ASSIGNMENT CLASS XII DIFFERENTIATION

1. Find $\frac{dy}{dx}$ for the following:

(a) $y=\frac{1}{\sqrt{a^2-x^2}}$ (b) $y=\frac{5x}{\sqrt[3]{1-x^2}}+\sin^2(2x+3)$ (c) $y=\frac{\cos x+\sin x}{\cos x-\sin x}$

(d) $y=\log\sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$ (e) $y=\log(x+\sqrt{1+x^2})$ (f) $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$

2. Show that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$.

3. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx}+y=0$.

4. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$.

5. Find $\frac{dy}{dx}$ for the following:

(a) $\sin^{-1}(\cos x)+\cos^{-1}(\sin x)$ (b) $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ (c) $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

(d) $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$ (e) $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

6. Find $\frac{dy}{dx}$ for the following:

(a) $\cos^{-1}(4x^3-3x)$ (b) $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ (c) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

(d) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ (e) $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

7. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

8. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, prove that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

9. If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$, show that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$.

10. If $y\log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

11. If $\log\left(\sqrt{x^2+y^2}\right) = \tan^{-1}\frac{y}{x}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

12. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $\left(1-x^2\right)\frac{dy}{dx} = xy+1$.

13. If $y = x^{\cos x} + \cos x^{\sin x}$, find $\frac{dy}{dx}$.

14. If $x^a y^b = (x+y)^{(a+b)}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

15. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

16. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

17. If $x = a\sin 2t(1+\cos 2t)$, $y = b\cos 2t(1-\cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$.

18. If $x = a\left(\frac{1+t^2}{1-t^2}\right)$, $y = \frac{2t}{1-t^2}$, show that $\frac{dy}{dx} = \frac{1+t^2}{2at}$.

19. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, find $\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}$.

20. If $y = A\cos nx + B\sin nx$, prove that $\frac{d^2y}{dx^2} + n^2 y = 0$.

21. If $y = e^x(\sin x + \cos x)$, prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

22. If $y = \tan^{-1}x$, show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$.

23. If $y = \left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

ANSWERS

1.(a) $\frac{x}{(a^2-x^2)^{3/2}}$ (b) $\frac{15-5x^2}{3(1-x^2)^{4/3}} + 2\sin(4x+6)$ (c) $\sec^2(x+\frac{\pi}{4})$ (d) $\frac{-\sin x\cos x}{1+\cos^2 x} + \frac{e^{2x}}{1-e^{2x}}$ (e) $\frac{1}{\sqrt{1+x^2}}$

(f) $-\sec^2(\frac{\pi}{4}-x)$ **5**. (a) -2 (b) $\frac{1}{2}$ (c) -1 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$ **6**.(a) $\frac{-3}{\sqrt{1-x^2}}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{1}{2(1+x^2)}$

(d) $\frac{x}{\sqrt{1-x^4}}$ (e) $\frac{-1}{\sqrt{1-x^2}}$ **13**. $x^{\cos x}\left\{\frac{\cos x}{x} - (\log x)\sin x\right\} + \cos x^{\sin x}\left\{-\sin x\tan x + \cos x\log(\cos x)\right\}$

15. $27\log 3 - 12$ **16**. 1 **19**. $\frac{-3}{2}$