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02 July 2025 20:27

INTEGRATION

7.2 Integration as an Inverse Process of Differentiation

Integration is the inverse process of differentiation. Instead of differentiating a function, we are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called *integration* or *anti differentiation*.

$\frac{d}{dx} \sin x = \cos x$ (derived)

$\frac{d}{dx} x^2 = 2x$

x^2 is antiderivative of $2x$

$\frac{d}{dx} e^x = e^x$

$\sin x$ is anti derivative (Integral of) $\cos x$

* $\frac{d}{dx}(\sin x + C) = \cos x$

$\frac{d}{dx} \sin x + 20 = \cos x$

$\frac{d}{dx}(e^x + C) = e^x$

Anti derivative of $\cos x$ is not unique

Actual

There exist infinitely many anti derivative of functions

which can be obtained by using C

C is arbitrary set of Real no.

C is parameter varying

* If F is a function

such that $\frac{d}{dx} F(x) = f(x)$, $\forall x \in I$

$\frac{d}{dx}(F(x) + C) = f(x)$ $\forall x \in I$

Thus $\{F + C, C \in R\}$ is family of Anti derivative of f

* $\int$ Sum

$\int f(x)\,dx \Rightarrow$ Indefinite Integral of f w.r.t. x

* $\int f(x)\,dx = F(x) + C$

f(x) is integrand

x is variable of integration

*

Symbols/Terms/Phrases	Meaning
$\int f(x)\,dx$	Integral of f with respect to x
$f(x)$ in $\int f(x)\,dx$	Integrand
x in $\int f(x)\,dx$	Variable of integration
Integrate	Find the integral
An integral of f	A function F such that $F'(x) = f(x)$
Integration	The process of finding the integral
Constant of Integration	Any real number C, considered as constant function

Derivation	Integral (Anti Derivation)
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
$\frac{d}{dx}(x) = 1$	$\int dx = x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(-\cos x) = \sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$
$\frac{d}{dx}(\cos^{-1} x) = \frac{+1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = -\cos^{-1} x + C$
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$
$\frac{d}{dx}(-\cot^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$
* $\frac{d}{dx}\log\lvert x\rvert = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log\lvert x\rvert + C$
* $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$	$\int a^x\,dx = \frac{a^x}{\log a} + C$

* $\frac{d}{dx}\log \sec x + c = \tan x$ *

$\boxed{\log \sec x + c = \int \tan x}$

Some Properties of Indefinite Integral

1. The process of Diff. and integration are inverse of each other

$\frac{d}{dx}\int f(x)\,dx = f(x)$

and $\int f'(x)\,dx = f(x) + C$

2. Two indefinite integral with same derivative lead to same family of curves so they are equivalent.

3. $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

4. $\int k\,f(x)\,dx = k\int f(x)\,dx$

* Find integral of

(1) $I = \int \frac{x^3 - 1}{x^2}\,dx$

$= \int \frac{x^3}{x^2}\,dx - \int \frac{1}{x^2}\,dx$

$= \int x\,dx - \int x^{-2}\,dx$ $\int x^n = \frac{x^{n+1}}{n+1} + C$

$= \frac{x^2}{2} - \frac{x^{-2+1}}{-2+1} + C$

$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + C$

$= \frac{x^2}{2} + \frac{1}{x} + C$

* $\int (x^{\frac{2}{3}} + 1)\,dx$ * $\int 2e^x\,dx$

* $\int (\sin x + \cos x)\,dx$ * $\int \frac{1 - \sin x}{\cos^2 x}\,dx$

* $\int \operatorname{cosec} x(\operatorname{cosec} x + \cot x)\,dx$