MATHEMATICS

QUADRATIC EQUATIONS_SYNOPSIS

•	**An equation of the form** $ax^2+bx+c=0$ **where** $a(\neq 0),b,c\in C$ **is called a quadratic equation in** x **and** a,b,c **are called coefficients.**
•	**The roots of the quadratic equation** $ax^2+bx+c=0$ **are given by** $x=\frac{-b\pm\sqrt{\Delta}}{2a}$ **where** $\Delta=b^2-4ac$ **is the discriminant of the Quadratic Equation.**
•	**The quadratic equation whose roots are** α,β **is** $x^2-(\alpha+\beta)x+\alpha\beta=0$ **a) If** α,β **are the roots of** $ax^2+bx+c=0$**, then** **1.** $\alpha+\beta=-\frac{b}{a},\alpha\beta=\frac{c}{a}$ **2.** $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{b^2-2ac}{a^2}$ **3.** $\lvert\alpha-\beta\rvert=\frac{\sqrt{b^2-4ac}}{\lvert a\rvert}$
•	**In the quadratic equation** $ax^2+bx+c=0$ **i) If** a **and** c **have the same sign, then the product of the roots** $\frac{c}{a}$ **is positive** **ii) If** a **and** c **are of opposite signs, the product of the roots** $\frac{c}{a}$ **is negative** **iii) If** $a+b+c=0$**, then the roots are 1 and** $\frac{c}{a}$ **iv) If** $a-b+c=0$**, then the roots are -1 and** $-\frac{c}{a}$ **v) If the roots are negative, then** a,b,c **will have the same sign** **vi) If the roots are positive, then** a,c **will have the same sign different from the sign of** b

•	If α, β are the roots of $ax^2+bx+a=0$, then the roots are reciprocals to each other Nature of roots of $ax^2+bx+c=0$
•	<u>Case - I</u> a, b, c are rational i) $\Delta > 0$ and is a perfect square $\Leftrightarrow$ Rational and unequal ii) $\Delta > 0$ and not a perfect square $\Leftrightarrow$ Irrational and unequal iii) $\Delta = 0$ $\Leftrightarrow$ Rational and equal iv) $\Delta < 0$ $\Leftrightarrow$ Imaginary
•	<u>Case – II</u> a, b, c are real i) $\Delta > 0 \Leftrightarrow$ Real and unequal ii) $\Delta = 0$ $\Leftrightarrow$ Real and equal iii) $\Delta < 0 \Leftrightarrow$ Imaginary
•	If $f(x)=0$ is a quadratic equation, then the equation whose roots are i) The reciprocals of the roots of $f(x)=0$ is $f\left(\frac{1}{x}\right)=0$ ii) Increased by K than that of $f(x)=0$ is $f(x-k)=0$ ii) Decreased by K than that of $f(x)=0$ is $f(x+k)=0$ iii) Negative of the roots of $f(x)=0$ is $f(-x)=0$ iv) K times the roots of $f(x)=0$ is $f\left(\frac{x}{k}\right)=0$ v) Square of the roots of $f(x)=0$ is $f\left(\sqrt{x}\right)=0$ and (eliminate square root) vi) Cubes of the roots of $f(x)=0$ is $f\left(\sqrt[3]{x}\right)=0$ and (eliminate cube root)
•	If α, β are the roots of the quadratic equation $f(x)=0$, then the equation whose roots are $a\alpha+b, a\beta+b$ is $f\left(\frac{x-b}{a}\right)=0$
•	Condition for the roots of $ax^2+bc+c=0$ to be in the ratio $m:n$ is $(m+n)^2 ac = mnb^2$
•	Conditions for one root of $ax^2+bc+c=0$ may be the square of the other is

	$b^3 + a^2c + ac^2 = 3abc$
•	**If one root of the quadratic equation** $ax^2 + bc + c = 0$ **is equal to the nth power of the other, then** $\left(ac^n\right)^{\frac{1}{n+1}} + \left(a^n c\right)^{\frac{1}{n+1}} + b = 0$
•	**If the difference of the roots of** $ax^2 + bx + c = 0$ **is same as the difference of the roots of** $px^2 + qx + r = 0$ **then** $\frac{\Delta_1}{\Delta_2} = \frac{a^2}{p^2}$ **(where** Δ_1, Δ_2 **are the discriminants)**
•	**In a Q.E with rational coefficients, irrational roots occur as conjugate pairs**
	In a Q.E with real coefficients, imaginary roots occur as conjugate pairs **a) The equations** $a_1x^2 + b_1x + c_1 = 0, a_2x^2 + b_2x + c_2 = 0$ **where** $a_1b_2 - a_2b_1 \neq 0$ **have a common root if** $\left(c_1a_2 - c_2a_1\right)^2 = \left(a_1b_2 - a_2b_1\right)\left(b_1c_2 - b_2c_1\right)$ **and the common root is** $\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$ **or** $\frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$ **b) If a Quadratic equation** $ax^2 + bx + c = 0$ **in** x **has more than two roots then it is an identity in** x **i.e.** $a = b = c = 0$
•	**If** $a_1x^2 + b_1x + c_1 = 0$ **and** $a_2x^2 + b_2x + c_2 = 0$ **have the same roots then** $a_1 : a_2 = b_1 : b_2 = c_1 : c_2$
•	**If the equation** $ax^2 + bx + c = 0$ **has complex roots, then** $\forall x \in R, ax^2 + bx + c$ **and** a **will have the same sign**
•	**If the equation** $ax^2 + bx + c = 0$ **has equal roots, then** $\forall x \in R - \left\{\frac{-b}{2a}\right\}, ax^2 + bx + c$ **and** a **will have the same sign**
•	**a) If the equation** $ax^2 + bx + c = 0$ **has real roots** α **and** $\beta\left(\alpha > \beta\right)$ **then** **i)** $\beta < x < \alpha \Rightarrow ax^2 + bx + c$ **and a will have opposite signs.** **ii)** $x > \alpha$ **or** $x < \beta \Rightarrow ax^2 + bx + c$ **and a will have the same sign**
•	**b) The expression** $y = ax^2 + bx + c$ **always represents a parabola whose vertex is at**

	$\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$. **It is also called the turning point of the graph. Length of the latusrectum of the parabola is** $\frac{1}{\lvert a \rvert}$
•	**a) If** $a>0$, **then the minimum value of** ax^2+bx+c **occurs at** $x=\frac{-b}{2a}$ **and it is** $\frac{4ac-b^2}{4a}, \forall x \in R$
•	**b) If** $a<0$, **then the maximum value of** ax^2+bx+c **occurs at** $x=\frac{-b}{2a}$ **and it is** $\frac{4ac-b^2}{4a}, \forall x \in R$
•	**a) If** $f(x)=\frac{ax^2+bx+c}{ax^2-bx+c}$ **or** $\frac{ax^2-bx+c}{ax^2+bx+c}$, $(b^2-4ac<0)$, **then the minimum and maximum values of** $f(x)$ **are given by** $f\left(\pm\sqrt{\frac{c}{a}}\right)$
•	**b) The minimum and maximum values of** $f(x)=\frac{(x+a)(x+b)}{(x+c)}$ **are respectively** $\left(\sqrt{a-c}+\sqrt{b-c}\right)^2$ **and** $\left(\sqrt{a-c}-\sqrt{b-c}\right)^2$ **and they are obtained at** $x=-c\pm\sqrt{(a-c)(b-c)}\,(a<c, b<c)$
•	**The necessary and sufficient condition for the second degree expression** $ax^2+2hxy+by^2+2gx+2fy+c$ **to be expressed as a product of two linear factors is** $abc+2fgh-af^2-bg^2-ch^2=0$ **or** $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0, h^2\ge ab, g^2\ge ac, f^2\ge bc$ **.**

QUADRATIC EQUATIONS_ASSIGNMENT

1. If α, β are the roots of $ax^2+bx+c=0$ then $(a\alpha+b)^{-2}+(a\beta+b)^{-2}=$

1) $\frac{b^2-2ac}{a^4}$ 2) $\frac{b^2-2ac}{a^3c}$ 3) $\frac{b^2-2ac}{a^2c^2}$ 4) $\frac{b^2-2ac}{c^4}$

2. If α, β are the roots of $x^2-p(x+1)-c=0$ then $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}=$

1) 3 2) 2 3) 1 4) 0

3. If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30^0$ and $\tan 15^0$, respectively then the value of $2+q-p$ is

1) 0 2) 1 3) 2 4) 3

4. If α, β are the roots $ax^2+2bx+c=0$ and $\alpha+\delta, \beta+\delta$ are the roots of $Ax^2+2Bx+C=0$ then $\frac{b^2-ac}{B^2-AC}=$

1) a/A 2) A/a 3) $(a/A)^2$ 4) $(A/a)^2$

5. If the ratio of the roots of $x^2+bx+c=0$ and $x^2+qx+r=0$ are the same, then

1) $r^2c=qb^2$ 2) $r^2b=qc^2$ 3) $rb^2=cq^2$ 4) $rc^2=bq^2$

6. In a triangle PQR, $\angle R=\pi/2$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2+bx+c=0 (a\neq 0)$, then

1) $a+b=c$ 2) $b+c=0$ 3) $a+c=b$ 4) $b=c$

7. All the values of m for which both roots of he equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4, lie in the interval

1) $-1<m<3$ 2) $1<m<4$ 3) $-2<m<0$ 4) $m>3$

8. If both the roots of the quadratic equation $x^2-2kx+k^2+k-5=0$ are less than 5, then k lies in the interval

1) $(5,6]$ 2) $(6,\infty)$ 3) $(-\infty,4)$ 4) $[4,5]$

9. If roots of the equation $x^2-bx+c=0$ be two consecutive integers, then b^2-4c equals

1) -2 2) 3 3) 2 4) 1

10. If $p(q-r)x^2+q(r-p)x+r(p-q)=0$ has equal roots then $\frac{2}{q}=$

1) $\frac{1}{p}+\frac{1}{r}$ 2) $\frac{1}{p}-\frac{1}{r}$ 3) $p+r$ 4) pr

11. If $x^2-cx+d=0$, $x^2-ax+b=0$ have one common root and second has equal roots then $2(b+d)=$

1) 0 2) ac 3) a + c 4) a – c

12. For the equation $|x^2|+|x|-6=0$, the root are

1) One and only one real number 2) real with sum one

3) real with sum zero 4) real with product zero

13. If $20^{3-2x^2}=\left(40\sqrt{5}\right)^{3x^2-2}$, then x =

1) $\pm\sqrt{\frac{13}{12}}$ 2) $\pm\sqrt{\frac{12}{13}}$ 3) $\pm\sqrt{\frac{4}{5}}$ 4) $\pm\sqrt{\frac{5}{4}}$

14. The number of real solutions of $e^{\sin x}-e^{-\sin x}=4$ is

1) 0 2) 1 3) 4 4) ∞

15. The expression $2x^2+4x+7$ has minimum value m at $x=\alpha$. The order pair (α,m) is

1) $(1,5)$ 2) $(1,-5)$ 3) $(-1,-5)$ 4) $(-1,5)$

16. If α,β are the roots of $ax^2+bx+c=0$ then $\left(\frac{\alpha}{a\beta+b}\right)^3-\left(\frac{\beta}{a\alpha+b}\right)^3=$

1) 0 2) 1 3) $(a+b)^2$ 4) $(a-b)^2$

17. If α,β are the roots of $x^2+px-q=0$ and γ,δ **are the roots of** $x^2+px+r=0$ then $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$ =

1) $2q^2$ 2) $2p^2$ 3) $(q+r)^2$ 4) $(q-r)^2$

18. x_1 and x_3 are the roots of the equation $Ax^2-4x+1=0$ and x_2 and x_4 are the roots of the equation $Bx^2-6x+1=0$. If x_1,x_2,x_3,x_4 form a HP, then $(A,B)=$

1) $(3,3)$ 2) $(8,8)$ 3) $(3,8)$ 4) $(8,3)$

19. If $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$, $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$ then α,β are roots of the equation

1) $x^2+x+1=0$ 2) $x^2+x+2=0$ 3) $x^2+2x+2=0$ 4) $x^2+2x+3=0$

20. If $p,q\in\{1,2,3,4\}$, the number of equation of the form $px^2+qx+1=0$ having real roots is

1) 15 2) 9 3) 7 4) 8

21. The quadratic equation whose roots are the x and y intercept of the line passing through (1.1) and making a triangle of area A with the axes may be

1) $x^2+Ax+2A=0$ 2) $x^2-2Ax+2A=0$ 3) $x^2-Ax+2A=0$ 4) $x^2-2Ax^2-2A=0$

22. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2 + b|x| + c = 0$ is

1) 2 2) 4 3) 0 4) -1

23. If x is real, then the greatest and least values of the expression $\frac{x^2 - 2x + 2}{x^2 + 3x + 9}$

1) $2 \text{ and } -\frac{2}{27}$ 2) $2 \text{ and } \frac{2}{27}$ 3) $2 \text{ and } \frac{1}{27}$ 4) 2 or 3

24. If the equation $(\lambda^2 - 5\lambda + 6)x^2 + (\lambda^2 - 3\lambda + 2)x + (\lambda^2 - 4) = 0$ is satisfied by more than two values of x then $\lambda =$

1) 1 2) -2 3) 3 4) 2

25. If $\sin\theta, \sin k\theta$ are the roots of $4x^2 + 2x - 1 = 0$ then $\theta =$

1) -18^0 2) 18^0 3) 36^0 4) -36^0

26. The equation $(\cos p - 1)x^2 + (\cos p)x + (\sin p) = 0$, in the variable x has real roots. Then p can take any value in the interval

1) $(0, 2\pi)$ 2) $(-\pi, 0)$ 3) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ 4) $(0, \pi)$

27. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is.

1) $[0,1]$ 2) $\left[0, \frac{1}{2}\right]$ 3) $\left[\frac{1}{2}, 1\right]$ **4)** $(0,1]$

28. If $x^2 + x + 1 = 0$. Then the value of $\left(x + \frac{1}{x}\right) + \left(x^2 + \frac{1}{x^2}\right) + \ldots\ldots + \left(x^{52} + \frac{1}{x^{52}}\right)$ equals

1) 1 2) 0 3) -1 4) -52

29. Given that, for all real x, the expression $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ lies between $\frac{1}{3}$ and 3. The values between which the expression $\frac{9.3^{2x} + 6.3^x + 4}{9.3^{2x} - 6.3^x + 4}$ lies are

1) $\frac{1}{3}$ and 3 2) -2 and 0 3) -1 and 1 4) 0 and 2

30. If $a + b + c = 0$ then the equation $3ax^2 + 2bx + c = 0$ has at least one root in

1) $(1, 2)$ 2) $(0, 1)$ 3) $(-1, 1)$ 4) $(1, 3)$

31. Let α and β be the roots of he equation $ax^2+bx+c=0$. Observe the lists given below

List – I	List – II
i) $\alpha=\beta$	A) $\left(ac^2\right)^{1/3}+\left(a^2c\right)^{1/3}+b=0$
ii) $\alpha=2\beta$	B) $2b^2=9ac$
iii) $\alpha=3\beta$	c) $b^2=6ac$
iv) $\alpha=\beta^2$	D) $3b^2=16ac$
	E) $b^2=4ac$
	F) $\left(ac^2\right)^{1/3}+\left(a^2c\right)^{1/3}=b$

The correct match of List – I from List – II is

	i	ii	iii	iv
1)	E	B	D	F
2)	E	B	A	D
3)	E	D	B	F
4)	E	B	D	A

32. Observe the following lists $\left(x\in\mathbb{R}\right)$

List – I (Inequation)	**List – II (Solution set)**
A: $x^2-4x+3>0$	1. $(3,4)$
B: $x^2-5x+6\le 0$	2. $(1,1)\cup(2,4)$
C: $x^2+6x-27>0,\ -x^2+3x+4>0$	3. $(-\infty,1)\cup(3,\infty)$
D: $x^2-3x-4<0, x^2-3x+2>0$	4. $[3,4]$
	5. $[2,3]$

Matching List – I, from List – II

	A	B	C	D
1)	1	2	3	4
2)	3	5	1	2
3)	1	3	5	4
4)	3	2	4	1

KEY SHEET

1. 3	**2.3**	**3.4**	**4.3**	**5.3**	**6.1**	**7.1**	**8.3**	**9.4**	**10.1**
11.2	**12.3**	**13.2**	**14.1**	**15.4**	**16.1**	**17.3**	**18.3**	**19.2**	**20.3**
21.2	**22.3**	**23.2**	**24.4**	**25.2**	**26.4**	**27.4**	**28.3**	**29.1**	**30.2**
31.4	**32.2**								