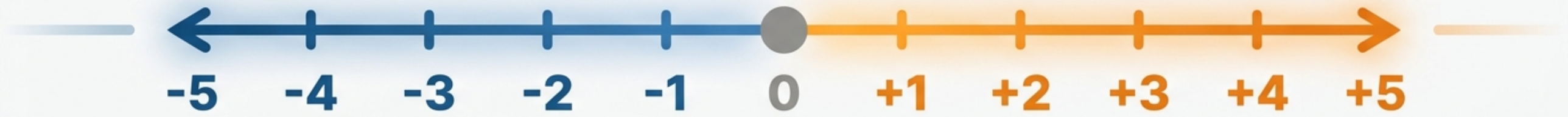
Mastery of Integers

A Visual Guide to Directed Numbers and Operations



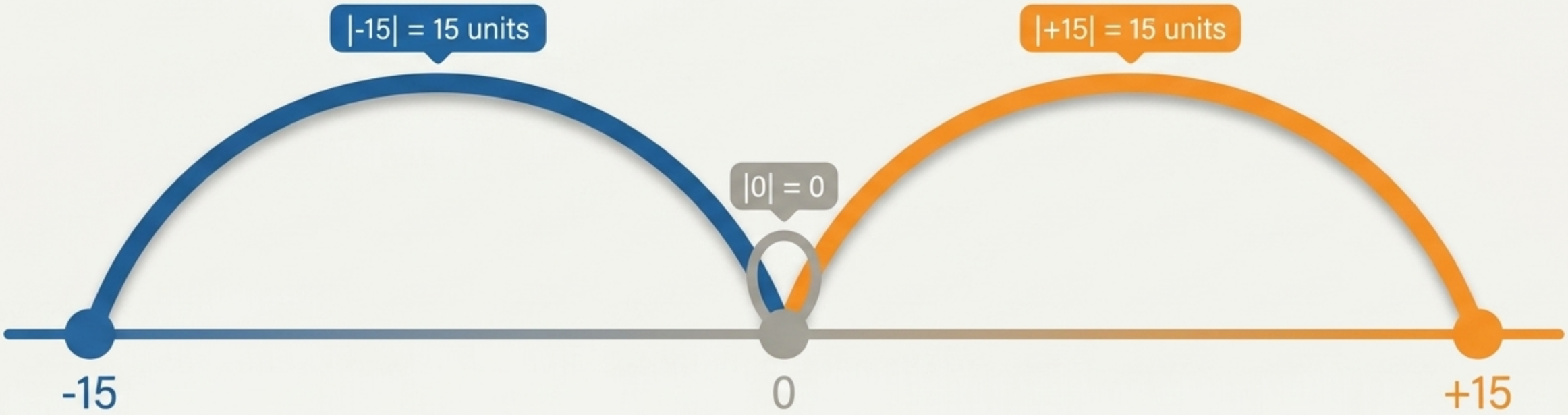
A visual cheat sheet designed for instant comprehension.

The Integer Universe

$$\mathbb{I} = \{ \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$$

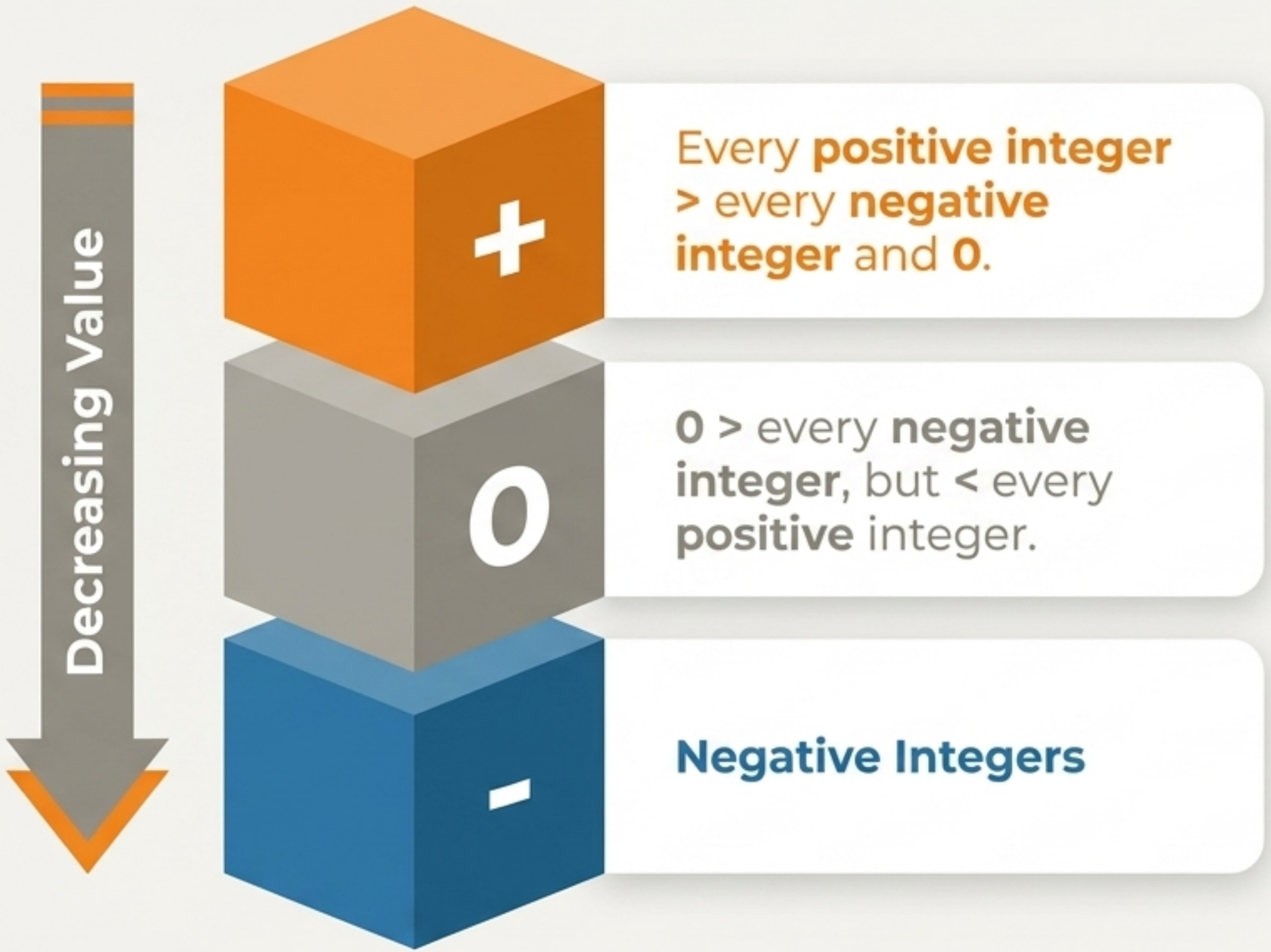
Absolute Value is Simply Distance

The absolute value of an integer is its numerical value regardless of its sign.
It is always non-negative because distance cannot be less than zero.



The absolute value of a positive number is always equal to the number itself.
The absolute value of a negative number is always greater than the number.

The “Greater Than” Hierarchy



Every **positive integer** **>** every **negative integer** and **0**.

0 > every **negative integer**, but **<** every **positive** integer.

Negative Integers

Comparing Two Negatives

Out of two negative integers, the one with the smaller absolute value is greater.

Since |-15| < |-23|, therefore -15 > -23

Directed Numbers in the Real World

Positive and negative integers together are called **directed numbers**. If a positive integer represents a particular direction or state, the negative integer represents its exact opposite.

Positive (+)		Negative (-)
Moving East **(+5 m)**		Moving West **(-5 m)**
Profit of ₹80 **(+80)**		Loss of ₹80 **(-80)**
Heights above sea level		Depths below sea level
Rise in temperature by 32°C **(+32)**		Fall in temperature by 15°C **(-15)**

The Addition Protocol

Are the integers sharing the same sign?

YES (Like Signs)

Add their **absolute values** and assign the same sign to the sum.

$$(+57) + (+112) = +169$$

$$(-32) + (-83) = -115$$

NO (Unlike Signs)

Determine the difference of their absolute values. Assign the sign of the integer with the greater absolute value.

$$(+53) + (-29) = +24 \text{ (since } 53 > 29\text{)}$$

$$(-53) + (+29) = -24 \text{ (since } 53 > 29\text{)}$$

The Subtraction Transformation

Core Rule: Change the sign of the integer to be subtracted and then add.

Subtract -5 from 8

$$(+8) - (-5)$$

Signs change

$$(+8) + (+5) = +13$$

Subtract +5 from -8

$$(-8) - (+5)$$

Signs change

$$(-8) + (-5) = -13$$

The Universal Matrix for Multiplication & Division

The rules for dividing integers are exactly the same as the rules for multiplying them.

Sign Matrix	(+)	(−)
(+)	(+) Positive	(−) Negative
(−)	(−) Negative	(+) Positive

Pattern Callout:
Same Signs = Positive Product/Quotient.
Unlike Signs = Negative Product/Quotient.

Multiply:
(−30)
× (+5)
= −150

Divide:
(−12)
÷ (−3)
= +4

Special Properties of Zero

Addition

The sum of any integer and zero is the integer.

$$a + 0 = a$$

Subtraction

Subtracting zero changes nothing, but subtracting an integer from zero inverses it.

$$a - 0 = a$$

$$a - 0 \neq 0 - a$$

0

Multiplication

The product of any integer and zero is always zero.

$$a \times 0 = 0$$

Division

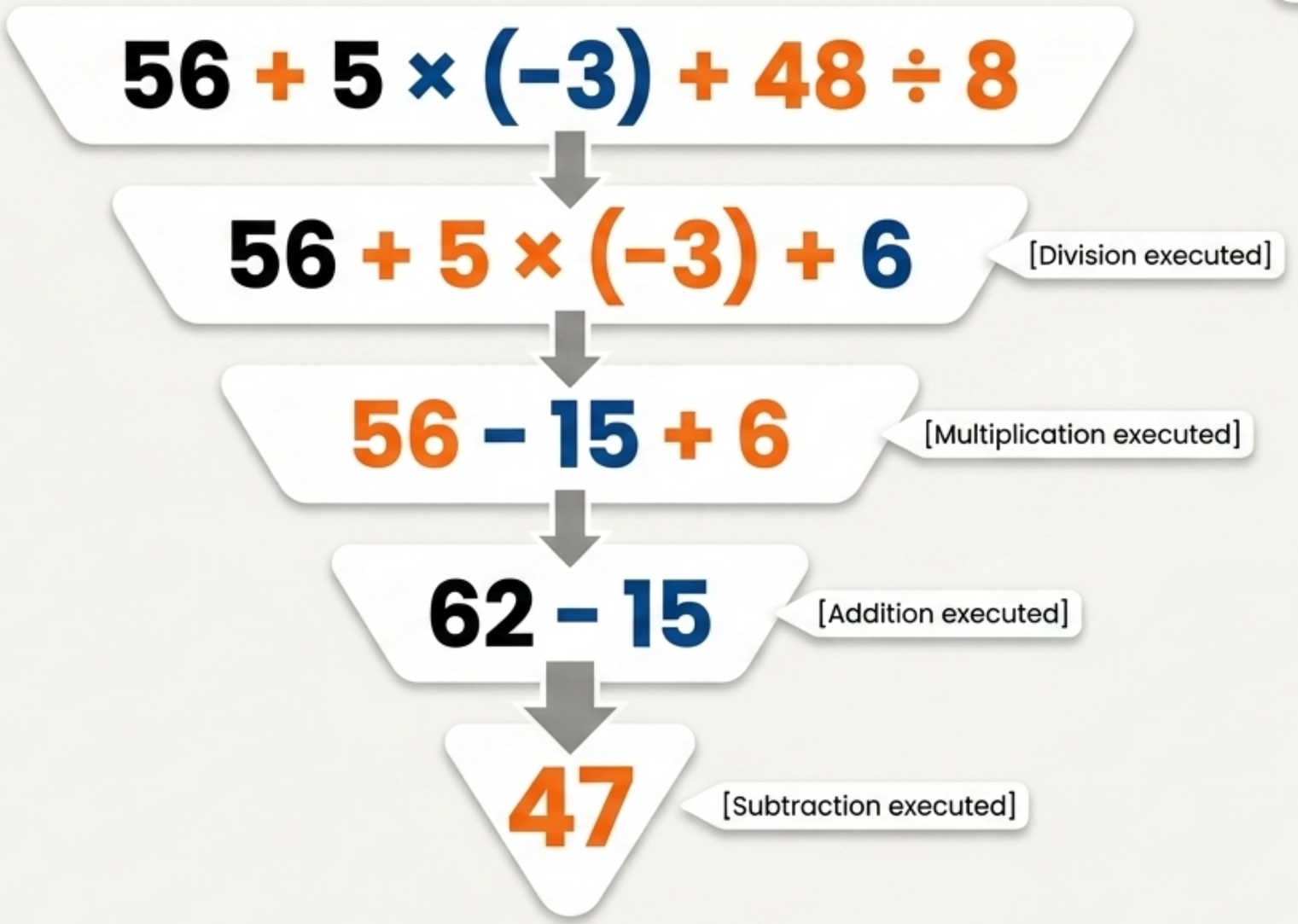
Zero divided by a non-zero integer is zero.
But dividing an integer BY zero is NOT DEFINED.

$$0 \div a = 0$$

$$a \div 0 \text{ is NOT DEFINED}$$

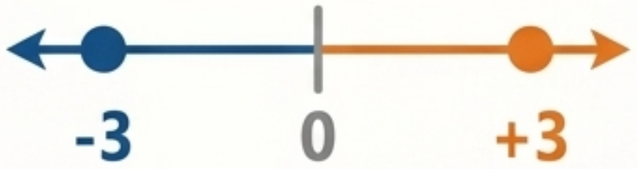
Putting It Together: The DMAS Protocol

D stands for Division
M stands for Multiplication
A stands for Addition
S stands for Subtraction



Integer Operations Reference Guide

The Integer Line



Absolute Value

$|-a| = a$ and $|+a| = a$

Always measures distance.

Addition Rules

Like signs = **Add** values, keep **sign**

Unlike signs = **Subtract** values, keep **sign** of larger absolute value.

Subtraction Rule

Change the sign of the subtracted integer, then add.

(+8) – (-5) becomes (+8) + (+5)

Mult/Div Matrix

(+)(+) = +	(-)(+) = -
(+)(-) = -	(-)(-) = +

Order of Ops

DMAS:
Division → **Multiplication** → → **Addition** → **Subtraction.**