LIMIT OF A FUNCTION OF SINGLE VARIABLE

Let $f(x) = \frac{x^2-4}{x-2}$

When $x = 2$, $f(x) = \frac{(2)^2-4}{2-2} = \frac{4-4}{2-2} = \frac{0}{0}$

When $x = 1.9$, $f(x) = \frac{(1.9)^2-4}{1.9-2} = \frac{3.61-4}{-0.1} = \frac{-0.39}{-0.1} = 3.9$

When $x = 1.99$, $f(x) = \frac{(1.99)^2-4}{1.99-2} = \frac{3.9601-4}{-0.01} = \frac{-0.0399}{-0.01} = 3.99$

When $x = 1.999$, $f(x) = \frac{(1.999)^2-4}{1.999-2} = \frac{3.996001-4}{-0.001} = \frac{-0.003999}{-0.001} = 3.999$

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Now let's try to use the value of x slightly greater than 2

When $x = 2.1$, $f(x) = \frac{(2.1)^2-4}{2.1-2} = \frac{4.41-4}{0.1} = \frac{0.41}{0.1} = 4.1$

When $x = 2.01$, $f(x) = \frac{(2.01)^2-4}{2.01-2} = \frac{4.0401-4}{0.01} = \frac{0.0401}{0.01} = 4.01$

When $x = 2.001$, $f(x) = \frac{(2.001)^2-4}{2.001-2} = \frac{4.004001-4}{0.001} = \frac{0.004001}{0.001} = 4.001$

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x: **1.9 → 1.99 → 1.999 → ------------------- → 2**

$f(x)$: **3.9 → 3.99 → 3.999 → -------------------- → 4**

2 ← -----------------← 2.001 ← 2.01 ← 2.1 : x

4 ← -----------------← 4.001 ← 4.01 ← 4.1 : $f(x)$

So we observe that if the value of x approaches to 2 either from left hand side or from right hand side on the number line, the value of the function $f(x)$ approaches to 4 from left hand side and as well as from right hand side on the number line while at x = 2 it is meaningless.

In Mathematics this typical situation is denoted as

$$\lim_{x\to 2} f(x) = 4.$$