NEWTONS LAWS OF MOTION AND FRICTIONS STRAIGHT LINES

INTRODUCTION

In this chapter, we shall study the motion of bodies along with the causes of their motion assuming that mass is constant. In addition, we are going to treat the translational motion of rigid bodies. Before we start doing anything else and since this chapter is about the Laws of Motion, we are going to state these laws first. Later we are going to discuss its applications in Statics and Dynamics.

FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

(1) may change only speed

(2) may change only direction of motion.

(3) may change both the speed and direction of motion.

(4) may change size and shape of a body

Unit of force : newton and $\frac{kg \cdot m}{s^2}$ (MKS System)dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 newton = 10^5 dyne

Dimensional Formula of force : $[M\ L\ T^{-2}]$

Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

Gravitational Force

The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.

Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.

(b) These are long range forces

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

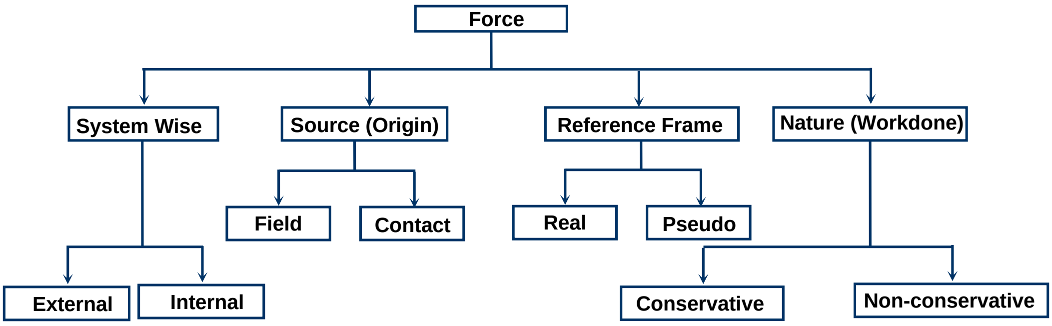
Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

Weak Force

It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

KINDS OF FORCES



The first classification is self explanatory.

All forces in case of dynamics of a particle can be classified in two ways w.r.t. its origin as:

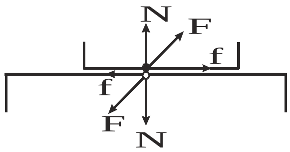
(a) contact forces, and (b) non-contact forces.

Non-contact force

Those forces which do not require contact between the bodies to act, for example gravitational force, electro-magnetic force etc., are known as non-contact forces.

Contact Force (F)

Forces which act when bodies are in contact are known as contact forces. It is usually convenient to resolve contact forces into components, one parallel to the surface of contact, the other perpendicular to the surface of contact.



Normal Force or Normal Reaction (N):

The component of the contact force that is perpendicular to the surface of contact is known as the Normal reaction force. It measures how strongly the surfaces in contact are pressed together.

Friction Force (f):

It is the component of the contact force parallel to the surface of contact. The direction of the force of friction is opposite to the direction of relative motion between the surfaces, or is such as to oppose any tendency of relative motion between the surfaces.

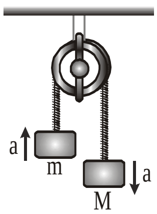
Example 1 : Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

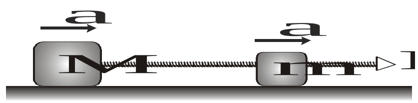
Solution : Block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

TENSION:

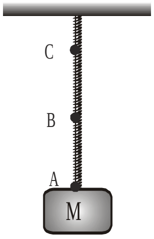
When a body is connected by means of a string or a rope, a force may be exerted on the body by the string or the rope. This force is called tension.

(i) If a string is ideal (inextensible and massless), then the magnitude of the accelerations of any number of masses connected through this string is always same.



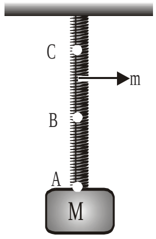


(ii) If a string is massless, the tension in it is same everywhere; on the other hand if a string is not massless (i.e. its mass is not negligible), tension at different points may be different.



String is massless i.e

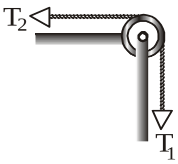
$T_A = T_B = T_C$



String is not massless i.e.

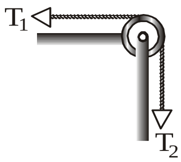
$T_A \neq T_B \neq T_C$ and $T_A < T_B < T_C$

(iii) If friction is present between the pulley and the string, tension is different on the two sides of the pulley But if the friction is absent between the pulley and the string, tension will be same on both sides of the pulley, provided the pulley is massless.



There is no friction between pulley and string i.e.,

$T_1 = T_2$



There is friction between pulley and string i.e.,

$T_1 \neq T_2$

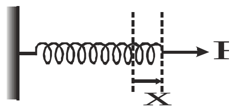
ELASTIC FORCE

An ideal spring follows Hooke's law which says that the force applied by a springs on bodies connected to it is proportional to its extension or compression (change in length from its natural length).

Consider a light spring which is connected to a vertical wall as shown in the figure. Suppose that it is pulled to the right by means of a force *F,* which causes the spring to get elongated by *x* over its natural length instantaneous rest to ring force f will be equal to applied force and opposite direction..

Then, $f \propto -x$

$\Rightarrow \quad f = -kx$



where *k* is a constant that is characteristic of the spring also known as the spring constant.

NEWTON'S LAWS OF MOTION

First Law of Motion:

Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled by an external impressed force to change that state.

Second Law of Motion:

The rate of change of momentum of a body is proportional to the impressed force, and takes place in the direction in which the force acts.

$$\vec{F} \propto \frac{d\vec{P}}{dt}$$

$$\vec{F} = \frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$, if *m* is a constant xz

or, $$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$, if *m* is not constant.

REMARKS	
1.	The direction of motion of a particle does not in general coincide with the **direction of the force acting** on it. It is the rate of change of velocity, which is related to the force.
2.	The acceleration must be measured with respect to an **inertial reference frame.**

Example 2 : A body of mass m = 1 kg falls from a height h = 20 m from the ground level. What is the magnitude of total change in momentum of the body before it strikes the ground?

Solution : Since the body falls from rest (u = 0) through a distance h before striking the ground, the speed v of the body is given by kinematical equation.

$v^2 = u^2 + 2as$; Putting $a = g$ and $s = h$, we obtain

$v = \sqrt{2gh}$

$\Rightarrow$ The magnitude of total change in momentum of the body

$\Delta p = | mv - 0 | = mv$, where $v = \sqrt{2gh}$

$\Rightarrow \Delta p = m\sqrt{2gh} = (1)\sqrt{(2 \times 10 \times 20)}$ kg-m/sec

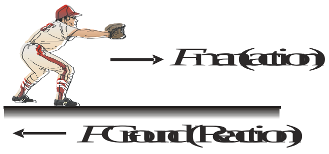
$\Rightarrow \Delta p = 20$ kg-m/sec.

Third Law of Motion:

To every action there is an equal and opposite reaction.

Here by action and reaction, one means force only, if an object exerts a force *F* on a second, then the second object exerts an equal but opposite force in the opposite direction.

$$|F_{man}| = |F_{ground}|.$$



Significance of Newton's Laws:

(a) The first law tells us about the natural state of a body, which is in motion along a straight line with constant speed or rest. It is also known the law of inertia.

(b) The second law tells us that if a body does not follow its natural state of motion then it is under the influence of other bodies, that is, a net unbalanced force must be acting on it.

(c) The third law tell us about the nature of force, that is, force exist in pairs.

REMARKS

1. The acceleration must be measured with respect to an inertial reference frame.
2. The direction of motion of a particle does not in general coincide with the direction of the force acting on it. It is the rate of change of velocity, which is related to the force.

Example 3 : A horse refuses to pull a cart. The horse reasons, "according to Newton's third law, whatever force I exert on the cart, the cart will exert an equal and opposite force on me so the resultant force will be zero and I will have no chance of accelerating the cart." What is wrong with this reasoning?

Solution :

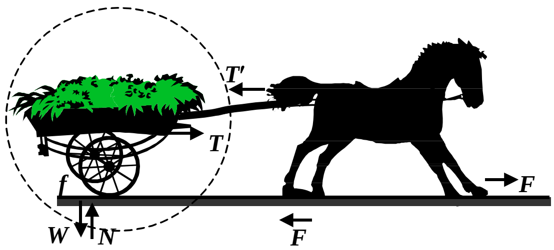


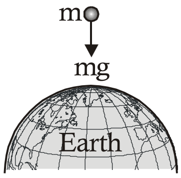
Figure Horse pulling a cart. The cart will accelerate to the right if the force ***T*** exerted on it by the horse is greater than the frictional force ***f*** exerted on the cart by the ground. The force ***T'*** is equal and opposite to ***T***, but because it is exerted on the horse it has no effect on the motion of the cart.

Figure is a sketch of a horse pulling the cart. Since we are interested in the motion of the cart, we have *circled it* and indicated the forces acting on it. The force exerted by the horse is labelled ***T***. Other forces on the cart are its weight *W*, the vertical support force of ground ***N***, and the horizontal force exerted by the ground labeled ***f*** (for friction).

The vertical forces ***W*** and ***N*** balance each other. The horizontal forces are ***T*** to the right and ***f*** to the left. The cart will accelerate if ***T*** is greater than ***f***. ***T'*** is the reaction force exerted on the horse, not the cart. It has no effect on the motion of the cart. It does effect the motion of the horse. If the horse is to accelerate to the right, there must be a force ***F*** (to the right) exerted by the ground on the horse's feet that is greater than ***T'*** . This example illustrates the importance of a simple diagram (free body diagram) in solving mechanics problems. Had the horse drawn a simple diagram, he would have seen that he need only push back hard against the ground so that the ground would push him forward.

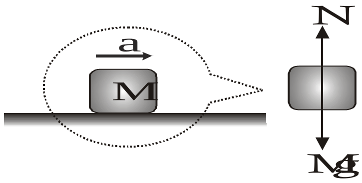
FREE BODY DIAGRAM

In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.



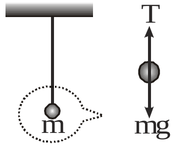
(A)

F.B.D. of a particle in gravitational field. The earth pulls the particle of mass *m* by a force mg.



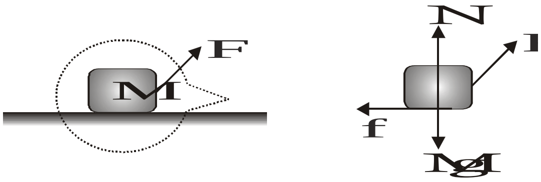
(B)

F.B.D. of a block placed on a horizontal surface. Two vertical forces act on the block : The earth pulls the block downward by mg, and the surface pushes the block upward by *N*.



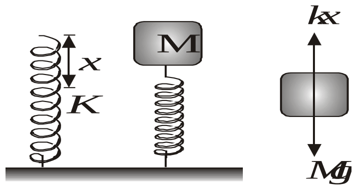
(C)

F.B.D. of a ball suspended by a string. Two vertical forces act on the ball : The earth pulls the ball downward by mg and the string pulls the ball upwards by *T*.



(D)

F.B.D. of a block placed on a rough surface being pulled by an external force. There are four forces acting on the block: the gravitational pull Mg; the normal reaction *N*; the external force *F*; and the tangential force of friction *f.*



(E)

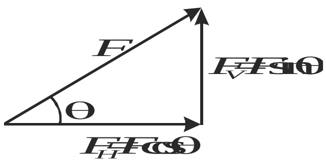
F.B.D. of a block supported by a spring of stiffness *k*. Two vertical forces act on the block: the gravitational pull, and the spring force *kx*.

EQUILIBRIUM

A system is said to be in equilibrium if it does not tend to undergo any further change of its own, i.e. any further change must be produced by external means (e.g. forces) i.e., forces which have zero linear resultant and zero torque. Such forces (and the object) are said to be in equilibrium.

Resolution of a Force:

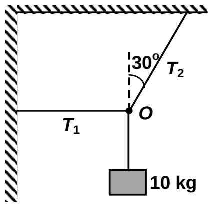
When a force is replaced by an equivalent set of components, it is said to be resolved.



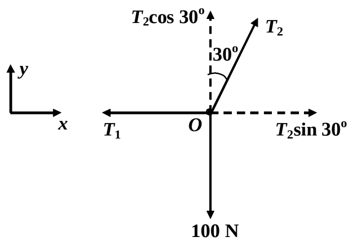
REMARKS
Mathematically a body is said to be in equilibrium if (a) net force acting on it is zero i.e. $\vec{F}_{net} = 0$ (b) net moments of all the forces acting on it about any axis is also zero. If for a body $\vec{F} = 0$ i.e. the body is said to be in translational equilibrium $\vec{F} = 0$ i.e., $m\vec{a} = 0$ $m.\frac{d\vec{v}}{dt} = 0$ (as *m* is assumed to be a constant). Or $\vec{v} = \text{const.}$ i.e., if a body is in translational equilibrium it will be either at rest or in uniform motion in a straight line. If it is at rest, the equilibrium is called static.

STEPS TO SOLVE THE PROBLEM
1. Make a simple sketch showing the body under consideration. 2. Identify the forces acting on the body, draw arrows on your sketch to show the direction of each force acting on the body. 3. Choose a coordinate system and resolve the forces into components that are parallel to the coordinate axes. 4. Write the equation for equilibrium along each axis of the co-ordinate system. 5. Solve the equation for the required unknown(s).

Example 4 : A block of mass 10 kg is suspended with two strings, as shown in the Figure. Find the tension in each string. ($g = 10\ m/s^2$)



Solution : The free body diagram of the joint O is drawn as shown in the Figure



F.B.D. of the joint *O*

Applying ***equations for equilibrium.***

$\Sigma F_x = 0$ $\quad T_2 \sin 30° - T_1 = 0$ $\quad$ ..(i)

$\Sigma F_y = 0$ $\quad T_2 \cos 30° - 100 = 0$ $\quad$ ..(ii)

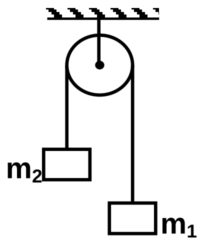
Thus, $\quad T_2 = \dfrac{100}{\cos 30^0} = \dfrac{200}{\sqrt{3}}$ N

Substituting the value of T_2 in equation (i), we get

$T_1 = T_2 \sin 30° = \dfrac{100}{\sqrt{3}}$ N

Example 5 :

Two blocks of mass m_1 and m_2 are attached at the ends of an inextensible string, which passes over a smooth massless pulley. If $m_1 > m_2$, find

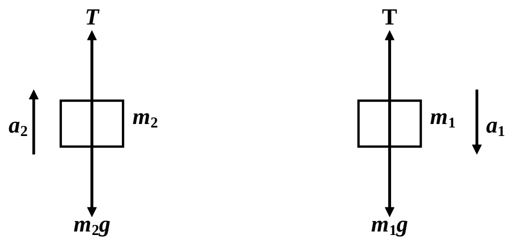


(a) the acceleration of each block

(b) the tension in the string.

Solution :

The *free body diagram* of each block is shown in the figure.



Note

- The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction of motion. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the bodies move in the opposite directions.
- Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

Applying Newton's second Law on

Block m_1 $\quad m_1g - T = m_1a_1$ $\quad$ ..(i)

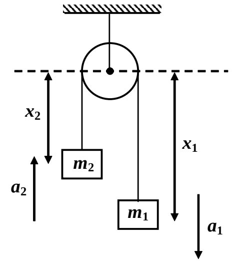
Block m_2 $\quad -m_2g + T = m_2a_2$ $\quad$ ..(ii)

Number of unknowns : T, a_1 and a_2 (**three**)

Number of equations : only **two**

Obviously, we require one more equation to solve the problem. **Note** that the whenever one finds the number of equations less than the number of unknowns, one must think about the **constraint relation**. In the previous problems we have obtained the constraint relation by experience and judgment. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?



Position of each block is located w.r.t. centre of the pulley.

1. Assume the direction of acceleration of each block, e.g. a_1 (*downward*) and a_2 (*upward*) in this case.
2. Locate the position of each block from a fixed point (depending on convenience), e.g. *centre of the pulley* in this case.
3. ***Identify the constraint*** and write down the equation of constraint in terms of the distance assumed.

For example, in the chosen problem, ***the length of string remains constant*** is the constraint or restriction**.**

Thus, $x_1 + x_2 = \text{constant}$

Differentiating both the sides w.r.t. time, we get

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Each term on the left side represents velocity of the block.

Since we have to find a relation between accelerations, therefore, we differentiate it once again w.r.t. time.

Thus $$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

Since the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1, \qquad a_1 > 0$$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

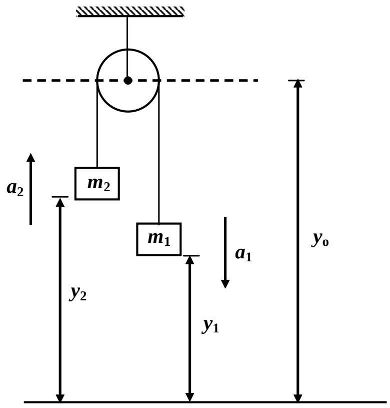
$$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2, \qquad a_2 > 0$$

Thus $a_1 - a_2 = 0$

or $a_1 = a_2 = a$

Alternatively, one can also assume the distance of each block from the ground as shown in the figure.

One can easily see that an expression for length of the string can not be written unless we locate the centre of the pulley w.r.t. ground.



If y_0 be the distance of the ground from the centre of the pulley then the length of string is

$$(y_0 - y_1) + (y_0 - y_2) = \text{constant}$$

$$2y_0 - y_1 - y_2 = \text{constant}$$

Differentiating twice w.r.t. time, we get

$$0 - \frac{d^2y_1}{dt^2} - \frac{d^2y_2}{dt^2} = 0$$

Now, $\frac{d^2y_1}{dt^2} = -a_1$ and $\frac{d^2y_2}{dt^2} = +a_2$

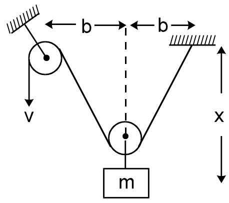
Thus $a_1 - a_2 = 0$

$a_1 = a_2 = a$

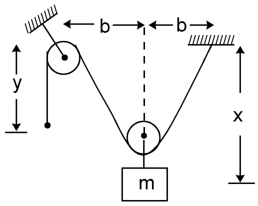
Substituting $a_1 = a_2 = a$ in equations (i) and (ii) and after solving them, we get

$$a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]g \qquad \text{also} \qquad T = \left[\frac{2m_1 m_2}{m_1 + m_2}\right]g$$

Example 6 : The figure shows one end of a string being pulled down at constant velocity v. Find the velocity of mass 'm' as a function of 'x'.



Solution :



Using constraint equation

$2\sqrt{x^2+b^2}$ + y = length of string = constant

Differentiating w.r.t. time : $\frac{2}{2\sqrt{x^2+b^2}} \cdot 2x \left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right) = 0$

$\left(\frac{dy}{dt}\right) = v$

$\therefore \quad \left(\frac{dx}{dt}\right) = -\frac{v}{2x}\sqrt{x^2+b^2}$

REFERENCE FRAMES

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) Inertial reference frame: Frame of reference moving with constant velocity.

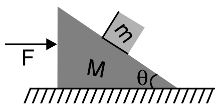
(b) Non-inertial reference frame: A frame of reference moving with non-zero acceleration.

$\vec{F} - m\vec{\alpha} = m\vec{a}$

where *a* is the acceleration of the body relative to the non-inertial frame. The term $-m\alpha$ is an example of a pseudo-force.

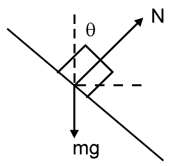
In general, the 'Second Law' takes the form: $\vec{F} + \vec{F}_P = m\vec{a}$, in a non-inertial frame.

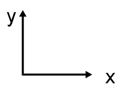
Example 7 : All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.



Solution :

Acceleration of (block + wedge) is a = $\frac{F}{(M+m)}$





Let us solve the problem by using both frames.

From inertial frame of reference (Ground)

F.B.D. of block w.r.t. ground (Apply real forces):

with respect to ground block is moving with an acceleration 'a' .

$\therefore \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ......(i)

and $F_x = ma \Rightarrow N\sin\theta = ma$ .....(ii)

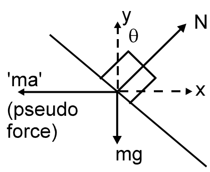
From Eqs. (i) and (ii)

a = g tan θ

$\therefore$ F = (M + m) a = (M + m) g tan θ

From non–inertial frame of reference (Wedge) :

F.B.D. of block w.r.t . wedge (real forces + pseudo force)



w.r.t. wedge, block is stationary

$\therefore \Sigma Fy = 0 \Rightarrow N\cos\theta = mg$ ....(iii)

$Fx = 0 \Rightarrow N\sin\theta = ma$ ....(iv)

From Eqs. (iii) and (iv) , we will get the same result

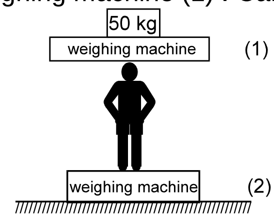
i.e. F = (M + m) g tan θ .

WEIGHING MACHINE :

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Example 7 :

A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another same weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1) . Calculate the readings of weighing machines (1) and (2) .



Answer : 500 N, 1150 N.

FRICTION

Whenever the surface of a body slides over that of another, each body exerts a force of friction on the other parallel to the surfaces. The **force of friction** on each body is in a direction opposite to its motion relative to the other body.

The **force of friction** comes into action only when there is a relative motion between the two contact surfaces or when an attempt is made to have it.

It is a self adjusting force, it can adjust its magnitude to any value between zero and the limiting (maximum) value i.e

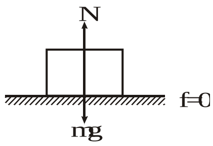
$0 \le f \le f_{max}$

The frictional force acting between any two surfaces at rest with respect to each other is called the force of **static friction.** And the frictional force acting between surfaces in relative motion with respect to each other is called the force of kinetic **friction or sliding friction.**

TYPES OF FORCE OF FRICTION

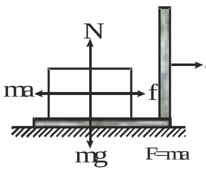
Friction acting between two bodies the relative motion is known as kinetic friction while the force of friction acting between bodies, opposing the onset of relative motion is known as static friction.

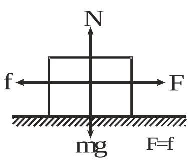
The force of friction acting between layer of fluids in known as viscous force. It is worth noting that :



(i) If a body is at rest on a rigid horizontal surface and no pulling force is acting on it, force of friction on it is zero.

(ii) Now if a force is applied to pull the body and it does not move, the force of friction which acts on it is equal in magnitude and opposite in direction to the applied force i.e. friction is a self-adjusting force. Since the body is at rest, the friction is called static friction.





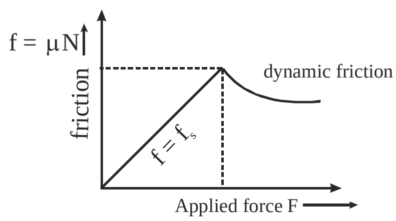
(iii) If the applied force is increased, the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum force of static friction upto which body does not move is called limiting friction. Thus, the force of static friction is a self-adjusting force with an upper limit which is known as the limiting force of static friction.

(iv) This limiting force of friction (F_L) is found experimentally to proportional to the force of normal reaction i.e.

$$f_L \propto N$$

or, $$f_L = \mu_s N$$

where μ_s is a dimensionless constant called coefficient of static friction which depends upon the nature of surfaces in contact.



If the applied force is increased above its limiting value, it is observed that relative motion occurs. The force of friction opposing motion is known as kinetic friction (sliding friction).

$$f_K = \mu_K N$$

where μ_K is coefficient of kinetic friction and is less than μ_s.

(v) According to laws of friction, the force of kinetic friction is independent of the area of contact and independent of the relative velocities of the bodies.

REMARKS

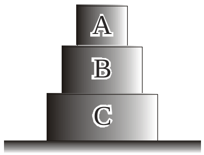
1. In problems if μ_s and μ_K are separately not given but only a single μ is given then use

$$f_K = \mu_K N = \mu N \text{ and } f_L = \mu_S N = \mu N$$

2. If more than two blocks are placed one over the other on a the ground then normal reaction between the two blocks will be equal to the weight of the blocks over the common surface.

For example $N_1 = m_A g$ (Normal reaction between *A* and *B*)

$N_2 = (m_B + m_A)g$ (Normal reaction between *B* and *C*)

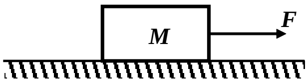


Example 8 :

A block of mass M = 10 kg is placed at rest on a horizontal surface as shown in the figure. The coefficient of friction between the block and the surface is μ_s = 0.3 and μ_k = 0.2. It is pulled with a horizontal force F.

Find the magnitude of the friction if

(a) F = 20 N (b) F = 40 N



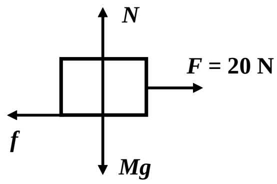
Solution :

The maximum value of friction force is

$f_{s(max)} = \mu_s N = \mu_s Mg$

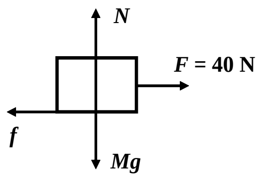
or $f_{s(max)}$ = (0.3) (10) (10) = 30 N

(a) To keep the block *stationary* the magnitude of friction force should be = F = 20 N since $f < f_{max}$. Therefore the force of friction is f =20 N



(b) $\because$ $f_{s(max)} < F$

so the body moves and now force of friction $f = \mu_k N$ = 0.2(10)(g) = 20 N



Note that in this case friction force is unable to keep the block stationary and the block accelerates with

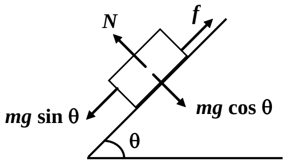
$$a = \frac{F - f_k}{M} = \frac{40-20}{10} = 2 \text{ m/s}^2$$

Note *that friction force* is not always equal to $\mu_s N$. It is the *limiting* or *maximum value* of static friction. At any stage friction may attain any value between 0 and $\mu_s N$.

$$0 \le f \le \mu_s N$$

ANGLE OF FRICTION

Suppose a body is placed on an inclined surface whose angle of inclination θ varies between 0 to $\pi/2$. The *coefficient of friction* between the body and the surface is μ_s. Let the initial value of θ be zero and if we slowly start increasing the value of θ, then at a particular value of $\theta = \phi$ the block *just starts to move.* This value of $\theta = \phi$ is called the ***angle of friction.***



A block of mass m is placed on an incline whose inclination may be varied between 0 to $\pi/2$. When $\theta = \phi$ the friction force is maximum and block just starts sliding

Mathematically, if the block is just about to move, then $mg \sin\theta = f$

When $\theta = \phi$, $mg \sin\phi = f_{max}$

Or $mg \sin\phi = \mu_s N = \mu_s mg \cos\phi$ or $\tan\phi = \mu_s$

Thus $\phi = \tan^{-1}\mu_s$

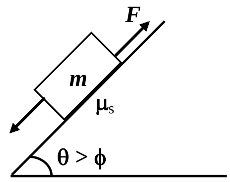
The angle of friction is that minimum angle of inclination of the inclined plane at which a body placed at rest on the inclined plane is about to slide down.

When $\theta \leq \phi$ (or $\tan^{-1}\mu_s$) the body is in equilibrium.

When the angle of inclination is more than the angle of friction ($\theta > \phi$) the block starts sliding down with acceleration. And, if we wish to keep it in equilibrium an external force has to be applied.

Example 9 :

A block of mass m is placed at rest on an inclined plane whose angle with the horizontal is more than the angle of friction ($\theta > \phi$). An external force parallel to the inclined plane is applied in the upward direction to keep it in equilibrium. Find the magnitude of the force F.

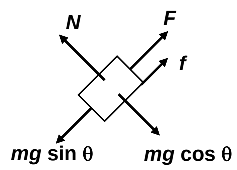


Solution

Since the block has a tendency to move downward, the force of friction acts upward on the block as shown in its *free body diagram.*

Applying equation of equilibrium,

$$N = mg \cos\theta$$



Free Body Diagram of the block when friction force is acting at its maximum value

$$F + f_{max} = mg \sin\theta$$

or $\quad F = mg \sin\theta - f_{max}$

$$= mg \sin\theta - \mu_s mg \cos\theta$$

or $\quad F = mg(\sin\theta - \mu_s \cos\theta)$

Note that this is the minimum magnitude of force required to keep it in equilibrium. If we apply a force slightly more than this the block does not start moving up but the magnitude of the friction force gets reduced. It becomes equal to zero when the external force attain a value equal to $F = mg \sin\theta$, as shown in Figure.

If the magnitude of F is further increased then the block has a tendency to move upward; the direction of friction force gets reversed. The block will not start moving up unless the external force attains the maximum value.

The free body diagram of the body is shown in **Figure**

Applying the equations of equilibrium

$$N = mg \cos\theta$$

$$F_{max} = mg \sin\theta + f_{max}$$

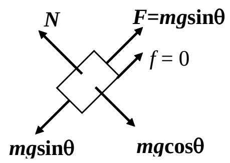
or $\quad F_{max} = mg(\sin\theta + \mu_s \cos\theta)$

Conclusion

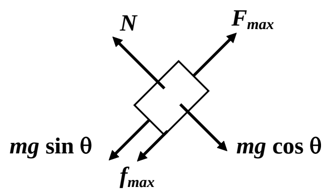
The block remains stationary if

$$F_{min} \leq F \leq F_{max}$$

or $\quad mg(\sin\theta - \mu_s \cos\theta) \leq F \leq mg(\sin\theta + \mu_s \cos\theta)$



When $F = mg \sin\theta$, $f = 0$



The body has a tendency to move upward. The friction force acts downward.