WORK, ENERGY AND POWER STRAIGHT LINES

INTRODUCTION :

The term 'work' as understood in everyday life has a different meaning in scientific sense. If a coolie is carrying a load on his head and waiting for the arrival of the train, he is not performing any work in the scientific sense. In the present study, we shall have a look into the scientific aspect of this most commonly used term i.e., work.

WORK

(a) Work in terms of rectangular components

$$\therefore \quad \vec{F}.\vec{S} = F_x S_x + F_y S_y + F_z S_z$$

(b) Work done by a variable force

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_B}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

Example 1 : A particle moving in the *xy* plane undergoes a displacement $\vec{s}$ = (8.0 $\hat{i}$ + 6.0 $\hat{j}$) m while *a* constant force *F* = (4.0 $\hat{i}$ + 3.0 $\hat{j}$) *N* acts on the particle.

(a) Calculate the magnitude of the displacement and that of the force.
(b) Calculate the work done by *the force*.

Solution: (a) $s = \sqrt{x^2 + y^2} = \sqrt{(8.0)^2 + (6.0)^2}$ = 10 m

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.0)^2 + (3.0)^2} =$ 5 N

(b) Work done by force, *W* = $\vec{F} \cdot \vec{s}$

= (4.0 $\hat{i}$ + 3.0 $\hat{j}$) · (8.0 $\hat{i}$) + 6.0 $\hat{j}$) N.m

= 32 + 0 + 0 + 18 = 50 N.m = 50J

Example 2 : A force *F* = (4.0 *x* $\hat{i}$ + 3.0 y$\hat{j}$) N acts on a particle which moves in the x-direction from the origin to *x* = 5.0 m. Find the work done on the object by the force.

Solution: Here the work done is only due to *x* component of force because displacement is along *x*-axis.

i.e., $W = \int_{x_1}^{x_2} F_x dx$

$$= \int_0^5 4x \, dx = \left[2x^2\right]_0^5 = 50 \text{ J}$$

POWER

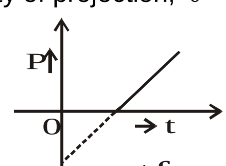
A unit power is the power of an agent which does unit work in unit time.
The power of an agent is said to be one watt if it does one joule of work in one second.
1 watt = 1 joule/section = 10^7 erg/second

Also, 1 watt = $\frac{1\,\text{newton} \times 1\,\text{metre}}{1\,\text{second}}$ = 1 N m s^{-1} .

Dimensional formula of power : $[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2\,T^{-3}]$

Power has 1 dimension in mass, 2 dimensions in length and – 3 dimensions in time.

S.No.	Human Activity	Power (W)
1	Heart beat	1.2
2	Sleeping	83
3	Sitting	120
4	Riding in a car	140
5	Walking (4.8 km h^{-1})	265
6	Cycling (15 km h^{-1})	410
7	Playing Tennis	440
8	Swimming (breaststroke, 1.6 km h^{-1})	475
9	Skating	535
10	Climbing Stairs (116 steps min^{-1})	685
11	Cycling (21.3 km h^{-1})	700
12	Playing Basketball	800
13	Tube light	40
14	Fan	60

UNIT OF WORK / POWER	POWER
1 Joule = 10^7 erg 1 erg = 10^{-7} joule 1 ev = 1.6 × 10^{-19} J 1 kwh = 3.6 × 10^6 J 1 joule = 6.25 × 10^{12} Mev 1 Joule sec^{-1} = 1 watt 746 watt = 1 hp	Power = $P = \frac{dw}{dt} = \frac{dk}{dt} = \vec{F} \times \vec{V}$ = rate of doing work; power delivered in projectile motion P is given by $P = mg^2t + (-\,mgu \sin\theta)$ Where u = velocity of projection, θ = angle of projection. $P_{av} = \frac{\int dw}{time}$  pvs. t forprojectile

Example 3 : What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep mine?

Solution : Mass, m = 20 metric ton = 20 × 1000 kg; Distance, S = 20 m; Time, t = 1 hour = 3600 s

Power = $\frac{\text{Work}}{\text{Time}} = \frac{mg \times S}{t} = \frac{20 \times 1000 \times 9.8 \times 20}{3600}$ watt = $\mathbf{1.09 \times 10^3\ W}$

ENERGY

Definition: Energy is defined as internal capacity of doing work. When we say that a body has energy we mean that it can do work.

Energy appears in many forms such as mechanical, electrical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear etc., and can change from one form to the other.

In this section, we restrict ourselves to mechanical energy which comprises of two forms:

(i) kinetic energy

(ii) potential energy

(i) Kinetic Energy : Kinetic energy is the internal capacity of doing work of the object by virtue of its motion.

$K = \frac{1}{2}m(\vec{v}.\vec{v})$, m = mass, v = velocity

Typical kinetic energies (K)

S.No.	Object	Mass (kg)	Speed (m s^{-1})	K(J)
1	Air molecule	»10^{-26}	500	» 10^{-21}
2	Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
3	Stone dropped from 10 m	1	14	10^{2}
4	Bullet	5×10^{-5}	200	10^{3}
5	Running athlete	70	10	3.5×10^{3}
6	Car	2000	25	6.3×10^{5}

Relation Between Momentum and Kinetic Energy : $p = \sqrt{2mE_K}$

(ii) Potential Energy : Potential energy is the internal capacity of doing work of a system by virtue of its configuration.

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

(a) Important points for P.E.

(i) Potential energy can be defined only for conservative forces.

(ii) Potential energy can be positive or negative, depending upon choice of frame of reference.

(iii) Potential energy depends on frame of reference but change in potential energy is independent of reference frame.

(iv) Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

(b) Types of potential energy

(i) Elastic potential energy

$$U = \frac{1}{2}ky^2$$

where *k* is force constant and '*y*' is the stretch or compression. Elastic potential energy is always positive.

(ii) Electric potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

As charge can be positive or negative, therefore electric potential energy can also be positive or negative.

(iii) Gravitational potential energy

$$U = -G\frac{m_1m_2}{r}$$

Which for a body of mass '*m*' at height '*h*' relative to surface of the earth reduces to $U = mgh$. Gravitational potential energy can be positive or negative.

Mechanical Energy

Definition : Mechanical energy '*E*' of an object or a system is defined as the sum of kinetic energy '*K*' and potential energy '*U*', i.e., $E = K + U$

(a) It is a scalar quantity having dimensions $[ML^2T^2]$ and SI units joule.

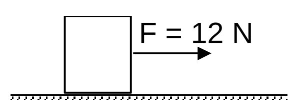
(b) It depends on frame of reference.

(c) A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if $E = 0$ either both PE and KE are zero or PE may be negative and KE may be positive such that $KE + PE = 0$.

(d) As mechanical energy $E = K + U$, i.e., $E - U = K$. Now as *K* is always positive, $E - U \geq$ 0, for existence of a particle in the field, $E \geq U$.

(e) As mechanical energy $E = K + U$ and *K* is always positive, so if '*U*' is positive '*E*' will be positive. However, if potential energy *U* is negative, '*E*' will be positive if $K > |U|$ and *E* will be negative if $K < |U|$ i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around the planet are in bound state.

Example 4 : A 2 kg block initially at rest is pulled to the right along a horizontal frictionless surface by a constant force of 12 N, as shown in the figure. Find



(a) The speed of the block after it has moved 3.0 m.

(b) The acceleration of the block and its final speed using the kinematic equation $\boldsymbol{v_f^2 = v_i^2 + 2as}$.

Solution: The normal force balances the weight of the block, and neither of these forces does work since the displacement is horizontal. Since there is no friction, the resultant external force is the 12 N force. The work done by this force is

$W = Fs$ = (12 N) (3.0 m) = 36 N.m = 36 J

Using the work-energy theorem and noting that the initial kinetic energy is zero, we get

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\text{J})}{2\text{kg}} = 36\ m^2/s^2$$

v_f = 6 m/s

(b) a = **6.0 m/s^2**; **v_f = 6 m/s.**

Note that result calculated in two ways are same.

Difference between Conservative force and non-conservative force

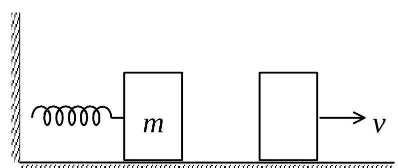
Conservative Force		Non-Conservative Force	
1.	Work done by conservative force depends on initial and final positions, not on path followed.	1.	Work done by non-conservative force does depend on the actual path traversed.
2.	Work done by conservative force around a closed loop is zero.	2.	Work done by non-conservative force around a closed loop is non-zero.
3.	Conservative force is related with P.E. as dw = –dU and $F(x) = -\frac{dU}{dx}$ where dU = change in P.E. and dw is the work done by conservative force	3.	Work done by non-conservative force is related with heat energy and kinetic energy.
4.	Gravitational force/spring force/ electrostatic force are examples of conservative force.	4.	Friction in liquid or solid are non-conservative force.

Example 5 : A block of mass m = 1kg is pushed against a spring of spring constant k =100 N/m fixed at one end to a wall. The block can slide on a frictionless table as shown in Figure. The natural length of the spring is L_0 = 1m and it is compressed to half its natural length when the block is released. Find the velocity of the block.

Solution: When the block is released the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter the block loses contact with the spring and moves with constant velocity. Initially the compression in the spring $=\frac{L_o}{2}$.

When the distance of block from the wall becomes x where $x < L_0$ the compression is $(L_0 - x)$. Using the principle of conservation of energy

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{2}k(L_o - x)^2 + \frac{1}{2}mv^2$$



Solving this, $v = \sqrt{k/m}\left[\frac{L_o^2}{4} - (L_o - x)^2\right]^{\frac{1}{2}}$

When the spring acquires its natural length $x = L_0$, we have

then $v = \sqrt{\frac{k}{m}}\frac{L_o}{2}$ = **5 m/s**

Energy Laws

1. Work Energy Therom :

W_{net} = work done by all the forces(external, internal, conservatives, non consevatives)

= $\Delta K = K_f - K_i$ = change in K.E

Where K_f is final kinetic energy and K_i is the initial kinetic energy.

2. $W_{ext} + W_{nc} + W_c = \Delta K$ where $W_c = -\Delta U$

Where W_{ext} = work done by external forces.

W_{nc} = work done by non conservative force.

W_c = work done by conservative force.

ΔK = Change in K.E

ΔU = change in P.E

3. $W_{ext} + W_{nc} = (K_f + U_f) - (K_i + U_i)$ = change in potential energy

4. Conservation of mechanical energy (if $W_{ext} + W_{nc} = 0$, $\Delta ME = 0$)

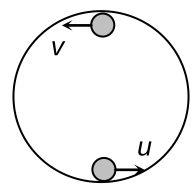
   $U_i + k_i = U_f + K_f$, under conservative force total mechanical energy remains conserved

   U_i = initial P.E and U_f = Final P.E

MOTION IN A VERTICAL CIRCLE

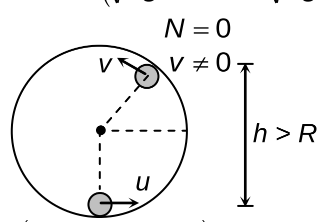
(a) Condition of looping the loop is $(u \geq \sqrt{5gR})$

$$v = \sqrt{gR}, N = 0$$

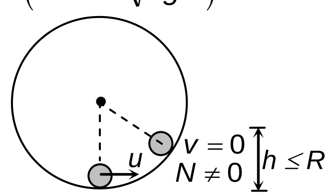


$$u = \sqrt{5gR},\ N = 6mg$$

(b) Condition of leaving the circle is $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$

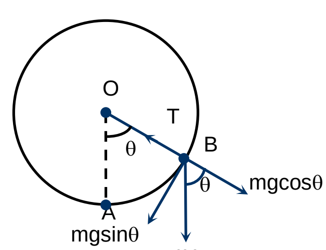


(c) Condition of oscillation is $\left(0 < u \leq \sqrt{2gR}\right)$



Example 6 : A heavy particle hanging from a fixed point by a light inextensible string of length ℓ is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Solution:



Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$$F_R = T - mg\cos\theta \qquad (i)$$

If v_B be the speed of the particle at B, then

$F_R = \frac{mv_B^2}{\ell}$ (ii)

From (i) and (ii), we get,

$T - mg\cos\theta = \frac{mv_B^2}{\ell}$ (iii)

Since at B, T = mg

$\Rightarrow$ $mg(1 - \cos\theta) = \frac{mv_B^2}{\ell}$

$\Rightarrow$ $v_B^2 = g\ell(1 - \cos\theta)$ (iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$\frac{1}{2}mv_A{}^2 = mg\ell(1-\cos\theta) + \frac{1}{2}mv_B^2$;

where $v_A = \sqrt{g\ell}$ and $v_B = \sqrt{g\ell(1-\cos\theta)}$

$\Rightarrow$ $g\ell = 2g\ell(1 - \cos\theta) + g\ell(1 - \cos\theta)$

$\Rightarrow$ $\cos\theta = \frac{2}{3}$ (v)

Putting the value of $\cos\theta$ in equation (iv), we get : $v = \sqrt{\frac{g\ell}{3}}$