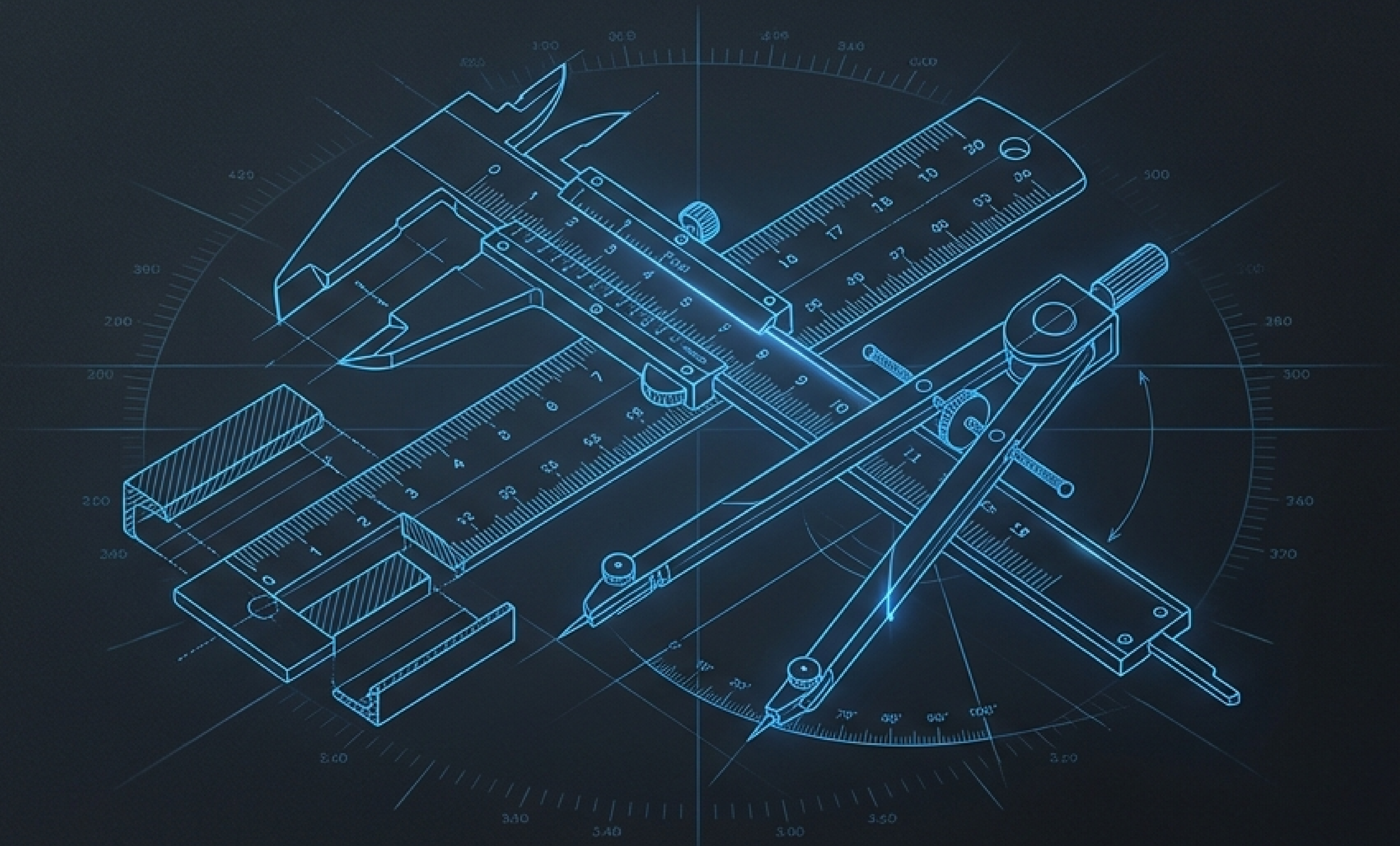
Forging the Tools of Discovery

A Scientist’s Guide to Units and Measurement



A World of Confusion

Before a global standard, scientists used competing systems.
Collaboration was difficult. Results were hard to verify.
Science needed a common language.

CGS System

- Centimetre
- Gram
- Second

FPS System

- Foot
- Pound
- Second

MKS System

- Metre
- Kilogram
- Second

How can we build on each other's work if we can't agree on how to agree on how to measure?

Tool #1: The Universal Ruler

The International System of Units (SI)

The globally accepted standard for science, industry, and commerce.
A single, coherent system for measurement.

The 7 Pillars of Measurement

All of physics rests on seven fundamental base quantities.
Every other unit is derived from these.

Length
metre
(m)

Mass
kilogram
(kg)

Time
second
(s)

Electric Current
ampere
(A)

Thermodynamic Temperature
kelvin
(K)

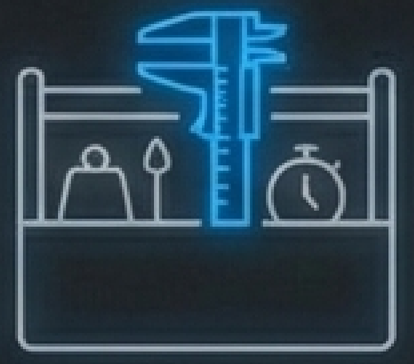
Amount of Substance
mole
(mol)

Luminous Intensity
candela
(cd)

Tool #2: The Precision Caliper

Communicating Certainty with Significant Figures

A measurement is more than just a number; it's a statement about precision. Significant figures tell us which digits are reliable and which are uncertain.



The Rules of Precision

Rule 1

All non-zero digits are significant.

287.5 cm
(4 significant figures)

Rule 2

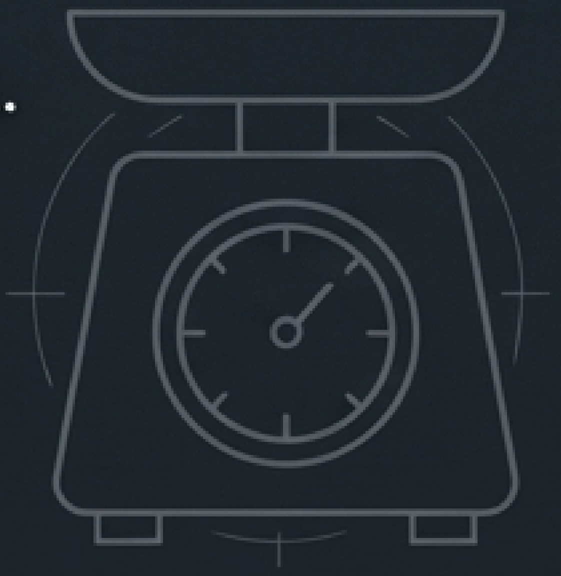
Zeros between non-zero digits are significant.

4.700 m
(4 significant figures, indicating instrument precision)

Rule 3

Leading zeros are NOT significant.

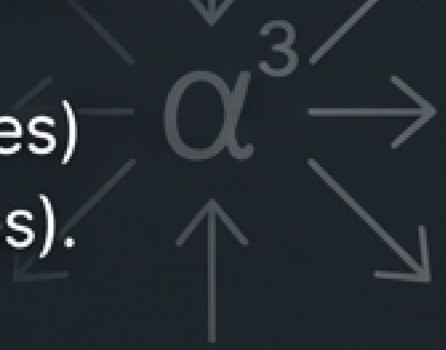
0.00**2308** kg
(4 significant figures: 2, 3, 0, 8)



Rule 4

For numbers without a decimal, trailing zeros are ambiguous. Use scientific notation.

4.700 x 10^3 m (4 significant figures)
vs **4.7** x 10^3 m (2 significant figures).



Calculating with Confidence

Multiplication & Division

Result has the same number of sig figs as the measurement with the *least* sig figs.

Case Study: Calculating Density

Mass = 4.237 g (4 sig figs)
Volume = 2.51 cm^3 (3 sig figs)

$$\text{Density} = \frac{4.237\ \text{g}}{2.51\ \text{cm}^3} = 1.688047\ldots\ \text{g/cm}^3$$

1.69 g/cm^3
(rounded to 3 sig figs)

Addition & Subtraction

Result has the same number of decimal places as the measurement with the *least* decimal places.

Case Study: Sum of masses

436.32 g
+ 227.2 g
+ 0.301 g
= 663.821 g

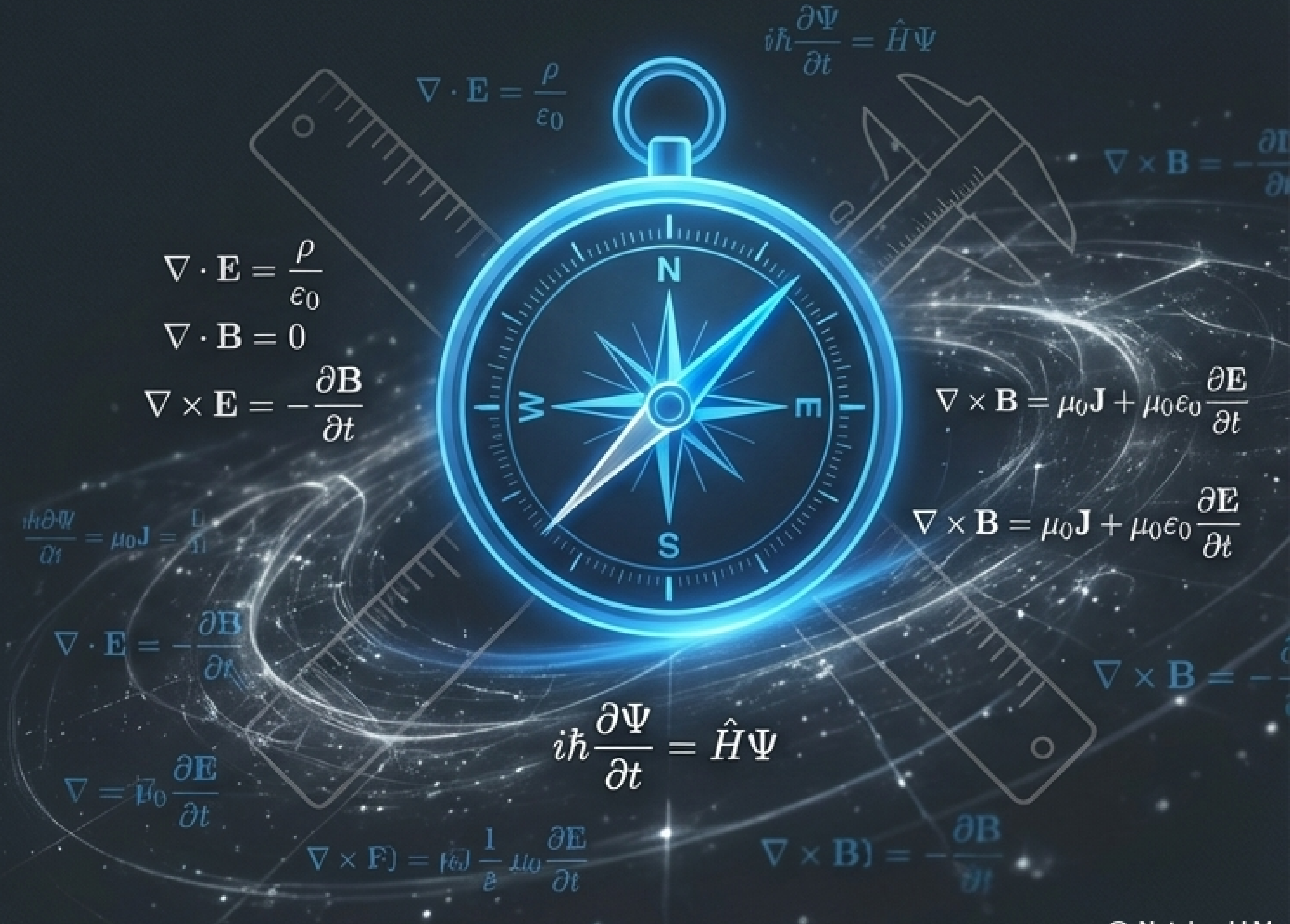
663.8 g
(rounded to one decimal place)

NotebookLM

Tool #3: The Logic Compass

Navigating Physics with Dimensional Analysis

Beyond numbers and units lies the fundamental ***nature*** of a physical quantity—its dimension. This is the key to checking our work and even deriving new equations.



The Alphabet of the Universe

Any physical quantity can be expressed as a combination of seven fundamental dimensions.

Length	→	[L]
Mass	→	[M]
Time	→	[T]
Electric Current	→	[A]
Temperature	→	[K]
Amount of Substance	→	[mol]
Luminous Intensity	→	[cd]

Example: Velocity = $\frac{\text{distance}}{\text{time}}$ ⟶ Dimension of Velocity = $\frac{[L]}{[T]}$ = $[L\,T^{-1}]$

Application 1: The Sanity Check

The Principle of Homogeneity: The dimensions on both sides of a valid equation must be the same. You can't add apples and oranges.

Is the equation `$\frac{1}{2}mv^2 = mgh$` dimensionally correct?

LHS (Kinetic Energy)	RHS (Potential Energy)
$[M] * ([L\ T^{-1}])^2$	$[M] * [L\ T^{-2}] * [L]$
↓	↓
$[M\ L^2\ T^{-2}]$	$[M\ L^2\ T^{-2}]$

✓ **LHS = RHS. The equation is dimensionally consistent.**

Application 2: Deriving Relationships

We can deduce the relationship between physical quantities by ensuring the dimensions match.

How does the period (T) of a simple pendulum depend on its mass (m), length (l), and gravity (g)?

Assumption: $T \propto m^x l^y g^z$

Dimensions: $[T^1] = [M]^x [L]^y [L T^{-2}]^z$

Equate Powers (comparing exponents on both sides):

M: $0 = x$ L: $0 = y + z$ T: $1 = -2z$

Solution (solving for x, y, z):

$x = 0$ → $z = -1/2$ → $y = 1/2$

$$T \propto l^{1/2} g^{-1/2} \text{ or } T \propto \sqrt{(l/g)}$$

Challenge: Fix Einstein's Equation

A student recalls the famous special relativity mass equation but forgets where to put the speed of light, *c*. Can we use dimensional analysis to place it correctly?

$$m = \frac{m_0}{\sqrt{1 - v^2}} \; ?$$

The term $\sqrt{1 - v^2}$ is dimensionally incorrect. We can't subtract a velocity-squared ($[L^2 T^{-2}]$) from a dimensionless number (1).

The term inside the square root must be dimensionless.

How can we combine v^2 with c (speed of light, $[L T^{-1}]$) to make a dimensionless ratio?

The Solution: A Dimensionless World

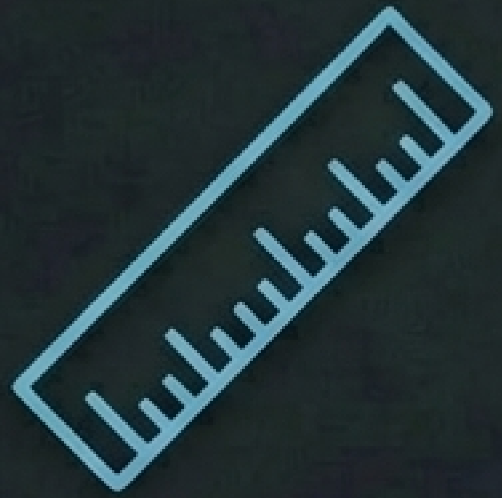
To make v^2 dimensionless, we must divide it by another velocity-squared. The only other relevant velocity is **c**, the speed of light.

$$m = \frac{m_0}{\sqrt{1 - v^2}} \quad \frac{v^2}{c^2}$$

The Correct Equation: $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

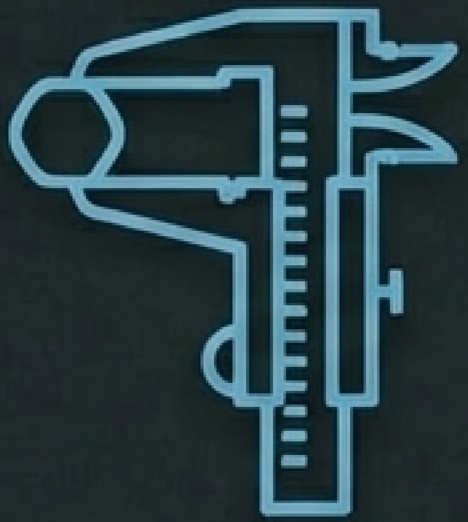
Takeaway: Dimensional analysis ensures that our most fundamental equations are logically sound.

Your Scientific Toolkit is Complete



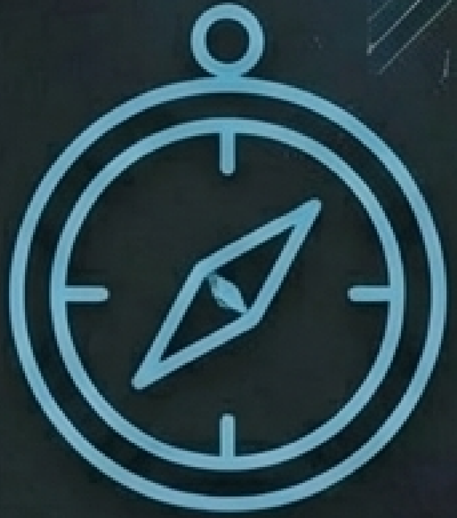
The Universal Ruler (SI Units)

A common language for unambiguous measurement.



The Precision Caliper (Significant Figures)

A system for communicating the true precision of your work.



The Logic Compass (Dimensional Analysis)

A powerful method to check, validate, and derive physical laws.

From Here, You Measure the Universe

The rules of measurement are not limitations.
They are the instruments that make discovery possible.