Mechanical Properties Of Fluids

Fluids

At Rest → Hydrostatics

In Motion → Hydrodynamics

Surface Tension

Fluid: Anything that can flow. (Liquids & Gases)

Hydro-Statics:

The branch of physics which deals with the fluids at rest.

(I) Pressure (P):

The normal force acting per unit area of a fluids is called as $\boxed{\text{Pressure}}$

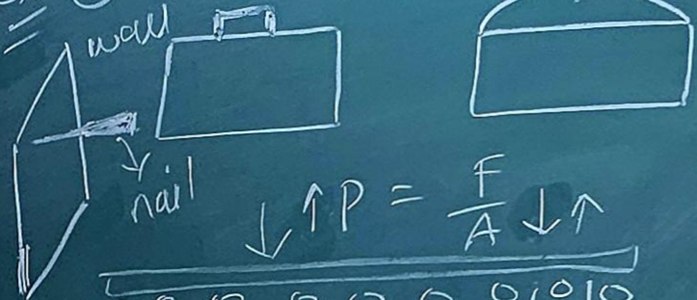
$$\text{Pressure} = \frac{\text{Normal Force}}{\text{Area}}$$

$$\boxed{P = \frac{F_N}{A}}$$

$$= \frac{N}{m^2}$$

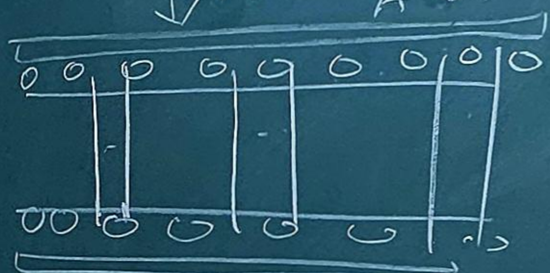
$$[P] = \frac{[F]}{[A]} = \frac{[L^1 M^1 T^{-2}]}{[L^2]} = [L^{-1} M^1 T^{-2}]$$

ex. ① Suitcase handles are made thick.

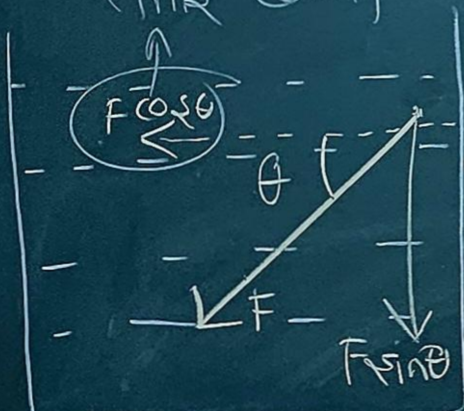


②

$$\downarrow\uparrow P = \frac{F}{A} \downarrow\uparrow$$

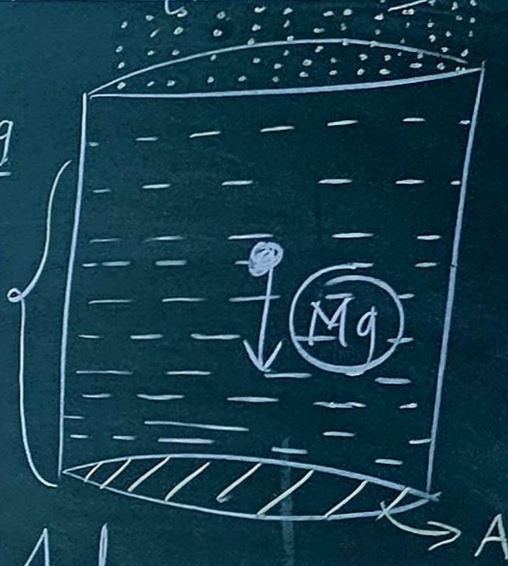


This component has to be absent in order to keep water at rest.



$$\boxed{P = \frac{F_N}{A} = \frac{F\sin\theta}{A}}$$

Pressure Due To Liquid Column



$$\text{Pressure} = \frac{F_N}{\text{Area}}$$

$$P = \frac{W}{A} = \frac{Mg}{A}$$

also,

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{M}{V}$$

$$M = \rho . V = \rho A d$$

$$\therefore P = \frac{\rho \cancel{A} d g}{\cancel{A}}$$

$$\boxed{\therefore P_L = \rho d g}$$

Also, one atmospheric pressure is acting on area A of

$$1\,atm = P_0 = 1.013 \times 10^5\ Pa.$$

Now, total pressure will be

$$\boxed{P_T = P_0 + P_L}$$

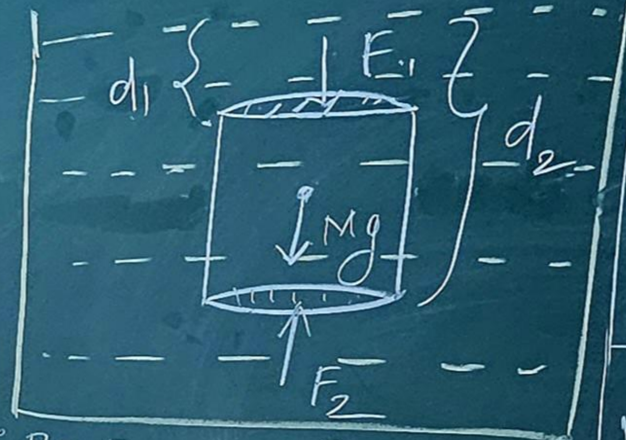
absolute 'p' = atmospheric p' + gauge 'p'

$$\boxed{P = P_0 + \rho d g}$$

Absolute pressure, Gauge Pressure:

At equilibrium

$$\boxed{F_{net} = 0}$$



$$F_1 + Mg = F_2$$

$$P_1 A + Mg = P_2 A \ldots \left[\because P = \frac{F}{A} \Rightarrow F = PA\right]$$

also, $\frac{M}{V} = \rho$

$$M = \rho V = \rho A(d_2 - d_1)$$

$$P_1 \cancel{A} + \rho \cancel{A}(d_2 - d_1)g = P_2 \cancel{A}$$

$$P_1 + \rho(d_2 - d_1)g = P_2$$

$$\therefore P_2 - P_1 = \rho(d_2 - d_1)g \ldots [P_2 > P_1]$$

$$\boxed{\therefore \Delta P = \rho \, \Delta d \, g}$$

change in absolute pressure

change in pressure due to liquid

Case (1) if $\Delta P = 0$

$$\therefore P_2 - P_1 = 0$$

$$\boxed{\therefore P_2 = P_1}$$

then $d_1 = d_2$

$\Delta d = 0$

At the same horizontal level pressure has to be the same

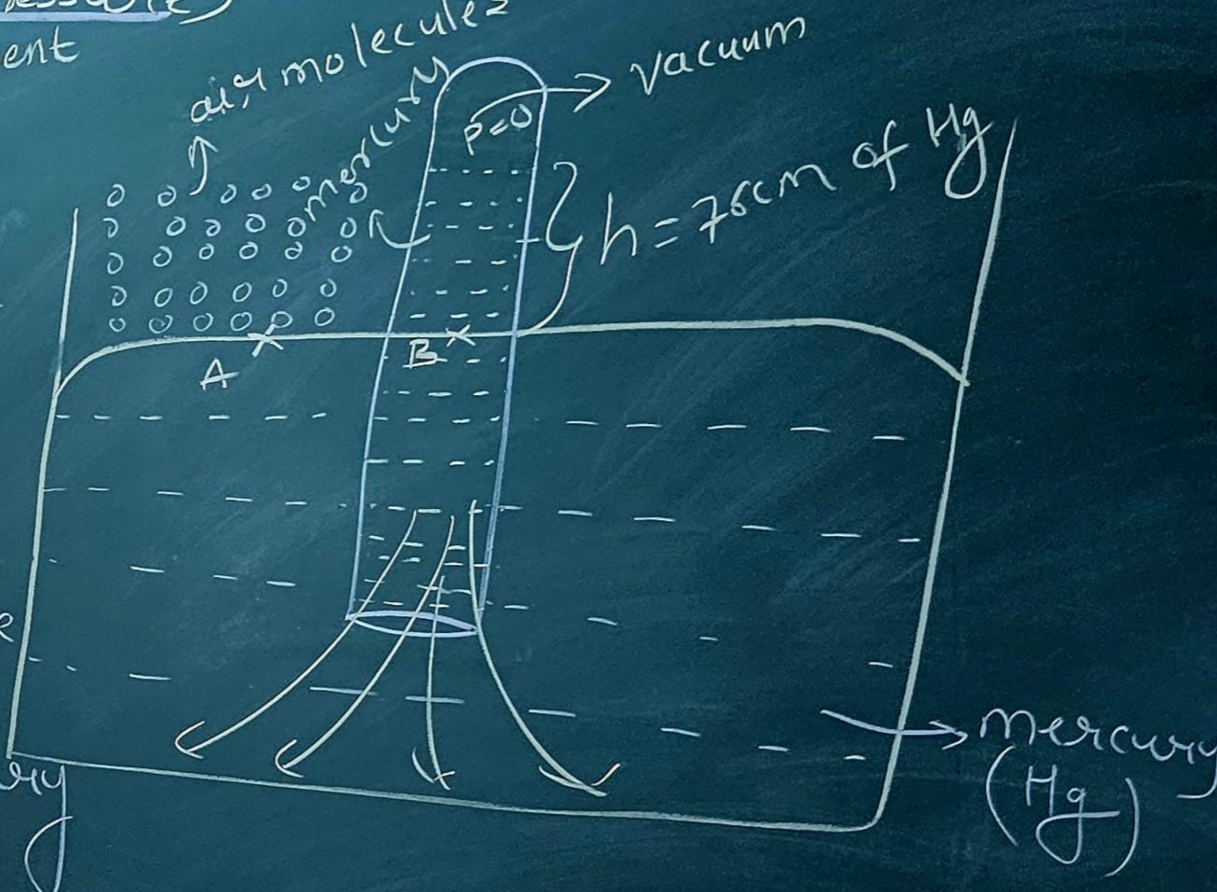
Barometer: [Gas Pressure]

Baro → pressure

meter → measurement

At the same Level, pressure has to be the same

$$\boxed{\therefore\ \underbrace{P_A}_{\text{pressure due to atmosphere}} = \underbrace{P_B}_{\text{pressure due to mercury}}}$$



$$\therefore P_0 = \rho_m d g$$

$$= 13600 \times 76\times10^{-2} \times 9.8$$

$$\boxed{\therefore P_0 = 1.013\times10^{5}\ \text{pa.}}$$

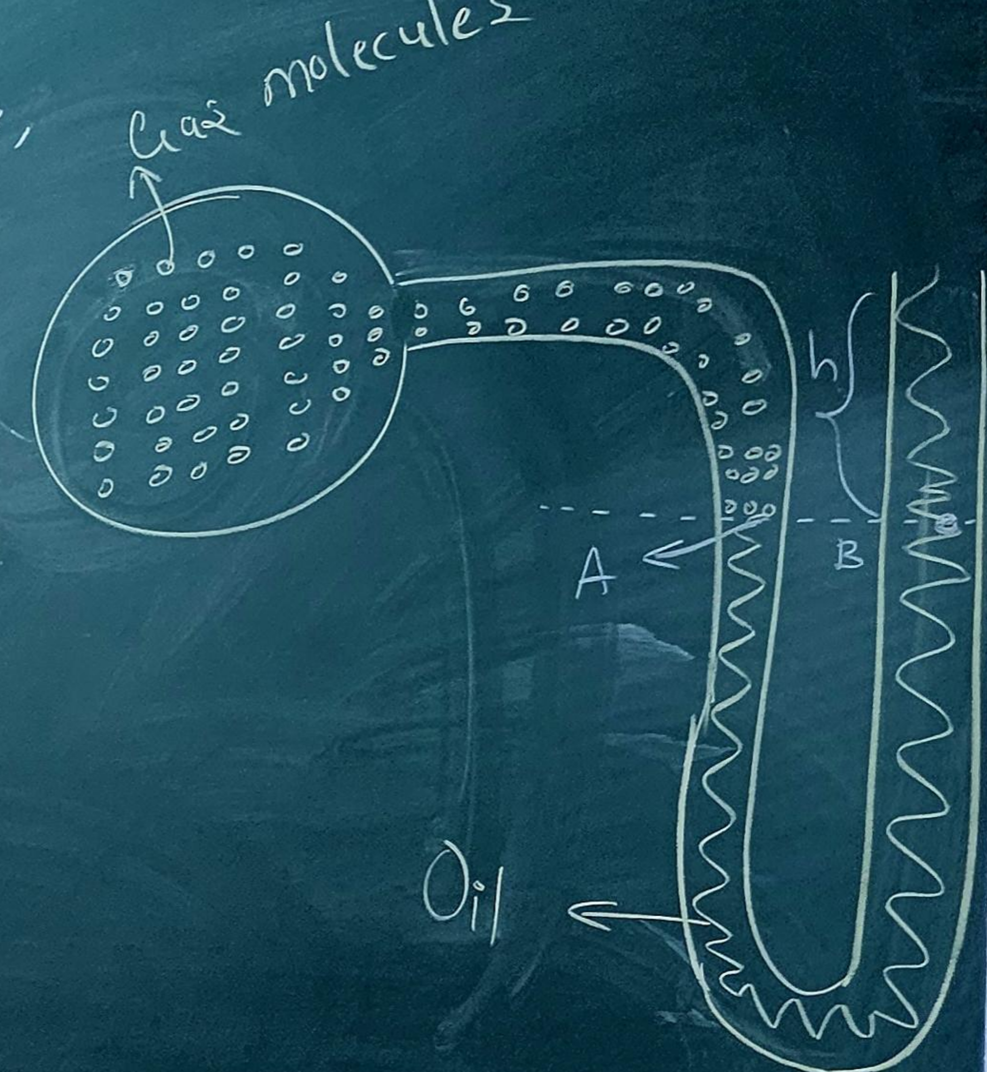
Open Tube Manometer [ ]

At the same Level,
pressure has to be the same,

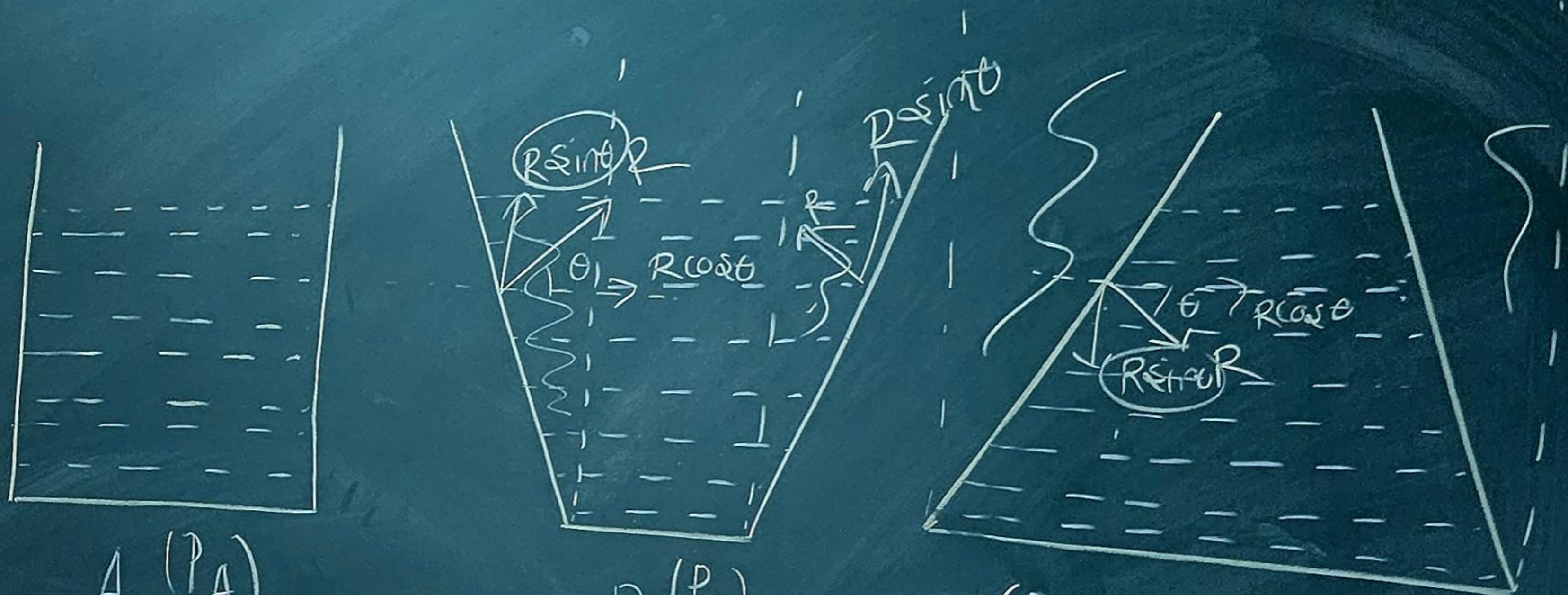
$$P_A = P_B$$

$$P_{gas} = \rho d g$$

$$\boxed{P_{gas} = \rho_{oil} \cdot h g}$$



Hydrostatic Paradox :



A (P_A) B (P_B) (P_C) C

(a) $P_A = P_B = P_C$

(b) $P_B - P_A > P_C$

(c) $P_C > P_B > P_A$

(d) $P_A > P_B > P_C$

Provided height of liquid column is same.

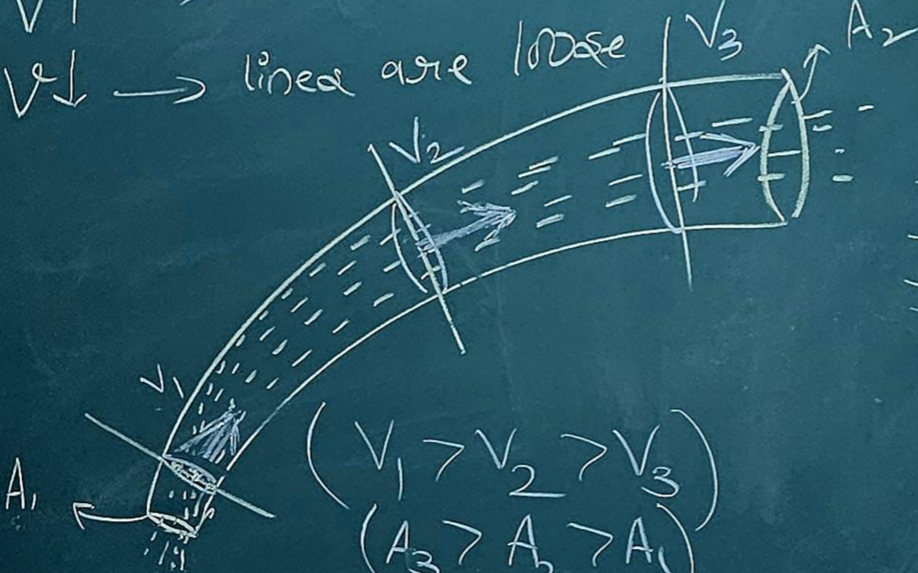
Hydrodynamics

→ study of fluid in motion.

→ Streamline Motion

$v\uparrow$ → lines are crowded

$v\downarrow$ → lines are loose

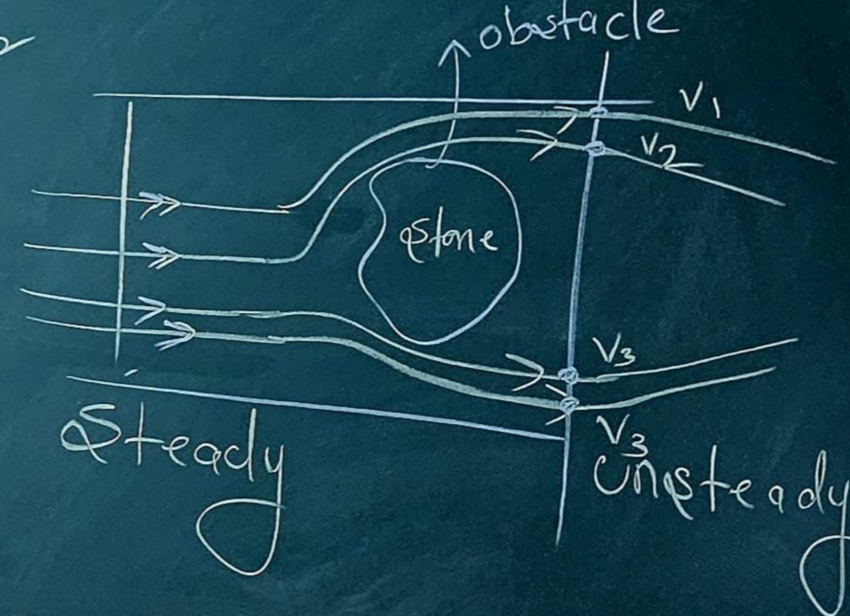


$(V_1 > V_2 > V_3)$

$(A_3 > A_2 > A_1)$

Velocity profile is different at different points but same at a single point/line

Turbulent Motion



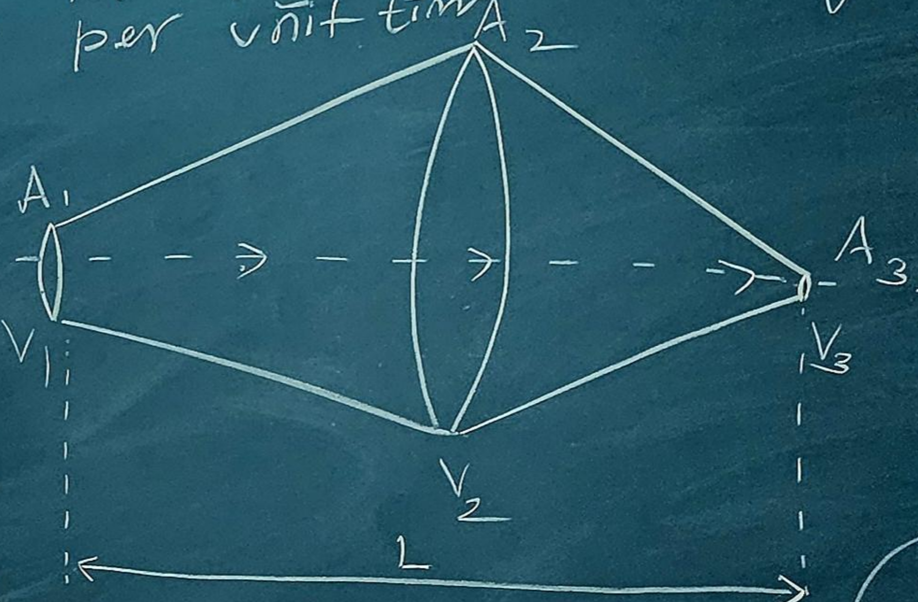
Hydrodynamics

Equation Of Continuity

For a steady Flow ---

mass entering per unit time = mass leaving per unit time

$$\boxed{\frac{\text{Mass}}{\text{time}} = \text{Constant}}$$



$$V = m^3 = m^2 \cdot m = A\,L$$

$$\frac{M}{t} = K$$

$$\therefore \rho = \frac{M}{V}$$

$$\therefore M = \rho V = \rho \cdot A \cdot L.$$

$$\therefore \frac{\rho A L}{t} = K$$

$$\therefore \rho A v = K$$

$$\therefore A v = K \quad (\rho = k)$$

$$\therefore A = \frac{K}{v}$$

$$\therefore A \propto \frac{1}{v}$$

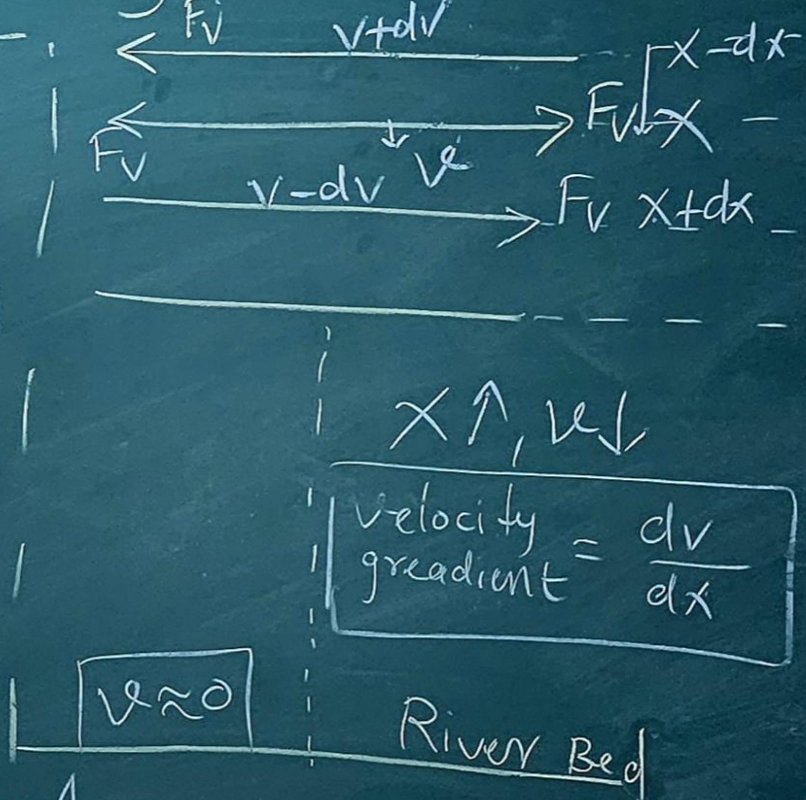
Velocity

$$\therefore A_1 v_1 = A_2 v_2 = A_3 v_3 = K$$

More is the area less will be the Velocity

Viscous Force (F_v) Hydrodynamics

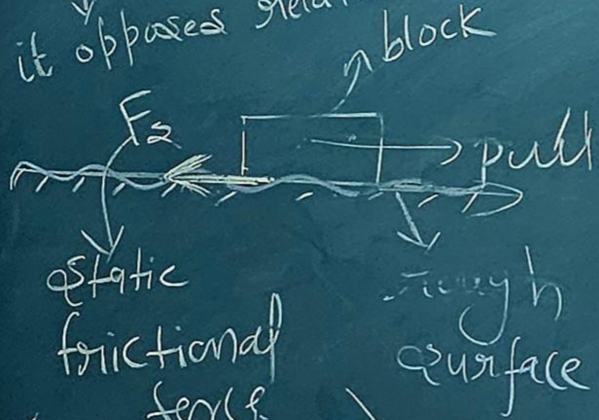
→ Liquid always flows in layers.



velocity gradient $= \frac{dv}{dx}$

As we move deep inside the OCEAN, velocity of water decreases

Friction in solids
it opposes relative motion



(when body is just about to move)

$$\boxed{F_{Limiting} = \mu_s N}$$

Newton's Formula for Viscous force

$F_v \propto A$

$F_v \propto \frac{dv}{dx}$

$\therefore F_v \propto A \frac{dv}{dx}$

$\therefore F_v = \eta A \frac{dv}{dx}$

η → Coefficient of Viscosity

$$F_v = \eta A \frac{dv}{dx}$$

$$\eta = \frac{F_v . dx}{A . dv}$$

$$= \frac{N \cancel{m}}{m^2 . \cancel{m}} . s$$

$$\eta = \frac{N.s}{m^2} \text{ (SI unit)}$$

$$= \frac{dyne . s}{cm^2} \text{ (CGS unit)}$$

$= 1$ Poise

$$\frac{1 Ns}{m^2} = \frac{10^5 \, dyne . s}{10^4 \, cm^2}$$

$$\boxed{\frac{1 Ns}{m^2} = 10 \text{ poise}}$$

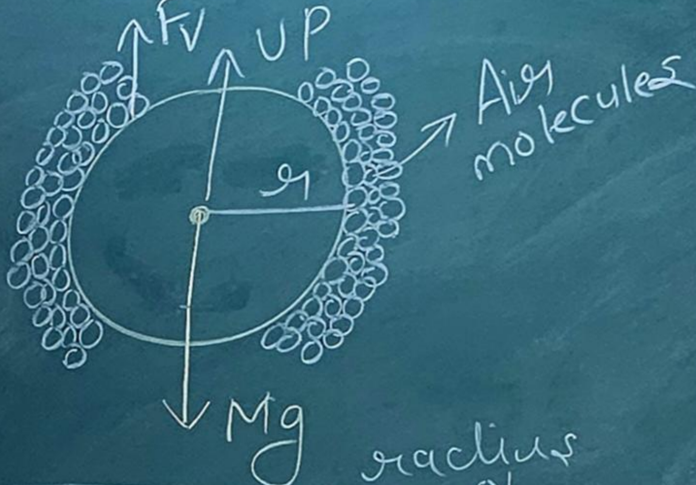
$$\boxed{1 \text{ SI unit} = 10 \text{ (CGS unit)}}$$

$$[\eta] = \frac{[F_v] \, [dx]}{[A] \, [dv]}$$

$$= \frac{[L^1 M^1 T^{-2}] [\cancel{L}]}{[L^2] . [\cancel{L^1} T^{-1}]}$$

$$\boxed{[\eta] = [L^{-1} M^1 T^{-1}]}$$

Stokes Law:



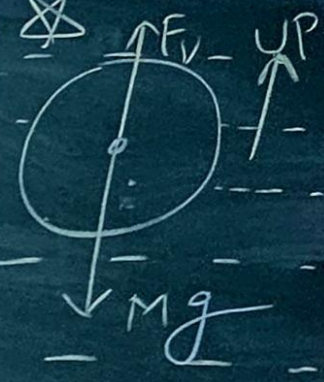
$$\boxed{F_V = 6\pi\eta r V_T}$$

radius of drop

coefficient of viscosity

terminal velocity

Terminal velocity (V_T)



At terminal speed

$$\boxed{F_{net} = 0}$$

i.e.

total upward force = total downward force

$$F_V + \text{upthrust} = Mg$$

$$6\pi\eta r V_T + Up = V\rho_L g$$

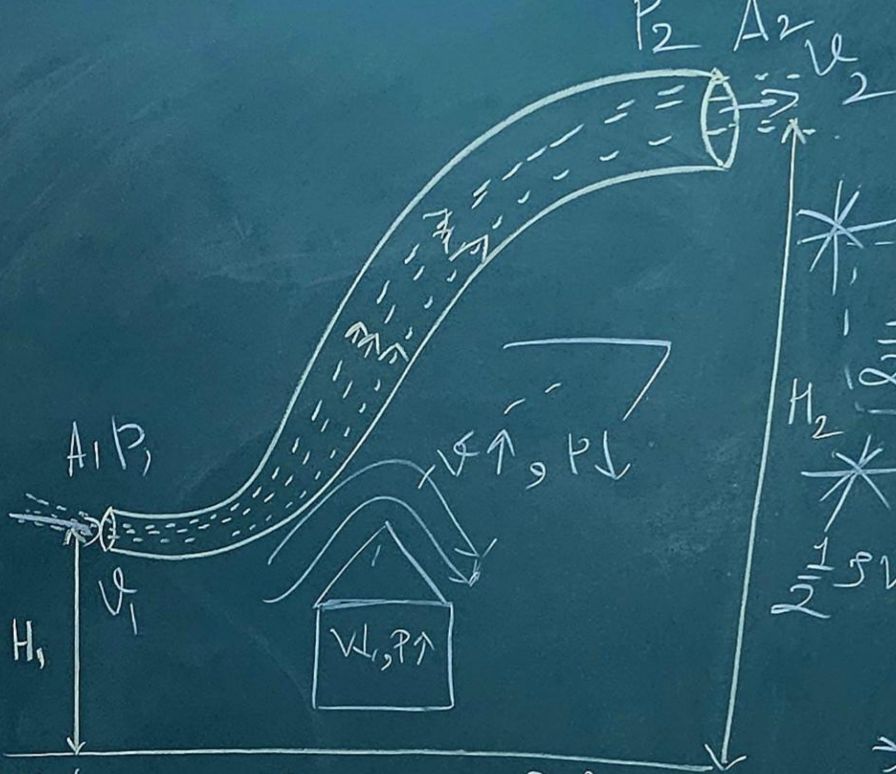
$$6\pi\eta r V_T + V\rho_a g = V\rho_L g$$

$$6\pi\eta r V_T = Vg(\rho_L - \rho_a)$$

$$\cancel{6}^{3}\cancel{\pi}\eta \cancel{r} V_T = \frac{\cancel{4}^{2}}{3}\cancel{\pi} r^{\cancel{3}} g(\rho_L - \rho_a)$$

$$\boxed{V_T = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_L - \rho_a) = \frac{2 r^2 g}{9\eta}(\rho_{object} - \rho_{medium})}$$

Bernoulli's Equation:



$$\frac{TE}{volume} = \text{Constant}$$

$$\frac{KE}{V} + \frac{PE}{V} + \frac{\text{pressure energy}}{V} = K$$

$$\frac{1}{2}\frac{Mv^2}{V} + \frac{MgH}{V} + \frac{P\cancel{V}}{\cancel{V}} = K$$

$$\frac{1}{2}\rho v^2 + \rho gH + P = K$$

$$\frac{1}{2}\rho v_1^2 + \rho gH_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho gH_2 + P_2 = K$$

if $H_1 = H_2 = H$ then

$$\boxed{\frac{1}{2}\rho v^2 + P = K}$$

(TTB)

Statement: At any given point, the sum of kinetic energy, potential energy & pressure energy per unit volume always remain constant i.e.

(V) D

(1) (i)

$$P = \frac{F}{A} = \frac{Mg}{A}$$

$$= \frac{2\times10^{3}\times9.8}{225\times10^{-4}}$$

$$= \frac{\overset{8}{\cancel{200}}\times98}{\underset{9}{\cancel{225}}\times10^{-4}}$$

$$= \frac{8\times98}{9}\times10^{4}$$

$$= \frac{784\times10^{4}}{9}$$

$$= \frac{87.1\times10^{4}\times100}{100}$$

$$= 0.871\times10^{6}\ (N/m^2)$$ (A)

(10)

$$P_L = \rho dg$$

$$= 1060\times2\times10^{2}\times9.8 \qquad d = 200m$$

$$= 1.06\times2\times9.8\times10^{5}$$

$$= 20.776\times10^{5}\ pa.$$

$$P_T = P_0 + P_L$$

$$P_T = P_0 + \rho dg$$

$$= 1.013\times10^{5} + 20.776\times10^{5}$$

$$= 21.789\times10^{5}\ pa$$

$$\boxed{P_T = 21.79\times10^{5}\ pa}$$

(12) $d = 1mm$

$r = 0.5\ mm = 5\times10^{-4}m$

$v_T = 2m/s$

$\eta = 1.8\times10^{-5}\ Ns/m^2 = 18\times10^{-6}\ Ns/m^2$

$F_V = ?$

By Stoke's Law

$$F_V = 6\pi\eta r v_T$$

$$= 6\times3.14\times18\times10^{-6}\times5\times10^{-4}\times2$$

$$= \cancel{108}\times\frac{22}{\cancel{7}}\times10^{-9}$$

$$= 15.428\times22\times10^{-9}$$

$$= 15.43\times22\times10^{-9}$$

$$= 339.46\times10^{-9}$$

$$\boxed{F_V = 3.3946\times10^{-7}\ N}$$

(13) $F_V = 1N$

$A = 10^{-2} m^2$

$dv = 2\times10^{-2} m/s$

$dx = 1.5\times10^{-3} m$

$= 15\times10^{-4} m$

$\eta = ?$

By newton's formula for viscosity

$$F_V = \eta A \frac{dv}{dx}$$

$$\frac{F_V . dx}{A dv} = \eta$$

$$\eta = \frac{1\times 15\times10^{-4}}{10^{-2}\times 2\times10^{-2}}$$

$$\boxed{\eta = 7.5 \left(\frac{Ns}{m^2}\right)}$$

(14) $v_T = ?$

$d = 0.4 mm$

$r = 0.2 mm = 2\times10^{-4} m$

$$\boxed{\eta = 0.1 \left(\frac{Ns}{m^2}\right)}$$

σ = specific gravity $= 0.9 =$ Relative density

$\rho_a = 1.29 (kg/m^3)$

Liquid

$\rho_{Liq} = 900$

$$V_T = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_{object} - \rho_{med})$$

$$= \frac{2}{9}\frac{\times 4\times10^{-8}\times 9.8}{0.1}(\rho_{bubble} - \rho_{Liq})$$

$$= \frac{8\times 9.8\times10^{-8}}{9\times 0.1}[1.29 - 900]$$

$$= \frac{784\times10^{-8}}{9}(-898.71)$$

$$= 87.1\times10^{-8}(-898.71)$$

$$= -78277.641\times10^{-8}$$

$$\boxed{V_T = -7.8278\times10^{-4} m/s}$$

(15)

$V_1 = 2\ m/s$

$d_1 = 10\ cm$

$r_1 = 5\ cm$

$V_2 = 4\ m/s$

$r_2 = ?$

$r_2 = \frac{5}{2} \times 1.414$

$= 5 \times 0.707$

$\boxed{r_2 = 3.535\ cm}$

$\boxed{d_2 = 7.070\ cm}$

$= 7.07 \times 10^{-2}\ m$

By Eqn of Continuity

$$\boxed{A_1 V_1 = A_2 V_2}$$

$$\cancel{\pi} r_1^2 V_1 = \cancel{\pi} r_2^2 V_2$$

$$25\,(cm)^2 \times 2 = r_2^2 \times 4$$

$$\frac{25 \times 2}{4} = r_2^2$$

(16)

WKT

$P_T = P_0 + P_L$

$P_T - P_0 = P_L$

$\Delta P = P_L$

$\therefore 4 \times 10^5 = \rho H g$ — (1)

At start

$u = 0$

$\therefore v^2 = \cancel{u^2} + 2as$

$\therefore v^2 = 2gH$

$\therefore \frac{v^2}{2g} = H$ — (2)

From (1) & (2)

$\therefore 4 \times 10^5 = 10^3 \times \frac{v^2}{2g} \times \cancel{g}$

$\therefore 8 \times 10^2 = v^2$

$\therefore v = 10 \times 2 \times \sqrt{2}$

$= 20 \times 1.414$

$= 2 \times 14.14$

$\boxed{\therefore v = 28.28\ m/s}$

(17)

$P_1 = 3\times10^5 \,(pa.)$

$P_2 = 2\times10^5 \,(pa.)$

$\rho = 1000 \left(\frac{kg}{m^3}\right)$

$v_1 = 0$

$v_2 = ?$

By Using Bernoulli's Eqn

$$\boxed{\frac{1}{2}\rho v^2 + \rho g H + P = K}$$

For uniform density & same height

$$\boxed{\frac{1}{2}\rho v^2 + P = K} \rightarrow \text{TTB}$$

$$\therefore \cancel{\frac{1}{2}\rho v_1^2}^{\,0} + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

$$3\times10^5 = \frac{1}{2}(10^3)v_2^2 + 2\times10^5$$

$$1\times10^5 = \frac{1}{2}\times10^3\, v_2^2$$

$$2\times10^2 = v_2^2$$

$$v_2 = \sqrt{2}\times10$$

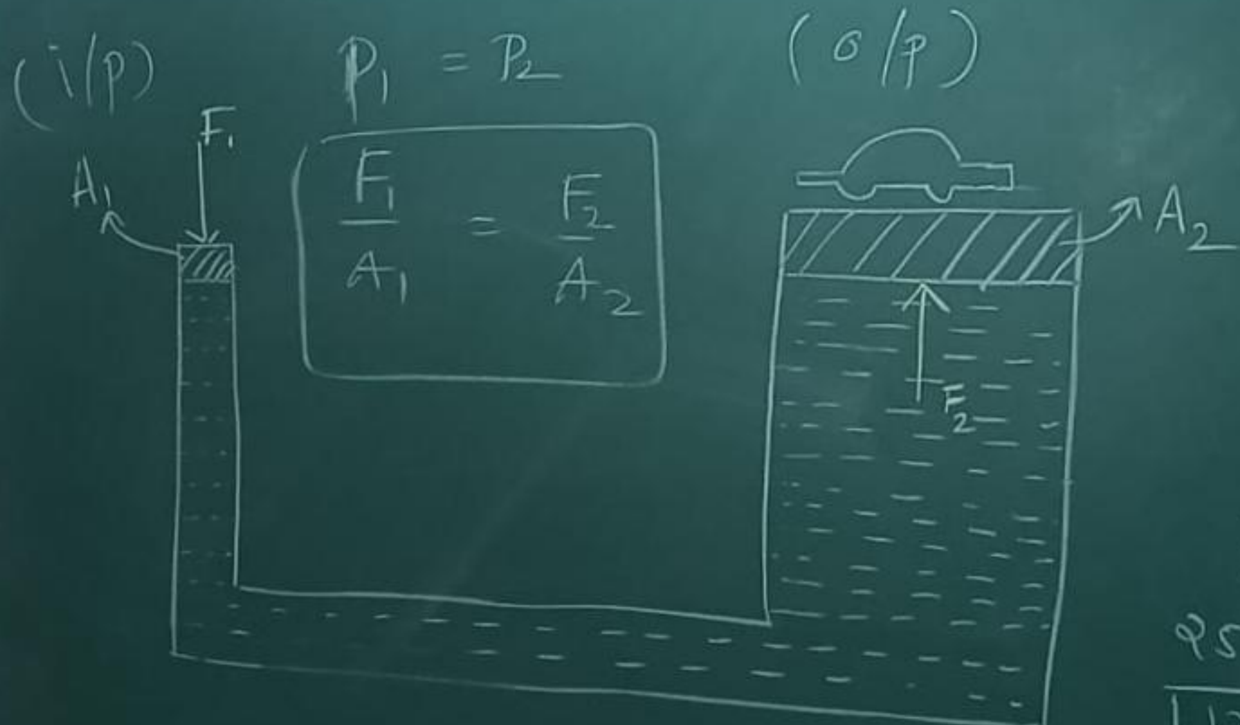
$$= 1.414\times10$$

$$\boxed{v_2 = 14.14\, m/s}$$

Pascal's Law

ex. $\frac{1}{1} = \frac{F_2}{100}$

$\boxed{100\,N = F_2}$



$P_1 = P_2$

$\boxed{\frac{F_1}{A_1} = \frac{F_2}{A_2}}$

(11) $A_1 = 30\,cm^2$

$A_2 = 1500\,cm^2$

$F_1 = 25\,N$

$F_2 = ?$

By Pascal's Law

$P_1 = P_2$

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{25}{\cancel{30\,cm^2}} = \frac{F_2}{\cancel{1500\,cm^2}}$ 50

$25 \times 50 = F_2$

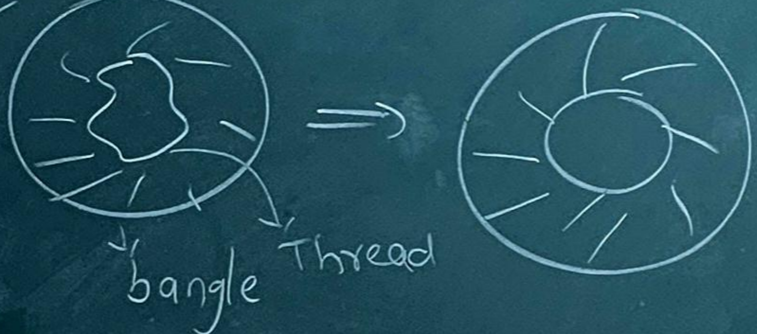
$\boxed{1250\,N = F_2}$

Surface Tension (T)

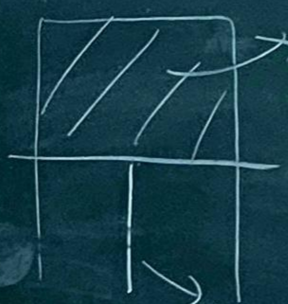
① Liquid always tries to minimise its surface area

ex.

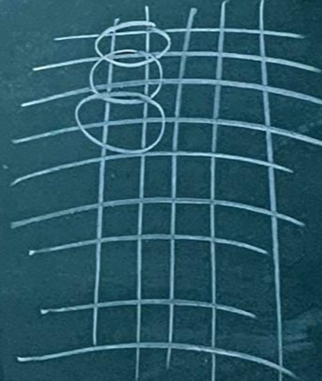
(i)



(ii)



③ Defn: The tangential force acting per unit length of an imaginary line on the free surface of liquid.



$$T = \frac{F_t}{\text{Length}}$$

$$T = \frac{F}{L} = \frac{N}{m} = \frac{[L^1 M^1 T^{-2}]}{[L^1]} = [L^0 M^1 T^{-2}]$$

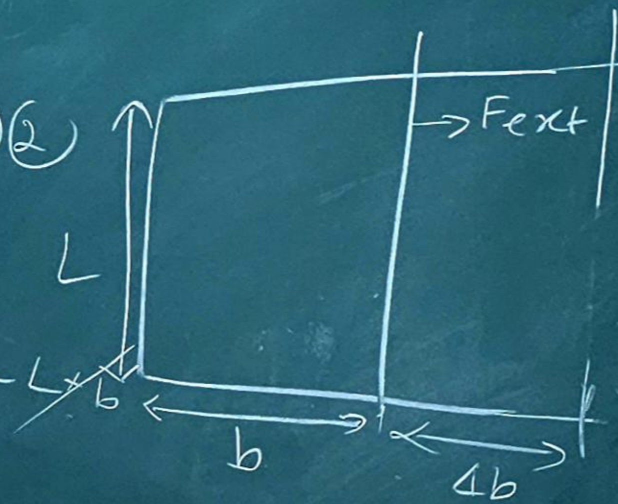
④ As the temperature ↑ surface tension decreases.

Relationship b/w Surface Tension (T) & Surface energy (U)

$A_i = L \times b$ ①

$A_f = L(b + \Delta b)$

$= (L \times b) + (L \times \Delta b)$ ②



change in area $= A_f - A_i$

$= \cancel{L \times b} + L \times \Delta b - \cancel{L \times b}$

$= L \times \Delta b$

Total change in area (dA) $= 2(L \times \Delta b)$ (front & back)

Now,

Surface Tension $= \dfrac{\text{Tange. Force}}{\text{Length}}$

$T = F_t / 2L$

$\boxed{F_t = T \cdot 2L}$

Now,

$WD = \text{force} \times \text{displa}$

$dw = F_t \times \Delta b$

$= T \cdot (2L \cdot \Delta b)$

$dw = T \cdot dA$

This workdone is stored in the form of surface energy

$\boxed{du = T \cdot dA}$ *

23

$A_i = 4\,cm^2$

$A_f = 9\,cm^2$

$dA = A_f - A_i$

$= 5\,cm^2$

$= 5 \times 10^{-4}\,m^2$

$T = 3 \times 10^{-2}\,\frac{N}{m}$

$dw = du = dE = ?$

$dw = T \cdot dA$

$= 3 \times 10^{-2} \times (5 \times 10^{-4})\,2$

$\boxed{dw = 3 \times 10^{-5}\,J}$

22

$r = 2\,cm$

$= 2 \times 10^{-2}\,m$

$T = 0.07\,N/m$

$= 7 \times 10^{-2}\,N/m$

$dw = T \cdot dA$

$= 7 \times 10^{-2} \times 4\pi r^2 \cdot 2$

$= \cancel{7} \times 8 \times \frac{22}{\cancel{7}} \times 10^{-2} \times 4 \times 10^{-4}$

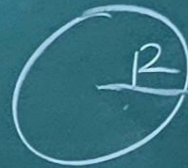
$= 176 \times 4 \times 10^{-6}$

$= 704 \times 10^{-6}$

$\boxed{dw = 7.04 \times 10^{-4}\,N/m}$

(20) 27 droplets

A Drop



$r = 0.1\,mm$
$= 0.1 \times 10^{-3}\,m$
$= 10^{-4}\,m$

$\boxed{dw = du = ?}$

$T = 0.072\,N/m$
$= 72 \times 10^{-3}\,N/m$

Initial area of 27 droplets $= 27 \times 4\pi r^2$
(A_i) $= 108\pi r^2$

Final area of one big drop $= 1 \times 4\pi R^2$
$= 4\pi R^2$

By Volume Conservation

$V_i = V_f$

$27 \times \frac{4}{\cancel{3}}\cancel{\pi} r^3 = \frac{4}{\cancel{3}}\cancel{\pi} R^3$

$27 r^3 = R^3$

$\boxed{R = 3r}$

$A_f = 36\pi r^2$

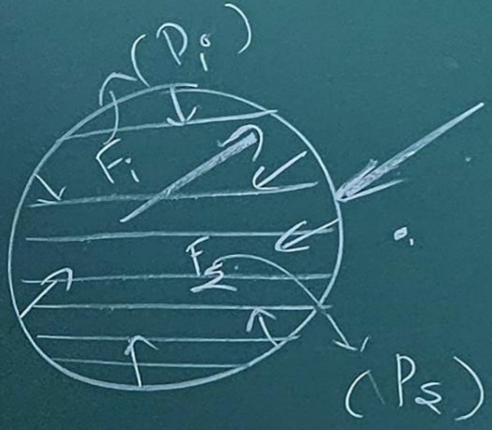
$dA = A_i - A_f$

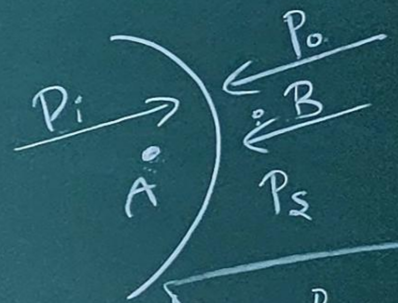
$dA = 72\pi r^2$

$dw = T \cdot dA$
$= 72 \times 10^{-3} \times 72 \times 3.142 \times 10^{-8}$
$= 5184 \times 3.141 \times 10^{-11} =$

16282.9×10^{-11}
$= 1.628 \times 10^{-7}\,J$

Excess pressure Inside A Drop of Liquid





$$\boxed{P_i - P_0 = \frac{2T}{R}}$$

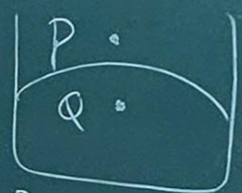
$$P_{concave} - P_{convex} = \frac{2T}{R}$$

$P_i \longrightarrow$ $\longleftarrow P_0$
$\longleftarrow P_S$

($P_i > P_0$)

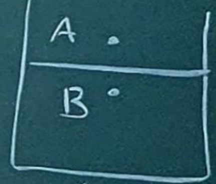
$$\boxed{P_i - P_0 = \frac{2T}{R}}$$

For Drop



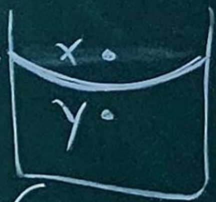
($P_Q > P_P$)

Plane



($P_B > P_A$)

Concave



($P_x > P_y$)

For Bubble

$$\boxed{P_i - P_0 = \frac{4T}{R}}$$

Excess pressure Inside A Drop of Liquid (3M)

WD = Force × displacement

$dW = (F_i - F_0) \times \Delta R$

$\frac{F}{A} = P$

$F = PA$

$\therefore dW = (P_i A - P_0 A)\Delta R$

$= A(P_i - P_0)\Delta R$

$= 4\pi R^2 \Delta R (P_i - P_0)$ — ①

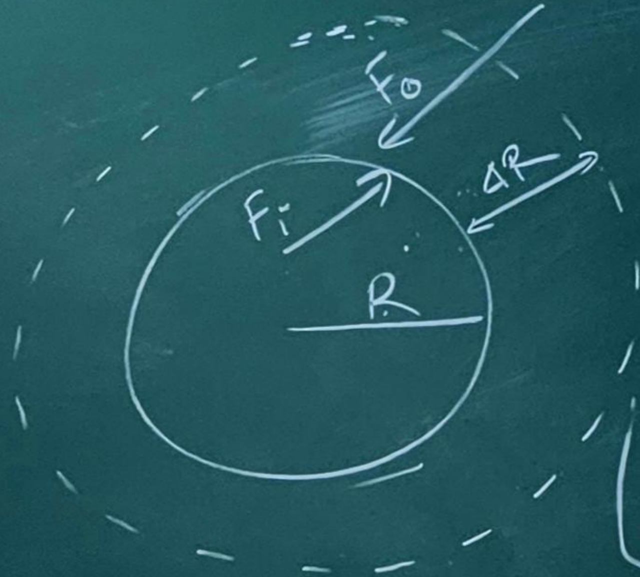
$A_i = 4\pi R^2$

$A_f = 4\pi (R + \Delta R)^2$

$dA = A_f - A_i = 4\pi (R+\Delta R)^2 - 4\pi R^2$

$= 4\pi [\cancel{R^2} + 2R\Delta R + \Delta R^2 - \cancel{R^2}]$

$= 4\pi [2R\Delta R + \Delta R^2]$



$\because \Delta R$ is small

so $\Delta R^2 \simeq 0$

$\therefore dA = 8\pi R \Delta R$

$\therefore dW = T \cdot dA$

$= T\, 8\pi R \Delta R$ — ②

From ① & ②

$\cancel{4\pi} R^{\cancel{2}} \cancel{\Delta R} (P_i - P_0) = T\, \underset{2}{\cancel{8}} \cancel{\pi} \cancel{R} \cancel{\Delta R}$

$R(P_i - P_0) = 2T$

$$\boxed{\therefore \underset{(\Delta P)}{P_i - P_0} = \frac{2T}{R}}$$

For Bubble

$\left(\Delta P = \frac{4T}{R}\right)$

Capillarity & Capillary Action

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

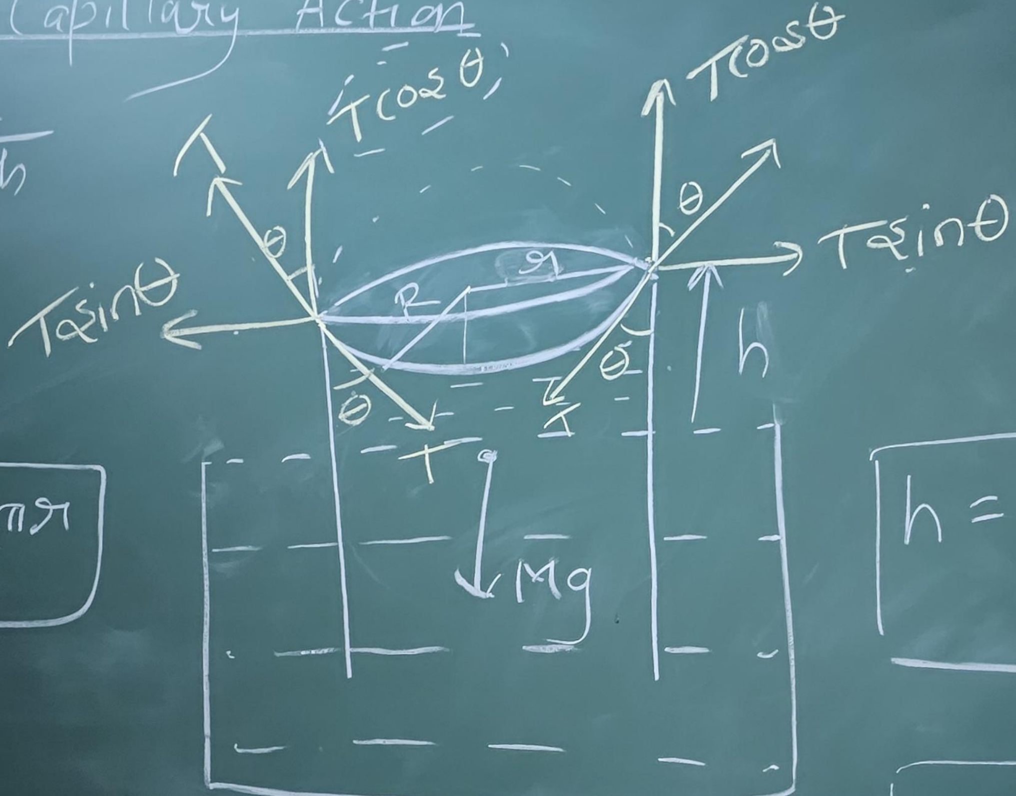
$$T\cos\theta = \frac{F_{up}}{2\pi r}$$

$$\boxed{F_{up} = T\cos\theta \cdot 2\pi r}$$

$$F_{down} = Mg$$

$$= V\rho g$$

$$F_{down} = \pi r^2 h \rho g$$



In Equilibrium

$$F_{up} = F_{down}$$

$$T\cos\theta \cdot 2\cancel{\pi}\cancel{r} = \cancel{\pi} r^{\cancel{2}} h \rho g$$

$$2T\cos\theta = r h \rho g$$

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

or

$$\boxed{r = \frac{2T\cos\theta}{\rho h g}}$$

$$\boxed{\frac{1}{9.8} = 0.1020}$$

(18) $h = ?$

$r = 0.1\ mm$
$= 1 \times 10^{-4}\ m$

$T = 7 \times 10^{-2}\ \frac{N}{m}$

$\theta = 0^\circ$

$\rho = 1000\ kg/m^3$

$g = 9.8\ m/s^2$

$$r = \frac{2T\cos\theta}{\rho h g}$$

$$h = \frac{2T\cos\theta}{\rho r g}$$

$$= \frac{2 \times 7 \times 10^{-2} \times 1}{10^3 \times 10^{-4} \times 9.8}$$

$$= \frac{14 \times 10^{-1}}{9.8} = \frac{1.4}{9.8} = 0.1020 \times 1.4$$

$=$

$= 0.14280\ m$
$= 0.1428\ m$