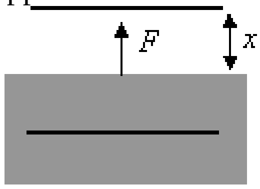
PHYSICS-capacitors
Assignment-2

1)

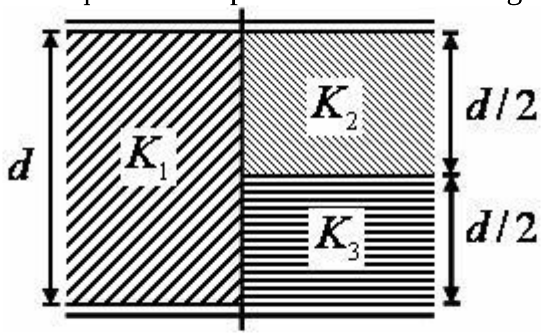
A parallel plate capacitor having interplanar seperation 'd' is lowered in water as shown in the figure. The distance between upper plate and free surface of water is x as shown.The plates of the capacitors are then fixed and maintained at constant charge. The value of x can be changed from 0 to d.The electrostatic force on the free upper surface of the water as shown



1) Increases as x increases
2) Decreases as x increases
3) Remains constant as x changes
4) Data insufficient for drawing inference.

2)

The equivalent capacitance of the arrangement shown in figure, if A is the area of each plate, is

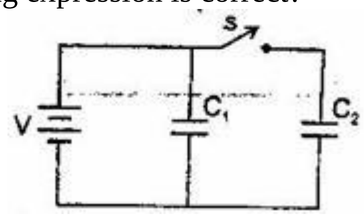


3)

The plates of a parallel plate capacitor have surface area A and are initially separated by a distance d. They are connected to a battery of voltage V_0. Now, the plates of the capacitor are pulled apart to a separation 2d. Then, increase in the energy of the battery is

4)

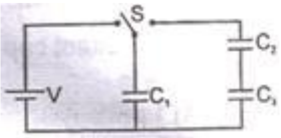
Two capacitors of equal capacitance $(C_1 = C_2)$ are as shown in figure. Initially, while the switch is open (as shown) one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometime after the switch is closed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2 respectively; The energies stored in the capacitors are U_1 and U_2 respectively. Which of the following expression is correct?



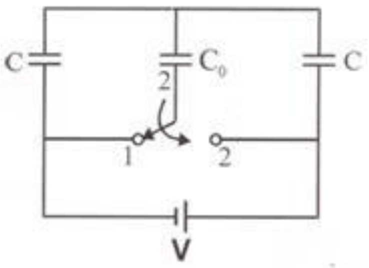
1) $Q_0 = \frac{1}{2}(Q_1 + Q_2)$ 2) $U_0 = U_1 + U_2$ 3) $Q_0 = \frac{1}{2}\left(\frac{U_1}{Q_1} + \frac{U_2}{Q_2}\right)$ 4) $Q_1 = Q_2$

5)

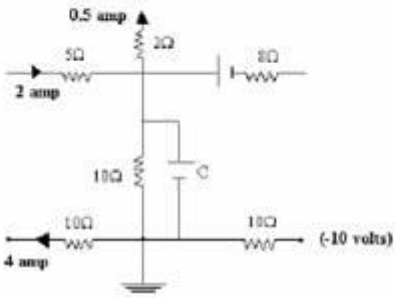
When the switch S in the figure is thrown to the left, the plates of capacitors C_1 acquire a potential difference V=2.5 V. Initially the capacitors C_2 and C_3 are uncharged. The switch is now thrown to the right. What are the final charges on C_2 and C_3 capacitors . Given $C_1 = 5\mu C, C_2 = 5\mu C, C_3 = 10\mu C$.



6)
How much heat will be generated in the circuit shown in the adjoining figure after the switch is shifted from position 1 to position 2? Given : $V = \sqrt{\frac{10}{C_0} + \frac{5}{C}}$. All the values are in M.K.S system.



7)
In the adjoining figure , the cell shown is of unknown e.m.f . The energy stored in the capacitor at steady state is 5 joules . Find the value of C(in milli farad)



8)

Two capacitors $C_1 = 1\mu F$ and $C_2 = 4\mu F$ are charged to a potential difference of 100 volts and 200 volts respectively. The charged capacitors are now connected to each other with terminals of opposite sign connected together. If final charge on capacitor $C_1 = 1\mu F$ in steady state is $20x\,\mu C$, then find the value of *x* .

9)

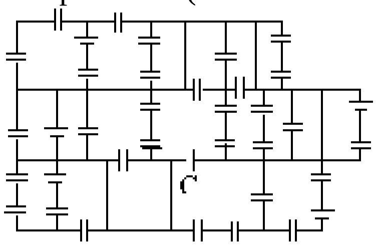
A parallel plate capacitor with air as a dielectric is arranged horizontally.
The lower plate is fixed and the other connected with a perpendicular spring. The area of each plate is A. In the steady positions, the distance between the plates is d_0.When the capacitor is connected with an electric source with the plates as d_1. Mass of the upper plate is m.

Given: d_0=2cm, d_1=1.5cm, $\varepsilon_0 A = 10^{-11}$ farad metre, m=1 g,g=10 m/s^{2,} v=1 kilovolt. Then match the items tin Column I with those in Column II.

Column I		**Column II**	
(a)	Spring constant (in n/m)	(p)	$\frac{9}{4}$
(b)	Initial extension in the spring(in mm)	(q)	$\frac{200}{3\sqrt{3}}$
(c)	Maximun voltage for a given k in which an equilibrium is possible(in kilovolt)	(r)	$\frac{40}{9}$
(d)	Angular frequency of the oscillating system around the equilibrium value d_1 (amplitude of the oscillation << d_0) (in rad/s)	(s)	$\frac{16}{9\sqrt{3}}$

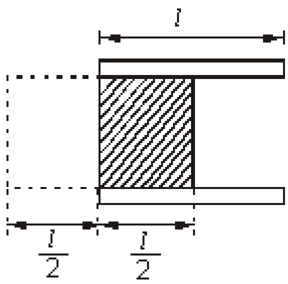
10)

In figure all the capacitors have a capacitance of 6.0 μF, and all the batteries have an emf of 1 V. What is the charge on capacitor C (in microcoulombs).



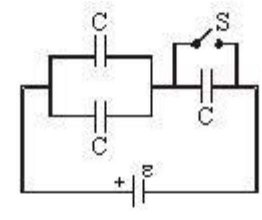
11)

A capacitor of capacitance C is charged by a battery of emf V and then disconnected. The work done by an external agent to insert a dielectric of dielectric strength k of half the length of the capacitor is



12)

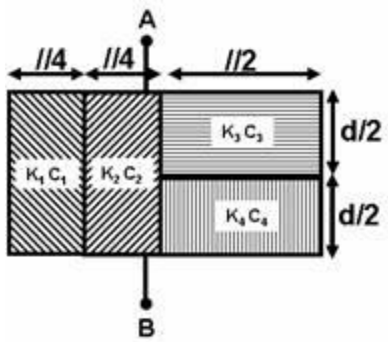
In the circuit shown, each capacitor has a capacitance C. The emf of the cell is e. If the switch S is closed



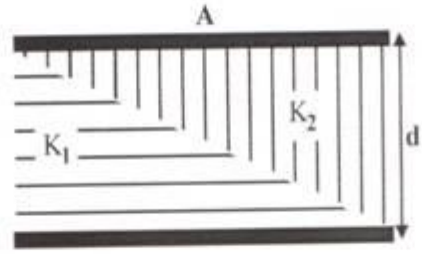
1) some charge will flow out of the positive terminal of the cell
2) some charge will enter the positive terminal of the cell
3) the amount of charge flowing through the cell will be $C\varepsilon$
4) the amount of charge flowing though the cell will be $\frac{4}{3}C\varepsilon$

13)
The plates of the capacitor formed by inserting four-dielectric slabs (as shown) have an area equal to S.

Find the equivalent capacitance between A and B if $K_1 = 2K_2 = K_3 = K_4 = 5$

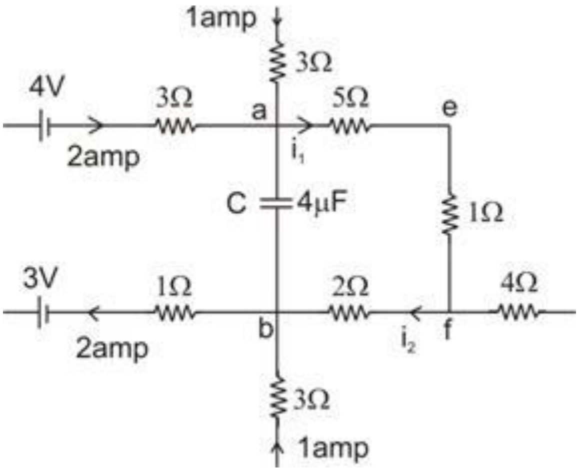


14)



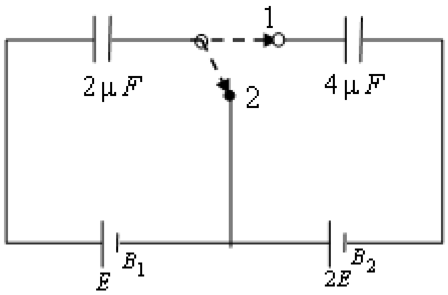
A parallel plate capacitor has the space between the plates filled with a medium whose dielectric constant increases uniformly with distance. If d is the distance between the plates and $K_1$and K_2 are the dielectric constant of the two plates (squares each of area A) respectively, determine the capacity of the capacitor.

15)
A part of circuit in steady state along with the currents flowing in the branches, the values of resistances etc. is shown in figure. The energy stored in the capacitor $(C = 4\mu F)$ is $n \times 10^4$ Joules. Find n?



MULTI PARAGRAPH QUESTIONS

In the circuit shown, initially the switch is in position 1 for a long time, then it has been shifted to position 2. Due to shifting of switch, some charges flow in the circuit.



16)

The charges on both capacitors $2\mu F$ and $4\mu F$ in final steady state respectively :

1) $2E\ \mu C, 3E\ \mu C$ 2) $2E\ \mu C, 4E\ \mu C$ 3) $3E\ \mu C, 4E\ \mu C$ 4) $4E\ \mu C, 3E\ \mu C$

17)

Find the extra charge flown through B_1 and B_2 due to shifting of switch.

1) zero 2) $1\ \mu C$ 3) $2\ \mu C$ 4) $3\ \mu C$

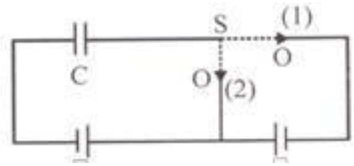
18)

Find heat dissipated in the circuit.

1) $0.5E^2\ \mu J$ 2) $1.0E^2\ \mu J$ 3) $1.5E^2\ \mu J$ 4) $2.0E^2\ \mu J$

19)

A capacitor of capacitance C is connected through a switch S to two batteries of emfs ε_1 and ε_2 as shown. Initially the switch is in position (1). The switch can be shifted from position (i) to position (2) and heat energy dissipation can be calculated. Now answer the given questions.



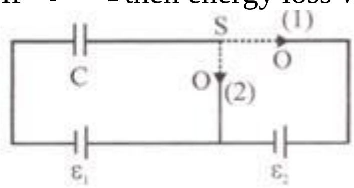
20)

If $\varepsilon_1 > \varepsilon_2$ the energy loss when switch is shifted from position (1) to position (2) is

1) $\frac{1}{2}C\varepsilon_1^2$ 2) $\frac{1}{2}C\varepsilon_2^2$ 3) $\frac{1}{2}C[\varepsilon_1 - \varepsilon_2]^2$ 4) Zero

21)

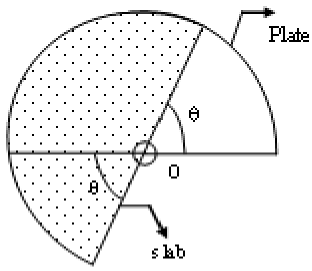
If $\varepsilon_1 < \varepsilon_2$ then energy loss when switch is shifted from position (1) to position (2) is



22)

Now, the terminals of ε_2 is connected as shown below. If $\varepsilon_1 > \varepsilon_2$, the energy loss when switch is shifted from position (1) to position (2) is

A capacitor consists of two fixed semicircular plates of radius R and separation d. A movable semicircular slab of thickness d made of dielectric with the dielectric constant K placed between them. The slab can freely rotate about the axis O as shown in the figure A constant voltage V is maintained between the plates. Then answer the following question.



23)

The equivalent capacitance of the given system as shown in the figure

24)

Find the magnitude of torque of electrostatic forces acting on the slab about point O, in the given configuration

25)

Find the magnitude of the angular acceleration of the movable slab if the mass of the slab is m

1) $\frac{\varepsilon_0 V^2 (K-1)}{2md}$ 2) $\frac{\varepsilon_0 V^2 (K-1)}{4dm}$ 3) $\frac{\varepsilon_0 R^2 V^2 (K+1)}{2md}$ 4) $\frac{\varepsilon_0 V^2 (K+1)}{4dm}$

PHYSICS – ASSIGNMENT – KEY HINTS

1.3 2.2 3.2 4.1,4 5.5 6.5 7.4 8.7 9. A- r, B –p C – s D- q 10.6 11.2

12.1 13.1 14.1 15. 8 16. 2 17.1 18.2 19.2 20.2 21.2 22.1 23.1 24.2 25.1

1)

Hint

2)

Hint)

3)

Hint)

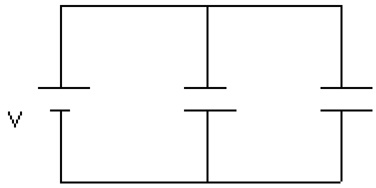
The force of attraction acting between the plates $F = \frac{q^2}{2\epsilon_0 A}$

$F = \frac{C^2V^2}{2\epsilon_0 A} = \frac{\epsilon_0 AV^2}{2d^2}$; $W = \int_d^{2d} F dx = \frac{\epsilon_0 AV^2}{4d}$

When charge ΔQ leaves the plates of the capacitor it increases the energy of the battery by $\Delta QV = \Delta CV^2$ $= \frac{\epsilon_0 AV^2}{2d}$.

4)

Hint)



$Q_0 = CV$

$V_0 = \frac{1}{2}CV^2$

$Q_1 = CV$

$Q_2 = CV$

$V_1 = \frac{1}{2}CV^2$

$V_2 = \frac{1}{2}CV^2$

$\frac{V}{2} + \frac{V}{2}$

$\frac{Q^2}{2C} + \frac{Q^2}{u}$

$\frac{Q}{2C} + \frac{Q}{2C} = \left(\frac{v}{2} + \frac{v}{2}\right) = \frac{v}{2}$

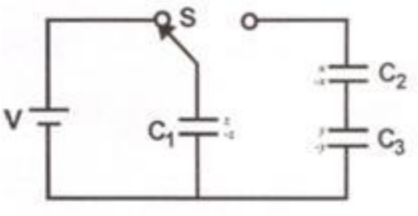
5)

Hint) Charge on the capacitor C_1 before switch is the right is C_1 V=Q.

Let charge on the capacitor after switch is thrown to the right are X, Y and Z as shown in figure. For isolated plate between C_2 and C_3

Y - X = 0 ----------(1)

And between C_1 and C_2



C_1 and C_2 ----------(2)

Applying Kirchhoff's second law in loop

$$\frac{X}{C_2}+\frac{Y}{C_3}=\frac{2}{C_1} \text{ ------------(3)}$$

From equation (1), (2) and (3)

$$X=Y=\frac{C_1C_2C_3V}{C_1C_2+C_2C_3+C_3C_1}$$

Putting the values of C_1C_2 and C_3 and V than X=5.

6)

Hint) When the switch is in the position 1, C and C_0 are in parallel and C is in series with the combination

Hence $C_{eq}=\frac{C(C+C_0)}{(2C+C_0)}$

Charge on $C_{eq}=V\frac{C(C+C_0)}{(2C+C_0)}$

By the charge distribution principle

$$q_2=\frac{VC(C+C_0)C_0}{(2C+C_0)(C+C_0)}=\frac{VCC_0}{(2C+C_0)}$$

$$q_1=\frac{VC(C+C_0)C}{(2C+C_0)(C+C_0)}=\frac{C^2V}{(2C+C_0)}$$

In position 2

$$q_3'=\frac{C^2V}{2C+C_0}, q_2'=q_2, q_1'=\frac{VC(C+C_0)}{2C+C_0}$$

Heat produced = loss on stored energy + extra energy drawn from the battery.

Here loss in stored energy is zero because C_{eq} is the same in both the position of the key.

Energy drawn from the battery = $V\Delta q=V(q_1'-q_1) or V(q_3-q_3')$

$\Rightarrow$Heat produced = $V\left[\frac{VC(C+C_0)}{2C+C_0}-\frac{VC^2}{2C+C_0}\right]=\frac{V^2CC_0}{2C+C_0}$

Putting the given condition then heat produced is 5J.

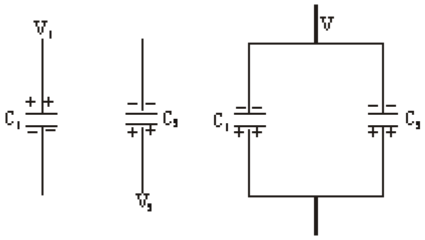
7)

Hint) Hint : p.d across the capacitor = 50 volts

Energy stored = $\frac{1}{2}C(50)^2 = 5 \Rightarrow C = 4\times10^{-3}$ farad

8)

Hint) putting the given condition then heat produced is 5J



Initial charge on $C_1 = C_1V_1 = 100\mu C$; Initial charge on $C_2 = C_2V_2 = 800\mu C$

when the terminals of opposite polarity are connected together, the magnitude of net charge finally is equal to the difference of magnitude of charges before connection.

(charge on C_2)$_i$ - (charge on C_1)$_i$

(charge on C_2)$_i$ - (charge on C_1)$_i$

= (charge on C_2)$_f$ - (charge on C_1)$_f$

$$C_2V_2 - C_1V_1 = C_2V_1 + C_{1V}$$

$$V = \frac{C_1V_2 - C_1V_1}{C_2 + C_1} = \frac{800-100}{5} = 140\,\text{volts}$$

9)

Hint) (A). Let x_0 be the initial extension in the spring, in equilibrium, $kx_0=mg$

When voltage source is connected, the plate separation changes from d_0 to d_1.

The extension of the spring is (d_0-d_1).

When equilibrium attained again, we have

$$k\left[x_0 + (d_0 - d_1)\right] = mg + \frac{1}{2}\varepsilon_0 E^2 A$$

Or $$k(d_0 - d_1) = \frac{1}{2}\varepsilon_0\left(\frac{V}{d_1}\right)^2 A$$

$$k = \frac{\varepsilon_0 AV^2}{2d_1^2(d_0 - d_1)} \quad ....(1)$$

Given: $\varepsilon_0 A = 10^{-11}$ farad metre,T=1000 volt, d_0=2 cm, d_1=1.5 cm

$$k = \frac{10^{-11}\times10^6}{2\left(\frac{9}{4}\right)\times(10^{-2})^2\times(2-1.5)\times10^{-2}} = \frac{40}{N/m}$$

(B). The initial extension in the spring,

$$x_0 = \frac{mg}{k} = \frac{10^{-3}\times10\times9}{40} = 2.25\times10^{-3}m = 2.25mm$$

(C). From equation (1), we have $$v^2 = \frac{2k_1^2(d_0 - d_1)}{A\varepsilon_0}$$

Differentiating with respect to d_1,we get

$$2v\left(\frac{dv}{dd_1}\right) = \frac{2k}{A\varepsilon_0}\left[2d_1d_0 - 3d_1^2\right]$$

For v to maximum, $\left(\frac{dv}{dd_1}\right) = 0$, on solving for d, we get

$$d_1 = \left(\frac{2}{3}d_0\right)$$

Hence, $v_{max}^2 = \frac{2k\left(\frac{2}{3}d_0\right)^2\left[d_0 - \frac{2}{3}d_0\right]}{A\varepsilon_0}$

Or $v_{max} = \sqrt{\frac{k}{A\varepsilon_0}}\left(\frac{2}{3}d_0\right)^{3/2}$ ....(2)

$$v_{max} = \sqrt{\frac{40\times8\times8\times10^{-6}}{9\times10^{-11}\times9\times3}} = \frac{16\times10^3}{9\times\sqrt{3}}$$ volt

$= \frac{16}{9\sqrt{3}}$ kilovolt.

(D). Let the small displacement of the upper plate be x downwards from equilibrium positions. Hen the net force on the plate is

$$F = -k\left[l_0 + (d_0 - d) + x\right] + mg + \frac{1}{2}\varepsilon_0 A\left[\frac{V^2}{(d_1 - x)^2}\right]$$

$$= -k(d_0 - d) - kx + \frac{1}{2}\frac{\varepsilon_0 AV^2}{d_1^2}\left[1 - \frac{x}{d_1}\right]^{-2}$$

$$= -k(d_0 - d) - kx + \frac{1}{2}\frac{\varepsilon_0 AV^2}{d_1^2}\left(1 + \frac{2x}{d_1}\right)$$

$$= -k(d_0 - d) - kx + k(d_0 - d_1)\left(1 + \frac{2x}{d_1}\right)$$

$$= -kx\left[\frac{3d_1 - 2d_0}{d_1}\right]$$

Acceleration = $\frac{F}{m} = -\frac{k}{m}\left(\frac{3d_1 - 2d_0}{d_1}\right)x$

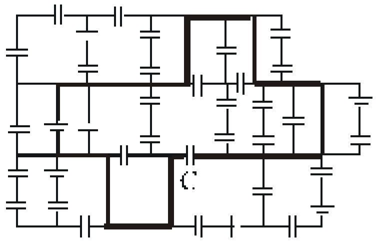
Angular frequency of SHM is

$$\omega = \sqrt{\left[\frac{k}{m}\left(\frac{3d_1 - 2d_0}{d_1}\right)\right]}$$

$$= \sqrt{\frac{40}{9\times10^{-3}}\frac{(3\times1.5\times-2\times2)}{1.5}}$$

$= \frac{200}{3\sqrt{3}}$ rad/s.

10)

Hint) 

11)

Hint) Charge remains conserved

$$q = CV \left(C_1 = \frac{KC}{2}, \ \& C_2 = \frac{C}{2} \right)$$

$$C^1 = \frac{C(K+1)}{2}, V^1 = \frac{2V}{K+1}$$

$$U_1 = \frac{1}{2}CV^2$$

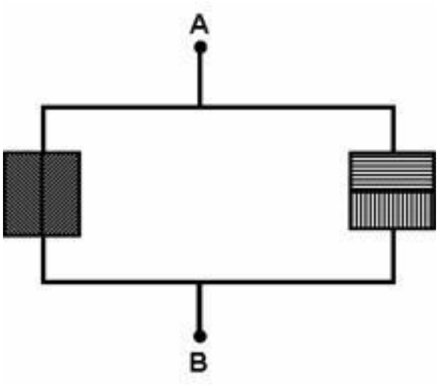
$$U_2 = \frac{1}{2}CV^2\left(\frac{2}{K+1}\right)$$

$$\Delta U = \frac{1}{2}CV^2\left(\frac{K-1}{K+1}\right)$$

12)

Hint)

13)

Hint) 

Consider the capacitor as a parallel combination of C_1 and C_2 and series combination of C_3 and C_4 Equ. Capacitance

$$= C_1 + C_2 + C_3C_4(C_3 + C_4)$$

$$\frac{\varepsilon_o}{d}\left(K_1\frac{S}{4} + K_2\frac{S}{4}\right) + \frac{\varepsilon_o S/2}{\frac{d/2}{K_3} + \frac{d/2}{K_4}}$$

$$= \frac{\varepsilon_o S}{4d}(K_1 + K_2) + \frac{35}{8}\frac{\varepsilon_o S}{d}\left[\frac{K_3K_4}{K_3 + K_4}\right]$$

14)

Hint) Case (i) when no dielectric

Given $C = \frac{\varepsilon_0 A}{d}$

Case (ii) when dielectric is filled : a small dotted element is considered of thickness dx

The small capacitance of the dotted portion

$\frac{1}{dc} = \frac{1}{dc_1} + \frac{1}{dc_2}$ where dc_1 =capacitance of capacitor with dielectric k_1

dc_2 = capacitor of capacitor with dielectric k_2.

Let l, b the length and breadth of the capacitor plate. Therefore, $l \times b = A$

$$dc_1 = \frac{k_1(bdX)\varepsilon_0}{d'}$$

Therefore,
$$dc_1 = \frac{k_1 bdX\varepsilon_0}{d\left[1-\frac{X}{l}\right]} = \frac{k_1 A\varepsilon_0 dx}{d(l-X)}$$

Similarly,
$$dc_2 = \frac{k_2\varepsilon_0(bdX)}{d-d'} = \frac{k_2\varepsilon_0 bdX}{d-d+\frac{dx}{l}} = \frac{k_2\varepsilon_0 Adx}{xd}$$

To find the capacitance of the whole capacitor, we integreate the above equation

$$C = \int_0^1 \frac{K_1K_2A\varepsilon_0 dx}{K_2ld + d(K_1-K_2)X}$$

$$= \frac{K_1K_2A\varepsilon_0}{d(K_1-K_2)}\left[\log\{K_2ld + dK_1l - dK_2l\} - \log K_2ld\right]$$

$$C = \frac{K_1K_2A\varepsilon_0}{d(K_1-K_2)}\log\frac{K_1}{K_2}$$

15)

Hint) In steady state the branch containing capacitance acts as the open circuit since capacitance offers infinite resistance to d.c. The capacitance simply collects the charge.

Applying Kirchhoffs I law to junction a and b, we note that i_1 = 3 amp. And i_2 = 1 amp.

Applying Kirchhoffs II law to the part of circuit aefb, we get

$$V_a - i_1\times 5 - i_1\times 1 - i_2\times 2 = V_b$$

Where V_a and V_b are potentials at points a and b respectively,

Or $V_a - V_b = 6i_1 + 2i_2$

$= 6\times 3 + 2\times 1 = 20\,\text{Volt}$

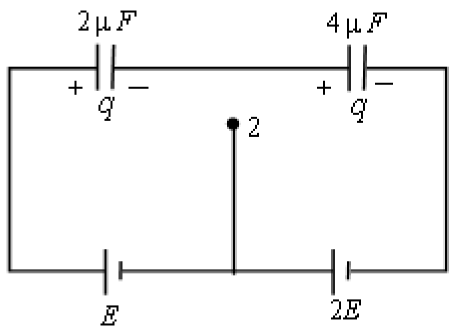
That is p.d. across capacitor, V = 20 Volt.

Therefore Energy stored in capacitor C,

<u>MULTI PARAGRAPH QUESTIONS</u>

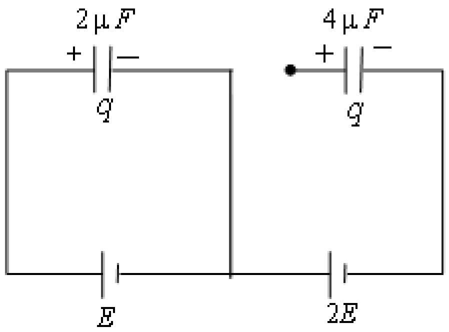
16)

Hint) When switch is in position 1, the circuit is as shown below :



So, the charge o both capacitors would be same, $q_1 = q_2 = 4E\ \mu C$

When switch is shifted to 2, the capacitor of $4\mu F$ gets isolated as it is not a part of complete circuit and hence its charge remains constant, i.e., further charging or discharging of capacitor doesnt take place.



Charge on capacitor of $2\mu F,\ q' = 2E\mu C$

Charge on $2\mu F$ capacitor, $q_1' = 2E\mu C$

Charge on $4\mu F$ capacitor, $q_2' = 4E\mu C$

17)

Hint) Extra charge flown through B_1 is, $q_{flown} = q_1 - q_2$ entering into $+ve$ terminal so $q_{flown} = 2E\mu C,$ i.e., charging of B_1 takes place with a charge of $2\mu C$.

Extra charge flown through B_2 is 0.

18)

Hint) $\sum U_i + W_{By\ the\ battery} = \sum U_f + W_{on\ the\ battery} + \Delta H$

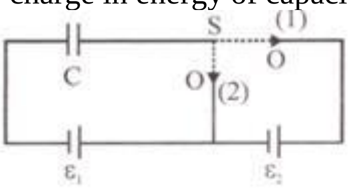
$$\Rightarrow \quad \frac{(4E)^2}{2\times 2} + \frac{(4E)^2}{2\times 4} + 0$$

$$= \frac{(2E)^2}{2\times 2} + \frac{(4E)^2}{2\times 4} + (2E\times E) + \Delta H$$

$\Delta H + E^2 \mu J.$

19)

Hint) Initially $q_i = C(\varepsilon_1 - \varepsilon_2)$ finally $q_f = c\varepsilon_1$. Therefore, charge flown through battery is $\Delta q = c\varepsilon_2$ therefore, work done by battery $\omega = \Delta q\,\varepsilon_1 = C\varepsilon_1\varepsilon_2$ Now charge in energy of capacitor $= \frac{1}{2}C[\varepsilon_2^2 - 2\varepsilon_1\varepsilon_2]$

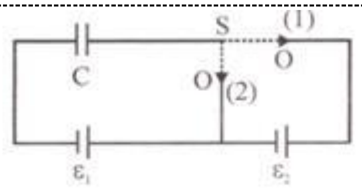


$\therefore$ heat lost $= \frac{1}{2}C[\varepsilon_2^2 - 2\varepsilon_2] + C\varepsilon_1\varepsilon_2 = \frac{1}{2}C\varepsilon_2^2$

20)

Hint)

Energy loss will be same as above $= \frac{1}{2}C\varepsilon_2^2$



21)

Hint)

Energy loss is same as above and that is $= \frac{1}{2} C \varepsilon_2^2$

22)

23)

Hint)

24)

Hint) $\tau = -\frac{d\upsilon}{d\theta}, \qquad \upsilon = \frac{1}{2} CV^2$

25)

Hint) $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}, \qquad I = \frac{mR^2}{2}$