Exponentials and logarithms

04 March 2026 23:06

exponentials and log...

Exponentials and logarithms

$2^3 = 8$

Here 2 is base and 3 is exponent

$(a^b)^c = a^{bc}$

$a^b . a^c = a^{b+c}$

Find x.

$16^3 = 2^x$

$27^6 . 9^4 = 3^x$

According to legend, two girls, Meena and Arial, once did a favor for a powerful king. The king asked them to choose their reward. Both girls knew the king loved chess, and both girls came from very hungry villages.

Arial asked to have one grain of rice placed on the first square of her chessboard. She asked that each day thereafter, the king place an amount of rice on the next square of her chessboard that was 10 more grains than the amount of rice placed on the previous square. The king was a little surprised at how much Arial sought, but he agreed.

Meena also asked that the king place a single grain of rice on the first square of her chessboard. She asked that on each day thereafter, the king place on the next square of her chessboard twice the amount of rice he had placed on the board the day before. The king quickly agreed, and congratulated her on not being as greedy as Arial.

On the second day, the king placed $1 + 10 = 11$ grains on the second square of Arial's board and $1 \cdot 2 = 2$ grains on Meena's board. Arial teased Meena for getting so little, but Meena didn't mind. On the third day, Arial chuckled as the king gave her $11 + 10 = 21$ grains of rice but only gave Meena $2 \cdot 2 = 4$ grains. And so on.

Which girl got the better deal?

Problem 19.1 Jump to Solution

In this problem we build functions to describe how much rice Arial receives on day n and how much rice Meena receives on day n.

(a) The king paid Arial 1 grain on day 1. What is Arial's payment on day 2? On day 3? On day 10? Let the function $A(n)$ represent the amount of rice Arial receives on day n. Find $A(n)$.

(b) The king paid Meena 1 grain on day 1. How many times must he double this initial payment to determine Meena's payment on day 2? On day 3? On day 10? Let the function $M(n)$ represent the amount of rice Meena receives on day n. Find $M(n)$.

(c) Which girl got the better deal?

Problem 19.2 Jump to Solution

Consider the function $f(x) = 2^x$.

(a) Plot several points on the graph of the function by choosing various values of x. Make sure you use both negative and positive values for x.

(b) Use your points from the first part to draw the graph of $f(x)$.

(c) What is the range of $f(x)$?

Problem 19.3 Jump to Solution

(a) Use the graph from the previous problem to explain why we know that $x = 3$ if $2^x = 8$.

(b) Use your explanation in part (a) to solve the equation $3^{2x} = 3^{x-5}$.

Problem 19.3 Jump to Solution

(a) Use the graph from the previous problem to explain why we know that $x = 3$ if $2^x = 8$.

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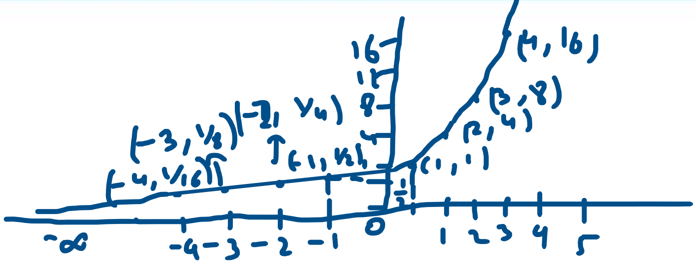


$(0, \infty)$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$= \frac{2^n}{2}$

$2x = x - 5$

$2^{2^{4x}} = 2^2 =$

$2^{4\cdot 2^x}$

$2^{2^{x+2}}$

Problem 19.4 Source: Mandelbrot Jump to Solution

In this problem, we solve the equation $2^{(16^x)} = 16^{(2^x)}$.

(a) Write 16 as a power of two in the equation $2^{(16^x)} = 16^{(2^x)}$.

(b) Write each side of your equation in part (a) in the form 2^{2^a}, where a is some expression in terms of x.

(c) Solve for x.

$(3^4)^3 = 3^{12}$ $3^{4^3} = 3^{64}$

$2^{4\times 2^x} = 2^{2^2\cdot 2^x} = 2^{2^{(x+2)}}$

$4x = x+2$

$x = \frac{2}{3}$

2 $(2^4)^{2^x}$ $(2^4)^x$ $2^{(4\times 2^x)}$

$2^{2^{4x}}$ + RHS

$(2^4)^{2^x} \rightleftarrows (16)^{2^x})$

$2^{4\cdot 2^x}$

$2^{2^2\cdot 2^x}$

$= 2^{2^{x+2}}$

Problem 19.5 Jump to Solution

Suppose that $2^x = 6$. In this problem we evaluate 2^{3x-1}.

(a) Let $a = 2^x$. Rewrite 2^{3x-1} as an expression whose only variable is a.

(b) Use your expression from (a) to solve the problem.

Problem 19.6 Jump to Solution

In this problem we find all solutions to the equation

$$4^x - 33\cdot 2^{x-1} + 8 = 0.$$

(a) Write all the terms with variables in their exponents as powers of 2.

(b) What makes this problem difficult is that there are variables in the exponents. What substitution could we make to turn this into an equation we know how to solve?

(c) Solve the equation your substitution produces, then solve the original equation. Make sure you test your solutions!

$2^{3x-1} = 2^{3x}\cdot 2^{-1}$

$= (2^x)^3/2$

$= a^3/2$

exponentials homew...

$2^x = 6$

$2^{3x} = (2^x)^3 = 6^3 = 216$

$2^{3x-1} = 2^{3x}\cdot 2^{-1} = 216/2$

$= 108$

Exponentials homework

domain = x value
range = f(x)

What are the domain and range of the function $f(x) = 3 \cdot 5^x - 4$?

$x \in R$
$0 - 4 = -4$ +ve
$\infty - 4 = \infty$
$3.5^x \to 0$
$[-4, \infty)$
$0 - 4 \to -4$

Type your solution, notes and/or work here.

19.1.2:

Find all values of r such that $5^{2r-3} = 25$.

$2r - 3 = 2$

Type your solution, notes and/or work here.

19.1.3:

( )

Find all values of t such that $6^{3t-1} = 36^{t-3}$.

Type your solution, notes and/or work here.

19.1.4:

Suppose $5^x = 3$. Find 5^{2x+3}.

$5^{2x+3} = 5^{2x} \cdot 5^3$

Type your solution, notes and/or work here.

19.1.5:

Find the value of x that satisfies the equation $25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$.

Type your solution, notes and/or work here.

19.1.6★:

Find all solutions to the equation $3 \cdot 9^t - 82 \cdot 3^t + 27 = 0$.

Type your solution, notes and/or work here.

19.1.7★:

If $9^{x-1} = 7$, then what is 3^{2x+3}?

Type your solution, notes and/or work here.

$3^{2x-2+5} = 3^{2x+3}$

$3^{2x-2} \cdot 3^5 = 3^{2x+3}$

Show Me the Money

You want to buy a brand new car. Unfortunately, the car costs $\$27,951$, and you only have a few thousand dollars. However, all is not lost. You can take out a loan! Through a loan, a bank will give you the money you need for the car, and you agree to pay them back later. A bank won't do this for free, of course. When you pay the bank back, you have to give them some extra money in return for letting you borrow the money in the first place. This extra money is the **interest** on the loan.

Interest is typically stated as a percentage of the loan amount that you must pay each year in extra money. For example, suppose the bank loaned you $\$25{,}000$ to buy your car at a 10% interest rate. This initial amount of money you borrow is called the **principal** on the loan. If you don't make any payments during the first year after you get the loan, at the end of that first year you owe the original $\$25{,}000$ principal, plus an additional $(0.10)(\$25{,}000) = \$2{,}500$ in interest. So you owe a total of $\$25{,}000 + \$2{,}500 = \$27{,}500$ at the end of the first year.

If the loan is a **simple interest** loan, then in the second year, the interest rate is only applied to the principal. So, at the end of the second year, you will owe an additional $(0.10)(\$25{,}000) = \$2{,}500$, bringing the total amount you owe to $\$30{,}000$. However, most loans don't work this way!

For most loans, you are charged interest on the total amount of money you owed at the end of the first year, including the interest from the first year. For such a loan, the amount of interest in the second year is 10% of the $\$27{,}500$ owed at the end of the first year, or $(0.10)(\$27{,}500) = \$2{,}750$. This brings the total owed at the end of two years to $\$27{,}500 + \$2{,}750 = \$30{,}250$. When the interest on a loan is charged on both the original principal as well as the interest that has accumulated in the past, we say that the loan is a **compound interest** loan.

If you take out a compound interest loan, you need to know more than just the interest rate. You need to know how frequently the interest is **compounded**. For example, if you take out a $\$1000$ loan at 6% interest and the interest is compounded annually, then at the end of one year, $(\$1000)(0.06) = \$60$ is added to the amount you owe. So, at the end of the first year, you owe $\$1000 + \$60 = \$1060$.

However, if the 6% loan is compounded semi-annually, then interest is added to the loan every 6 months. After the first six months of the loan, $1/2$ the interest that you would owe on an annual loan is added to the amount you owe. An annually compounded loan would result in $\$60$ interest in one year, so at the six-month point in the semi-annually compounded loan, $(\$60)/2 = \$30$ is added to the amount you owe. This means you owe $\$1030$ after six months. Over the next six months, the interest *is charged on this new amount, not just on the first* $\$1000$*!*

The first full year of interest on annually compounded 6% loan of $\$1030$ is $(0.06)(\$1030) = \$61.80$. So, for the final six months of our semi-annually compounded loan, the interest is $\$61.80/2 = \$30.90$. Therefore, the amount you owe at the end of the first year of the semi-annually compounded loan is $\$1030 + \$30.90 = \$1060.90$.

In this section, you should feel free to use a calculator to perform routine arithmetic.

Problem 19.7

Jayne puts $\$5,000$ in an investment that earns 9% simple interest. How much will she have at the end of:

(a) One year?

(b) Two years?

(c) Three years?

(d) n years? (Answer in terms of n.)

$5000(1+\frac{9}{100})$ $\quad 5000(1+\frac{9}{100}\times 2)$

$5000(1.18)$

SI

CI

Problem 19.8

Jayne puts $\$5,000$ in an investment that earns 9% interest. If the interest in the investment is compounded annually, ... have at the end of:

(a) One year?

(b) Two years?

(c) Three years?

(d) n years? (Answer in terms of n.)

How do your answers compare to the simple interest investment in the previous problem?

$5000(1.09)^2$

Problem 19.9 Jump to Solution

Brad takes a $\$10,000$ loan that has an interest rate of 14%. How much does Brad owe (including his initial $\$10,000$) at the end of the year if the interest is compounded:

(a) Annually?

(b) Semi-annually? (Two times a year.)

(c) Quarterly? (Four equally spaced times a year.)

(d) m evenly spaced times a year? (Answer in terms of m.)

(e) Investigate your answer to part (d) for higher and higher values of m. Does there appear to be a limit to the amount of money Brad can owe?

half year = 7 %

$10000(1.07)^2 =$

$\left(1+\frac{0.14}{m}\right)^{m}$

$\log_e j = \ln\left(1+\frac{0.14}{m}\right)^m \quad \lim_{m\to\infty}$

$m\left(\frac{0.14}{m}\right)$

$\log j = 0.14$

$j = e^{0.14}$

$2.71^{0.14}$

19.2.1:

Paula invests $\$10,000$ for 5 years at an interest rate of 10%. At the end of those 5 years, how much is her investment ... the following cases:

(a) The interest is simple interest.

Type your solution, notes and/or work here.

(b) The interest is compounded annually.

Type your solution, notes and/or work here.

(c) The interest is compounded quarterly.

Type your solution, notes and/or work here.

19.2.2:

Joanie takes a $\$6,000$ loan to pay for her car. The interest rate on the loan is 12%. She makes no payments for 4 years, but has to pay back all the money she owes at the end of 4 years. How much more money will she owe if the interest compounds quarterly than if the interest compounds annually?

Type your solution, notes and/or work here.

Show Solution

19.2.3:

Bill invests $\$2,500$ in an investment that pays 5% interest compounded annually for the first two years, then 8% interest compounded annually for three years after that. Debbie invests $\$2,500$ in an investment that pays 8% interest compounded annually for the first three years, then 5% interest compounded annually for two years after that. Which investment is worth the most money after 5 years?

Type your solution, notes and/or work here.

Show Solution

19.2.4:

Beth invests $\$4,200$ at an annually compounded interest rate of 6%. Josey invests $\$4,200$ at a simple interest rate of $r\%$.

(a) What is r if their investments are worth the same amount after 1 year?

Type your solution, notes and/or work here.

Show Solution

(b) What is r if their investments are worth the same amount after 2 years?

Type your solution, notes and/or work here.

Show Solution

(c) What is r if their investments are worth the same amount after 10 years?

Type your solution, notes and/or work here.

Show Solution

19.2.5:

Show that if $\$k$ is invested at an interest rate of $r\%$ for n years, compounded annually, then the total amount at the end of n years is

$$\left(1+\frac{r}{100}\right)^n (\$k).$$

19.2.6:

Show that if $\$k$ is invested at an interest rate of $r\%$ for n years, compounded m times a year, then the total amount at the end of n years is

$$\left(1+\frac{r}{100m}\right)^{nm} (\$k).$$

Type your solution, notes and/or work here.

Show Solution

Problem 19.10 Jump to Solution

Sayeed borrowed $\$2,300$. The interest on the loan was compounded annually for 5 years, and the interest rate remained the same throughout that time. At the end of the five years, Sayeed owed $\$3,301.95$ (including the original $\$2,300$). To the nearest tenth of a percent, what was the interest rate?

Problem 19.11 Jump to Solution

I want to have $\$1,000,000$ in the bank when I retire in 10 years. I can invest money today at a rate of 10%, compounded semi-annually. I will not invest any more money between now and retirement.

(a) Suppose I invest $\$x$ today. In terms of $\$x$, how much will this be worth in 10 years?

(b) How much should I invest right now to have $\$1,000,000$ in 10 years?

Problem 19.12 Jump to Solution

I take a three-year loan for $\$10,000$. The interest rate is 6% compounded annually. At the end of each year, I make a payment right when the annual interest is added to the amount I owe. For the following year, I am then charged interest on the amount I still owe after my payment. All three of my payments are the same and my loan is completely paid off after the third payment.

(a) Suppose each payment is $\$x$. In terms of x, what is the present value of my first payment? Of my second payment? Of my third payment?

(b) What is the amount of each payment?

$(1.06)^n 10000 = \text{future value}$

$\frac{x}{(1.06)^n} = \text{present value}$

$$\frac{x}{1.06} + \frac{x}{(1.06)^2} + \frac{x}{(1.06)^3} = 10000$$

$$x\underbrace{\left(\frac{1}{1.06} + \frac{1}{(1.06)^2} + \frac{1}{(1.06)^3}\right)}_{c} = \frac{10000}{c}$$

19.3.1:

Billy takes a $\$3,000$ loan that compounds semi-annually (twice a year). He makes no payments for the first 4 years, and after 4 years he owes $\$3,950.43$. What is the interest rate of the loan?

Type your solution, notes and/or work here. Show Solution

1.05 + 1.+0.05 / 00

$500,000 \div (1+0.05)^{10}$

19.3.2:

How much money should I invest at an annually compounded interest rate of 5% so that I have $\$500,000$ in ten years?

Type your solution, notes and/or work here. Show Solution

19.3.3:

The annually compounded interest rate for the next 20 years is 8%. You win a lottery and are allowed to choose one of the following four options. Order the options from most valuable to least valuable.

(a) Receive $\$100,000$ in 20 years.

(b) Receive $\$50,000$ in 10 years.

(c) Receive $\$30,000$ in 10 years and another $\$50,000$ in 20 years.

(d) Receive $\$25,000$ right away.

$1 + \frac{r}{200}$ $3000\left(1+\frac{r}{200}\right)^8 = 3950.43$

d c b a

value after 20 yrs FV

value today PV

of all 4 cases to compare

$x(1.08)^{20} = 100000$

$x = $ ____

$\left(1+\frac{r}{200}\right)^8 = \left(\frac{3950.43}{3000}\right)^1$

$\left(1+\frac{r}{200}\right)^{8 \cdot 1/8} = \left(\frac{3950.43}{3000}\right)^{1/8}$

$= 1.4151$ $10000(1+r/100)^6$

$(1.4151)^{1/6} = 1+r$

$10000(1.03)^3(1.09)^3$

1.0596

$r = 5.96\%$

19.3.4:

Greta invests $\$10,000$ in an investment that pays 3% interest, compounded annually, for the first three years, then 9% interest, compounded annually, for the last three years. Rui invests $\$10,000$ in an investment that pays $r\%$ for all six years. The two investments are worth the same amount after 6 years. Is r greater than, equal to, or less than 6?

Type your solution, notes and/or work here. Show Solution

19.3.5★:

Stefan takes out a four-year loan of $\$12,000$. The interest rate is 8.5% compounded annually. Stefan makes a payment at the end of each year right when the annual interest is added to the amount he owes. For the following year, he is charged interest on the amount he still owes after his payment. If all four payments are the same, what is the amount of each payment?

Type your solution, notes and/or work here. Show Solution

$\frac{x}{(1.085)^4}$ (4) $+ \frac{x}{(1.085)^3}$ (3) $+ \frac{x}{(1.085)^2}$ (2) $+ \frac{x}{1.085}$ (1) $= 12000$

3663.46

PV or today value

$a \times (1.08)^4 = x$

$x/(1.08)^4 = a$

What is a Logarithm?

We use an exponent to indicate that we multiply a number by itself repeatedly. For example, 2^6 means the product of six twos. However, suppose we want to write "What power of 2 equals 64?" One way to write this is to let the exponent be x, and write the equation $2^x = 64$. Mathematicians so frequently refer to such exponents that there is a special name for them: **logarithms**. For example, we can write x as $x = \log_2 64$, and this means that x is the solution to the equation $2^x = 64$. Therefore, the two equations $x = \log_2 64$ and $2^x = 64$ are equivalent. Since $2^6 = 64$, we know that $\log_2 64 = 6$.

Just as 2 is the base of the expression 2^6, we say that 2 is the **base** of the logarithm $\log_2 64$. The base of a logarithm must be a positive number, and it cannot equal 1. When

- $\log_a b^n = n \times \log_a b$
- $5^{\log_5 n} = n^{\log_5 5} = n$
- $\log_a bc = \log_a b + \log_a c$

Just as 2 is the base of the expression 2^6, we say that 2 is the **base** of the logarithm $\log_2 64$. The base of a logarithm must be a positive number, and it cannot equal 1. When speaking, we say $\log_2 64 = 6$ as "The logarithm base 2 of 64 is 6," and this means "The exponent to which we must raise 2 to get 64 is 6."

- $5^{\log_5 n} = n^{\log_5 5} = n$
- $\log_a bc = \log_a b + \log_a c$
- $\log_{a^n} b = (1/n)\log_a b$

As we will see, much of understanding logarithms requires being able to convert between **logarithmic form**, $\log_a b = c$, and **exponential form**, $a^c = b$.

> **Important:** If $a > 0$ and $a \neq 1$, then
>
> $$\log_a b = c \quad \text{and} \quad a^c = b$$
>
> are equivalent.

Problems

Problem 19.13 Jump

Evaluate each of the following:

(a) $\log_3 81$

(b) $\log_{10} 100000$

(c) $\log_8 2$

(d) $\log_5 \sqrt[3]{25}$

(e) $\log_2 \frac{1}{8}$

(f) $\log_{1/2} \sqrt{2}$

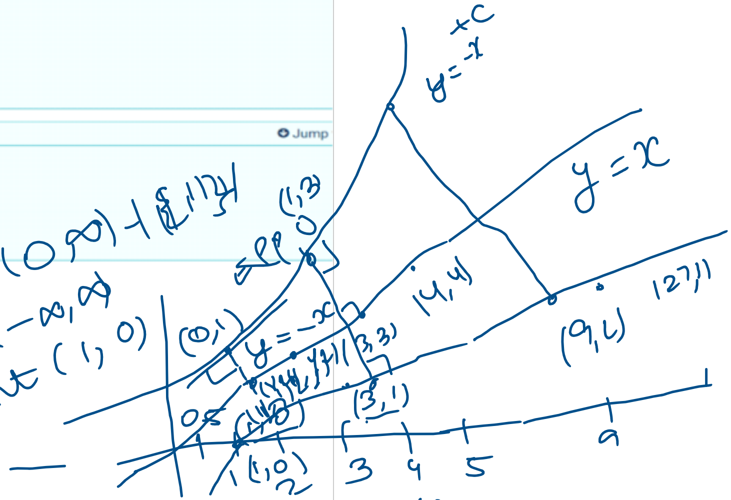
Problem 19.14 Jump

(a) Graph the function $f(x) = \log_3 x$.

(b) What are the domain and range of f?

(c) What is the x-intercept of the function?

(d) How is f related to the function $g(x) = 3^x$?



Problem 19.15

In this problem we find the base n such that $\log_n 3 = -3/2$.

(a) Write the equation as an exponential equation.

(b) Solve your equation from part (a).

Problem 19.16

In this problem we evaluate $\log_{3\sqrt{3}}(1/81)$.

(a) Set the logarithm equal to x and write the corresponding exponential equation.

(b) Solve the resulting equation.

Problem 19.17

In this problem we solve the equation $y = 2 + 3\log_5(2x+1)$ for x in terms of y.

(a) Isolate $\log_5(2x+1)$.

(b) Convert your equation from part (a) to exponential form.

(c) Isolate x in your equation from part (b).

Problem 19.18

If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, then what is r?

$\log_2 2^r = \log_2 5$

$r \log_2 2 = \log_2 5$

$r \cdot 1 = \log_2 5$

$r = \log_2 5$

$2^r = 5$

$r = \log_2 5$

19.4.1:

Evaluate each of the following:

(a) $\log_4 64$

Type your solution, notes and/or work here.

$\log_4 4^3 = 3 \log_4 4$

$= 3$

(b) $\log_6 1296$

Type your solution, notes and/or work here.

$\log_6 6^4 = 4 \log_6 6$

$= 4$

$\log_4 2$

$\log_{2^2} 2 = \frac{1}{2} \log_2 2$

$= \frac{1}{2}$

$\log_4 2 = a$

$4^a = 2$

$a = 1/2$

19.4.2:

Evaluate each of the following:

(a) $\log_2 \frac{1}{16}$

Type your solution, notes and/or work here.

(b) $\log_{1/3} 27$

Type your solution, notes and/or work here.

19.4.3:

Evaluate each of the following:

(a) $\log_2 8\sqrt{2}$

Type your solution, notes and/or work here.

(b) $\log_{\sqrt{3}} 9$

Type your solution, notes and/or work here.

19.4.4:

Evaluate each of the following:

(a) $\log_4 32$

Type your solution, notes and/or work here.

(b) $\log_{27} 3\sqrt{3}$

Type your solution, notes and/or work here.

19.4.5:

Find r such that $\log_{81}(2r-1) = -1/2$.

Type your solution, notes and/or work here.

19.4.6:

Solve for x in terms of y in each of the following:

(a) $y = \log_2(x-4)$

Type your solution, notes and/or work here.

(b) $y = 3\log_4(2x)$

Type your solution, notes and/or work here.

(c) $y = 4 - 2\log_7(3-x)$

Type your solution, notes and/or work here.

19.4.7:

(a) Find the domain and range of $f(x) = 2\log_3(5-x)$

Type your solution, notes and/or work here.

(b) Find the domain and range of $g(x) = 7 - 3\log_8(2x-5)$.

Type your solution, notes and/or work here.

19.4.8:

Find the base b such that $\log_b 5\sqrt{5} = \frac{5}{2}$.

Type your solution, notes and/or work here.

19.4.9:

Is it true that the x-intercept of $y = \log_a x$ is $(1, 0)$ for all positive constants a (except $a = 1$)?

Type your solution, notes and/or work here.

19.4.10:

(a) Evaluate $\log_2 4, \log_2 4^2, \log_2 4^3, \log_2 4^4$.

Type your solution, notes and/or work here.

(b) Let $x = \log_a b$. Write this equation in exponential form.

Type your solution, notes and/or work here.

(c) Let $y = \log_a b^c$. Write this equation in exponential form.

Type your solution, notes and/or work here.

(d) Use parts (b) and (c) to prove that $\log_a b^c = c \log_a b$.

Type your solution, notes and/or work here.

Review Problems

19.19:

Find the domain and range of the function $g(x) = 7 - 4^x$.

Type your solution, notes and/or work here.

19.20:

Find all values of c such that $6^{3c-1} = \frac{1}{36}$.

Type your solution, notes and/or work here.

19.21:

Find 4^{x+3} if $2^x = 9$.

Type your solution, notes and/or work here.

19.22:

Find all values of x such that $\frac{3^{x^2}}{3^{2x}} = 27$.

Type your solution, notes and/or work here.

19.27:

Gert invests all of her money at the end of 2000. At the end of each year, she checks the value of her investment. At the end of what year will she first find she's doubled her money in each of the following cases:

(a) Gert invests all of her money at 5% simple interest.

Type your solution, notes and/or work here. | Show Solution

(b) Gert invests all of her money at 5% interest, compounded annually.

Type your solution, notes and/or work here. | Show Solution

(c) Gert invests all of her money at 5% interest, compounded twice a year.

Type your solution, notes and/or work here. | Show Solution

(d) Gert invests all of her money at 5% interest, compounded monthly.

Type your solution, notes and/or work here. | Show Solution

19.28:

I have just won a lottery that will pay me $\$1,000,000$ in 10 years. A company offers to buy my winning ticket today for $\$300,000$.

(a) If the annually compounded interest rate is 9%, should I take the offer?

Type your solution, notes and/or work here. | Show Solution

(b) For what annually compounded interest rate is my lottery ticket worth $\$300,000$ today?

19.28:

I have just won a lottery that will pay me $\$1,000,000$ in 10 years. A company offers to buy my winning ticket today for $\$300,000$.

(a) If the annually compounded interest rate is 9%, should I take the offer?

Type your solution, notes and/or work here. | Show Solution

(b) For what annually compounded interest rate is my lottery ticket worth $\$300,000$ today?

Type your solution, notes and/or work here. | Show Solution

19.29:

Suppose the annually compounded interest rate is 8%. Order the following from most valuable to least valuable:

(a) $\$2,500$ paid today.

(b) $\$5,000$ paid 9 years from now.

(c) $\$4,500$ paid 8 years from now.

(d) $\$4,000$ paid 5 years from now.

Type your solution, notes and/or work here. | Show Solution

19.30:

Claire has borrowed $\$5,000$. She will repay the loan entirely by making one payment in 3 years, then another payment in 6 years. The second payment will be exactly double the amount of the first payment. How much is the first payment if the interest rate of the loan is 8.5%, compounded annually?

Type your solution, notes and/or work here. | Show Solution

19.23:

Source: AHSME

Find all values of x such that $2^{2x} - 8 \cdot 2^x + 12 = 0$.

Type your solution, notes and/or work here.

Show Solution

19.24:

Adisa borrows $\$5,000$ at 14% interest, compounded twice a year. How much does she owe at the end of 8 years?

Type your solution, notes and/or work here.

Show Solution

19.25:

Jake invested his whole life savings today in an investment that pays 6% interest, compounded annually. In ten years, this investment will be worth $\$531,402$. What is Jake's life savings today?

Type your solution, notes and/or work here.

Show Solution

19.26:

Rate these 10-year investments from best to worst:

(a) Receive 10% simple interest for 10 years.

(b) Receive 7.5% interest, compounded 4 times a year for 10 years.

(c) Receive 8% interest, compounded twice a year for 10 years.

(d) Receive 11% simple interest for the first 5 years, then receive 7% interest, compounded annually, for the next 5 years. (The latter interest is paid on the whole value of the investment after the first 5 years.)

Type your solution, notes and/or work here.

Show Solution

Evaluate each of the following:

(a) $\log_3 243$

Type your solution, notes and/or work here.

(b) $\log_{1/2} \frac{1}{8}$

Type your solution, notes and/or work here.

(c) $\log_2 4\sqrt[3]{2}$

Type your solution, notes and/or work here.

(d) $\log_9 27$

Type your solution, notes and/or work here.

(e) $\log_7 \frac{1}{343}$

Type your solution, notes and/or work here.

(f) $\log_{\sqrt{5}} 125\sqrt{5}$

Type your solution, notes and/or work here.

19.32:

Find the base n such that $\log_n 4\sqrt{2} = 10$.

Type your solution, notes and/or work here.

Show Solution

19.33:

Suppose a is a constant and f is a function such that $f(x) = \log_a x$. Why can we not have $a = 1$? (In other words, why are we not allowed to let 1 be the base of a logarithmic function?)

Type your solution, notes and/or work here.

Show Solution

19.34:

Find x if $\log_9(2x - 7) = \frac{3}{2}$.

Type your solution, notes and/or work here.

Show Solution

19.35: Source: AMC 12

For how many positive integers b is $\log_b 729$ a positive integer?

Type your solution, notes and/or work here.

Show Solution

Challenge problems

19.36:

Source: AHSME

If $\log_2(\log_2(\log_2(x))) = 2$, then how many digits are in x?

Type your solution, notes and/or work here.

Show Solution

19.37:

I invested all of my money in an investment that pays the same annually compounded interest rate for 30 years. At the end of the first 10 years, my investment had doubled in value.

(a) After how many years will my original investment have multiplied by 4 in value?

Type your solution, notes and/or work here.

Show Solution

(b) If I initially invested $\$5,000$, how much will my investment be worth at the end of the 30 years?

Type your solution, notes and/or work here.

Show Solution

19.38:

Find the domain and range of $f(x) = 2\log_3(x^2 - 4x - 5)$.

Type your solution, notes and/or work here.

Show Solution

19.39:

Find the domain and range of the function $f(x) = \dfrac{5}{2+3^x}$.

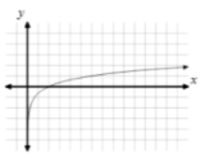
Type your solution, notes and/or work here.

Show Solution

19.40:

The graph at right is the graph of the equation $y = f(x)$, where $f(x) = \log_a(xb)$ and a and b are constants. Find a and b.

Hint



Type your solution, notes and/or work here.

Show Solution

19.41:

If $8^x = 27$, then what is 4^{2x-3}?

Hint

Type your solution, notes and/or work here.

Show Solution

19.42:

Source: AHSME

If $\dfrac{4^x}{2^{x+y}} = 8$ and $\dfrac{9^{x+y}}{3^{5y}} = 243$, where x and y are real numbers, then what is the ordered pair (x, y)?

Type your solution, notes and/or work here.

Show Solution

19.43:

(a) Evaluate $\log_2 8$, $\log_2 16$, and $\log_2(8 \cdot 16)$.

Type your solution, notes and/or work here. | Show Solution

(b) Evaluate $\log_3 \frac{1}{9}$, $\log_3 \sqrt{3}$, and $\log_3 \left(\frac{1}{9} \cdot \sqrt{3}\right)$.

Type your solution, notes and/or work here. | Show Solution

(c) Do you notice a relationship among $\log_a b$, $\log_a c$, and $\log_a(bc)$? Can you prove it?

Hint

Type your solution, notes and/or work here. | Show Solution

19.44★:

Alice invests some money at an annually compounded interest rate of $r\%$. Bob invests the same amount at a simple interest rate of $s\%$. If their investments are worth the same amount after 10 years, then which of their investments is worth more after 11 years?

Hint

Type your solution, notes and/or work here. | Show Solution

19.45★:

Source: ARML

Compute the number of real values of x such that $x^{100} - 4^x \cdot x^{98} - x^2 + 4^x = 0$.

Hint

Type your solution, notes and/or work here. | Show Solution

math hw

Problem 11.6

In this problem we find all solutions to the equation

$$4^x - 33 \cdot 2^{x-1} + 8 = 0$$

(a) Write all the terms with variables in their exponents as powers of 2.

(b) What makes this problem difficult is that there are variables in the exponents. What substitution could we make to turn this into an equation we know how to solve?

(c) Solve the equation your substitution produces, then solve the original equation. Make sure you test your solutions.

$(2^2)^x$

2^{2x}

$2^2 \cdot 2^x$

$= 2^2$

a) $2^{2x} - 33 \times 2^{x-1} + 8 = 0$

b) $y = 2^x$

$2^{2x} = (2^x)^2 = y^2$ $\quad 2^{x-1} = \frac{2^x}{2} = \frac{y}{2}$

$y^2 - 33\left(\frac{y}{2}\right) + 4 = 0$

$2y^2 - 33y + 16 = 0$

$2y^2 - 32y - y + 16 = 0$

$2y(y-16) - 1(y-16) = 0$

$(2y-1)(y-16) = 0$

$y = \frac{1}{2} \quad y = 16$

c) $2^x = \frac{1}{2}$ $\quad 2^x = 16 \rightarrow 2^x = 2^4$

$\boxed{x = -1}$ $\quad \boxed{x = 4}$

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3/18/2026 (2/5)

19.2.4:

Beth invests $4,200 at an annually compounded interest rate of 6% Josey invests $4,200 at a simple interest rate of r%.

(a) What is r if their investments are worth the same amount after 1 year?

compound | simple

$4200(1.06)^t$ | $4200(1+rt)$

$(1.06)^t = 1+r(t)$

$(1.06)^1 = 1+r\times1$ → $r = 1.06-1$, $r = 0.06$ → **r = 6%**

Type your solution, notes and/or work here.

(b) What is r if their investments are worth the same amount after 2 years?

$(1.06)^2 = 1+r\times2$

$1.1236 = 1+2r$ → $2r = 0.0618$ → **r = 6.18%**

Type your solution, notes and/or work here.

(c) What is r if their investments are worth the same amount after 10 years?

$(1.06)^{10} = 1+r\times10$

$1.79085 = 1+10r$ → $10r = 0.79085$

$r = 0.079085$

r = 7.91%

Type your solution, notes and/or work here.

19.2.5:

Show that if $k is invested at an interest rate of r% for n years compounded annually, then the total amount at the end of n years is

correct → $\left(1+\frac{r}{100}\right)^n (\$k)$

19.2.6:

Show that if $k is invested at an interest rate of r% for n years, compounded m times a year, then the total amount at the end of n years is

correct → $\left(1+\frac{r}{100m}\right)^{mn} (\$k)$

Type your solution, notes and/or work here.

Show Solution

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3/18/2026 (3/5)

Problem 19.10 Jump to Solution

Sayeed borrowed $2,300. The interest on the loan was compounded annually for 5 years, and the interest rate remained the same throughout that time. At the end of the five years, Sayeed owed $3,301.95 (including the original $2,300). To the nearest tenth of a percent, what was the interest rate?

$3301.95 = 2300(1+r)^5$

$\frac{3301.95}{2300} = (1+r)^5$

$\rightarrow 1.43563 = (1+r)^5$

$1+r = 1.0749$

$r = 1.0749 - 1$

$\rightarrow r = 0.0749$

$r = 7.49\%$

Problem 19.11 Jump to Solution

I want to have $1,000,000 in the bank when I retire in 10 years. I can invest money today at a rate of 10%, compounded semi-annually. I will not invest any more money between now and retirement.

(a) Suppose I invest $x today. In terms of $x, how much will this be worth in 10 years?

(b) How much should I invest right now to have $1,000,000 in 10 years?

(a) $x\left(1+\frac{0.1}{2}\right)^{2\times 10}$

$x(1.5)^{20}$

(b) $1{,}000{,}000 = x(1.5)^{20}$

$x = \frac{1{,}000{,}000}{2.653}$

$x = 376{,}889$

Problem 19.12 Jump to Solution

I take a three-year loan for $10,000. The interest rate is 6% compounded annually. At the end of each year, I make a payment right when the annual interest is added to the amount I owe. For the following year, I am then charged interest on the amount I still owe after my payment. All three of my payments are the same and my loan is completely paid off after the third payment.

(a) Suppose each payment is $x. In terms of x, what is the present value of my first payment? Of my second payment? Of my third payment?

(b) What is the amount of each payment?

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3/18/2026 (1/5)

19.2.1:

Paula invests $10,000 for 5 years at an interest rate of 10%. At the end of those 5 years, how much is her investment [illegible] the following cases

(a) The interest is simple interest

$10{,}000 \times (1 + (0.1)(5)) = 10000 \times (1 + 0.5) = \boxed{15{,}000}$

Type your solution, notes and/or work here

(b) The interest is compounded annually

$10000 (1.1)^5 = 10000 \times 1.61051 = \boxed{16{,}105.1}$

Type your solution, notes and/or work here

(c) The interest is compounded quarterly 4×5 5years

$10{,}000 \left(1 + \frac{0.1}{4}\right)^{20} = 10000 (1.025)^{20} = \boxed{16{,}386.20}$

19.2.2:

Joanie takes a $6,000 loan to pay for her car. The interest rate on the loan is 12%. She makes no payments for 4 years, but has to pay back all the money she owes at the end of 4 years. How much more [illegible] if the interest compounds quarterly than if the interest compounds annually? ← annually

$6000 (1.12)^4 = 6000 \times 1.57352 = 9{,}441.12$

Quarterly

$6{,}000 \left(1 + \frac{0.12}{4}\right)^{4 \times 4} = 6000 \times 1.60471 = 9{,}628.26$

$9628.26 - 9441.12 = \boxed{187.14}$

Type your solution, notes and/or work here

19.2.3:

Bill invests $2,500 in an investment that pays 5% interest compounded annually for the first two years, then 8% interest compounded annually for three years after that. Debbie invests $2,500 in an investment that pays 8% interest compounded annually for the first three years, then 5% interest compounded annually for two years after that. [illegible] worth the most money after 5 years?

Type your solution, notes and/or work here

Bill

first two years

$2500(1.05)^2 = 2756.25$

next three years

$2756.25(1.08)^3 = \boxed{3471.83}$

Debbie

first three years

$2500(1.08)^3 = 3149.28$

next 2 years

$3149.28 \times (1.05)^2 = \boxed{3471.83}$

both are the same

Home work
3-25-2026

3/25/2026 (1/3)

What is a Logarithm?

We use an exponent to indicate that we multiply a number by itself repeatedly. For example, 2^6 means the product of six twos. However, suppose we want to write "What power of 2 equals 64?" One way to write this is to let the exponent be x, and write the equation $2^x = 64$. Mathematicians so frequently refer to such exponents that there is a special name for them: **logarithms**. For example, we can write x as $x = \log_2 64$ and this means that x is the solution to the equation $2^x = 64$. Therefore, the two equations $x = \log_2 64$ and $2^x = 64$ are equivalent. Since $2^6 = 64$, we know that $\log_2 64 = 6$.

Just as 2 is the base of the expression 2^x, we say that 2 is the **base** of the logarithm $\log_2 64$. The base of a logarithm must be a positive number, and it cannot equal 1. When speaking, we say $\log_2 64 = 6$ as "The logarithm base 2 of 64 is 6," and this means "The exponent to which we must raise 2 to get 64 is 6."

As we will see, much of understanding logarithms requires being able to convert between **logarithmic form**, $\log_a b = c$, and **exponential form**, $a^c = b$.

Important: If $a > 0$ and $a \neq 1$, then

$$\log_a b = c \quad \text{and} \quad a^c = b$$

are equivalent.

Problem 19.13

Evaluate each of the following

(a) $\log_3 81 \rightarrow 3^x = 81 \Rightarrow \boxed{x = 4}$

(b) $\log_{10} 100000 \rightarrow 10^x = 100000 \Rightarrow \boxed{x = 5}$

(c) $\log_8 2 \rightarrow 8^x = 2 \rightarrow (2^3)^x = 2 \rightarrow 2^{3x} = 2^1 \rightarrow 3x = 1 \rightarrow \boxed{x = \frac{1}{3}}$

(d) $\log_5 \sqrt[3]{25} \rightarrow 5^x = 25^{\frac{1}{3}} \rightarrow 5^x = (5^2)^{\frac{1}{3}} \rightarrow 5^x = 5^{\frac{2}{3}} \rightarrow \boxed{x = \frac{2}{3}}$

(e) $\log_2 \frac{1}{8} \rightarrow 2^x = 2^{-3} \rightarrow \boxed{x = -3}$

(f) $\log_{1/2} \sqrt{2} \rightarrow \frac{1}{2}^x = \sqrt{2} \rightarrow (2^{-1})^x = 2^{\frac{1}{2}} \rightarrow 2^{-x} = 2^{\frac{1}{2}} \rightarrow \boxed{x = -\frac{1}{2}}$