drag co-efficient remains practically constant until a Reynold's number of 2×10^5 is reached. At this stage the C_d drops steeply by a factor of 5.

This is due to the fact that the laminar boundary layer turns turbulent and stays unseparated over a longer distance, then reducing the wake considerably.

IES-45. Which one of the following is correct? [IES-2008]
For flow of an ideal fluid over a cylinder, from the front stagnation point,

- (a) Pressure first decreases then increases
- (b) Velocity first decreases then increases
- (c) Pressure remains the same
- (d) Velocity remains the same

IES-45. Ans. (a) At the stagnation point pressure is maximum
 \therefore For flow past ideal fluid over a cylinder from front stagnation point pressure first decreases then increases.

IES-46. What is the commonly used boundary layer control method to prevent separation? [IES-2009]

- (a) Use of smooth boundaries
- (b) Using large divergence angle in the boundary
- (c) Suction of accelerating fluid within the boundary layer
- (d) Suction of retarded fluid within the boundary layer

IES-46. Ans. (d) Following are some of the methods generally adopted to retard separation:

1. Streamlining the body shape.
2. Tripping the boundary layer from laminar to turbulent by provision roughness.
3. Sucking the retarded flow.
4. Injecting high velocity fluid in the boundary layer.
5. Providing slots near the leading edge.
6. Guidance of flow in a confined passage.
7. Providing a rotating cylinder near the leading edge.
8. Energizing the flow by introducing optimum amount of swirl in the incoming flow.

Thermal Boundary Layer

IES-47. Which non-dimensional number relates the thermal boundary layer and hydrodynamic boundary layer? [IES-2008]

- (a) Rayleigh number
- (b) Peclet number
- (c) Grashof number
- (d) Prandtl number

IES-47. Ans. (d) Prandtl number relates the thermal boundary layer and hydrodynamic boundary layer.

$$\frac{\delta}{\delta_t} = \frac{\text{Hydrodynamic boundary layer}}{\text{Thermal boundary layer}} = (\text{Pr})^{1/3}$$

IES-48. Thermal boundary layer is a region where [IES-1993]

- (a) Inertia terms are of the same order of magnitude as convection terms
- (b) Convection terms are of the same order of magnitude as dissipation terms
- (c) Convection terms are of the same order of magnitude as conduction terms
- (d) Dissipation is negligible

IES-48. Ans. (b)

IES-49. The thickness of thermal and hydrodynamic boundary layer is equal if (Pr = Prandtl number, Nu = Nusselt number) [IES-1993]

- (a) $P_r = 1$ (b) $P_r > 1$ (c) $P_r < 1$ (d) $P_r = N_u$

IES-49. Ans. (a) $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$ if $P_r = 1$, $\delta = \delta_{th}$

IES-50. Hydrodynamic and thermal boundary layer thickness is equal for Prandtl number [IES-1992]

- (a) Equal (b) Less than (c) Equal to 1 (d) More than 1

IES-50. Ans. (c) $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$ if $P_r = 1$, $\delta = \delta_{th}$

IES-51. In a convective heat transfer situation Reynolds number is very large but the Prandtl number is so small that the product $Re \times Pr$ is less than one in such a condition which one of the following is correct?

- (a) Thermal boundary layer does not exist [IES-2004, 2007]
 (b) Viscous boundary layer thickness is less than the thermal boundary layer thickness
 (c) Viscous boundary layer thickness is equal to the thermal boundary layer thickness
 (d) Viscous boundary layer thickness is greater than the thermal boundary layer thickness

IES-51. Ans. (c) $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$ if $P_r < 1$, $\delta < \delta_{th}$

IES-52. The ratio of the thickness of thermal boundary layer to the thickness of hydrodynamic boundary layer is equal to (Prandtl number)ⁿ, where n is: [IES-1994]

- (a) $-1/3$ (b) $-2/3$ (c) 1 (d) -1

IES-52. Ans. (a) $\frac{\text{Thickness of thermal boundary layer}}{\text{Thickness of hydrodynamic layers}} = (\text{Prandtl Number})^{-1/3}$

IES-53. For flow over a flat plate the hydrodynamic boundary layer thickness is 0.5 mm. The dynamic viscosity is 25×10^{-6} Pa s, specific heat is 2.0 kJ/(kgK) and thermal conductivity is 0.05 W/(m-K). The thermal boundary layer thickness would be: [IES-2001]

- (a) 0.1 mm (b) 0.5 mm (c) 1 mm (d) 2 mm

IES-53. Ans. (b)

IES-54. Prandtl number of a flowing fluid greater than unity indicates that hydrodynamic boundary layer thickness is: [IES-2002]

- (a) Greater than thermal boundary layer thickness
 (b) Equal to thermal boundary layer thickness
 (c) Greater than hydrodynamic boundary layer thickness
 (d) Independent of thermal boundary layer thickness

IES-54. Ans. (a)

IES-55. Consider the following conditions for heat transfer (thickness of thermal boundary layer is δ_t , velocity of boundary layer is δ and Prandtl number is P_r): [IES-2000]

1. $\delta_t = \delta$ if $P_r = 1$ 2. $\delta_t \gg \delta$ if $P_r \ll 1$ 3. $\delta_t \ll \delta$ if $P_r \gg 1$

Which of these conditions apply for convective heat transfer?

- (a) 1 and 2 (b) 2 and 3 (c) 1 and 3 (d) 1, 2, and 3

IES-56. Ans. (d) We know that $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$

Previous 20-Years IAS Questions

Boundary Layer Definitions and Characteristics

IAS-1. Velocity defect in boundary layer theory is defined as [IAS-2003]

- (a) The error in the measurement of velocity at any point in the boundary layer
 (b) The difference between the velocity at a point within the boundary layer and the free stream velocity
 (c) The difference between the velocity at any point within the boundary layer and the velocity nearer the boundary
 (d) The ratio between the velocity at a point in the boundary layer and the free stream velocity

IAS-1. Ans. (b)

IAS-2. Assertion (A): The thickness of boundary layer cannot be exactly defined. [IAS-1996]

Reason (R): The Velocity within the boundary layer approaches the inviscid velocity asymptotically.

- (a) Both A and R are individually true and R is the correct explanation of A
 (b) Both A and R are individually true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

IAS-2. Ans. (a)

IAS-3. Velocity distribution in a turbulent boundary layer follows [IAS-2003]

- (a) Logarithmic law (b) Parabolic law (c) Linear law (d) Cubic law

IAS-3. Ans. (a)

Displacement Thickness (δ^*)

IAS-4. How is the displacement thickness in boundary layer analysis defined?

- (a) The layer in which the loss of energy is maximum [IAS-2007]
 (b) The thickness up to which the velocity approaches 99% of the free stream velocity.
 (c) The distance measured perpendicular to the boundary by which the free stream is displaced on account of formation of boundary layer.
 (d) The layer which represents reduction in momentum caused by the boundary layer.

IAS-4. Ans. (c)

IAS-5. The displacement thickness at a section, for an air stream ($\rho = 1.2 \text{ kg/m}^3$) moving with a velocity of 10 m/s over flat plate is 0.5mm. What is the loss mass rate of flow of air due to boundary layer formation in kg per meter width of plate per second? [IAS-2004]

- (a) 6×10^{-3} (b) 6×10^{-5} (c) 3×10^{-3} (d) 2×10^{-3}

IAS-5. Ans. (a) Q (loss per meter) = $\rho \times \delta^* \times \text{velocity} = 1.2 \times \left(\frac{0.5}{1000}\right) \times 10 \text{ kg/ms} = 6 \times 10^{-3} \text{ kg/ms}$

IAS-6. If the velocity distribution in a turbulent boundary layer is given by

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9} \text{ then the ratio of displacement thickness to nominal layer}$$

thickness will be:

[IAS-1998; IES-2006]

- (a) 1.0 (b) 0.6 (c) 0.3 (d) 0.1

IAS-6. Ans. (d) Displacement thickness (δ^*) = $\delta \int_0^1 (1 - z^{1/9}) dz = 0.1\delta$

IAS-7. The velocity distribution in the boundary over the face of a high

spillway found to have the following from $\frac{u}{u_a} = \left(\frac{y}{\delta}\right)^{0.25}$. An a certain

section, the free stream velocity u_a was found to be 20m/s and the boundary layer thickness was estimated to be 5cm. The displacement thickness is: [IAS-1996]

- (a) 1.0 cm (b) 2.0 cm (c) 4.0 cm (d) 5.0 cm

IAS-7. Ans. (a) Displacement thickness (δ^*) = $\delta \int_0^1 (1 - z^{0.25}) dz = 0.2\delta = 0.2 \times 5 = 1.0 \text{ cm}$

Momentum Thickness (θ)

IAS-8. If U_∞ = free stream velocity, u = velocity at y and δ = boundary layer thickness, then in a boundary layer flow, the momentum thickness θ is given by: [IES-1997; IAS-2004]

$$(a) \theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy \quad (b) \theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

$$(c) \theta = \int_0^\delta \frac{u^2}{U_\infty^2} \left(1 - \frac{u}{U_\infty}\right) dy \quad (d) \theta = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

IAS-8. Ans. (a)

Energy Thickness (δe)

IAS-9. Which one of the following is the correct relationship between the boundary layer thickness δ , displacement thickness δ^* and the momentum thickness θ ? [IAS-2004; IES-1999]

- (a) $\delta > \delta^* > \theta$ (b) $\delta^* > \theta > \delta$ (c) $\theta > \delta > \delta^*$ (d) $\theta > \delta^* > \delta$

IAS-9. Ans. (a) $\delta > \delta^* > \theta > \delta^{**}$

Momentum Equation for Boundary Layer by Von-karman

IAS-10. Match List-I (Variables in Laminar Boundary layer Flow over a Flat Plate Set Parallel to the Stream) with List-II (Related Expression with usual notations) and select the correct answer using the codes given below: [IES-2004; IAS-1999]

List-I				List-II					
A.	Boundary layer thickness	1.	$1.729 / \sqrt{Ux / \nu}$						
B.	Average skin-friction coefficient	2.	$0.332 \rho U^2 / \sqrt{Ux / \nu}$						
C.	Shear stress at boundary	3.	$5\sqrt{\nu x / U}$						
D.	Displacement thickness	4.	$0.664 \sqrt{\nu / Ux}$						
		5.	$1.328 / \sqrt{UL / \nu}$						
Codes:	A	B	C	D	A	B	C	D	
(a)	3	5	4	2	(b)	2	4	1	3
(c)	3	5	2	1	(d)	5	4	1	2

IAS-10. Ans. (c)

IAS-11. The equation of the velocity distribution over a plate is given by $u = 2y - y^2$ where u is the velocity in m/s at a point y meter from the plate measured perpendicularly. Assuming $\mu = 8.60$ poise, the shear stress at a point 15 cm from the boundary is: [IES-2002]
 (a) 1.72 N/m² (b) 1.46 N/m² (c) 14.62 N/m² (d) 17.20 N/m²

IAS-11. Ans. (b)

Laminar Boundary Layer

IAS-12. The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]

- (a) X (b) $X^{\frac{1}{2}}$ (c) $X^{\frac{1}{5}}$ (d) $X^{\frac{4}{5}}$

IAS-12. Ans. (b) $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ or $\delta \alpha \frac{5x}{\sqrt{\frac{\rho \nu x}{\mu}}}$ or $\delta \alpha \sqrt{x}$

IAS-13. For laminar flow over a flat plate, the thickness of the boundary layer at a distance from the leading edge is found to be 5 mm. The thickness of the boundary layer at a downstream section, which is at twice the distance of the previous section from the leading edge will be: [IES-1994]

- (a) 10 mm (b) $5\sqrt{2}$ mm (c) $5\sqrt{2}$ mm (d) 2.5 mm

IAS-13. Ans. (b) Thickness of boundary layer for laminar flow over a flat plate is proportional to square root of ratio of distances from the leading edge. Thus new thickness = $5 \times \sqrt{2}$ mm.

Turbulent Boundary Layer

IAS-14. The thickness of turbulent boundary layer at a distance x from the leading edge over a flat plate varies as

[IAS-2003; 2004; 2007; IES-1996, 1997; 2000]

IAS-14. Ans. (a) $\frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}}$ or, $\delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5}$ or, $\delta \propto x^{4/5}$

IAS-15. Consider the following statements comparing turbulent boundary layer with laminar boundary layer: [IAS-1997]

1. Turbulent boundary layers are thicker than laminar boundary layer
2. Velocity in turbulent boundary layers is more uniform
3. In case of a laminar boundary layer, the thickness of the boundary layer increases more rapidly as the distance from the leading edge increases.
4. For the same local Reynolds number. Shear stress at the boundary is less in the case of turbulent boundary layer.

Of these statements:

- | | |
|-----------------------------|-------------------------|
| (a) 1,2,3 and 4 are correct | (b) 1 and 3 are correct |
| (c) 3 and 4 are correct | (d) 1 and 2 are correct |

IAS-15. Ans. (a)

Total Drag Due to Laminar and Turbulent Layers

IAS-16. In a laminar boundary layer over a flat plate, what would be the ratio of wall shear stresses τ_1 and τ_2 at the two sections which lie at distances $x_1=30$ cm and $x_2=90$ cm from the leading edge of the plate? [IAS-2004]

- | | | | |
|-----------------------------------|---|---|---|
| (a) $\frac{\tau_1}{\tau_2} = 3.0$ | (b) $\frac{\tau_1}{\tau_2} = \frac{1}{3}$ | (c) $\frac{\tau_1}{\tau_2} = (3.0)^{1/2}$ | (d) $\frac{\tau_1}{\tau_2} = (3.0)^{1/3}$ |
|-----------------------------------|---|---|---|

IAS-16. Ans. (c) $\tau_o = 0.323 \frac{\mu u}{x} \times \sqrt{\text{Re } x}$ i.e. $\tau_o \propto \frac{1}{\sqrt{x}}$

$$\therefore \frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}} = \sqrt{\frac{90}{30}} = (3)^{1/2}$$

IAS-17. Match List-I (Device) with List-II (Use) and select the correct answer using the codes given below the Lists: [IAS-2002]

List-I	List-II
A. Pitot	1. Boundary shear stress
B. Preston tube	2. Turbulent velocity fluctuations
C. Flow nozzle	3. The total head
D. Hot wire anemometer	4. Flow rate
Codes:	A B C D
(a) 4 2 3 1	(b) 3 1 4 2
(c) 4 1 3 2	(d) 3 2 4 1

IAS-17. Ans. (b)

Boundary Layer Separation and Its Control

IAS-18. Assertion (A): In an ideal fluid, separation from a continuous surface would not occur with a positive pressure gradient. [IAS-2000]

Reason (R): Boundary layer does not exist in ideal fluid.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-18. Ans. (a) In Ideal fluid viscosity is zero so no boundary layer is formed.

IAS-19. Flow separation is caused by: [IAS-1996; IES-1994, 1997;2000; GATE-2002]

- (a) Reduction of pressure to local vapour pressure
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Thinning of boundary layer thickness to zero.

IAS-19. Ans. (c) i.e. an adverse pressure gradient.

When the pressure goes increasing $\left(\frac{\partial P}{\partial x} > 0\right)$ in the direction of flow, the pressure force acts against the direction of direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickening the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as "Boundary Layer Separation".

IAS-20. Flow separation is caused by **[IAS-2002]**

- (a) Thinning of boundary layer thickness to zero
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Reduction of pressure to local vapour pressure

IAS-20. Ans. (c) Separation takes place where $\frac{dp}{dx} > 0$ and $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

IAS-21. Boundary layer separation takes place when **[IAS-2007]**

- | | |
|--|--|
| (a) $\left(\frac{du}{dy}\right)_{y=0} = +ve \text{ value}$ | (b) $\left(\frac{du}{dy}\right)_{y=0} = -ve \text{ Value}$ |
| (c) $\left(\frac{du}{dy}\right)_{y=\delta} = 0$ | (d) $\left(\frac{du}{dy}\right)_{y=0} = 0$ |

IAS-21. Ans. (d) But $\frac{\partial p}{\partial x} > 0$

IAS-22. Flow separation is likely to take place when the pressure gradient in the direction of flow is: **[IAS-1998]**

- | | |
|------------------------|------------------------|
| (a) Zero | (b) Adverse |
| (c) Slightly favorable | (d) Strongly favorable |

IAS-22. Ans. (b)

Question from Conventional Paper

To solve the problems below use "Algorithm" from 'Highlight'

1. Explain displacement and momentum boundary layer thickness. Assume that the shear stress varies linearly in a laminar boundary layer such that $\tau = \tau_0 \left[1 - \frac{y}{\delta} \right]$. Calculate the displacement and momentum thickness in terms of δ . [IES-1998]

2. Derive the integral momentum equation for the boundary layer over a flat plate and determine the boundary layer thickness δ , at a distance x from the leading edge assuming linear velocity profile $(u/U) = y/\delta$ where u is the velocity at the location at a distance y from the plate, and U is the free stream velocity. [IAS-1998]

3. When a fluid flows over a flat plate, the velocity profile within the boundary layer may be assumed to be $V_x = U \left[\frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right]$ for $y \leq \delta$. [IES-1995]

Where U is a constant and the boundary layer thickness δ is a function of x given by $\delta = 5 \left(\frac{\mu x}{\rho U} \right)^{1/2}$. Here μ and ρ denote the viscosity and density of the fluid respectively. Derive an expression for the variation of V_y across the boundary layer. i.e. calculate displacement thickness.

4. The velocity profile for laminar flow in the boundary layer of a flat plate is given by $\frac{u}{U} = \sin \left(\frac{\pi}{2} - \frac{y}{\delta} \right)$. Where u is the velocity of fluid in the boundary layer at a vertical distance y from the plate surface and U is the free stream velocity. Prove that the boundary layer thickness δ may be given by the expression, $\delta = \frac{4.795x}{\sqrt{\text{Re}_x}}$ [IES-1992]

5. Explain briefly the Boundary Layer Theory as propounded by Prandtl. Obtain an expression for the thickness of the boundary layer for laminar flow assuming the velocity distribution law as $\frac{u}{U} = 2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2$ [IAS-1990]

Where U = approach velocity of the stream, u = velocity of the stream in the boundary layer at a distance y from the boundary and δ = thickness of the boundary layer.



Laminar Flow

Contents of this chapter

1. Relationship between Shear Stress and Pressure Gradient
2. Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law
3. Flow of Viscous Fluid between Two Parallel Plates

Question: Show that the friction factor is inversely proportional to the Reynold's number in case of laminar flow in circle pipes.

[IES-2003 AMIE (Summer)-1998]

Answer: Let, u = Velocity of flow at radial distance.
 μ = Dynamic viscosity of flow fluid.

$$\frac{\partial P}{\partial x} = \text{Pressure gradient}$$

R = Radius of pipe

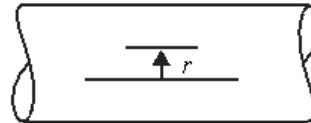
\bar{U} = Average velocity

$$\therefore U_{\max} = -\frac{1}{4\mu} \frac{\partial P}{\partial x} \cdot (R^2 - r^2)$$

$$\bar{U} = \frac{U_{\max}}{2} = \frac{1}{2} \times \left(-\frac{1}{4\mu} \times \frac{\partial P}{\partial x} \times R^2 \right) = \frac{1}{8\mu} \frac{\partial P}{\partial x} R^2$$

$$-\partial P = \frac{8\mu \bar{U}}{R^2} \partial x$$

Integrating both side we get



$$-\int_{P_1}^{P_2} dP = \int_{x_1}^{x_2} \frac{8\mu\bar{U}}{R^2} dx$$

$$\therefore (P_1 - P_2) = \frac{8\mu\bar{U}}{R^2} (x_2 - x_1) \quad \text{Let } x_2 - x_1 = L \text{ (Length of pipe)}$$

$$\therefore \Delta P = \frac{8\mu\bar{U}L}{R^2}$$

$$\therefore h_f \rho g = \frac{8\mu\bar{U}L}{R^2} \quad \text{where } (h_f) = \text{head loss}$$

$$\text{or, } h_f = \frac{8\mu\bar{U}L}{\rho g \times \left(\frac{D}{2}\right)^2} \quad \text{or, } h_f = \frac{32\mu\bar{U}L}{\rho g \times D^2}$$

Comparing with $h_f = \frac{fLV^2}{2Dg}$ for $V = \bar{U}$ and $f = \text{friction factor}$

$$\therefore \frac{fLV^2}{2Dg} = \frac{32\mu\bar{U}L}{\rho g \times D^2} \Rightarrow f = \frac{64\mu}{\rho V D} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64}{R_e}$$

$$\therefore f \propto \frac{1}{R_e}$$

i.e. friction factor is inversely proportional to Reynold's number.

OBJECTIVE QUESTIONS (GATE, IES, IAS)

Previous 20-Years GATE Questions

GATE-1. In flow through a pipe, the transition from laminar to turbulent flow does not depend on [GATE-1996]

- (a) Velocity of the fluid
- (b) Density of the fluid
- (c) Diameter of the pipe
- (d) Length of the pipe

GATE-1. Ans. (d) It is totally depends on Reynolds number = $\frac{\rho V D}{\mu}$

Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law

GATE-2. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u=u_0 (1-4r^2/D^2)$, where r is the radial distance from the centre. If the viscosity of the fluid is μ , the pressure drop across a length L of the pipe is: [GATE-2006]

- (a) $\frac{\mu u_0 L}{D^2}$
- (b) $\frac{4\mu u_0 L}{D^2}$
- (c) $\frac{8\mu u_0 L}{D^2}$
- (d) $\frac{16\mu u_0 L}{D^2}$

GATE-2. Ans. (d) By Hagen-Poiseuille law, for steady laminar flow in circular pipes

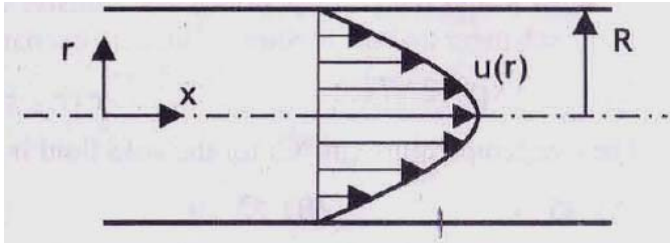
$$\tau = -\mu \frac{\partial u}{\partial r} \Rightarrow \tau = \frac{-\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\mu \frac{\partial u}{\partial r} = \frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\mu u_0 \left(\frac{-8r}{D^2} \right) = \frac{P}{L} \cdot \frac{r}{2} \quad \dots \dots \dots \left[\because u = u_0 \left(1 - \frac{4r^2}{D^2} \right) \right]$$

$$P = \frac{-16\mu L u_0}{D^2} \quad [(-) \text{ sign is due to drop}]$$

GATE-3. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression



$$u(r) = -\frac{R^2}{4\mu} \left(\frac{dp}{dx} \right) \left(1 - \frac{r^2}{R^2} \right)$$

[GATE-2009]

Where $\frac{dp}{dx}$ is a constant. The average velocity of fluid in the pipe is:

- (a) $-\frac{R^2}{8\mu} \left(\frac{dp}{dx} \right)$
- (b) $-\frac{R^2}{4\mu} \left(\frac{dp}{dx} \right)$
- (c) $-\frac{R^2}{2\mu} \left(\frac{dp}{dx} \right)$
- (d) $-\frac{R^2}{\mu} \left(\frac{dp}{dx} \right)$

GATE-3. Ans. (a)

GATE-4. A fully developed laminar viscous flow through a circular tube has the ratio of maximum velocity to average velocity as [IES-1994, GATE-1994]
 (a) 3.0 (b) 2.5 (c) 2.0 (d) 1.5

GATE-4. Ans. (c) Ratio = $\frac{\text{Maximum velocity}}{\text{Average velocity}}$ for fully developed laminar viscous flow through a circular tube has value of 2.0

GATE-5. For laminar flow through a long pipe, the pressure drop per unit length increases. [GATE-1996]

- (a) In linear proportion to the cross-sectional area
 (b) In proportion to the diameter of the pipe
 (c) In inverse proportion to the cross-sectional area
 (d) In inverse proportion to the square of cross-sectional area

GATE-5. Ans. (d) $\frac{\Delta P}{L} = \frac{128\mu Q}{\pi D^4} \propto \frac{1}{D^4} \text{ i.e. } \propto \frac{1}{A^2}$

GATE-6. In fully developed laminar flow in a circular pipe, the head loss due to friction is directly proportional to..... (Mean velocity/square of the mean velocity). [GATE-1995]

- (a) True (b) False (c) Insufficient data (d) None of the above

GATE-6. Ans. (a) $h_f = \frac{32\mu u L}{\rho g D^2}$

Previous 20-Years IES Questions

IES-1. Which one of the following statements is correct? [IES-1996]
Hydrodynamic entrance length for

- (a) Laminar flow is greater than that for turbulent flow
 (b) Turbulent flow is greater than that for laminar flow
 (c) Laminar flow is equal to that for turbulent flow
 (d) A given flow can be determined only if the Prandtl number is known

IES-1. Ans. (a) Hydrodynamic entrance length for laminar flow is greater than that for turbulent flow.

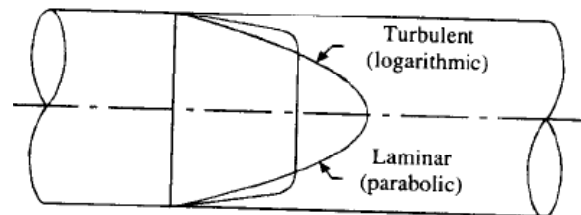


Fig. Velocity Distribution curves for laminar and turbulent flow

IES-2. Which one of the following statements is correct for a fully developed pipe flow? [IES-2009]

- (a) Pressure gradient balances the wall shear stress only and has a constant value.
 (b) Pressure gradient is greater than the wall shear stress.
 (c) The velocity profile is changing continuously.
 (d) Inertia force balances the wall shear stress.

IES-2. Ans. (a) Relationship between shear stress and pressure gradient

