

Section-A**I. Very Short Answer Questions. Answer all Questions.****Each Question carries 'Two' marks****10x2=20M**

1. If $A = \left[0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \right]$ and $f : A \rightarrow B$ is a surjection defined by $f(x) = \cos x$ then find B.
2. Find the domain of real valued function $f(x) = \frac{1}{\sqrt{x^2 - 3x + 2}}$.
3. A certain bookshop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs.60 and Rs. 40 each respectively. Using matrix algebra, find the total value of the books in the shop.
4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and $\det A = 45$, then find x.
5. If the position vectors of the points A, B and C are $-2\bar{i} + \bar{j} + \bar{k}$, $-4\bar{i} + 2\bar{j} + 2\bar{k}$ and $6\bar{i} - 3\bar{j} - 13\bar{k}$ respectively and $\overline{AB} = \lambda \overline{AC}$, then find the value of λ .
6. Is the triangle formed by the vectors $3\bar{i} + 5\bar{j} + 2\bar{k}$, $2\bar{i} - 3\bar{j} - 5\bar{k}$ and $-5\bar{i} - 2\bar{j} + 3\bar{k}$ equilateral.
7. Find the area of the parallelogram having $\bar{a} = 2\bar{j} - \bar{k}$ and $\bar{b} = -\bar{i} + \bar{k}$ as adjacent sides
8. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, prove that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.
9. Prove that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$.
10. Show that $\tanh^{-1} \left(\frac{1}{2} \right) = \frac{1}{2} \log_e 3$.

Section-B**II. Short Answer Questions. Answer any 'Five' Questions.****Each Question carries 'Four' marks.****5 x4 =20 M**

11. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then find $A^3 - 3A^2 - A - 3I$. Where I is unit matrix of order 3.

12. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar then prove that $\lambda = \frac{-146}{17}$.

13. Find a unit vector perpendicular to the plane determined by the points $P(1, -1, 2)$, $Q(2, 0, -1)$ and $R(0, 2, 1)$.

14. Prove that $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right) = \frac{1}{16}$.

15. Find all values of $x \neq 0$ in $(-\pi, \pi)$ satisfying $8^{1 + \cos x + \cos^2 x + \dots} = 4^3$.

16. If $\cos^{-1} \frac{p}{a} + \cos^{-1} \frac{q}{b} = \alpha$, then prove that $\frac{p^2}{a^2} - 2 \frac{pq}{ab} \cdot \cos \alpha + \frac{q^2}{b^2} = \sin^2 \alpha$

17. If $\sin \theta = \frac{a}{b+c}$, Then Show that $\cos \theta = \frac{2\sqrt{bc}}{b+c} \cdot \cos \frac{A}{2}$.

Section-C

III. Long Answer Questions. Answer any 'Five' Questions.
Each Question carries 'Seven' marks.

5 x 7 = 35 M

18. Let $f : A \rightarrow B, I_A$ and I_B be identity functions on A and B respectively. Then show that $f \circ I_A = f = I_B \circ f$.

19. Show that $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$ upto n terms $= \frac{n(n+1)^2(n+2)}{12}, \forall n \in N$.

20. A line makes an angle $\theta_1, \theta_2, \theta_3, \theta_4$, with diagonals of a cube show that $\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3 + \cos^2 \theta_4 = 4/3$.

21. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$.

22. Solve the system of linear equation by using Gauss-Jordan method
 $x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0$

23. If $A + B + C = \pi$ then prove that $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} - \cos^2 \frac{C}{2} = 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$.

24. Prove that $a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B) = 3abc$.