130pm04july

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Type of Relations

Empty Relation

A is non-empty set

$R : A \longrightarrow A$

is empty relation if no element of A is related to any element of A

i.e

$R = \phi \subset A \times A$

Example $A = \{1, 2, 3, 4, 5 \; - - - \; n\}$

$R : A \longrightarrow A$

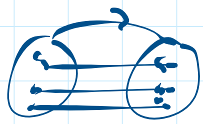
defined by

$R = \{(x, y) : \frac{x}{y} < 0 \quad x, y \in A\}$

$= \{ \;\text{\_\_\_\_\_\_\_\_}\; \}$

* Universal Relation

$R : A \longrightarrow A$



is said to be universal relation if each element of A is related to each element of A.

i.e

$\boxed{R = A \times A}$

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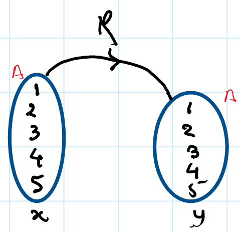
* $A = \{1, 2, 3, 4, 5\}$

$R : A \longrightarrow A$



defined by

$R = \{(x, y) : \boxed{x + y}\boxed{\in N}, x, y \in A\}$

$= \{(1,1) \; (1, 2), (1, 3) \; (1, 4) \; (1, 5) \; (2, 1) \; (2, 2) \; (2, 5)$

$(5, 5)\}$

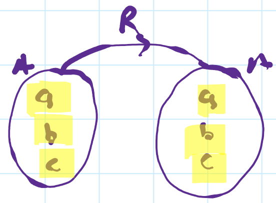
$\boxed{R = A \times A}$

Equivalence Relations

Let A be any non empty set

$A = \{a, b, c\}$

$R : A \longrightarrow A$



is

Ref Reflexive (x, y) (D, R) $\boxed{D = R}$

if $(a, a) \in R$ $\forall\ a \in A$

$(b, b) \in R$ $\forall\ b \in A$

$(c, c) \in R$ $\forall\ c \in A$

(ii) Symmetric

if $(a, b) \in R \Rightarrow (b, a) \in R$ for $a, b \in A\}$

(iii) Transitive :

if $(a, b) \in R$ and $(b, c) \in R$ for $a, b, c \in A$

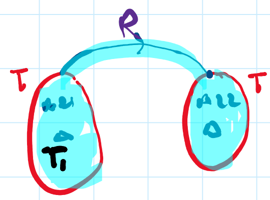
$\Rightarrow (a, c) \in R$

"A Relation R in set A is said to be equivalence Relation

if R is Reflexive
symmetric
transitive"

Example 1: Let T be the set of all triangles in a plane with R a relation in T given by

R = {(T_1, T_2) : (T_1 ~ T_2}. Show that R is an equivalence relation.



T = collection of all Δ

$R : T \longrightarrow T$

Relation given by

$R = \{(T_1\ T_2) : T_1 \sim T_2\}$

Sol: (i) $\boxed{\text{R is Reflexive}}$

Let $T_1 \in T$ $T_2 \in T$ $T_{10} \in T$

Every Δ is similar to itself

i.e

$T_1 \sim T_1$ $T_2 \sim T_2$ $T_{10} \sim T_{10}$

$(T_1\ T_1) \in R$ $(T_2, T_2) \in R$, $(T_{10}\ T_{10}) \in R$

Therefore R is Reflexive Relation

(ii) Symmetric

Let $T_1, T_2 \in T$

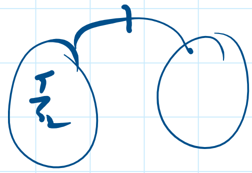
$(T_1, T_2) \in R \Rightarrow$

$T_1 \sim T_2 \Rightarrow T_2 \sim T_1$

$\Rightarrow (T_2, T_1) \in R$

$(T_1, T_2) \in R \Rightarrow (T_2, T_1) \in R$

Therefore R is symmetric



(iii) Transitive :

Let $T_1, T_2, T_3 \in T$

such that

$(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

$\Rightarrow T_1 \sim T_2$ and $T_2 \sim T_3$

$\Rightarrow T_1 \sim T_3$

$\Rightarrow (T_1, T_3) \in R$

R is transitive

Since R is Reflexive
Symmetric
and transitive

Therefore R is an equivalence Relation

* Example 2: Let S be any non empty set and R be a relation defined on power set of S i.e. on P(S) by A R B iff A ⊂ B for all A, B ∈ P(S). Show that R is reflexive and transitive but not symmetric.

S is a non empty set

Power set of S — $P(S)$

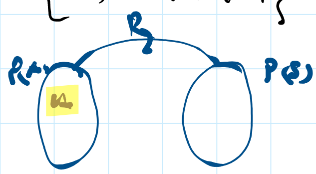
$R: P(S) \longrightarrow P(S)$

$A R B$ if $A \subset B$ $\forall$ $A, B \in P(S)$

$A = \{1, 2\}$

$A \subset A$ $\quad \phi \subset A$

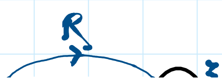
$P(A) = \{\phi, A, \{1\}, \{2\}\}$



$\boxed{\text{Reflexive}}$ Let $A \in P(S)$,

$A \subset A$

$A R A$ $\quad (A, A) \in R$



Example 3: Show that the relation R in the set Z of integers given by

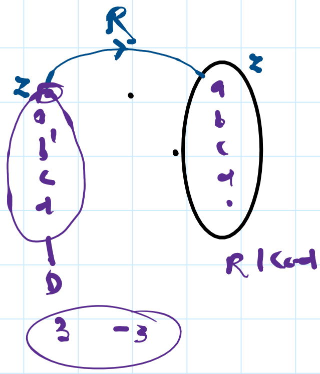
R = {(a, b) : 3 divides a – b}

is an equivalence relation.

$R: Z \longrightarrow Z$

Defined by

$R = \{(a,b) : 3 \text{ divides } a-b\}$



Reflexive: let $a \in Z$ (Domain) | let $3 \in Z$

3 divides $a-a$ | $3-3 = 0/3 = 0$

$(a,a) \in R$ | $(3,3) \in R$

R is reflexive | R is reflexive

Symmetric let $a, b \in Z$

$(a,b) \in R \Rightarrow$ 3 divides $a-b$

$\Rightarrow$ 3 divides $b-a$

$(a,b) \in R \Rightarrow (b,a) \in R$

Hence R is Symmetric

Transitive let $a, b, c \in Z$

and $(a,b) \in R$ & $(b,c) \in R \Rightarrow$ 3 divides $a-b$ and 3 divides $b-c$

$\Rightarrow$ 3 divides $a - \cancel{b} + \cancel{b} - c$

$\Rightarrow$ 3 divides $a-c$

$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

R is transitive

Since R is Ref. Sym. tran.

R is Equivalence Relation