Unit-I	Electrostatics	Marks
	Chapter1: Electric charge & fields	5
	Chapter 2: Electrostatic potential & capacitance	5
Unit-II	Current Electricity	
	Chapter-3: current Electricity	6

Syllabus of "Electric Charges and fields":

Electric Charges, conservation of charges, coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continous charge distribution. Electric field, electric field due to a point charge, electric field lines, electric dipoles electric field due to a dipole, torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and it's applications to find field due to infinitely long straight wires, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

II Electric Charge

It is an intrinsic property of the elementary particles like electrons, protons, etc. of which all the objects are made up of.

→ It is a scalar quantity.
→ It's SI unit is coulomb(C).
→ Symbol of charge (Q or q)

* There are two types of charges
+ve & -ve,

nature : (i) Same charges Repel
+ve +ve
-ve -ve
(ii) opposite charges ~~Attract~~
+ve -ve

→ The minimum charge that exists is the charge on an electron / proton.

$1e^- = 1.6 \times 10^{-19}$ C

$1p^+ = 1.6 \times 10^{-19}$ C

* Charge always comes with mass
→ means a body with charge must have charge but vice versa is not true.
→ A body can have +ve, -ve charge or can be neutral.
→ Neutral means total net charge zero.
ex:- neutron

→ But chargeless means there is no charge inside the particle/body.

ex:- photon, neutrino, Gluon

* masses of diff. particles

mass of e^- : 9.1×10^{-31} kg

mass of proton = 1.67×10^{-27} kg

mass of neutron = 1.67×10^{-27} kg

Charge	Mass
→ can be +ve/-ve	→ Always +ve
→ Independent of speed.	→ speed dependent $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ (m_0 → Rest mass; m → mass of body)

properties of charges:

(1) Additive nature of charges:

If a system contains three point charges q_1, q_2, q_3, then the total charge of the system will be algebraic addition of q_1, q_2 and q_3, i.e charge will add up

$q = q_1 + q_2 + q_3$

(2) Quantization of charge:

Electric charges is always quantized, i.e electric charge is integral multiple of charge 'e'.

$$\boxed{Q = \pm ne}$$

e → electronic charge

where $n = 0, \pm 1, 2, 3, \ldots$

Q → Total charge

n → no. of electrons

(3) Conservation of charge:

→ The total charge of an isolated system remains constant.

→ The electric charges can neither be created nor be destroyed, they can only be transfered from one body to another.

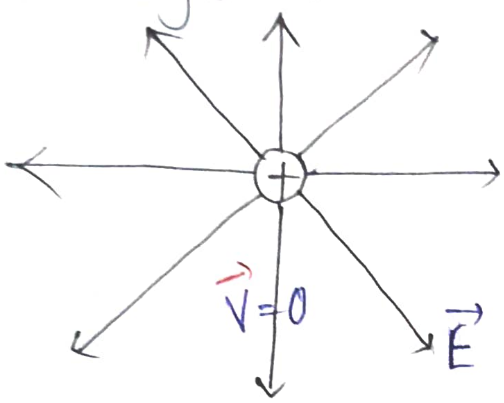
(4) Charge is invariant:

Charge is independent of frame of reference, i.e. charge on a body does not change whatever be its speed.

ex:- length, mass, time } → changes with speed comparable to C.

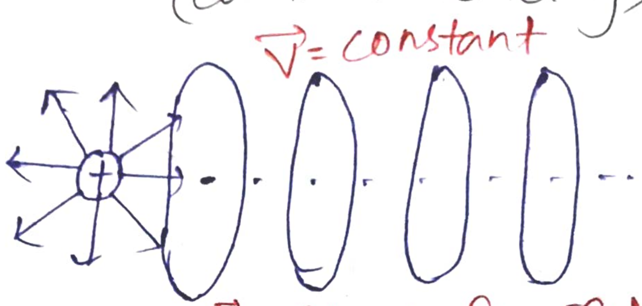
(5) Accelerated charge radiates energy:

(1) Charge at rest



Electric field

(2) Charge at uniform motion (constant velocity)

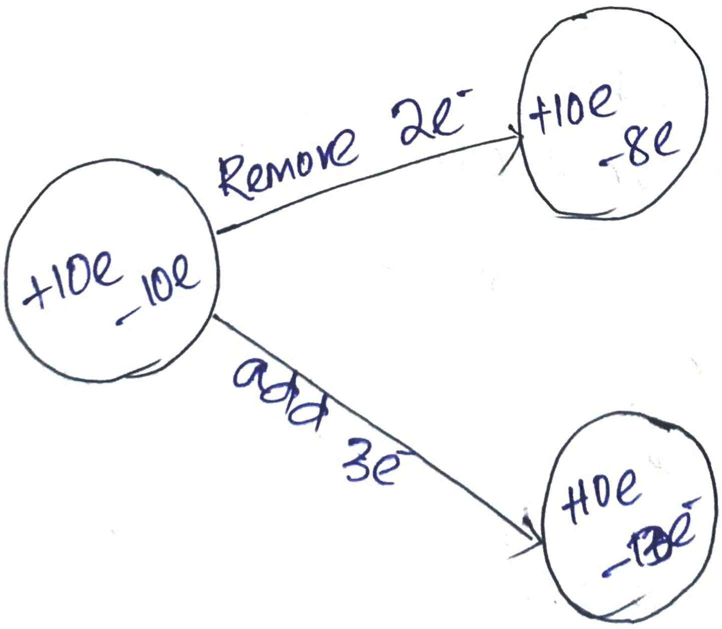


$\vec{E}$ = Electric field
$\vec{B}$ = magnetic field

(3) Charge in non-uniform (Accelerated)

$\vec{E}$ = Electric field
$\vec{B}$ = Magnetic field
EM waves

II Appearence of charge



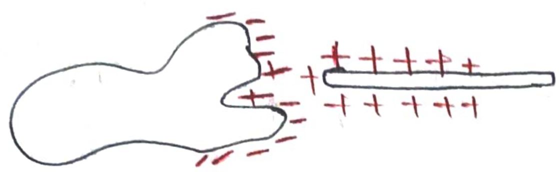
positively charged (+2e)

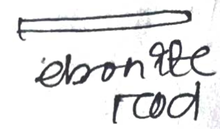
Negatively charged (-3e)

Methods of charging:

① By rubbing / friction

(i) Silk cloth & Glass rod (ii)

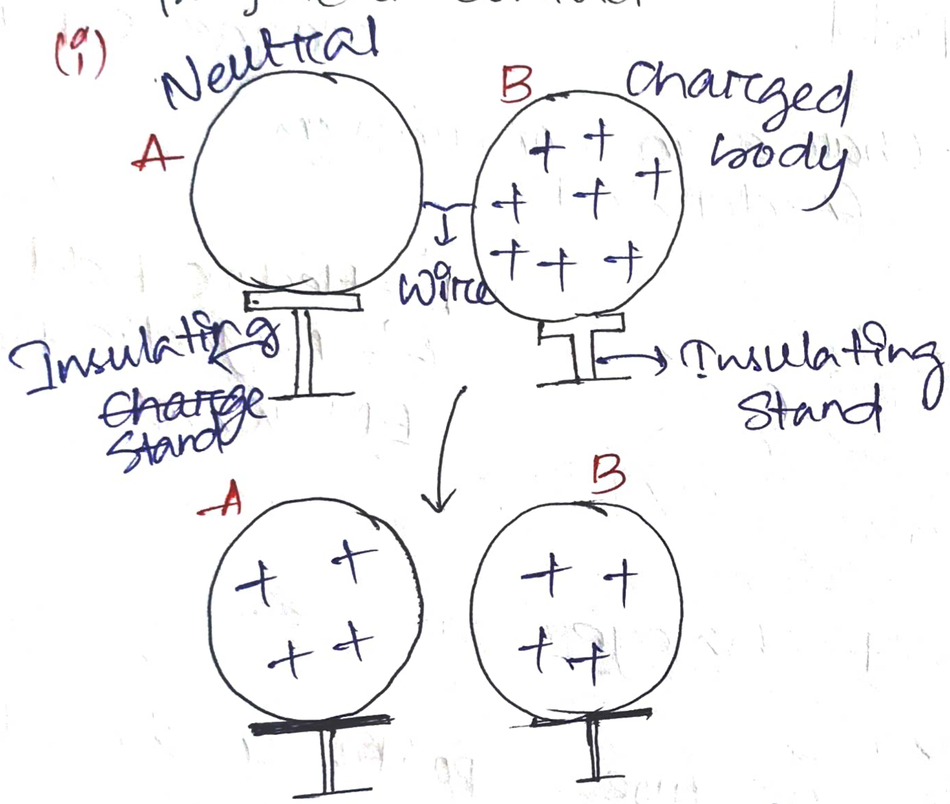




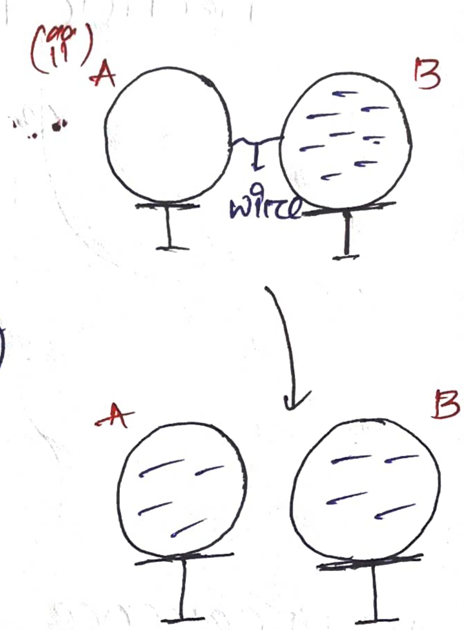
② By conduction (for conductors)

physical contact

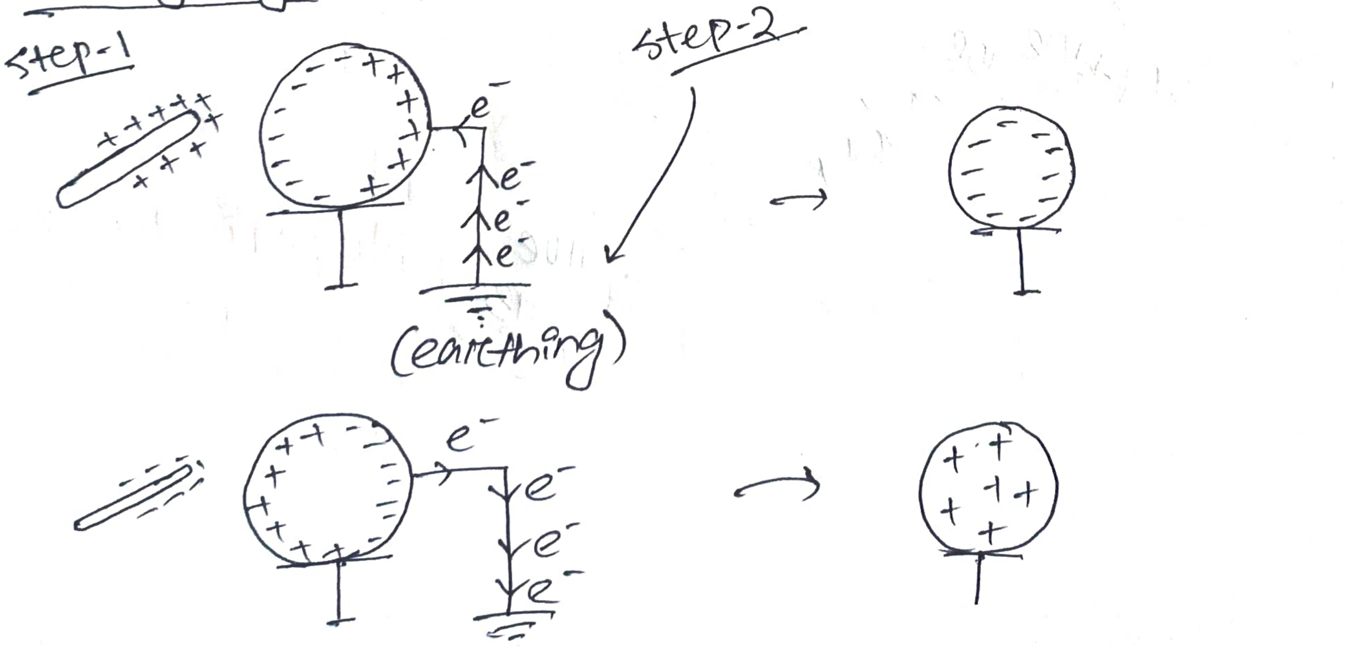
(i)



(ii)



③ Charging by induction (without contact)



~~Step by step~~

Qus: When a piece of polythene is with wool, a charge of -8×10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to the polythene?

Soln. $Q = -8\times10^{-7}$ C $M = ?$, $m_e = 9.1\times10^{-31}$ kg

We know

$$Q = ne$$

$$8\times10^{-7} = n\times1.6\times10^{-19}$$

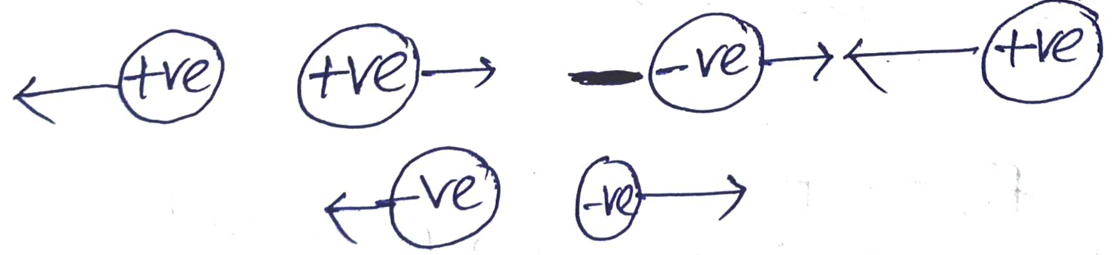
$$n = \frac{8\times10^{-7}}{1.6\times10^{-19}} = 5\times10^{12}$$

$$M = nm_e = 5\times10^{12}\times9.1\times10^{-31}\text{ kg}$$

$$\boxed{M = 45.5\times10^{-19}\text{ kg}}$$

Forces between charges:

→ Force between two charges is Electrostatic force (Repulsion ~~& Attraction~~).



Coulomb's law:

The force between two point charges is directly proportional to the product of the magnitude of the two charges q_1 & q_2 and inversely proportional to the square of distance between them.

$$F \propto q_1 q_2$$

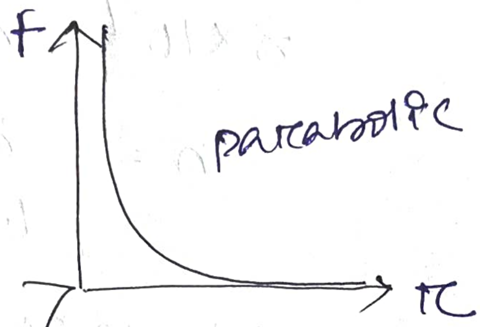
$$\& \quad F \propto \frac{1}{r^2} \Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = K\frac{q_1 q_2}{r^2}}$$

$K \rightarrow$ coulomb's const. ($9\times10^{9}\ Nm^2/C^2$)

* This is also known as coulomb's inverse square law as ($F \propto \frac{1}{r^2}$).

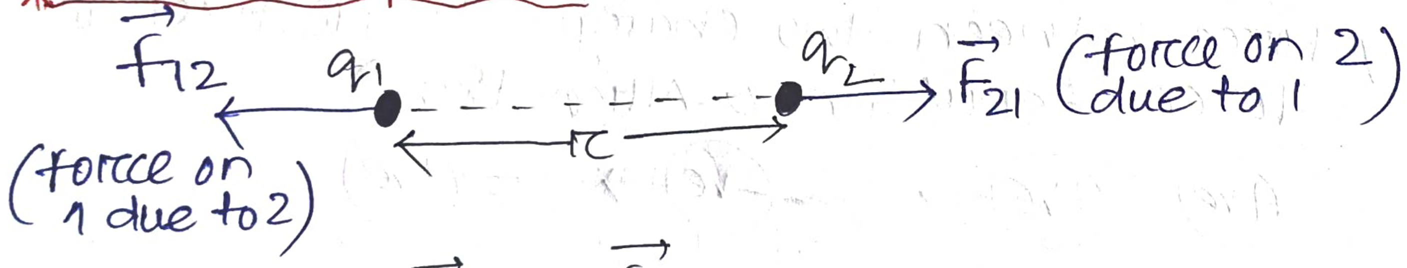
→ Graph betn F & r



$$K = \frac{1}{4\pi\varepsilon_0}$$

where $\varepsilon_0 = 8.854\times10^{-12}\ C^2/Nm^2$

* Vector form:



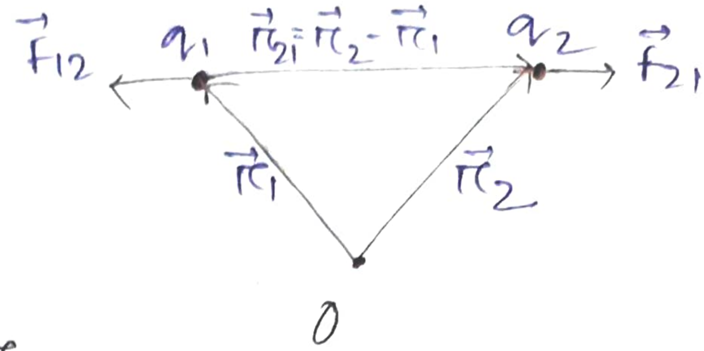
$$\vec{F}_{12} = -\vec{F}_{21}$$

or $|\vec{F}_{12}| = |\vec{F}_{21}|$ → magnitude only

and $\boxed{|\vec{F}_{12}| = |\vec{F}_{21}| = F = \frac{Kq_1q_2}{r^2}}$

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$



$\therefore \vec{r}_{21} = -\vec{r}_{12}$.

$|\vec{r}_{12}| = |\vec{r}_{21}| = r$

So, $\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$

$\vec{F}_{21} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$

$\therefore \vec{F}_{21} = -\vec{F}_{12}$

properties of Electrostatic force/coulomb's law

(i) Electrostatic force/coulomb's law is valid for point charges.

When the size of charged bodies are much smaller than the distance betn them then the charged bodies are point charges.

(ii) Two charges exert equal and opposite charges on each other.

They form Action-Reaction pair, Newton's 3rd law.

(iii) coulomb's force is a central force. They act along the line joining the two charges.

(iv) coulomb's law follows inverse square law.

(v) It's a long range force.

[from nuclear size 10^{-15}m - 10^{18}m]

⑥ coulomb's force depend on medium

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \longrightarrow \text{for free space}$$

$$F_m = \frac{1}{4\pi\varepsilon_m}\frac{q_1q_2}{r^2} \longrightarrow \text{for medium}$$

and $\boxed{\varepsilon_m = \varepsilon_0\varepsilon_r}$ → Relative permittivity

permittivity in medium

permittivity in free space

Now

$$F_m = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{q_1q_2}{r^2}$$

$$F_m = \frac{1}{\varepsilon_r}\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\right) = \frac{F_0}{\varepsilon_r}$$

$\therefore$ $\boxed{F_m = \frac{F_0}{\varepsilon_r}}$ → force reduced in medium

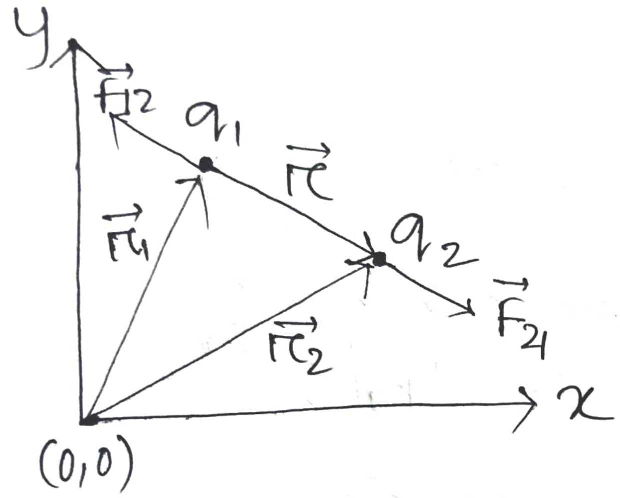
Imp.

Relative permittivity of different mediums

- vaccum – ($\varepsilon_r = 1$)
- Air – ($\varepsilon_r = 1.0006$)
- Glass – ($\varepsilon_r = 4.9$ to 7.5)
- water – ($\varepsilon_r = 80.4 \simeq 80$)
- Mica – (3–$7 = \varepsilon_r$)

(VII) Electrostatic force is a conservative force. (i.e The workdone against these forces does not depend on path)

#Vector form of coulomb's law (Board exam Style)



here $\vec{r}_1$ = position vector of q_1 charge.

$\vec{r}_2$ = position vector of chareg q_2.

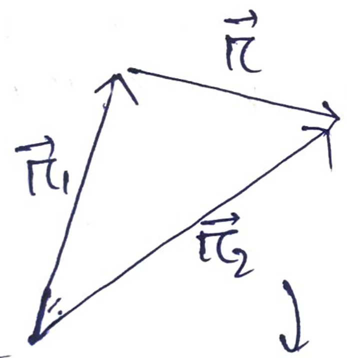
$\vec{r}$ = Displacement vector along force $\vec{F}_{21}$

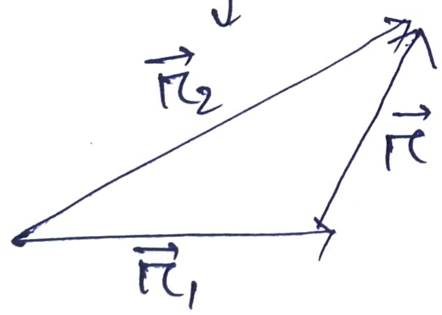
~~we know~~

$$\vec{F}_{21} = \frac{K q_1 q_2}{r^2}\hat{r}$$

$$\vec{F}_{21} = \frac{K q_1 q_2}{r^2}\frac{\vec{r}}{|\vec{r}|}$$

$$\boxed{\vec{F}_{21} = \frac{K q_1 q_2}{r^3}\vec{r}}$$





→ from triangle law of vector addition

$$\vec{r}_1 + \vec{r} = \vec{r}_2$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$r = |\vec{r}| = |\vec{r}_2 - \vec{r}_1|$$

Now

$$\boxed{\vec{F}_{21} = \frac{K q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)}$$

Similarly,

$$\boxed{\vec{F}_{12} = \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)}$$

(Ans:)

for understanding

For simplification we take

$$\vec{r}_2 - \vec{r}_1 = \vec{r}_{21}$$

$$\& \quad \vec{r}_1 - \vec{r}_2 = \vec{r}_{12}$$

So $$\vec{r}_{21} = -\vec{r}_{12}$$

but $$|\vec{r}_{21}| = |\vec{r}_{12}| = r$$

Hint

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

or $$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

And we can also write

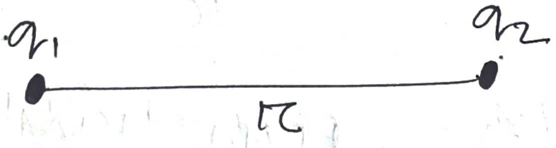
$$\vec{F}_{12} = \frac{Kq_1q_2}{(|\vec{r}_{21}|)^2}\hat{r}_{21} \quad \text{or} \quad \frac{Kq_1q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

$$\vec{F}_{21} = \frac{Kq_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} \quad \text{or} \quad \frac{Kq_1q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

—o—

Qus: Two charges of 1nC are separated by a distance of 10m. find the coulomb's force b/w them?

Ans:



Given

$q_1 = q_2 = 1nC = 10^{-9} C$

$r = 10m$

$$F = \frac{Kq_1q_2}{r^2}$$

$$F = 9\times10^{9} \times \frac{10^{-9}\times10^{-9}}{10^{2}} = 9\times10^{-11} N$$

$$\boxed{F = 9\times10^{-11} N}$$

Qus: Two charges q_1 and q_2 are now separated from r to $3r$. find the new force b/w them?

Ans:

$$f = \frac{Kq_1q_2}{r^2}$$

$$f' = \frac{Kq_1q_2}{(3r)^2} = \frac{Kq_1q_2}{9r^2}$$

$$f' = \frac{1}{9}\left(\frac{Kq_1q_2}{r^2}\right)$$

$$\boxed{f' = \frac{f}{9}}$$ Ans.

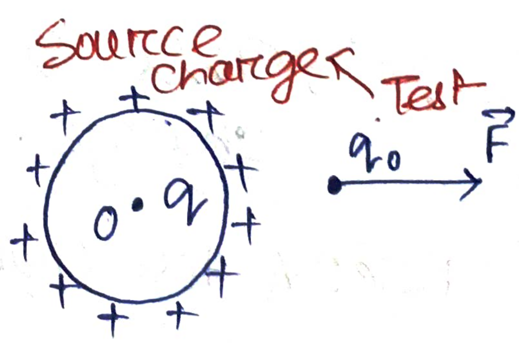
∴ new force f' is reduced by 9 times

Electric field:

Electric field: The space around a charge where electric force due to that charge can be experienced.

Electric field Intensity: The electric field at a point is defined as the electrostatic force per unit charge acting on a vanishingly small positive test charge placed at that point.

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}} \qquad \boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}}$$



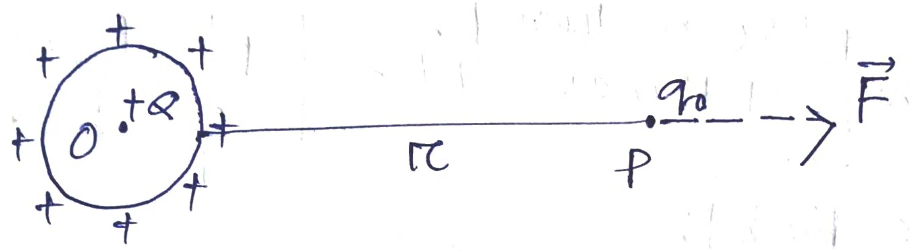
Note: The electric field $\vec{E}$ is a vector quantity whose dirⁿ is same as that of force $\vec{F}$ exerted.

→ Its S.I unit is N/C or V/m.

Electric field due to a point charge.

Let us consider a source charge $+Q$ placed in vaccum at origin O. place a point charge q_0 at point P where $|\vec{OP}| = r$.

The source charge $+Q$ exert a force($\vec{F}$) on the test charge(q_0). The force exerted on the test charge due to the Electric field ($\vec{E}$) of source charge ($+Q$).



$$\vec{F} = \frac{KQq_0}{r^2}\hat{r} ,$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

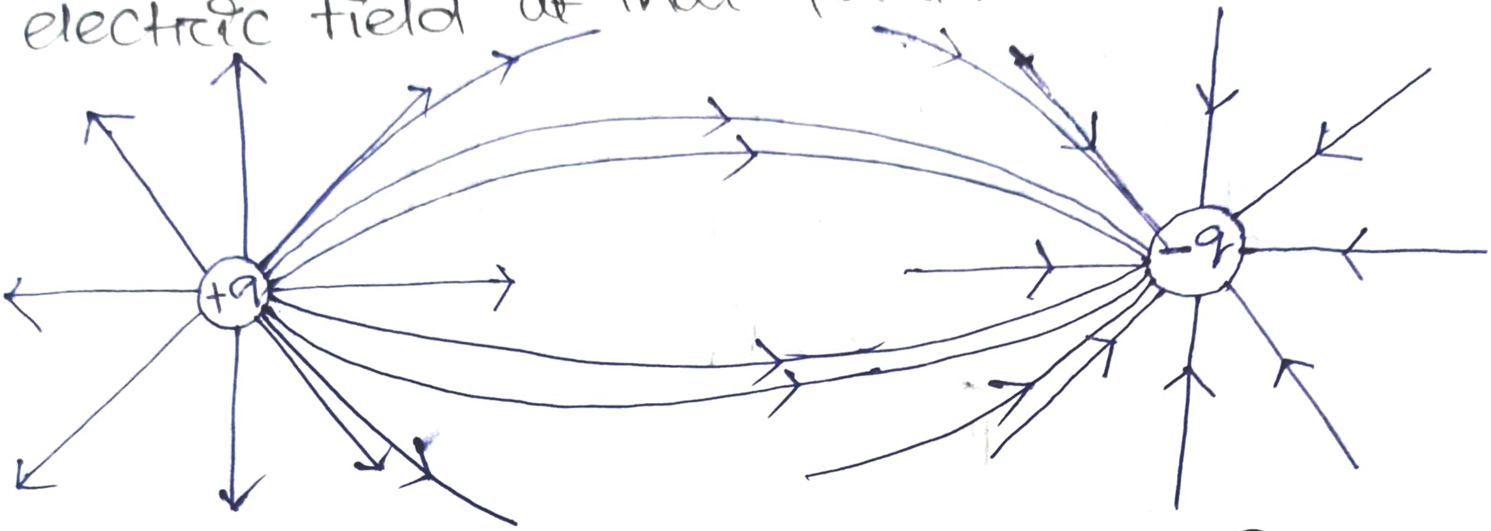
$$\vec{E} = \frac{KQq_0}{r^2}\Big/q_0 \ \hat{r} \Rightarrow \boxed{\vec{E} = \frac{KQ}{r^2}\hat{r}}$$

∴ This is Electric field due to a point charge.

Note: A test charge q_0 is positive in nature.

Electric lines of force/Electric field lines

An electric line of ~~m~~ force may be defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of electric field at that point.



Note: The lines of force do not really exist, they are imaginary curves. Yet the concepts of lines of force is very useful. Michael Faraday gave simple explanations for many of his discoveries (in electricity and magnetism) in terms of such lines of force.

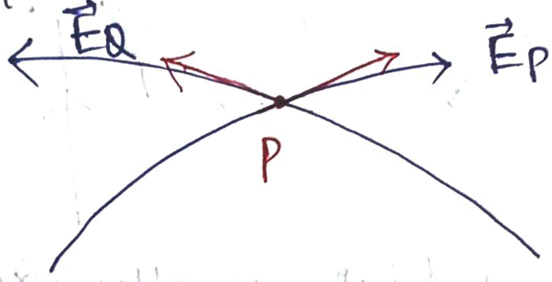
properties of electric lines of force:

1. The lines of force are continuous smooth curve without any breaks.
2. The line of force start at positive charge and end at negative charges they can not form closed loops. If there is a single charge, then the lines of force will start or end infinity.

3. The tangent to a line of force at any point gives the direction of the electric field at that point.

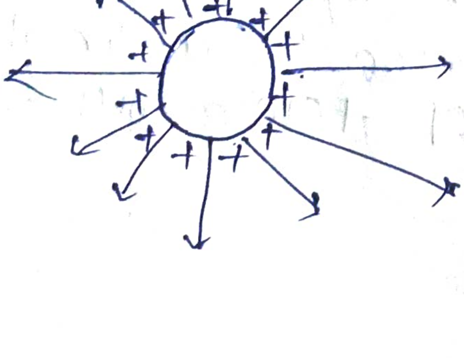
4. No two lines of force can cross each other.

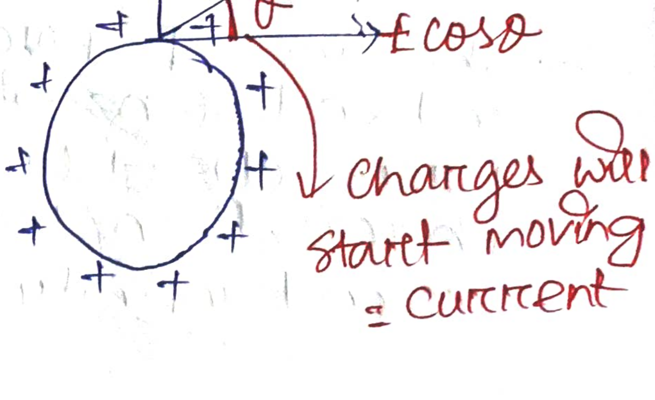
Reason: If they intersect, then there will be two tangents at the point of intersection (fig.) and hence two directions of the ~~these~~ electric field at the same point, which is not possible.



5. The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

Reason: If the lines of force are not normal to the conductor, the component of the field E parallel to the surface would cause the electrons to move and would set up a current on the surface. But no current flows in the equilibrium condition.

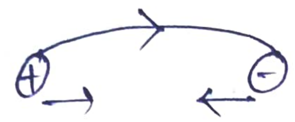




6. The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.

7. The lines of force have tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.

6.



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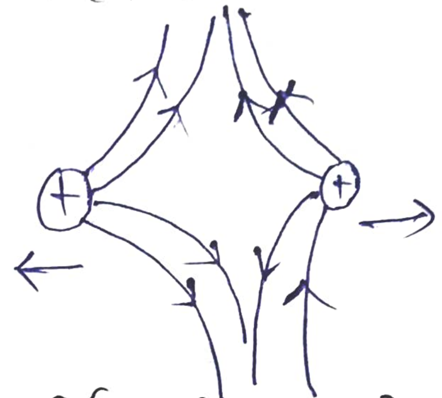
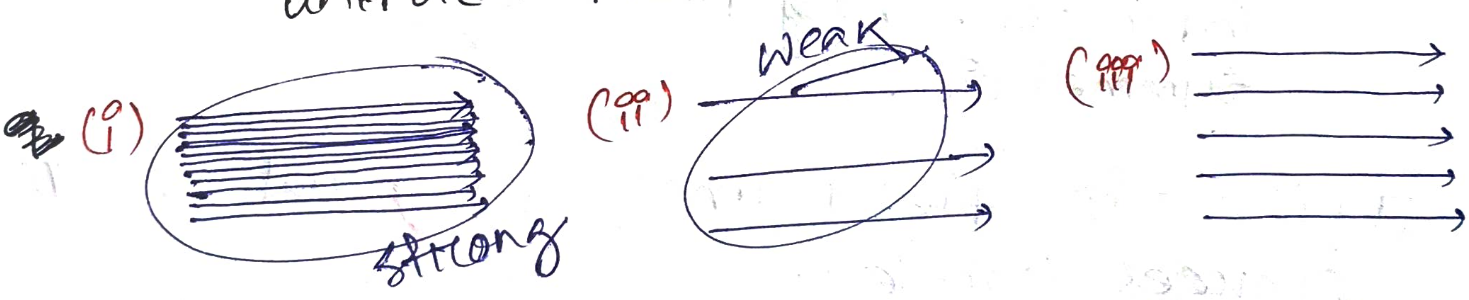


8. The relative closeness of lines of force gives a measure of the strength of the electric field in any region. The lines of force are

(i) close together in a strong field.

(ii) far apart in a weak field.

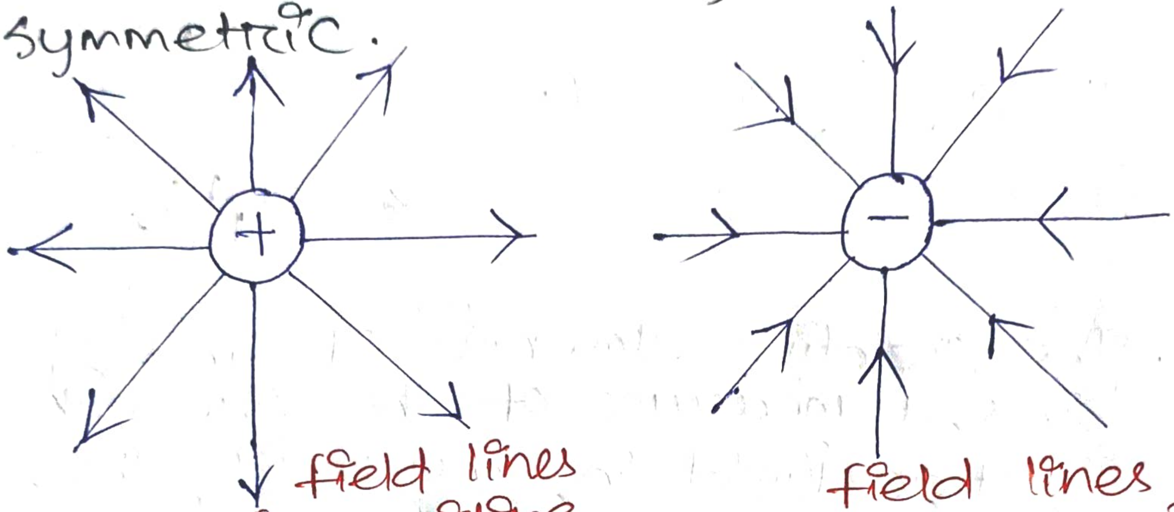
(iii) parallel and equally spaced in a uniform field.



9. The lines of force do not pass through a conductor because the electric field inside a conductor (charged) is zero.

II Electric field lines for different charged conductor:

(i) field lines of a positive point charge radially outwards.
They extend to infinity.
The field is spherically symmetric.



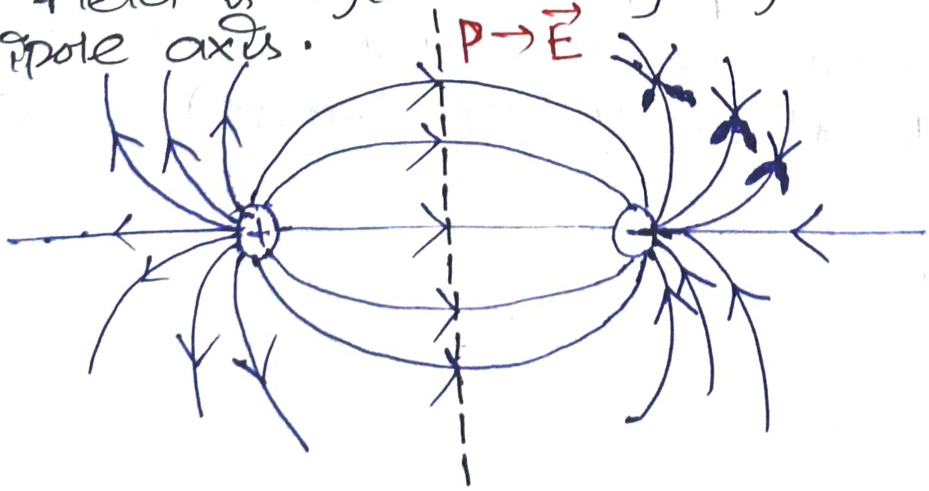
field lines of a positive point charge

field lines of a negative point charge.

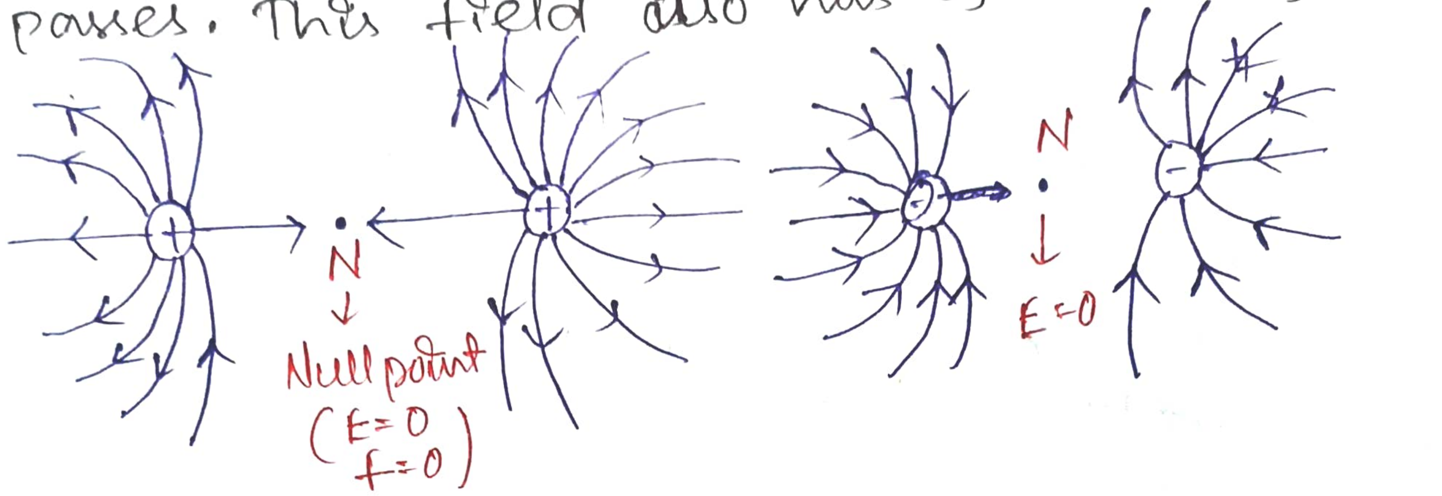
(ii) field lines of a negative point charge radially inwards. They start from infinity. The field is spherically symmetric.

II Electric field lines for equal and opposite charges (Dipole).

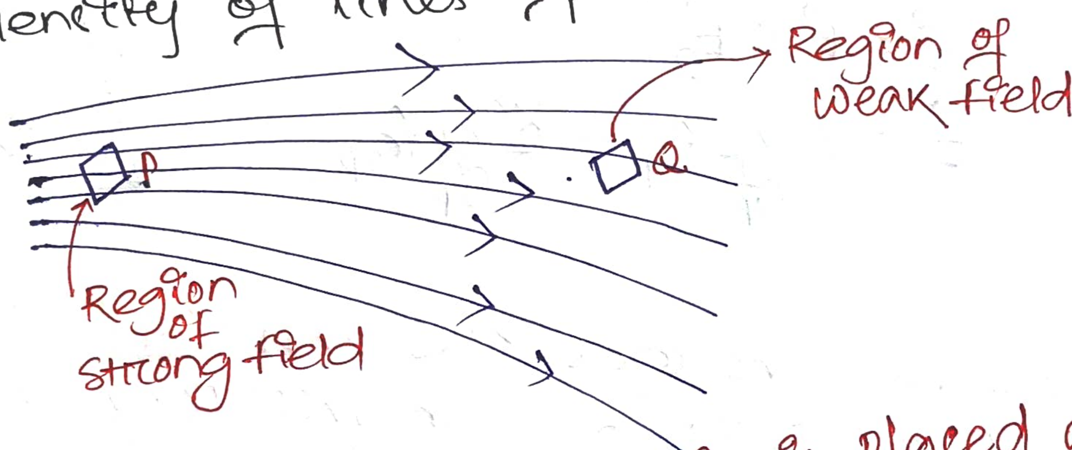
The field is cylindrically symmetric about the dipole axis.



* The field $\vec{E}$ is zero at the middle point N of the join of two same charges. The point is called neutral point from which no line of line of force passes. This field also has cylindrical symmetry.



Note: Electric field strength is proportional to the density of lines of force.



Qus: A point charge of 0.009μC is placed at the origin. Calculate the intensity of electric field due to this point $(\sqrt{2}, \sqrt{7}, 0)$m.

Ans: Given $Q = 0.009\mu C = 9\times10^{-9}C$

$$r = \sqrt{(\sqrt{2})^2 + (\sqrt{7})^2 + 0^2}$$

$$r = \sqrt{2+7}$$

$$r = \sqrt{9}$$

$$r = 3$$

$$E = \frac{KQ}{r^2}$$

$$E = \frac{9\times10^{9} \times 9\times10^{-9}}{3^2}$$

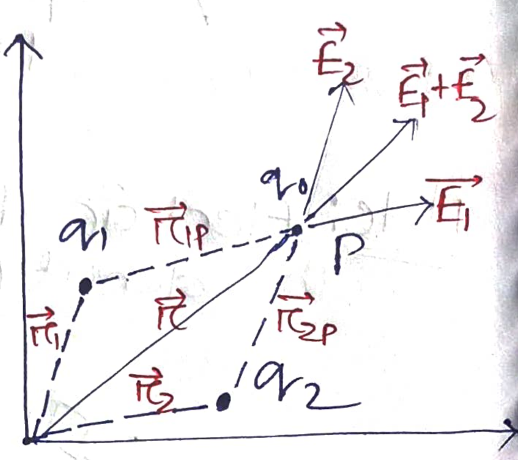
$$\boxed{E = 9 N/C}$$

Electric field due to A system of point Charges :

The electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.

Hence, the electric field at point P due to the system of N charges is

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \dots + \vec{E_N}$$

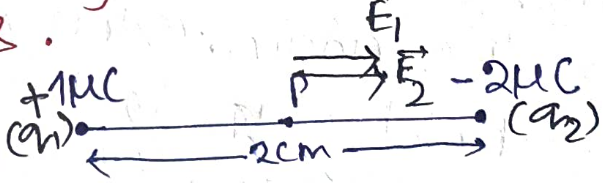


for two Charges

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} = \frac{Kq_1}{r_{1p}^2}\hat{r}_{1p} + \frac{Kq_2}{r_{2p}^2}\hat{r}_{2p}$$

Qus: Two charges $1\mu C$ and $-2\mu C$ are placed 2cm apart kept in vaccum. What is the Electric field intensity at the mid point of these two charges.



Ans: At point P

$$\vec{E}_{net} = \vec{E_1} + \vec{E_2} = E_1 + E_2$$

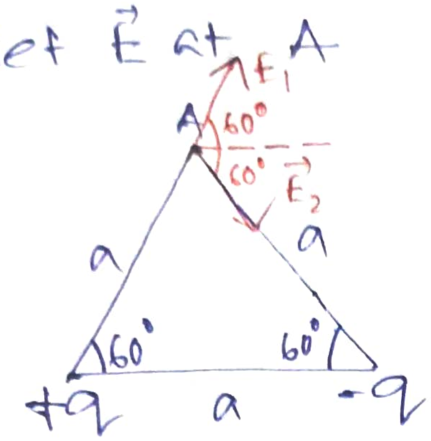
$$= \frac{Kq_1}{r^2} + \frac{Kq_2}{r^2}$$

$$= \frac{K}{r^2}(q_1 + q_2) = \frac{9\times10^9 \frac{Nm^2}{C^2}(10^{-6}C + 2\times10^{-6}C)}{(10^{-3})^2 m^2}$$

$$= \frac{9\times10^9}{10^{-6}} \times 3\times10^{-6} N/C = 27\times10^9 N/C$$

$$\boxed{\vec{E}_{net} = 27\times10^9 N/C}$$

(Q1) Net $\vec{E}$ at A



here angle betn $\vec{E}_1$ & $\vec{E}_2$ is $120°$

$$\vec{E}_1 = \frac{Kq}{a^2} = \vec{E}_2 = \frac{Kq}{a^2} \quad (E_1 = E_2 = E)$$

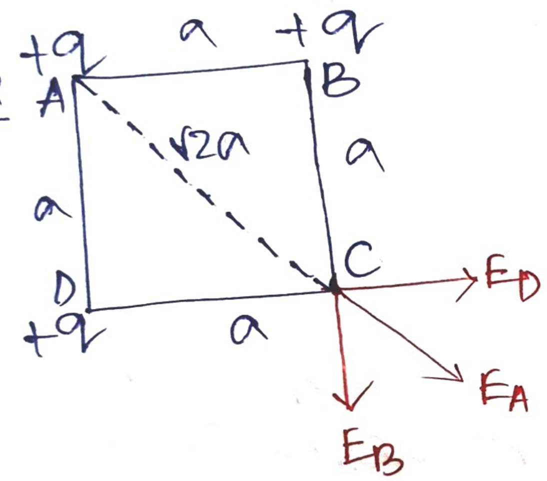
$$\vec{E}_{net} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 120°}$$

$$= \sqrt{E^2 + E^2 + 2E^2 \times -\tfrac{1}{2}}$$

$$= \sqrt{2E^2 - E^2}$$

$$\therefore \vec{E}_{net} = \sqrt{E^2} = E = \frac{Kq}{a^2} \quad (\text{dirn } +q \text{ to } -q)$$

Q.2



E_{net} at C.

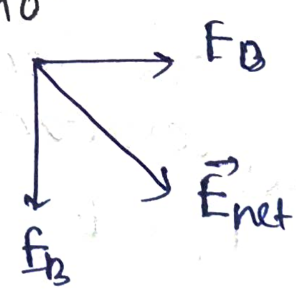
Ans: $E_B = \frac{Kq}{a^2} = E_D \quad (\theta = 90°;\ E_B \,\&\, E_D)$

$$E_A = \frac{Kq}{(\sqrt{2}a)^2} = \frac{Kq}{2a^2} = \frac{E}{2}$$

$$E_B = E_D = E$$

so now $\vec{E}_B + \vec{E}_D = \sqrt{E^2 + E^2 + 2E^2\cos 90°}$

$$= \sqrt{2}E$$



now E_{net} and E_A are in same dirn

so $E_{Net} = E_{net} + E_A$

$$\therefore \boxed{E_{Net} = \sqrt{2}E + E/2}$$

II Qus: Two point charges +16μC and −9μC are placed 8cm apart in air. Determine the position of the point at which the resultant field is zero.

Ans:

$$E_p^+ = E_p^-$$

$$\frac{Kq}{(8+x)^2} = \frac{Kq}{x^2} \Rightarrow \frac{K(+16\mu C)}{(8+x)^2} = \frac{K(9\mu C)}{x^2}$$

$$\Rightarrow \frac{16}{(8+x)^2} = \frac{9}{x^2} \Rightarrow \frac{4}{8+x} = \frac{3}{x} \Rightarrow 4x = 3(8+x)$$

$$\Rightarrow 4x = 24 + 3x$$

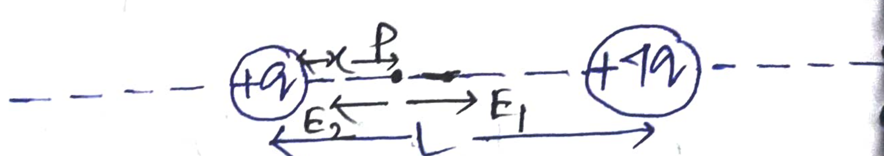
$$4x - 3x = 24$$

$$\therefore \boxed{x = 24\,cm}$$

II Qus: Two ~~like~~ positive charges +q & +4q C are placed L distance apart in air. Determine the position of the point at which the resultant field is zero.

Ans:

$$E_1 = E_2$$



$$\Rightarrow \frac{Kq}{x^2} = \frac{K(4q)}{(l-x)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{4}{(L-x)^2} \Rightarrow \left(\frac{L-x}{x}\right)^2 = 4$$

$$\frac{L-x}{x} = \pm 2 \,, \quad L - x = +2x \quad \text{or} \quad L - x = -2x$$

$$3x = L \qquad\qquad L = -x$$

$$\boxed{x = L/3} \qquad\qquad \boxed{x = -L}$$

not possible

Note: (i) If we have two like charges (+,+), (-,-) then E is zero between the charges near to the smaller charge.

$$\boxed{x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2}+\sqrt{q_1}}\right)L}$$

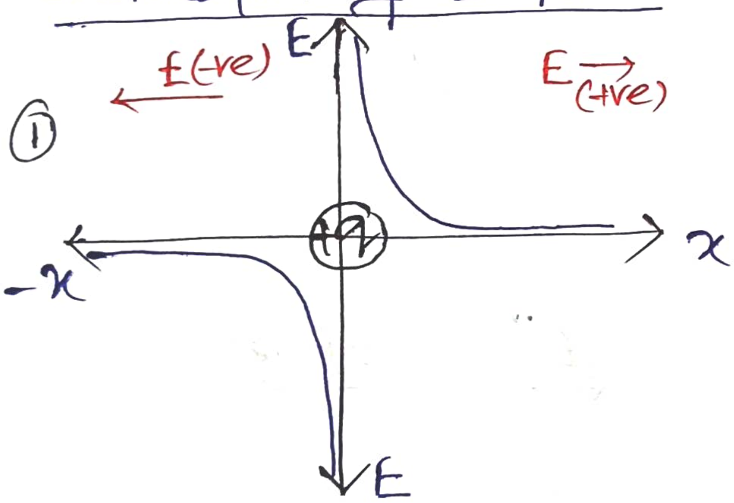
$\because q_1$ = smaller charge

(ii) If the charges are opposite then the E is zero outside the line and near to smaller charge.

$$\boxed{x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2}-\sqrt{q_1}}\right)L}$$

q = smaller

Graph of $\vec{E}$ v/s $\vec{x}$

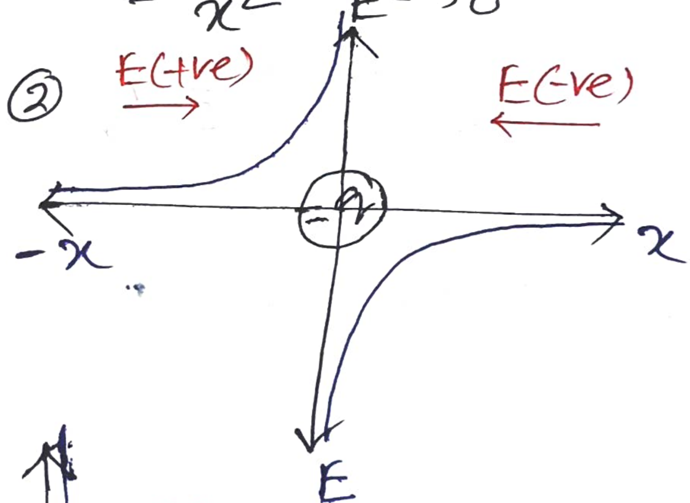
① 

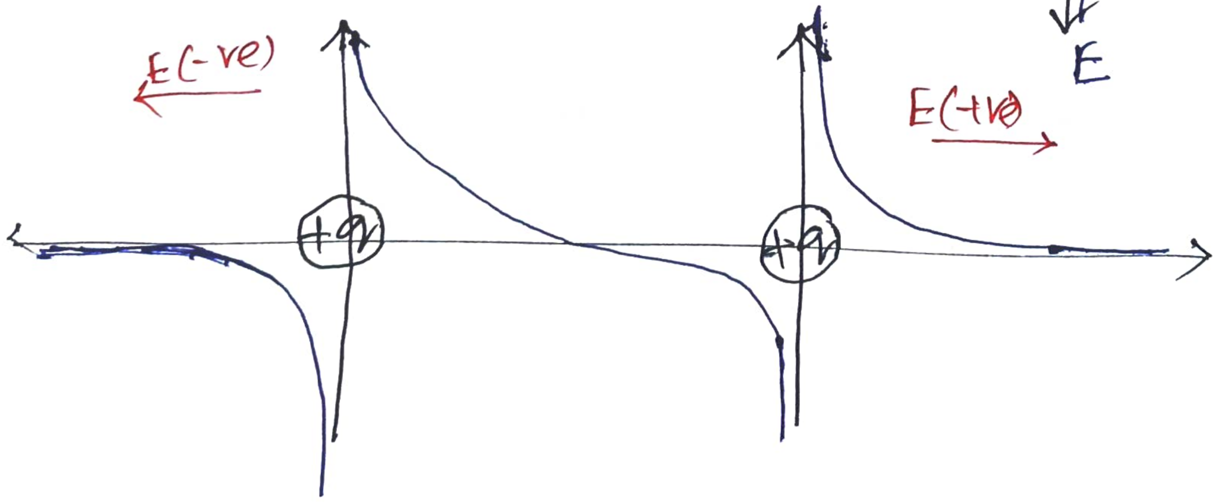
E(-ve) E(+ve)

$E \propto \frac{1}{x^2}$

$x \to \infty$

$E \to 0$

② 

③ 

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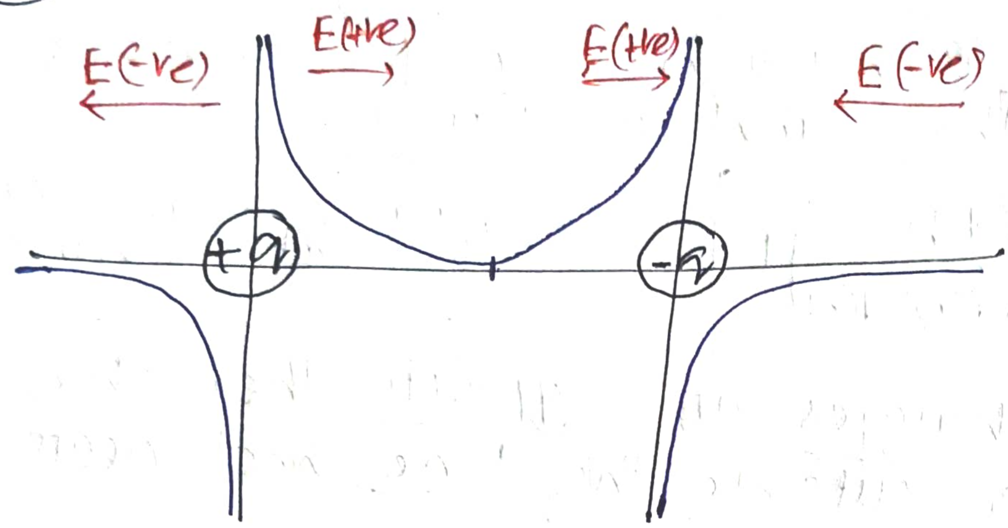

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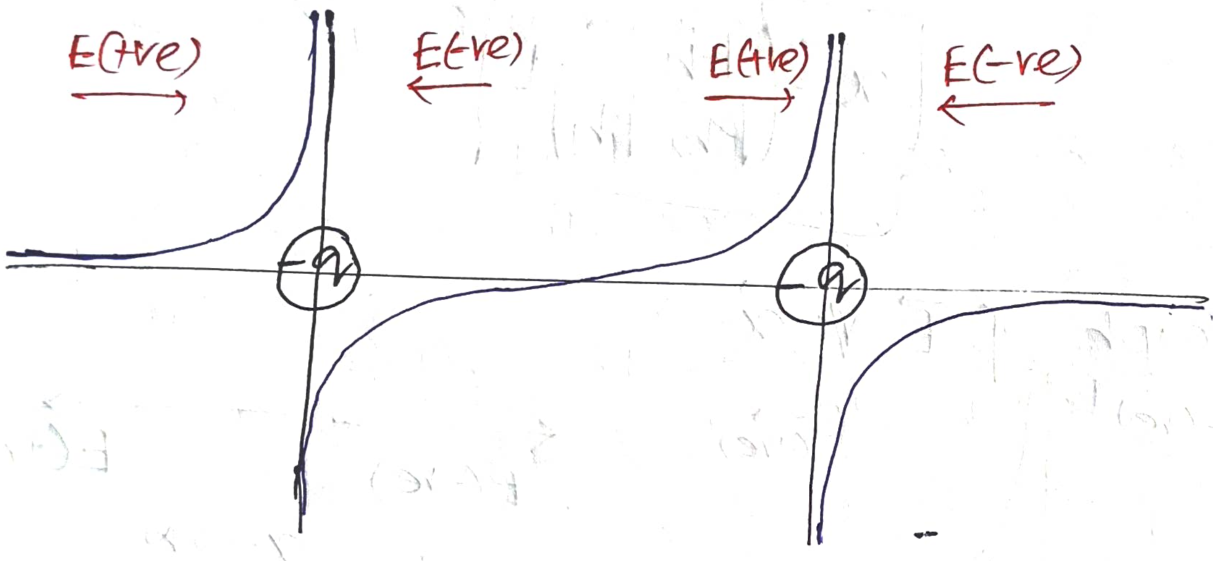

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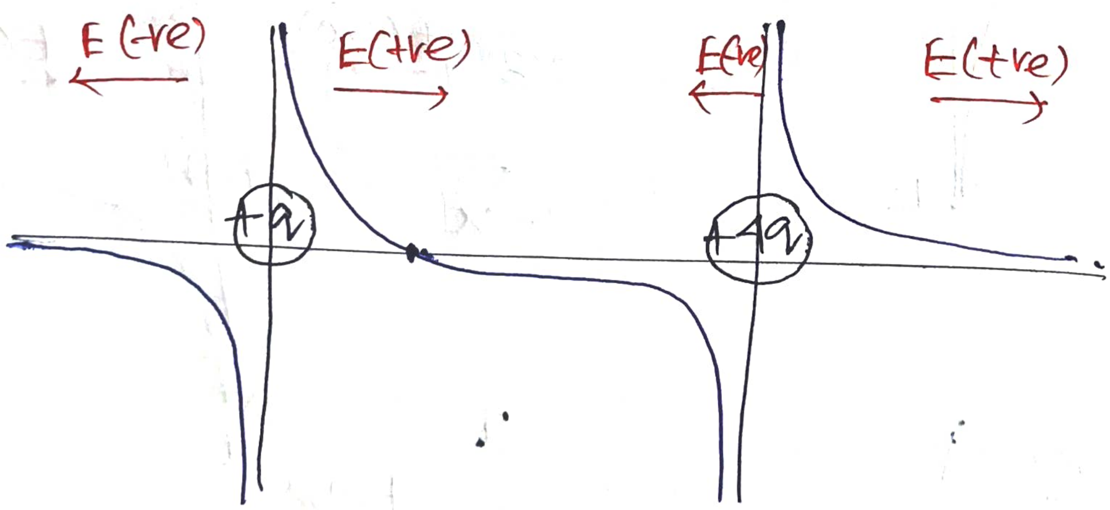


Some problems on force:

Qus: find the ratio of F_E & F_G for two electrons placed at same distance apart in air.

(a) 10^{10} (b) 10^{20} (c) 10^{30} (d) 10^{40}

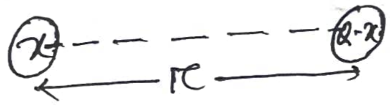
Ans: $F_E = \frac{Kq_1q_2}{r^2}$, $F_G = \frac{GM_1M_2}{r^2}$

$$\frac{F_E}{F_G} = \frac{Kq_1q_2}{GM_1M_2} = \frac{9\times10^9(1.6\times10^{-19})^2}{6.67\times10^{-11}\times(9.1\times10^{-31})^2} \simeq 10^{42}.$$

$\therefore$ Ratio $\simeq 10^{42} \simeq 10^{40}$.

Qus: A charge 'Q' is divided in two parts such that when thee two parts are kept at some separation the F_E between them is maximum. find charge on each part.

Ans: $F_E = \frac{Kx(Q-x)}{r^2}$



$$F_E = \frac{KQx}{r^2} - \frac{Kx^2}{r^2}$$

here $F_E = f(x)$,

So $\frac{dF_E}{dx} = 0 \longrightarrow$ for maximum

$$\Rightarrow \frac{KQ}{r^2} - \frac{2Kx}{r^2} = 0$$

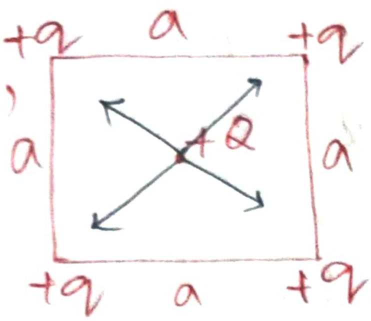
$$\Rightarrow KQ - 2Kx = 0$$

$$\Rightarrow Q = 2x \quad \text{or} \quad \boxed{x = Q/2}$$

$\therefore$ both (charge) parts are equal ($Q/2$).

(1)

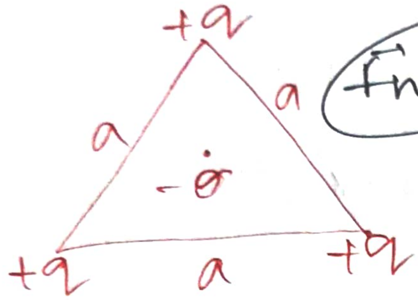
(a)



Find net force on +Q/−Q

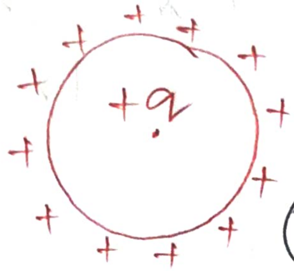
$\vec{F}_{net} = 0$

(b)



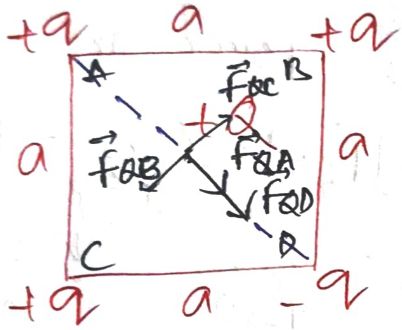
$\vec{F}_{net} = 0$

(c)



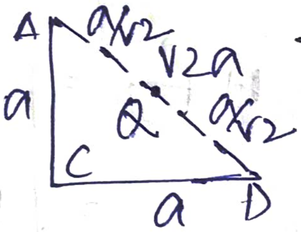
$\vec{F}_{net} = 0$

(d)



$\vec{F}_{QB}$ & $\vec{F}_{QC}$ equal and opposite so cancel each other

$\vec{F}_{QA} + \vec{F}_{QD} =$



so $F_{QA} = \dfrac{KQq}{(a/\sqrt{2})^2}$

$F_{QD} = \dfrac{KQq}{(a/\sqrt{2})^2}$

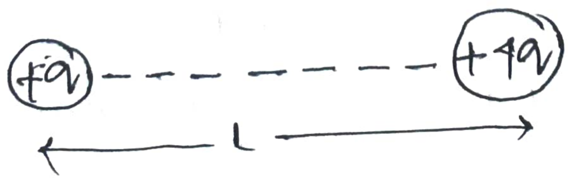
$$F_{net} = \frac{2KQq}{a^2} + \frac{2KQq}{a^2}$$

$$\boxed{F_{net} = \frac{4KQq}{a^2}}$$

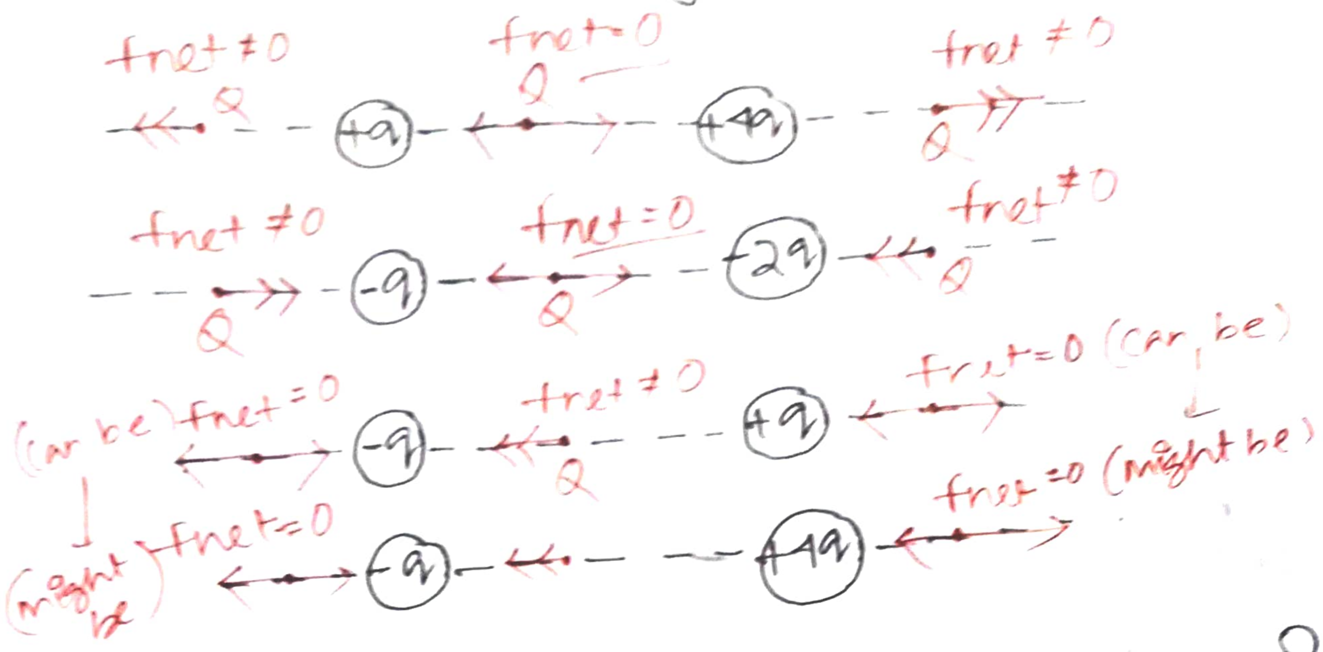
Ans.

II Third charge in equilibrium Type problem

Where should we keep a third charge on the line joining two charges such that 3rd charge is in equilibrium.



Let the third charge 'Q'.



* Net force is zero near smaller point charge

Charge Distribution:

We are studying the force/field in electro-static due to point charges. But in real world scenario we found charge body in which point charges are distributed over it.

Charge distribution can be three types

(I) Linear charge distribution (λ)
(II) Surface charge distribution (σ)
(III) Volume charge distribution (ρ)

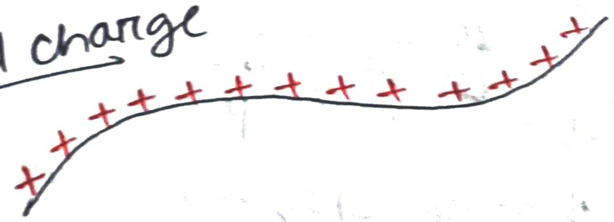
(I) Linear Charge Distribution: (LCD) - [λ]

If the total charge is distributed on a linear body then it is known as linear charge distribution.

total charge

L.C.D density $\boxed{\lambda = \frac{q}{l}}$ length

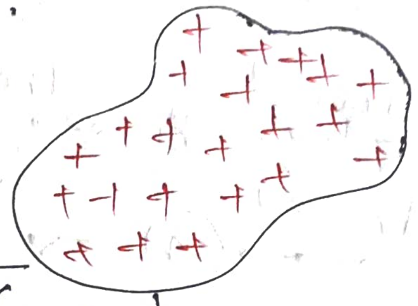
$\boxed{\lambda dl = dq}$



(II) Surface charge Distribution (σ)

If the total charge is distributed on the surface of a body then it is known as surface charge distribution.

It is represent by σ.



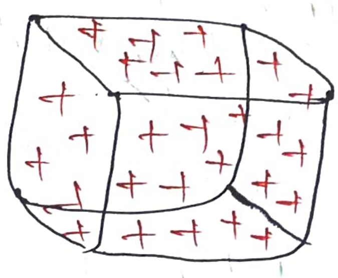
Surface charge density $\leftarrow$ $\boxed{\sigma = \frac{q}{A}}$

$d\sigma = \int \frac{dq}{dA} \Rightarrow \boxed{\int dq = \int \sigma \, dA}$

(III) Volume charge Distribution (ρ)

If the total charge is distributed over the volume of the body then it is known as volume charge distribution.

It is represented by ρ.

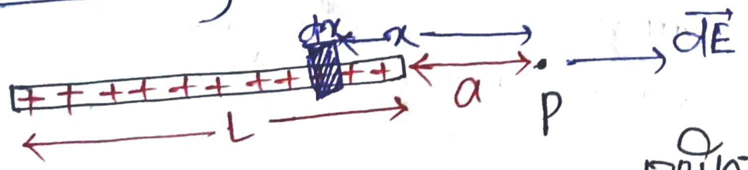


Volume charge density $\leftarrow$ $\boxed{\rho = \frac{q}{V}}$

or $\rho = \frac{dq}{dv} \Rightarrow \boxed{\int dq = \int \rho \, dv}$

Electric field due to continous charge distribution:

(a) Linearly charged rod



Small charge $dq \rightarrow$ point charge

small Electric field $\rightarrow d\vec{E}$

$$dE = \frac{K\,dq}{x^2}$$

$$E_{net} = \int dE = \int_a^{a+L} \frac{K\,dq}{x^2}$$

$$\left[\because \lambda = \frac{q}{L} \qquad \lambda = \frac{dq}{dx} \qquad dq = \lambda\,dx \right]$$

$$E_{net} = K \int_a^{a+L} \frac{\lambda\,dx}{x^2}$$

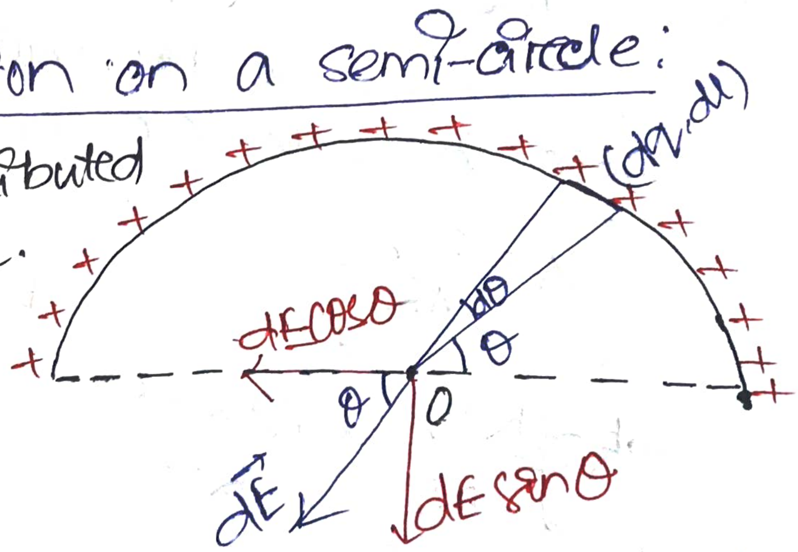
$$= K\lambda \int_a^{a+L} \frac{dx}{x^2}$$

$$= K\lambda \left[\frac{-1}{x}\right]_a^{a+L} = K\lambda \left[-\frac{1}{a+L} - \left(\frac{-1}{a}\right)\right]$$

$$\boxed{E_{net} = K\lambda \left[\frac{1}{a} - \frac{1}{a+L}\right]}$$

(b) Charge distribution on a semi-circle:

'q' charge is distributed over a curved line.



here $\boxed{\lambda = \frac{q}{\pi R}}$

dE cosθ will be canceled for each element due to mirror-core element. & all dE sin θ will add.

$$E_{net} = \int dE \sin\theta$$

$$= \int \frac{K\,dq}{R^2} \sin\theta$$

$$\left\{ \because \int dE = \int \frac{K\,dq}{R^2} \right.$$

~~dE/=~~

NOW

$$E_{net} = \int K\left(\frac{\lambda R d\theta}{R^2}\right)\sin\theta \qquad \left[\begin{array}{l} \because \lambda = \frac{dq}{dl} \\ dq = \lambda dl \\ dq = \lambda R d\theta \end{array}\right.$$

$$E_{net} = \frac{K\lambda}{R}\int_0^{\pi} \sin\theta \, d\theta$$

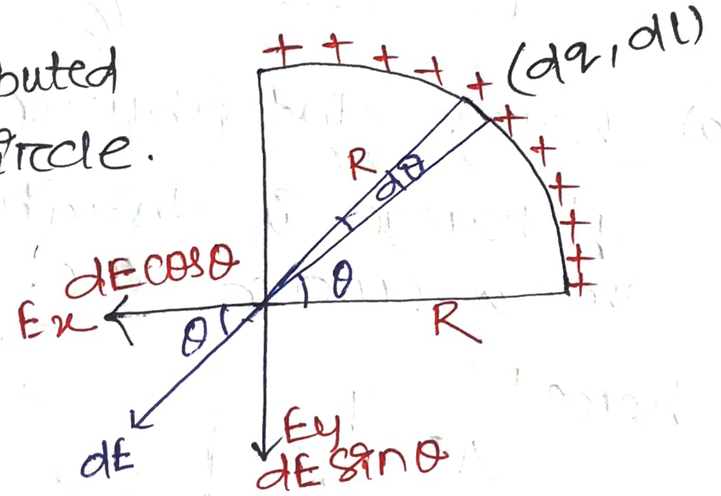
$$E_{net} = \frac{K\lambda}{R}\left[-\cos\theta\right]_0^{\pi} = \frac{K\lambda}{R}\left[-\cos\pi + \cos 0\right]$$

$$E_{net} = \frac{K\lambda}{R}\left[1+1\right] = \frac{2K\lambda}{R}$$

$$\therefore \boxed{E_{net} = \frac{2K\lambda}{R}\,(\hat{j})}$$

(c) Quarter circle :

'q' charge is distributed over the quarter circle.



$$\boxed{\lambda = \frac{2q}{\pi R}}$$

$$dE = \int \frac{K dq}{R^2} = \frac{K\lambda R d\theta}{R^2}$$

$$dq = \lambda dl$$
$$dq = \lambda R d\theta$$

NOW $E_x = \int dE \cos\theta$

$E_y = \int dE \sin\theta$

$$E_{net} = \sqrt{E_x^2 + E_y^2}$$

$$E_x = \int dE\cos\theta = \int_0^{\pi/2} \frac{K\lambda d\theta}{R}\cos\theta$$

$$= \frac{K\lambda}{R}\left[\sin\theta\right]_0^{\pi/2} = \frac{K\lambda}{R}$$

$$E_y = \int dE \sin\theta = \int_0^{\pi/2} \frac{K\lambda \, d\theta}{R} \sin\theta$$

$$E_y = \frac{K\lambda}{R}\left[-\cos\theta\right]_0^{\pi/2} = \frac{K\lambda}{R}\left[\cos 90 - (\cos 0)\right]$$

$$E_y = \frac{K\lambda}{R}\left[0 - (-1)\right] = \frac{K\lambda}{R}$$

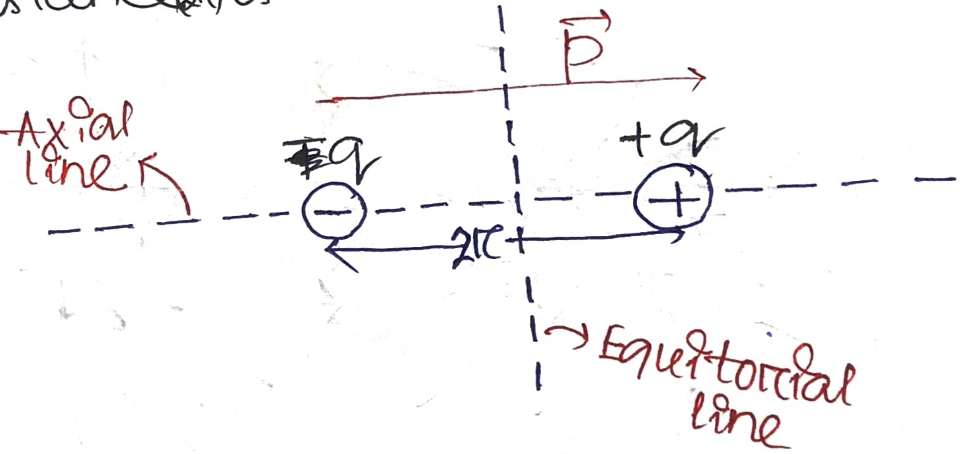
$$\therefore E_x = E_y = \frac{K\lambda}{R} = E$$

Now $E_{net} = \sqrt{E_x^2 + E_y^2 \cancel{+ 2E_x}}$

$$E_{net} = \sqrt{E^2 + E^2} = \sqrt{2}E$$

Electric Dipole:

A pair of equal and opposite charges separated by a small distance is called an electric dipole.



Dipole moment:

$$\boxed{\vec{p} = q \times 2a}$$

$$\boxed{\vec{p} = \text{charge}(q) \times \text{separation}(d)}$$

→ It is the product of either charges and the separation between them.

→ It's direction is from -ve charge to +ve charge.

→ It's a vector quantity. It's S.I unit is [C.M].

* Examples:

Dipoles are common in nature.
In molecules like HCl, H_2O, C_2H_5OH etc.

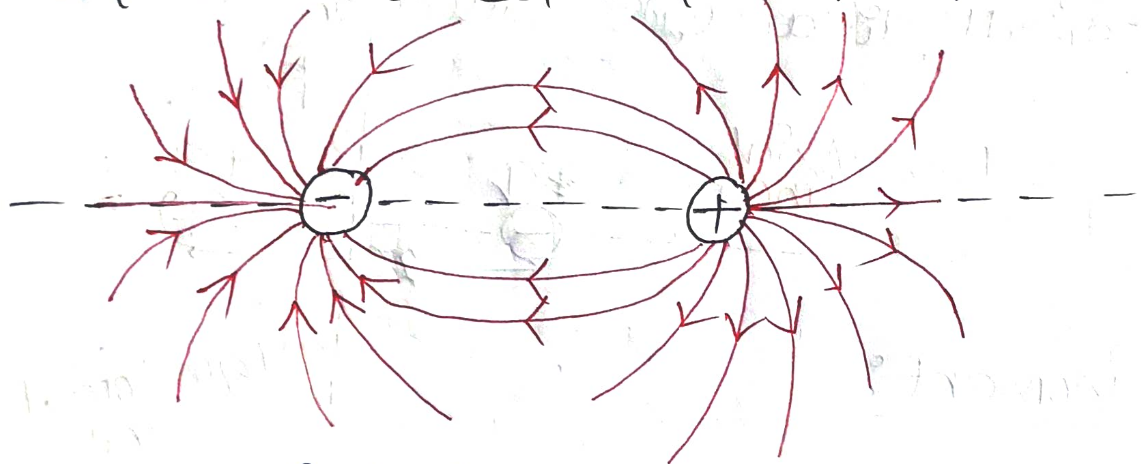
The centre of positive charge does not fall exactly over the centre of negative charges. Such molecules are electric dipoles. They have a permanent dipole moment.

Non-polar
⊖⊕

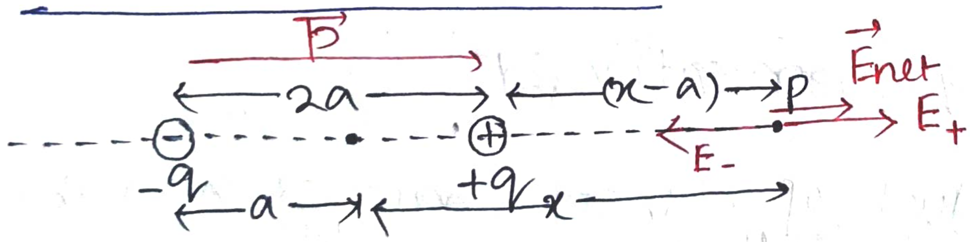
polar molecule
⊖ - - - ⊕

* Dipole field:

The electric field produced by an electric dipole is called dipole field.



E.F at axial line



Electric field at P due to +ve charge

$$|E_+| = \frac{Kq}{(x-a)^2}, \quad \vec{E}_+ = \frac{Kq}{(x-a)^2} \text{ in dir}^n \text{ of } \vec{P}$$

Electric field at P due to −ve charge

$$|E_-| = \frac{Kq}{(x+a)^2}, \quad \vec{E}_- = \frac{Kq}{(x+a)^2} \text{ opposite dir}^n \text{ of } \vec{P}$$

Now net electric field due to dipole at point P.

$$E_{net} = E_+ - E_-$$

$$= \frac{Kq}{(x-a)^2} - \frac{Kq}{(x+a)^2}$$

$$= Kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]$$

$$= Kq\left[\frac{(x+a)^2 - (x-a)^2}{(x-a)^2(x+a)^2}\right]$$

$$= Kq\left[\frac{x^2+a^2+2xa-(x^2+a^2-2xa)}{(x^2-a^2)^2}\right]$$

$$= Kq\left[\frac{4xa}{(x^2-a^2)^2}\right]$$

$$= 2Kx\left[\frac{q \times 2a}{(x^2-a^2)^2}\right]$$

$$\boxed{E_{net} = \frac{2Kpx}{(x^2-a^2)^2}}$$ along the dirc^n of $\vec{p}$

Axial position

* Special case: (for a short dipole)

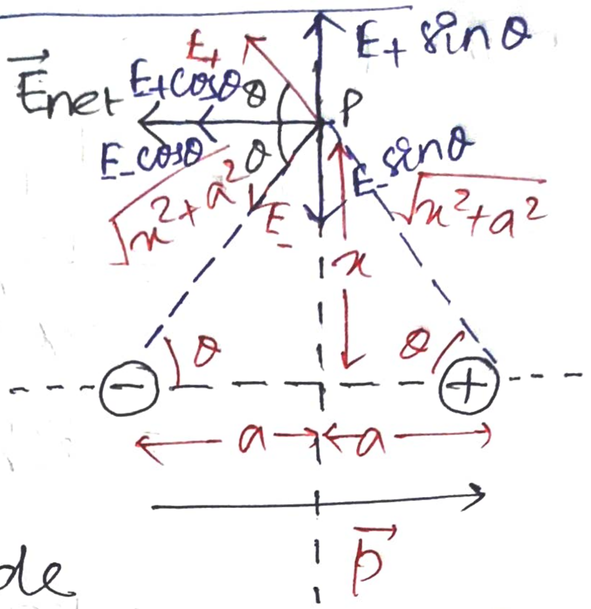
condition: $(x \gg a)$

$$x^2 - a^2 \simeq a^2$$

$$E_{net}(\text{axial}) = \frac{2KPx}{(x^2)^2} = \frac{2KP}{x^3}$$

$$\therefore \boxed{\vec{E}_{axial} = \frac{2K\vec{p}}{x^3}}$$

Electric field at equitorial point.



The electric fields are split into two components $E\cos\theta$ & $E\sin\theta$.

* The two $\sin\theta$ compo. $E_+\sin\theta$ and $E_-\sin\theta$ are opposite to each other and the magnitude is also same, so they cancel each other.

* Both $E_+\cos\theta$ and $E_-\cos\theta$ are in the same dirc^n (opposite to $\vec{p}$) ~~and~~ so the net electric field at point P.

$$\left[\because \cos\theta = \frac{b}{h} = \frac{a}{\sqrt{x^2+a^2}}\right.$$

and $E_+ = \frac{Kq}{\sqrt{x^2+a^2}} = E_-$

so $E_+ = E_- = E$

$$E_{net} = 2E\cos\theta$$

$$= 2\frac{Kq}{(\sqrt{x^2+a^2})^2}\cos\theta$$

$$= \frac{2Kq}{x^2+a^2} \cdot \frac{a}{\sqrt{x^2+a^2}}$$

$$= \frac{K(q \times 2a)}{(x^2+a^2)^{3/2}}$$

$$\boxed{E_{net} = \frac{Kp}{(x^2+a^2)^{3/2}} \text{ (opposite to } \vec{p})}$$

* special case : $(x >> a)$

$$x^2 + a^2 \simeq x^2$$

$$E_{net} = \frac{Kp}{(x^2)^{3/2}}$$

$$\boxed{E_{eq.} = \frac{Kp}{x^3}} \quad \text{or} \quad \boxed{\vec{E}_{eq.} = -\frac{K\vec{p}}{x^3}}$$

* Ratio E_{axial} & E_{equ}.

$$\frac{E_{axial}}{E_{eq.}} = \frac{\frac{2Kp}{x^3}}{\frac{Kp}{x^3}} = \frac{2}{1}$$

$$\therefore \boxed{E_{axial} : E_{eq} = 2 : 1}$$