Q

If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ find the value of x.

let $\sin^{-1}\frac{1}{5} = \alpha$ $\qquad$ $\cos^{-1}x = \beta$

$\sin\alpha = \frac{1}{5}$ $\qquad$ $\cos\beta = x$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{25}} = \sqrt{\frac{24}{25}} = \frac{\sqrt{24}}{5}$$

$$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-x^2}$$

$\sin(\alpha+\beta) = 1$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta = 1$

$$\frac{1}{5} \times x + \frac{\sqrt{24}}{5} \times \sqrt{1-x^2} = 1$$

$$\Rightarrow \left(1-\frac{x}{5}\right)^2 = \left(\frac{\sqrt{24}}{5}\sqrt{1-x^2}\right)^2$$

$$1 + \frac{x^2}{25} - \frac{2x}{5} = \frac{24}{25}(1-x^2)$$

$$1 + \frac{x^2}{25} - \frac{2x}{5} = \frac{24}{25} - \frac{24}{25}x^2$$

$$\frac{x^2}{25} + \frac{24}{25}x^2 - \frac{2x}{5} + 1 - \frac{24}{25} = 0$$

$$\Rightarrow \quad \frac{\cancel{25}}{\cancel{25}} x^2 - \frac{2x}{5} + \frac{25-24}{25} = 0$$

$$x^2 - 2 \times x \times \frac{1}{5} + \left(\frac{1}{5}\right)^2 = 0$$

$$\left(x - \frac{1}{5}\right)^2 = 0$$

$$\Rightarrow \quad x - \frac{1}{5} = 0 \quad \Rightarrow \quad \boxed{x = \frac{1}{5}}$$

Q Show that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$

Let $\sin^{-1}\left(\frac{3}{5}\right) = \alpha \qquad \sin^{-1}\frac{8}{17} = \beta$

now $\alpha - \beta = \cos^{-1}\frac{84}{85}$

$\cos(\alpha-\beta) = \frac{84}{85}$

$\sin\alpha = \frac{3}{5} \qquad \sin\beta = \frac{8}{17}$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$

LHS

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \frac{4}{5}\times\frac{15}{17} + \frac{3}{5}\times\frac{8}{17}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{84}{85}$$

$$= \underline{\underline{RHS}}$$

$$\cos\alpha = \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$\cos\alpha = \frac{4}{5}$$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$

$$= \sqrt{1-\frac{64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$= \frac{15}{17}$$

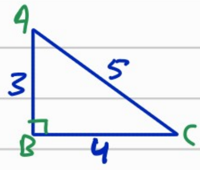
Q Prove $\underline{\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}} = \tan^{-1}\left(\frac{77}{36}\right)$

Q Prove $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Let $\sin^{-1}\frac{3}{5} = \alpha$

$\sin\alpha = \frac{3}{5}$ $\frac{P}{H}$

$\tan\alpha = \frac{3}{4} = \frac{P}{B}$



$2\alpha = \tan^{-1}\frac{24}{7}$

$\tan 2\alpha = \frac{24}{7}$

LHS $\quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16-9}{16}} = \frac{3}{\cancel{2}} \times \frac{\cancel{16}^{8}}{7} = \frac{24}{7} \text{ RHS}$$

Q Prove $\quad \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Common $\frac{9}{4}$

$$\cancel{\frac{9}{4}}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) = \cancel{\frac{9}{4}}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Put $\quad \sin^{-1}\frac{1}{3} = \alpha \Rightarrow \sin\alpha = \frac{1}{3}$

$$\frac{\pi}{2} - \alpha = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \frac{2\sqrt{2}}{3}$$

LHS

$$\sin\left(\frac{\pi}{2}-\alpha\right)$$

$\sin(90-\theta) = \cos\theta$

$$\Rightarrow \cos\alpha$$

$$\Rightarrow \sqrt{1-\sin^2\alpha}$$

$$= \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} = \text{RHS}$$

H.W

Q.1 $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

Q.2 $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$