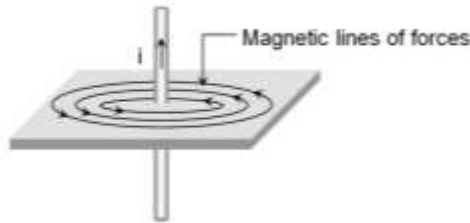


MAGNETIC EFFECT OF CURRENT & MAGNETISM

INTRODUCTION

The molecular theory of magnetism was given by Weber and modified later by Ewing. Oersted, in 1820 observed that a magnetic field is associated with an electric current. Since, current is due to motion of charges, it becomes mandatory to conclude that it is the moving charge which creates the field.



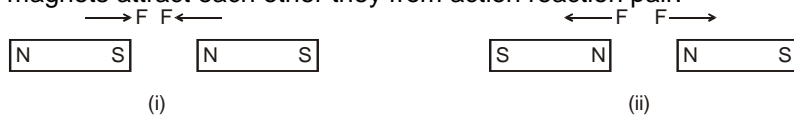
MAGNET

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. This force is called magnetic force. Those bodies are called magnets.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet.

Pole strength magnetic dipole and magnetic dipole moment :

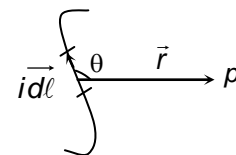
A magnet always has 'N' and 'S' and it is poles of two magnets repel each other and the unlike poles of two magnets attract each other they form an action reaction pair.



Magnetic Field

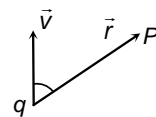
- (a) Biot-savart's law (\vec{B} due to a current element)

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right) \frac{id\vec{\ell} \times \vec{r}}{r^3}$$



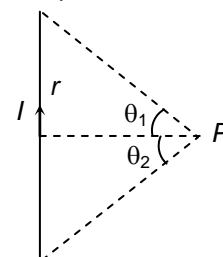
- (b) \vec{B} due to a moving point charge

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right) \frac{q\vec{v} \times \vec{r}}{r^3}$$

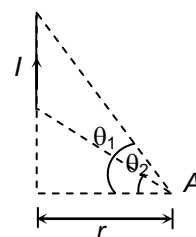


- (c) \vec{B} due to a straight current carrying wire

$$B = \frac{\mu_0 I}{4\pi r} (\sin\theta_1 + \sin\theta_2) \otimes$$



$$B \text{ at } A = \frac{\mu_0 I}{4\pi r} (\sin\theta_1 - \sin\theta_2) \otimes$$



- Case (i)** For infinite long straight current carrying wire
 $\theta_1 = \theta_2 = 90^\circ$

$$\therefore B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$

Case (ii) For semi infinite long straight current carrying wire

$$\theta_1 = 90^\circ, \theta_2 = 0 \quad \therefore B = \frac{\mu_0 I}{4\pi r}$$

Example 1 : Find the field strength at a distance R from an infinite straight wire that carries a current I.

Solution : Figure shows the infinitesimal contribution to the field, $d\vec{B}$ from an arbitrary current element. In order to integrate contributions of all the elements, we use the angle α , measure from the perpendicular, as the variable. From the diagram we see that

$$\begin{aligned} |d\vec{l} \times \hat{r}| &= dl \sin \theta \\ &= dl \sin \left(\frac{\pi}{2} + \alpha \right) = dl \cos \alpha \quad \dots (i) \end{aligned}$$

Since $l = R \tan \alpha$, we have

$$dl = R \sec^2 \alpha d\alpha. \quad \dots (ii)$$

Furthermore,

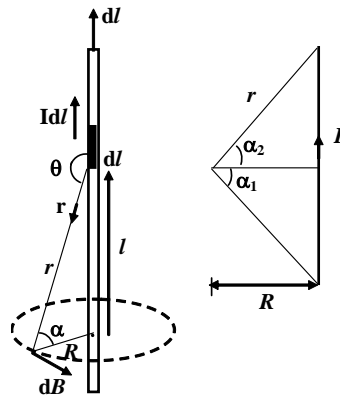
$$r = R \sec \alpha$$

On substituting these expression into equation and integrating we find

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi R} \int_{-\alpha_1}^{\alpha_2} \cos \alpha d\alpha \\ &= \frac{\mu_0 I}{4\pi R} (\sin \alpha_1 + \sin \alpha_2) \quad \dots (iii) \end{aligned}$$

Note: Those angles on opposite sides of the perpendicular are assigned opposite signs. This ensures that the contributions from either side of the wire add together rather than subtract. For an infinite wire, the limits of integration are $-\pi/2$ to $+\pi/2$. Thus

$$B = \frac{\mu_0 I}{2\pi R} \quad \dots (iv)$$



Calculating the magnetic field produced by a long straight wire we may use the angle α as the variable.

MAGNETIC FIELD DUE TO A CIRCULAR LOOP

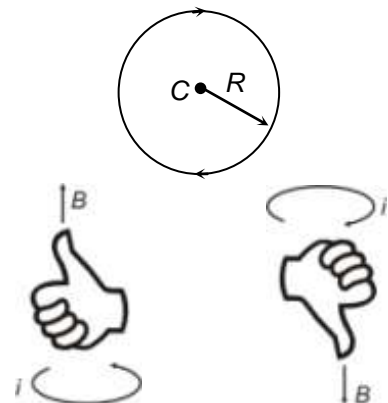
(d) \vec{B} due to a current carrying circular loop

(i) **At centre :** Due to each $d\vec{l}$ element of the loop \vec{B} at 'C' is inwards (in this case)

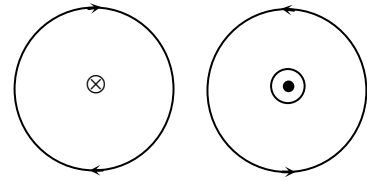
$$B = \frac{\mu_0 NI}{2R} \otimes$$

N = number of turns in the loop

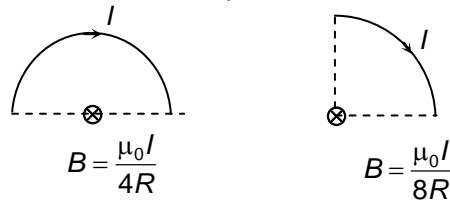
Direction of \vec{B} : The direction of the magnetic field at the centre of a circular wire can be obtained using the right hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.



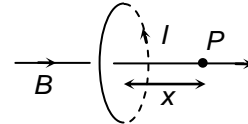
- (ii) Semicircular and quarter of a circle



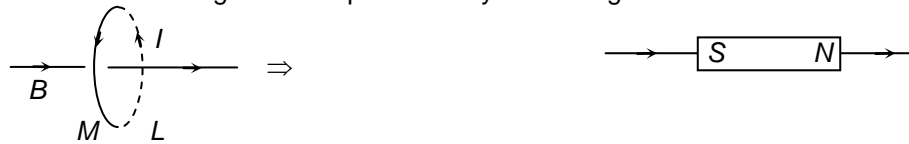
- (iii) On the axis of the loop

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

N = number of turns (integer)



- (e) **A loop as a magnet** : The pattern of the magnetic field due to current carrying circular loop is comparable with the magnetic field produced by a bar magnet.



The side 'L' (the side from which the \vec{B} emerges out) of the loop acts as 'NORTH POLE' and side M (the side in which the \vec{B} enters) acts as the 'SOUTH POLE'.

$$B_{\text{axis}} = \frac{\mu_0 N R^2}{2x^3} \text{ for } x \gg R$$

$$= \left(\frac{\mu_0}{4\pi} \right) \left(\frac{I N \pi R^2}{x^3} \right)$$

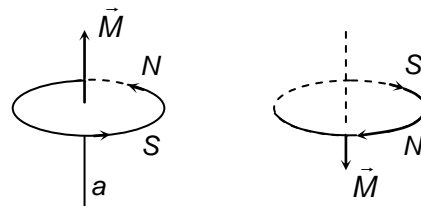
It is similar to B_{axis} due to magnet

$$= 2 \left(\frac{\mu_0}{4\pi} \right) \frac{m}{x^3}$$

Magnetic dipole moment of the loop

$$= I N \pi R^2 = I N A$$

(M)

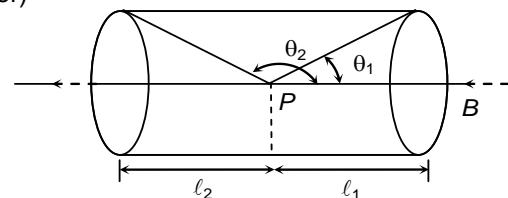


- (f) \vec{B} at the axis of a Solenoid

Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)

$$B = \frac{\mu_0 n i}{2} (\cos\theta_1 - \cos\theta_2)$$

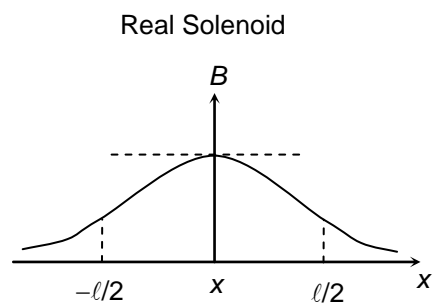
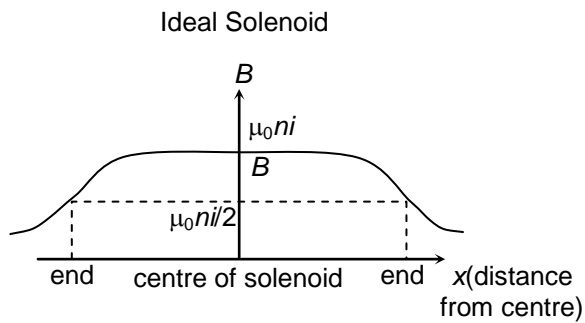
where n is number of turns per unit length



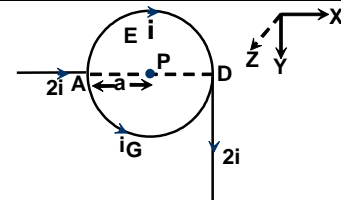
- (i) For Ideal solenoid (long solenoid)

$$B = \mu_0 n i$$

- (ii) Comparison between ideal and real solenoid



Example 2 : In the shown figure a current $2i$ is flowing in a straight conductor and entering along the diameter of the circular loop of similar conductor through the point A. The current is leaving the loop along another similar semi-infinite conductor parallel to the plane of the loop through the other opposite end D of the diameter.



What is the magnetic field at the centre of the loop P?

Solution : The magnetic field at the centre P due to the entering current along diameter is zero. The magnetic field at P due to the semicircular loop AED is

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{a} \hat{k} \quad \dots (1)$$

The magnetic field at P due to the semi-circular segment AGD is

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{a} (-\hat{k}) \quad \dots (2)$$

and the field at P due to the semi infinite straight conductor is given by

$$\vec{B}_3 = \frac{\mu_0}{4\pi} \cdot \frac{2i}{a} (-\hat{k}) \quad \dots (3)$$

The net field at P is

$$\begin{aligned} \therefore \vec{B} &= \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{a} \hat{k} - \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{a} \hat{k} - \frac{\mu_0}{4\pi} \cdot \frac{2i}{a} \hat{k} \\ &= -\frac{\mu_0}{4\pi} \cdot \frac{2i}{a} \hat{k} = \frac{\mu_0}{2\pi} \cdot \frac{i}{a} (-\hat{k}) \end{aligned}$$

\therefore The net magnetic field at P is $\frac{\mu_0 i}{2\pi a}$ along the perpendicular to the plane of the loop downward.

(g) Ampere's circuital law

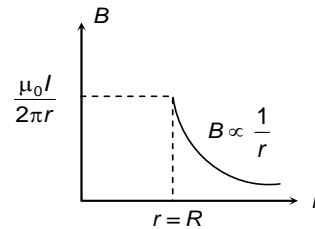
The line integral $\oint \vec{B} \cdot d\vec{\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

- (i) Line integral is independent of the shape of path and position of wire with in it.
- (ii) The statement $\oint \vec{B} \cdot d\vec{\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no nett current is passing through the path
- (iii) **Sign of current:** The current due to which \vec{B} is produced in the same sense as $d\vec{\ell}$ i.e. $\vec{B} \cdot d\vec{\ell}$ positive will be taken positive and the current which produces \vec{B} in the sense opposite to $d\vec{\ell}$ will be negative.

(h) \vec{B} due to hollow current carrying infinitely long cylinder : (I is uniformly distributed on the whole circumference)

- (i) for $r \geq R$; $B = \frac{\mu_0 I}{2\pi r}$
 (ii) $r < R$; $B_{in} = 0$



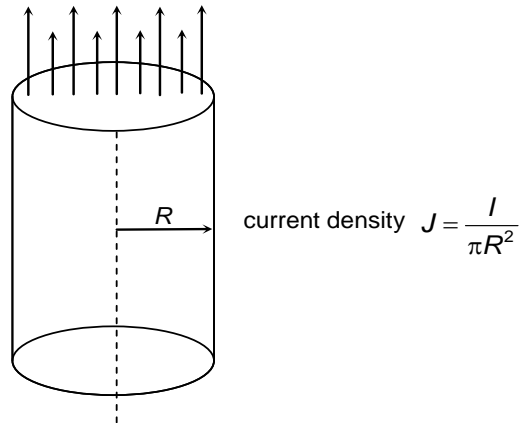
(i) \vec{B} due to solid infinite current carrying cylinder

Assume current is uniformly distributed on the whole section area

(i) $r \leq R$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2}$$

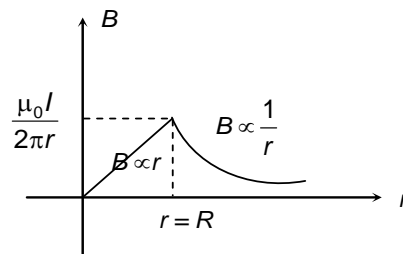
$$\Rightarrow \vec{B} = \frac{\mu_0 \vec{J} \times \vec{r}}{2}$$



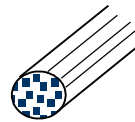
(ii) $r \geq R$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2} (\vec{J} \times \vec{r})$$



Example 3 : Suppose that the current density in a wire of radius a varies with r according to $J = Kr^2$, where K is a constant and r is the distance from the axis of the wire. Find the magnetic field at a point distance r from the axis when (a) $r < a$ and (b) $r > a$

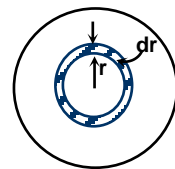


Solution: Choose a circular path centred on the axis of the conductor and apply Ampere's law.

(a) To find the current passing through the area enclosed by the path integrate

$$dI = J dA = (Kr^2) (2\pi r dr)$$

$$\Rightarrow I = \int dI = K \int_0^r 2\pi r^3 dr = \frac{K\pi r^4}{2}$$



Since $\int \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$$\Rightarrow B 2\pi r = \mu_0 \cdot \frac{\pi K r^4}{2} \Rightarrow B = \frac{\mu_0 K r^3}{4}$$

(b) If $r > a$, then net current through the Amperian loop is

$$I = \int_0^a Kr^2 2\pi r dr = \frac{\pi K a^4}{2}$$

$$\Rightarrow B = \frac{\mu_0 K a^4}{4r}$$

MAGNETIC FORCE ON MOVING CHARGE

(a) **Magnetic force acting on a moving charge particle in magnetic field**

When a charge q move with velocity \vec{v} , in magnetic field \vec{B} , then the magnetic force experienced by moving charge is given by formula:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$|\vec{F}| = qvB \sin \theta, \text{ where } \theta \text{ is angle between } \vec{v} \text{ and } \vec{B}$$

Note:

- (i) \vec{F} is perpendicular to both \vec{v} and \vec{B}
- (ii) Since \vec{F} is perpendicular to \vec{v} , power due to magnetic force on a charge particle is zero ($P = \vec{F} \cdot \vec{v}$).
- (iii) Since the work done by magnetic force is zero in every part of the motion, the magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. It can only change the direction of velocity.
- (iv) On a stationary charged particle, magnetic force is zero.
- (v) If $\vec{v} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

(b) Motion of charged particles under the effect of magnetic force

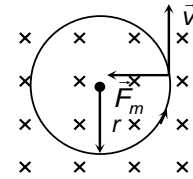
- (i) If particle is at rest
 $\therefore v = 0 \Rightarrow F_m = 0 \therefore$ particle will remain at rest

- (ii) $\vec{v} \parallel \vec{B}$ here $\theta = 0$ or $\theta = 180^\circ$
 $\therefore \vec{F}_m = 0 \Rightarrow \vec{a} = 0 \therefore \vec{v} = \text{constant}$
 \therefore particle will move in a straight line with constant velocity

- (iii) If \vec{v} is perpendicular to \vec{B} and \vec{B} is uniform

$$\vec{F}_m = qvB$$

Since \vec{F}_m is always perpendicular to \vec{v} path of charged particle will be a circle



$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2qVm}}{qB}$$

where, p = momentum of particle; K = kinetic energy of particle; V = accelerating potential.

$$T = \frac{2\pi m}{qB}, \omega = \frac{qB}{m}, f = \frac{qB}{2\pi m}$$

Note:

- (1) The plane of the circle is perpendicular to magnetic field. If the magnetic field is along z-direction, the circular path is in $x-y$ plane. The speed of the particle does not change in magnetic field.

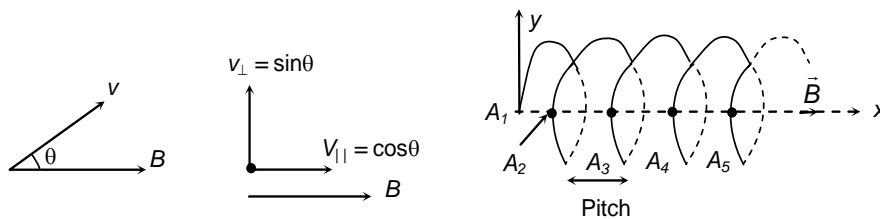
Hence, if v_0 be the particle, then velocity of particle at any instant of time will be

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \quad \text{where} \quad v_x^2 + v_y^2 = v_0^2$$

- (2) T , f and ω are independent of v while the radius is directly proportional to v .

- (iv) Angle between \vec{v} and \vec{B} is other than 0° , 90° , or 180° (Helical path)
 If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components $v_{||}$, parallel to the field and v_{\perp} , perpendicular to the field.

The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius r . The path is a helix.



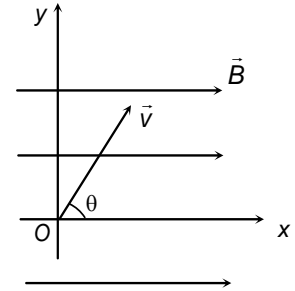
$$r = \frac{mv_{\perp}}{qB} = \frac{mv \sin \theta}{qB}$$

$$T = \frac{2\pi m}{qB}; \quad f = \frac{qB}{2\pi m}$$

$$\text{pitch (p)} = v_{\parallel} T = \frac{2\pi m v \cos \theta}{qB}$$

Note :

- Following points are worth noting in case of a helical path
- (1) The plane of the circle of the helix is perpendicular to magnetic field.
 - (2) The axis of the helix is parallel to magnetic field.
 - (3) The particle while moving in helical path in magnetic field touches the line passing through the starting point parallel to the magnetic field after every pitch.



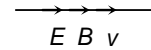
(c) Charged particle in uniform \vec{E} & \vec{B}

When a charged particle moves with velocity \vec{v} in an electric field \vec{E} and magnetic field \vec{B} , then. Net force experienced by it is given by following equation.

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

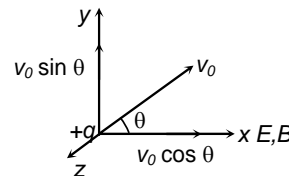
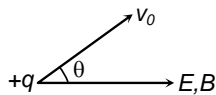
Combined force \vec{F} is known as Lorentz force

(i) $\vec{E} \parallel \vec{B} \parallel \vec{v}$

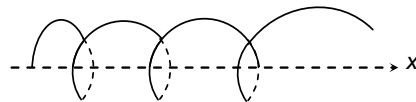


In above situation particle passes undeviated but its velocity will change due to electric field and magnetic force on it is zero.

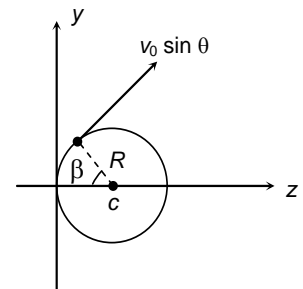
(ii) $\vec{E} \parallel \vec{B}$ and uniform, particle is released with velocity v_0 at an angle θ .



$$r = \frac{m v_0 \sin \theta}{qB}; \quad T = \frac{2\pi m}{qB}$$

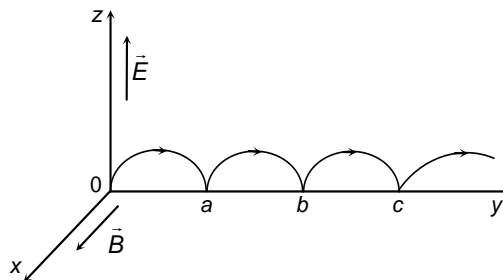


$$\vec{r} = \left\{ (v_0 \cos \theta)t + \frac{1}{2} \frac{qE}{m} t^2 \right\} \hat{i} + (R \sin \omega t) \hat{j} + (R - R \cos \omega t) (-\hat{k})$$



(d) Cycloid motion

Suppose that \vec{B} points in the x-direction, and \vec{E} in the z-direction.



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q(E\hat{z} + Bz\hat{y} - By\hat{z}) = m\vec{a} = m(\ddot{y}\hat{y} + \ddot{z}\hat{z})$$

$$\omega \equiv \frac{q\vec{B}}{m}$$

$$\ddot{y} = \omega \dot{z}, \quad \ddot{z} = \omega \left(\frac{E}{B} - \dot{y} \right)$$

Their general solution is

$$\left. \begin{aligned} y(t) &= C_1 \cos \omega t + C_2 \sin \omega t + (E/B)t + C_3 \\ z(t) &= C_2 \cos \omega t - C_1 \sin \omega t + C_4 \end{aligned} \right\}$$

$$y(t) = \frac{E}{\omega B} (\omega t - \sin \omega t), \quad z(t) = \frac{E}{\omega B} (1 - \cos \omega t)$$

$$R \equiv \frac{E}{\omega B}$$

$$(y - R\omega t)^2 + (z - R)^2 = R^2$$

$$v = \omega R = \frac{E}{B}$$

The particle moves as though it were a spot on the rim of a wheel, rolling down the y axis at speed, v . The curve generated in this way is called a cycloid. Notice that the overall motion is not in the direction of \vec{E} , but perpendicular to it.

Example 4 : A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge +ve and mass 1.67×10^{-27} kg is projected through the field in the + Y direction with a speed of 4.8×10^6 m/s.

- Find the force on the charged particle in magnitude and direction
- Find the force if the particle were negatively charged.
- Describe the nature of path followed by the particle in both the cases.

Solution : (a) $\vec{F} = q\vec{v} \times \vec{B} = e (\vec{v}\vec{j}) \times (B\vec{i}) = e v B (-\vec{k})$
 $= (1.6 \times 10^{-19}) (4.8 \times 10^6) (30 \times 10^{-3}) \sin 90^\circ$
 $= 2.3 \times 10^{-14}$ N.

The direction of the force is in the (-z) direction.

- If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.

In this case, $\vec{V} = -V\vec{j}$

- As $V \perp B$, the path described is a circle, where radius is given by

$$R = mV/qB = (1.67 \times 10^{-27}) (4.8 \times 10^6) / (1.6 \times 10^{-19}) (30 \times 10^{-3})$$

$$= 1.67 \text{ m.}$$

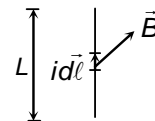
MAGNETIC FORCE ON A CURRENT CARRYING WIRE

Suppose a conducting wire, carrying a current i , is placed in a magnetic field \vec{B} . Consider a small current element $i \vec{d\ell}$ of the wire (figure).

The magnetic force acting on this current element $i \vec{d\ell}$ is given by

$$d\vec{F} = i \vec{d\ell} \times \vec{B}$$

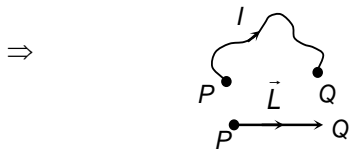
$$\vec{F} = i \int \vec{d\ell} \times \vec{B}$$



If \vec{B} is uniform then,

$$\vec{F} = i \left(\int \vec{d\ell} \right) \times \vec{B} \Rightarrow \vec{F} = i \vec{L} \times \vec{B}$$

Here $\vec{L} = \int \vec{d\ell}$ = vector length of the wire = vector connecting the end points of the wire



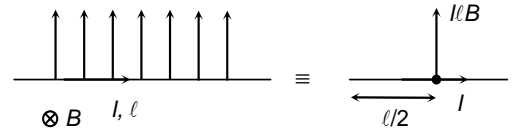
Note :

(a) If a current loop of any shape is placed in a uniform \vec{B} then \vec{F} on it is 0 ($\because \vec{L} = 0$)

(b) Point of application of magnetic force

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.

This can be used for calculation of torque.



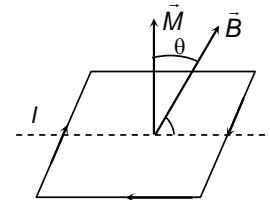
CURRENT LOOP IN UNIFORM MAGNETIC FIELD

$$\vec{\tau} = \vec{M} \times \vec{B}$$

$$|\vec{\tau}| = MB \sin \theta; \text{ where } M = NIA$$

Work done in rotating loop in uniform field from θ_1 to θ_2

$$W = MB(\cos \theta_1 - \cos \theta_2)$$



Example 5 : A uniformly charged disc whose total charge has magnitude q and whose radius is r rotates with constant angular velocity of magnitude ω . What is the magnetic dipole moment?

Solution : The surface charge density is $q/\pi r^2$. Hence the charge within a ring of radius R and width dR is

$$dq = \frac{q}{\pi r^2} (2\pi R dR) = \frac{2q}{r^2} (R dR)$$

The current carried by this ring is its charge divided by the rotation period,

$$di = \frac{dq}{2\pi/\omega} = \frac{q\omega}{\pi r^2} [R \cdot dR]$$

The magnetic moment contributed by this ring has the magnitude $dM = a |di|$, where a is the area of the ring.

$$dM = \pi R^2 |di| = \frac{q\omega}{r^2} \cdot R^3 \cdot dR$$

$$M = \int dM = \int_{R=0}^r q \frac{\omega}{r^2} (R^3 dR) = q\omega \frac{r^2}{4}$$

