

Prep ⚡ Express

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CAT Quant Formula Handbook

Every formula, identity & shortcut for QA — in one notebook.

handwritten with ♥ by Vikash Singh

CAT 2026 Edition

What's inside this notebook

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★ The Speed Sheet	30 results toppers over-use

— **Speed tip** — How to use this book: don't read it — **rebuild each box from memory** on rough paper once a week. A formula you can re-derive is a formula you'll never blank on at 10:47 am on exam day.

1

Number System

divisibility · HCF-LCM · factors · cyclicity · factorials · remainders · bases

1 The number family

Real $\mathbb{R}$

Rational $\mathbb{Q}$ ($p/q, q \neq 0$)

Integers $\mathbb{Z} \supset$ Whole $\mathbb{W} \supset$ Natural $\mathbb{N}$

Irrational

$\sqrt{2}, \sqrt{3}, \pi, e \dots$

non-terminating, non-repeating

Chain to remember

$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

2 Divisibility tests

By	Test	By	Test
2, 5, 10	last digit	8	last 3 digits $\div 8$
4, 25	last 2 digits	9	digit sum $\div 9$
3	digit sum $\div 3$	11	(alt. digit sums differ by 0 or 11k)
6	2 and 3 together	7 / 13	split into 3-digit groups, alternate $\pm$ (1001 = 7·11·13)

➡ **Speed tip** — For 7 alone: double the last digit, subtract from the rest, repeat. $343 \rightarrow 34 - 6 = 28 \checkmark$

3 HCF & LCM

Two numbers a, b

$\text{HCF} \times \text{LCM} = a \times b$

Fractions

$\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}, \text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Largest n leaving remainders r_1, r_2 with a, b

$\text{HCF}(a-r_1, b-r_2)$

Smallest n leaving same remainder r with a, b, c

$\text{LCM}(a, b, c) + r$

⚠ **CAT trap** — $\text{HCF} \times \text{LCM} = \text{product}$ holds for **two numbers only** — never apply it to three.

4 The factor toolkit ($N = p^a q^b r^c$, primes p, q, r)

Number of factors

$$d(N) = (a+1)(b+1)(c+1)$$

Sum of factors

$$\frac{p^{a+1}-1}{p-1} \cdot \frac{q^{b+1}-1}{q-1} \cdot \frac{r^{c+1}-1}{r-1}$$

Product of factors

$$N^{d(N)/2}$$

As product of two factors

$$d(N)/2 \quad ((d(N)+1)/2 \text{ if } N \text{ is a perfect square})$$

As product of two **co-prime** factors

$$2^{k-1}, k = \text{no. of distinct primes}$$

Co-primes to N below N (Euler)

$$\varphi(N) = N \left(1 - \frac{1}{p}\right) \left(1 - \frac{1}{q}\right) \left(1 - \frac{1}{r}\right)$$

Sum of those co-primes

$$N \cdot \varphi(N) / 2$$

— **Speed tip** — Worked once, remembered forever: $72 = 2^3 \cdot 3^2 \rightarrow$ factors $4 \cdot 3 = 12$, sum $(2^4-1) \cdot \frac{3^3-1}{2} = 15 \cdot 13 = 195$, product 72^6 , co-prime splits $2^1 = 2$ (1×72 , 8×9).

Perfect squares have an **odd** number of factors

$$d(N) \text{ odd} \Leftrightarrow N \text{ square}$$

5 Unit digit (cyclicity)

Digit	0,1,5,6	4	9	2	3	7	8
Cycle	itself	4,6	9,1	2,4,8,6	3,9,7,1	7,9,3,1	8,4,2,6

— **Speed tip** — Cycle length is 1, 2 or 4 — so reduce the power **mod 4** (remainder 0 $\rightarrow$ use the 4th term).
 7^{2026} : $2026 \bmod 4 = 2 \rightarrow$ unit digit 9.

Last two digits — the two rules that settle most questions

Number ending in 1: units stay 1 tens = (tens of base $\times$ units of power) mod 10 — $21^7 \rightarrow 2 \cdot 7 = 14 \rightarrow 41$

Powers of 2 chain $2^{10} = 1024 \rightarrow 24^{\text{even}}$ ends 76; and $76 \times (...ab)$ keeps ending of $(...ab \times 76)$

6 Factorials

Highest power of prime p in $n!$ (Legendre)

$$\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$$

Trailing zeros of $n!$

$$= \text{power of 5 in } n! \quad (100! \rightarrow 20+4 = 24 \text{ zeros})$$

⚠ **CAT trap** — For the highest power of a **composite** like $12 = 2^2 \cdot 3$, count powers of 2 and 3 separately, then take the binding constraint — don't apply Legendre to 12 directly.

7 Remainders toolkit

Product / sum rule

$\text{Rem}(a \cdot b) = \text{Rem}[\text{Rem}(a) \cdot \text{Rem}(b)]$, same for $+$, $-$

Fermat (p prime, $\gcd(a, p) = 1$)

$$a^{p-1} \equiv 1 \pmod{p}$$

Euler ($\gcd(a, n) = 1$)

$$a^{\varphi(n)} \equiv 1 \pmod{n}$$

Wilson (p prime)

$$(p-1)! \equiv -1 \pmod{p}$$

Always divisible

$a^n - b^n$ by $(a-b)$; by $(a+b)$ too when n even

When n is odd

$a^n + b^n$ divisible by $(a+b)$

— **Speed tip** — Negative remainders finish questions fast: $\text{Rem}(15^{20} \div 16) \rightarrow 15 \equiv -1$, so $(-1)^{20} = 1$.

8 Bases & successive division

Decimal $\rightarrow$ base b : divide by b , read remainders bottom-up. Base $b \rightarrow$ decimal:

$$(d_2 d_1 d_0)_b = d_2 b^2 + d_1 b + d_0$$

Successive division by a then b (remainders r_1, r_2)

$$N = a(b \cdot q + r_2) + r_1$$

2

Averages, Ratio & Mixtures

averages · weighted average · proportion · variation · partnership · alligation · replacement

1 Averages

Definition

$$\text{Avg} = \frac{\text{sum of observations}}{\text{count}}$$

Any A.P. (equally spaced values)

$$\text{Avg} = \frac{\text{first} + \text{last}}{2} = \text{middle term}$$

New member joins ($n \rightarrow n+1$), average changes by Δ

$$\text{new value} = \text{old avg} + (n+1)\Delta$$

One value replaced, average of n changes by Δ

$$\text{new} - \text{old value} = n\Delta$$

➡ **Speed tip** — First n naturals: $\text{avg} = \frac{n+1}{2}$. Consecutive evens/odds — average is just the middle number; don't sum anything.

2 Weighted average

Groups of sizes n_1, n_2 with averages A_1, A_2

$$A_w = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

⚠ **CAT trap** — The combined average always lies between A_1 and A_2 , pulled toward the bigger group. If an answer option sits outside $[A_1, A_2]$, eliminate it without calculating.

3 Ratio & proportion

If $\frac{a}{b} = \frac{c}{d}$ then (componendo-dividendo)

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Equal ratios can be merged

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

Mean proportional of a, b

$$\sqrt{ab}$$

Third proportional to a, b

$$b^2/a$$

Fourth proportional to a, b, c

$$bc/a$$

Duplicate / sub-duplicate ratio of $a : b$

$$a^2 : b^2 / \sqrt{a} : \sqrt{b}$$

➡ **Speed tip** — To combine $A:B = 2:3$ and $B:C = 4:5$, make B equal: $\times 4$ and $\times 3 \rightarrow A:B:C = 8:12:15$. (LCM the shared term.)

4 Variation & partnership

Direct: $y \propto x$

$$y = kx$$

Inverse: $y \propto 1/x$

$$xy = k$$

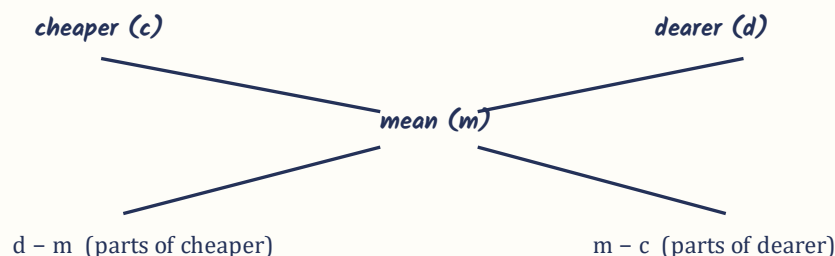
Joint

$$z \propto xy \Rightarrow z = kxy$$

Partnership — profit share

$$\propto \text{capital} \times \text{time invested}$$

5 Alligation — the cross



Mixing rule

$$\frac{\text{qty of cheaper}}{\text{qty of dearer}} = \frac{d - m}{m - c}$$

— **Speed tip** — Alligation works on **any** per-unit quantity — price, %, average marks, speed, concentration — whenever two pools blend into one mean.

6 Replacement (remove & refill)

Vessel V, replace x each time, n rounds — pure part left

$$\text{final} = \text{initial} \times \left(1 - \frac{x}{V}\right)^n$$

⚠ **CAT trap** — The formula tracks the fraction of the original liquid remaining — total volume returns to V after each refill, concentration is what decays.

3

Percentages, Profit-Loss & Interest

fraction table · multiplying factors · discounts · false weights · SI-CI · growth

1 The fraction-percentage table (non-negotiable memory work)

1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9
50%	33.33%	25%	20%	16.67%	14.28%	12.5%	11.11%
1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/20
10%	9.09%	8.33%	7.69%	7.14%	6.67%	6.25%	5%

— **Speed tip** — Read the table both ways: "37.5% of 64" = $3/8 \times 64 = 24$ — three seconds, no pen.

2 Multiplying factors & successive change

$+r\%$ means $\times$, $-r\%$ means $\times$ $(1 + r/100), (1 - r/100)$

Two successive changes $a\%$, $b\%$ (signs included) net = $a + b + \frac{ab}{100} \%$

$+r\%$ then $-r\%$ net = $-r^2/100 \%$ (always a loss)

A is $r\%$ more than B $\Rightarrow$ B is less than A by $\frac{100r}{100+r} \%$

Price $\uparrow r\% \Rightarrow$ cut consumption (same spend) by $\frac{100r}{100+r} \%$

3 Profit & loss

Profit% (on CP) $\frac{SP - CP}{CP} \times 100$

SP from CP $SP = CP (1 \pm g/100)$

Markup $m\%$ then discount $d\%$ profit% = $m - d - \frac{md}{100}$

Buy x , get y free — effective discount $\frac{y}{x+y} \times 100 \%$

False weight (sells w g as 1000 g at "cost") gain% = $\frac{1000 - w}{w} \times 100$

△ CAT trap — Two items sold at the same SP, one at $+x\%$, one at $-x\% \Rightarrow$ overall **loss of $x^2/100 \%$** . It is never break-even.

— **Speed tip** — Work through $CP = 100$ whenever the question is in percentages only — the arithmetic collapses.

4 Simple & compound interest

Simple interest

$$SI = \frac{P \cdot r \cdot t}{100} \text{ (same every year)}$$

Compound amount

$$A = P (1 + r/100)^t$$

Compounded k times a year

$$A = P (1 + r/100k)^{kt}$$

CI – SI, 2 years

$$P (r/100)^2$$

CI – SI, 3 years

$$P (r/100)^2 (3 + r/100)$$

Equal instalment X clearing debt P at CI

$$P = \frac{X}{(1+r/100)} + \frac{X}{(1+r/100)^2} + \dots$$

Growth intuition

Population / depreciation

$$P (1 \pm r/100)^n$$

Doubling time (rule of 72, approx.)

$$\text{years} \approx 72 / r$$

Money doubles in n yrs $\Rightarrow$ becomes $8\times$ in

$$3n \text{ yrs (CI: } 2^3)$$

⚠ CAT trap — At SI a sum doubling in n years **triples** in $2n$ years (interest is linear); at CI it doubles again every n years (growth is multiplicative). Don't mix the two mental models.

4

Time-Speed-Distance & Work

relative speed · trains · boats · races · circular tracks · clocks · pipes & work

1 Core relations

Distance = speed $\times$ time

$$d = s \cdot t$$

Unit change

$$\text{km/h} \rightarrow \text{m/s} : \times 5/18 \cdot \text{m/s} \rightarrow \text{km/h} : \times 18/5$$

Same distance both ways at x, y

$$\text{avg speed} = \frac{2xy}{x+y} \quad (\text{harmonic — never } (x+y)/2)$$

Always true

$$\text{avg speed} = \frac{\text{total distance}}{\text{total time}}$$

Fixed distance $\Rightarrow$ speed & time inverse

$$s_1/s_2 = t_2/t_1$$

— **Speed tip** — Speed becomes $3/4 \Rightarrow$ time becomes $4/3 \Rightarrow$ the extra time is $1/3$ of the original. Ratios finish TSD questions faster than equations.

2 Relative speed & trains

Opposite directions / same direction

$$s_1 + s_2 \quad / \quad |s_1 - s_2|$$

Train (length L) crosses pole / man

$$t = L / s$$

Crosses platform of length P

$$t = (L + P) / s$$

Two trains cross each other

$$t = (L_1 + L_2) / (\text{relative speed})$$

⚠ **CAT trap** — A train passing a **man sitting in another train** covers only its own length — use both speeds (relative), but one length.

3 Boats & streams

Downstream / upstream

$$b + w, b - w$$

Boat speed b

$$(\text{down} + \text{up})/2$$

Stream speed w

$$(\text{down} - \text{up})/2$$

Same distance, times

$$t_d, t_u$$

$$b/w = (t_u + t_d)/(t_u - t_d)$$

4 Races

"A beats B by x m in an L -metre race"

$$s_A : s_B = L : (L - x)$$

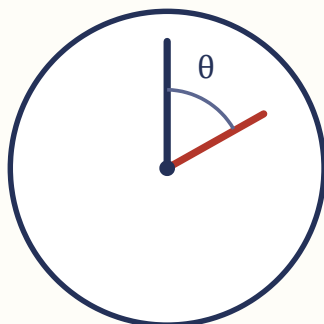
"A beats B by t seconds"

$$t = \text{time B needs for the beat distance}$$

5 Circular tracks (length L , speeds $a > b$)

First meeting — same direction	$L / (a - b)$
First meeting — opposite directions	$L / (a + b)$
First meeting at the start point	LCM (L/a , L/b)
Distinct meeting points (speeds $a : b$ lowest terms)	same dir: $a - b$ · opposite: $a + b$

6 Clocks



Angle at H:M	$\theta = 30H - 5.5M $ (take $360 - \theta$ if > 180)
Minute hand gains on hour hand	5.5° per minute
Hands coincide every	$65 \frac{5}{11}$ min $\rightarrow$ 11 times in 12 h
Right angles / straight line in 12 h	22 times / 11 times

Calendar in 60 seconds

Odd days: ordinary year / leap year	1 / 2
100 yrs / 400 yrs	5 odd days / 0
Century leap years	divisible by 400 only (1900 X, 2000 ✓)

7 Time & work

Rate = work per unit time	A alone in x days $\Rightarrow$ rate $1/x$
A + B together	$\frac{xy}{x+y}$ days
Efficiency $\propto 1/\text{time}$	twice as efficient $\Rightarrow$ half the days
Chain rule (men-days-hours $\propto$ work)	$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$
Wages split	$\propto$ work done = rate $\times$ time worked
Pipes: outlet = negative work	net rate = Σ fill - Σ drain

— **Speed tip** — Given pairs ($A+B$ in p , $B+C$ in q , $C+A$ in r): adding all three rates gives $2(A+B+C)$ — halve it, then subtract each pair to isolate one worker.

— **Speed tip** — Take total work = LCM of the day-counts, not 1 — integer units, zero fractions. A:12 d, B:18 d → work 36: rates 3 & 2 per day.

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1 Identities that pay rent

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$$

$$a^3+b^3+c^3 - 3abc = (a+b+c)(a^2+b^2+c^2 - ab - bc - ca)$$

$$\text{so if } a+b+c = 0$$

$$a^3+b^3+c^3 = 3abc$$

$$\text{Sophie Germain}$$

$$a^4 + 4b^4 = (a^2+2b^2+2ab)(a^2+2b^2-2ab)$$

The $x + 1/x$ family — if $x + 1/x = k$

$$x^2 + 1/x^2 = k^2 - 2$$

$$x^3 + 1/x^3 = k^3 - 3k$$

2 Quadratics $ax^2 + bx + c = 0$

Roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum · product of roots

$$\alpha + \beta = -b/a, \quad \alpha\beta = c/a$$

Build from roots

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Discriminant $D = b^2 - 4ac$ $D > 0$ real distinct · $D = 0$ equal · $D < 0$ imaginaryVertex (max if $a < 0$, min if $a > 0$)

$$\text{at } x = -\frac{b}{2a}, \text{ value } \frac{4ac - b^2}{4a}$$

Cubic (Vieta)

$$\Sigma\alpha = -b/a, \quad \Sigma\alpha\beta = c/a, \quad \alpha\beta\gamma = -d/a$$

Remainder theorem

$$p(x) \div (x - a) \text{ leaves } p(a)$$

⚠ CAT trap — "Roots are real" includes equal roots ($D \geq 0$). Read whether the question says real, distinct real, or rational — three different conditions.

3 Inequalities & maxima-minima

For positives

$$AM \geq GM \geq HM \text{ (equality } \Leftrightarrow \text{ all equal)}$$

Two numbers

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}, AM \times HM = GM^2$$

Sum fixed $\Rightarrow$ product max when

all parts equal

Product fixed $\Rightarrow$ sum min when

all parts equal

Triangle-type bound

$$|x+y| \leq |x| + |y|$$

Modulus as distance

$$|x-a| < r \Leftrightarrow a-r < x < a+r$$

$|x-a_1| + |x-a_2| + \dots$ minimised at

$x = \text{median of the } a_i$

— **Speed tip** — Multiplying or dividing an inequality by a negative flips the sign — the single most common silly error in QA. When the sign of a variable is unknown, **never divide by it**; move everything to one side and factor (wavy curve).

4 Indices, surds & logarithms

$$a^m \cdot a^n = a^{m+n}, (a^m)^n = a^{mn}, a^{-n} = 1/a^n, a^0 = 1$$

Rationalise

$$\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$$

$$\log(xy) = \log x + \log y, \log(x/y) = \log x - \log y, \log x^n = n \log x$$

Base change

$$\log_b a = \frac{\log a}{\log b}, \log_b a \cdot \log_a b = 1$$

Useful pair

$$a^{(\log_a x)} = x, a^{(\log_b c)} = c^{(\log_b a)}$$

Digits of N

$$[\log_{10} N] + 1 \quad (2^{10} = 1024: 10 \cdot 0.301 \rightarrow 4 \text{ digits})$$

⚠ **CAT trap** — For base $0 < b < 1$, log is **decreasing**: $\log_{1/2} x > \log_{1/2} y \Leftrightarrow x < y$. Inequality questions love hiding this.

5 Progressions

AP: n th term / sum

$$a + (n-1)d, S = \frac{n}{2}(2a + (n-1)d) = n \cdot (\text{avg of first \& last})$$

GP: n th term / sum

$$ar^{n-1}, S = a \cdot \frac{r^n - 1}{r - 1}$$

Infinite GP ($|r| < 1$)

$$S_\infty = \frac{a}{1-r}$$

Three in AP / GP / HP

$$2b = a+c, b^2 = ac, b = 2ac/(a+c)$$

Σ first n naturals / squares / cubes

$$\frac{n(n+1)}{2}, \frac{n(n+1)(2n+1)}{6}, \left(\frac{n(n+1)}{2}\right)^2$$

First n odd numbers

$$1 + 3 + \dots + (2n-1) = n^2$$

Telescoping seed

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

6 Functions – two-line reference

Even: $f(-x) = f(x)$, symmetric about y-axis · Odd: $f(-x) = -f(x)$, symmetric about origin

Composition is order-sensitive

$f \circ g(x) = f(g(x)) \neq g \circ f(x)$ in general

Graph shifts

$f(x-a)$: right a · $f(x)+k$: up k · $f(-x)$: mirror in y-axis

1 Triangle fundamentals

Angle sum / exterior angle

 $\Sigma = 180^\circ$ · ext. = sum of two remote interior

Triangle inequality

 $|b - c| < a < b + c$

Bigger side faces bigger angle

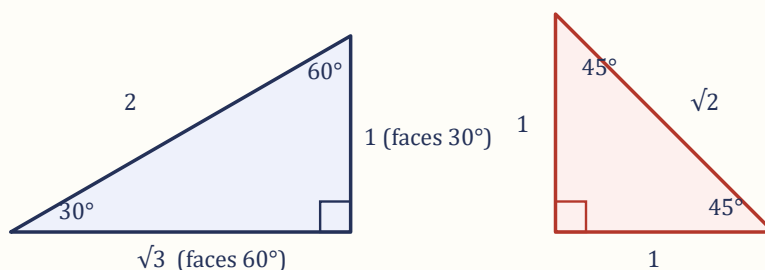
 $a > b \Leftrightarrow A > B$

Pythagoras (right angle at C)

 $a^2 + b^2 = c^2$

Triplets to spot instantly

(3,4,5) (5,12,13) (7,24,25) (8,15,17) (9,40,41) (20,21,29) + multiples



2 Area – five ways

 $\frac{1}{2} \times \text{base} \times \text{height}$ $\frac{1}{2} ab \sin C$ $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron, s = semi-perimeter) $r \cdot s$ (inradius $\times$ semi-perimeter) $\frac{abc}{4R}$ (circumradius)Equilateral, side a area $\frac{\sqrt{3}}{4}a^2$, height $\frac{\sqrt{3}}{2}a$

3 The four centres

Centre	Meeting of	Key fact
Centroid G	medians	divides median 2 : 1 from vertex
Incentre I	angle bisectors	$r = \text{Area} / s$
Circumcentre O	$\perp$ bisectors of sides	$R = abc / 4 \cdot \text{Area} = a / (2 \sin A)$
Orthocentre H	altitudes	inside acute Δ , at right-angle vertex if right Δ

Median length (to side a) $m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$ Angle bisector from A splits BC $BD / DC = AB / AC$ Right triangle (legs a , b , hyp c) $r = (a + b - c) / 2$, $R = c / 2$ Equilateral, side a $R = a / \sqrt{3}$, $r = a / (2\sqrt{3})$, $R = 2r$

— **Speed tip** — All four centres coincide for an equilateral triangle — if a question mixes centres, check whether the triangle is secretly equilateral.

4 Similarity & proportionality

Similarity tests	AA · SAS (ratio) · SSS (ratio)
Similar with ratio k	perimeters k , areas k^2
Basic proportionality ($DE \parallel BC$)	$AD/DB = AE/EC$
Midpoint theorem	midpoint segment $\parallel$ third side, = half of it
Altitude to hypotenuse	$h^2 = p \cdot q$ (p, q = hypotenuse pieces)

5 Polygons

Interior angle sum	$(n - 2) \times 180^\circ$
Each interior (regular) / exterior sum	$(n-2) \cdot 180^\circ / n$ · ext. always 360°
Diagonals	$n(n - 3)/2 \rightarrow$ pentagon 5, hexagon 9, octagon 20
Regular hexagon, side a	6 equilateral Δ s $\Rightarrow$ area $\frac{3\sqrt{3}}{2}a^2$

6 Quadrilaterals

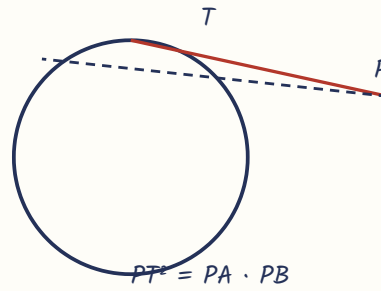
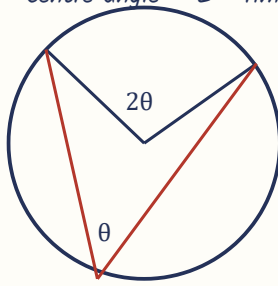
Figure	Area	Remember
Parallelogram	$b \times h$	diagonals bisect each other
Rhombus	$\frac{1}{2} d_1 d_2$	diagonals $\perp$ bisect
Rectangle	$l \times b$	diagonal $\sqrt{l^2 + b^2}$
Square	$a^2 = \frac{1}{2} d^2$	$d = a\sqrt{2}$
Trapezium	$\frac{1}{2} (a + b) h$	a, b = parallel sides
Kite	$\frac{1}{2} d_1 d_2$	one diagonal $\perp$ bisects other

Cyclic quadrilateral (vertices on a circle)

Opposite angles	$A + C = B + D = 180^\circ$
Ptolemy	$d_1 \cdot d_2 = ac + bd$ (products of opposite sides)
Brahmagupta area	$\sqrt{(s-a)(s-b)(s-c)(s-d)}$

7 Circles

centre angle = $2 \times$ rim angle



Angle in semicircle / same segment

90° / equal

Tangent $\perp$ radius; two tangents from a point

equal in length

Alternate segment

tangent-chord angle = angle in alternate segment

$\perp$ from centre to chord (distance d)

$$r^2 = d^2 + (\text{chord}/2)^2$$

Chords meeting at P (inside or outside)

$$PA \cdot PB = PC \cdot PD ; \text{ tangent: } PT^2 = PA \cdot PB$$

Circles touching externally / internally

$$d = r_1 + r_2 \quad / \quad d = |r_1 - r_2|$$

Common tangent lengths (centres d apart)

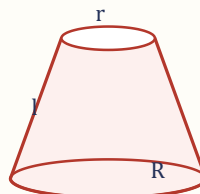
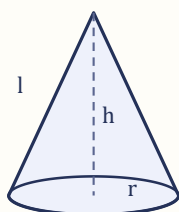
$$\text{direct } \sqrt{d^2 - (r_1 - r_2)^2}, \text{ transverse } \sqrt{d^2 - (r_1 + r_2)^2}$$

1 2D mensuration

Figure	Area	Boundary
Circle (radius r)	πr^2	$2\pi r$
Sector, angle θ	$(\theta/360) \cdot \pi r^2$	$\text{arc} = (\theta/360) \cdot 2\pi r$
Segment	$\text{sector} - \frac{1}{2} r^2 \sin \theta$	—
Ring (R, r)	$\pi(R^2 - r^2)$	—

2 3D solids — the master table

Solid	Volume	Curved/Lateral SA	Total SA
Cube (a)	a^3	$4a^2$	$6a^2 \cdot \text{diag } a\sqrt{3}$
Cuboid (l, b, h)	lbh	$2h(l+b)$	$2(lb+bh+hl) \cdot \text{diag } \sqrt{l^2+b^2+h^2}$
Cylinder (r, h)	$\pi r^2 h$	$2\pi r h$	$2\pi r(r+h)$
Cone (r, h)	$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r(l+r), l = \sqrt{r^2+h^2}$
Sphere (r)	$\frac{4}{3} \pi r^3$	—	$4\pi r^2$
Hemisphere	$\frac{2}{3} \pi r^3$	$2\pi r^2$	$3\pi r^2$
Frustum (R, r, h)	$(\pi h/3)(R^2+Rr+r^2)$	$\pi(R+r)l$	$+ \pi(R^2+r^2), l = \sqrt{h^2+(R-r)^2}$



Solids — the moves CAT actually tests

Melting / recasting

volume is conserved, surface area is not

Similar solids, length ratio k

areas k^2 , volumes k^3

Cone cut by plane $\parallel$ base at half height

small cone volume = $1/8$ of full

Rectangle rolled into cylinder

one side = $2\pi r$, other = h

Sector folded into cone

sector radius = l , arc = $2\pi r$

Painted cube cut into n^3 cubelets

3-face: $8 \cdot 2$ · 2-face: $12(n-2)$ · 1-face: $6(n-2)^2$ · none: $(n-2)^3$

3 Coordinate geometry

Distance

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section ($m : n$ internal)

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \text{ — external: minus signs}$$

Centroid

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Area of triangle (shoelace)

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Slope; line forms

$$m = (y_2 - y_1)/(x_2 - x_1); y = mx + c; y - y_1 = m(x - x_1); x/a + y/b = 1$$

$$ax + by + c = 0$$

$$\text{slope } -a/b; \parallel \Leftrightarrow \text{equal slopes}; \perp \Leftrightarrow m_1 m_2 = -1$$

Point $\rightarrow$ line distance

$$\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \cdot \text{parallel lines: } \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Circle, centre (h, k)

$$(x - h)^2 + (y - k)^2 = r^2$$

Reflections of (x, y)

$$\text{x-axis } (x, -y) \cdot \text{y-axis } (-x, y) \cdot y=x (y, x) \cdot \text{origin } (-x, -y)$$

4 Trigonometry – CAT dose

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Heights & distances

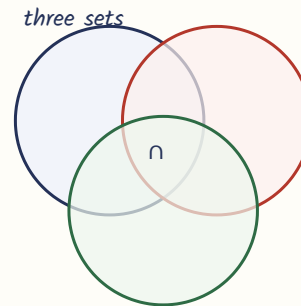
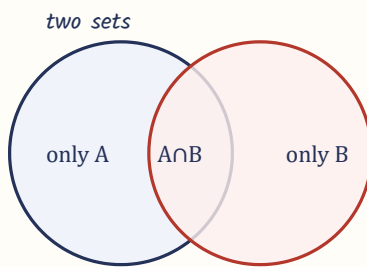
$$\tan \theta = \text{height} / \text{horizontal distance}$$

Elevation $30^\circ / 45^\circ / 60^\circ \Rightarrow$ distance

$$h\sqrt{3} / h / h/\sqrt{3}$$

⚠ CAT trap — Angle of depression is measured from the **horizontal at the top**, not from the vertical — it equals the elevation seen from the ground (alternate angles).

1 Set theory & Venn diagrams



Two sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Three sets

$$n(A \cup B \cup C) = \Sigma n(A) - \Sigma n(A \cap B) + n(A \cap B \cap C)$$

Exactly one of three

$$\Sigma n(A) - 2\Sigma n(A \cap B) + 3n(A \cap B \cap C)$$

Neither

$$n(U) - n(A \cup B)$$

Minimum possible overlap

$$n(A \cap B) \geq n(A) + n(B) - n(U)$$

Subsets of an n -set / proper & non-empty

$$2^n / 2^n - 1 \text{ each}$$

2 Counting — the core kit

AND $\Rightarrow$ multiply · OR $\Rightarrow$ add

$$m \times n \cdot m + n$$

Arrangements / selections

$${}^n P_r = \frac{n!}{(n-r)!}, {}^n C_r = \frac{n!}{r!(n-r)!}, {}^n P_r = {}^n C_r \cdot r!$$

Identities

$${}^n C_r = {}^n C_{n-r}; {}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$$

Words with repeated letters

$$\frac{n!}{p! q! \dots}$$

Circular / necklace-garland

$$(n-1)! / (n-1)!/2$$

 k people always together in a row of n

$$(n-k+1)! \times k!$$

 G girls no two adjacent (B boys placed first)

$$B! \times {}^{B+1} C_G \times G! \text{ (gap method)}$$

Functions from m -set to n -set / one-one

$$n^m / {}^n P_m \text{ (} n \geq m \text{)}$$

⚠ CAT trap — "At least one" almost always means **Total – None**. Counting "at least" cases directly is how three minutes disappear.

3 Selections & distributions

≥ 1 from n distinct things	$2^n - 1$
From n identical things (0 to n)	$n + 1$ ways
≥ 1 from groups of identical (p, q, r)	$(p+1)(q+1)(r+1) - 1$
$x_1 + x_2 + \dots + x_r = n$, non-negative integers	${}^{n+r-1}C_{r-1}$ (stars & bars)
...positive integers	${}^{n-1}C_{r-1}$
n distinct objects into r distinct boxes	r^n (each object picks a box)
Derangements (nothing in its place)	$D_1 \dots D_5 = 0, 1, 2, 9, 44$

4 Classic counts

Handshakes / round-robin games among n	${}^nC_2 = n(n-1)/2$
Lines from n points (no 3 collinear) / triangles	${}^nC_2$ / ${}^nC_3$
Squares on $n \times n$ board / rectangles	Σk^2 / $[n(n+1)/2]^2$ (8×8 : 204, 1296)
Grid paths, m steps right & n up	${}^{m+n}C_m$
Rank of a word	count words before it, letter by letter, +1

5 Probability

Definition	$P = \frac{\text{favourable outcomes}}{\text{total outcomes}}, 0 \leq P \leq 1$
Complement / at least one	$P(A') = 1 - P(A); 1 - P(\text{none})$
Addition rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Independent events	$P(A \cap B) = P(A) \cdot P(B)$
Conditional	$P(A B) = P(A \cap B) / P(B)$
Binomial: r successes in n trials	${}^nC_r p^r (1-p)^{n-r}$
Odds in favour $a : b$	$P = a/(a+b)$

■ **Speed tip** — Card deck in numbers: $52 = 4 \text{ suits} \times 13 \text{ ranks}$; 26 red / 26 black; 12 face cards (J, Q, K $\times$ 4); 4 aces. Dice pair: 36 outcomes, sum 7 is the most likely (6 ways).

6 Binomial theorem

Expansion ($n + 1$ terms)	$(a+b)^n = \Sigma {}^nC_r a^{n-r} b^r$
General term	$T_{r+1} = {}^nC_r a^{n-r} b^r$
Sum of coefficients	put every variable = 1
Row sums	${}^nC_0 + {}^nC_1 + \dots + {}^nC_n = 2^n$



The Speed Sheet

30 results toppers over-use — rebuild this page from memory every Sunday

1.	$1/8 = 12.5\%$, $1/16 = 6.25\%$	16.	$\text{HCF} \times \text{LCM} = a \times b$ (two numbers)
2.	$a\%$ then $b\%$: net = $a + b + ab/100$	17.	$x + 1/x = k \Rightarrow x^2 + 1/x^2 = k^2 - 2$
3.	same SP, $\pm x\% \Rightarrow$ loss $x^2/100\%$	18.	$\text{AM} \geq \text{GM}$: sum fixed $\Rightarrow$ product max at equality
4.	$\text{CI} - \text{SI (2 yrs)} = P(r/100)^2$	19.	quadratic extreme: $x = -b/2a$
5.	equal-distance avg speed = $2xy/(x+y)$	20.	$S_\infty = a/(1-r)$
6.	$\text{km/h} \rightarrow \text{m/s} : \times 5/18$	21.	$\Sigma n^2 = n(n+1)(2n+1)/6$, $\Sigma n^3 = (\Sigma n)^2$
7.	trains cross: $(L_1+L_2)/\text{relative speed}$	22.	digits of $N = \lfloor \log_{10} N \rfloor + 1$
8.	boat = $(\text{down}+\text{up})/2$, stream = $(\text{down}-\text{up})/2$	23.	right Δ : $r = (a+b-c)/2$, $R = c/2$
9.	clock angle = $ 30H - 5.5M $	24.	equilateral: area $\sqrt{3}a^2/4$, $R = 2r$
10.	$A+B$ together = $xy/(x+y)$ days	25.	similar figures: areas k^2 , volumes k^3
11.	$M_1D_1H_1/W_1 = M_2D_2H_2/W_2$	26.	centre angle = $2 \times$ rim angle
12.	$d(N) = (a+1)(b+1)(c+1)$	27.	$PT^2 = PA \cdot PB$
13.	$\phi(N) = N \cdot \prod (1 - 1/p)$	28.	diagonals = $n(n-3)/2$
14.	zeros in $n! = \lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \dots$	29.	$x_1 + \dots + x_r = n$ (≥ 0): ${}^{n+r-1}C_{r-1}$
15.	$a^n - b^n$ always $\div (a - b)$	30.	at least one = $1 - P(\text{none})$

— **Speed tip** — If you can rewrite this page blind in under 12 minutes, your formula recall is exam-ready. If not, the chapter numbers next to each miss tell you exactly where to go.

Prep ⚡ Express

*Formulas are the vocabulary of Quant.
Practice is the grammar.
This notebook gives you the vocabulary —
bring the grammar every single day.*

— Vikash Singh

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