# Final Revision 2025-26 exam-2

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**SECTION A (Multiple Choice Questions - 1 mark each)**

1. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then $A^3$ is:

2. If $P(A \cup B) = 0.9$ and $P(A \cap B) = 0.4$, then $P(\bar{A}) + P(\bar{B})$ is:

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is:

4. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of $x$ is:

5. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of $a$ is:

6. If $A = [a_{ij}]$ is a $3 \times 3$ diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is:

7. The principal value of $\cot^{-1}(-\frac{1}{\sqrt{3}})$ is:

8. If $A$ and $B$ are two square matrices of the same order, then $(A + B)(A - B)$ is equal to:

9. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is:

10. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at:

11. $\int \sqrt{1 + \sin x}dx$

12. $\int_0^{\pi/2} \cos xe^{\sin x}$

13. The area of the region enclosed between the curve $y = x|x|$, $x$-axis, $x = -2$ and $x = 2$ is:

14. The integrating factor of the differential equation $(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}})\frac{dx}{dy} = 1$ is:

15. The sum of the order and degree of the differential equation $[1 + (\frac{dy}{dx})^2]^3 = \frac{d^2y}{dx^2}$ is:

16. For a Linear Programming Problem (LPP) with objective function $Z = 3x + 2y$ and constraints $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0$, identify the correct feasible region from a given graph.

17. Let $\vec{a}$ be a position vector whose tip is the point $(2, -3)$. If $\overrightarrow{AB} = \vec{a}$, and the coordinates of A are $(-4, 5)$, then the coordinates of B are:

18. The respective values of $|\vec{a}|$ and $|\vec{b}|$, given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are:

19. **Assertion (A) and Reason (R) Type:** Based on a graph for the LPP: Min $Z = 50x + 70y$ subject to $2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$, with minimum value 380 at B(2,4). (A): The shaded portion is the feasible region. (R): The region $50x + 70y < 380$ has no common point with the feasible region.

20. **Assertion (A) and Reason (R) Type:** Let $A = \{x \in R: -1 \leq x \leq 1\}$. $f: A \to A$ is defined as $f(x) = x^2$. (A): $f$ is not an onto function. (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

21. The principal branch of $\cos^{-1} x$ is:

22. The values of $\lambda$ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real $x$ are:

23. If A and B are square matrices of the same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to:

24. If $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3\cos^2 x}, & x \neq \frac{\pi}{2} \\ k, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of $k$ is:

25. For real $x$, let $f(x) = x^3 + 5x + 1$. Then:

26. If the direction cosines of a line are $\lambda, \lambda, \lambda$, then $\lambda$ is equal to:

27. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, the product of the possible values of $x$ is:

28. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a:

29. If $A = kB$, where $A$ and $B$ are square matrices of order $n$, then:

30. Let $A$ and $B$ be two matrices of suitable orders. Then, which of the following is **not** correct? (Options involve transpose properties).

31. $\int e^x(\cos x - \sin x)dx$

32. The area of the region enclosed by the curve $y = \sqrt{x}$, the lines $x = 0$, $x = 4$ and the $x$-axis is:

33. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan\ x - \sec\ x = 0$ is:

34. The corner points of the feasible region of an LPP are $(0,2)$, $(3,0)$, $(6,0)$, $(6,8)$ and $(0,5)$. If $Z = ax + by$; $(a, b > 0)$ is the objective function, and maximum value of $Z$ is obtained at $(0,2)$ and $(3,0)$, then the relation between $a$ and $b$ is:

35. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is:

36. **Assertion (A) and Reason (R) Type:** (A): If $P(A \cap B) = 0$, then A and B are independent. (R): Two events are independent if the occurrence of one does not affect the other.

37. **Assertion (A) and Reason (R) Type:** (A): If the feasible region of an LPP is empty, then the LPP has no solution. (R): A feasible region is defined as the region that satisfies all constraints.

38. The principal value of $\sin^{-1}(\sin(-\frac{10\pi}{3}))$ is:

39. If P is a point on the line segment joining (3, 6, -1) and (6, 2, -2) and the y-coordinate of P is 4, then its z-coordinate is:

40. If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to:

41. If $f: N \to W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$, then $f$ is:

42. If the sides AB and AC of $\triangle ABC$ are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A is:

43. The function $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ is **not** continuous at:

44. If $f(x) = 2x + \cos\ x$, then $f(x)$:

45. Domain of $\sin^{-1}(2x^2 - 3)$ is:

46. If A and B are invertible matrices of order $3 \times 3$ such that det (A) = 4 and det $[(AB)^{-1}] = \frac{1}{20}$, then det (B) is equal to:

47. If $\int e^{-3\log x} dx = f(x) + C$, then $f(x)$ is:

48. The solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ represents a family of:

49. The order and degree of the differential equation $(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^2 = x\sin(\frac{dy}{dx})$ are:

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**SECTION B (Very Short Answer - 2 marks each)**

1. Find the domain of $\sec^{-1}(2x+1)$.
2. The radius of a cylinder is decreasing at 2 cm/s and its altitude is increasing at 3 cm/s. Find the rate of change of volume when radius is 4 cm and altitude is 6 cm.
3. **(a)** Find a vector of magnitude 5 perpendicular to both $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$.
   **OR**
   **(b)** Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}, \vec{a} \neq \vec{0}$. Show that $\vec{b} = \vec{c}$.
4. A man needs to hang two lanterns on a straight wire with endpoints A(4, 1, -2) and B(6, 2, -3). Find the coordinates of the points that trisect AB.
5. **(a)** Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to $x$.
   **OR**
   **(b)** If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.
6. Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.
7. The surface area of a spherical balloon increases at 5 mm$^2$/s. When the radius is 8 mm, find the rate at which the volume is increasing.
8. Using matrices and determinants, find the value(s) of k for which the system $5x - ky = 2; 7x - 5y = 3$ has a unique solution.
9. **(a)** Simplify $\sin^{-1}(\frac{x}{\sqrt{1+x^2}})$.
   **OR**
   **(b)** Find the domain of $\sin^{-1}\sqrt{x-1}$.
10. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.
11. **(a)** Find the least value of ‘a’ so that $f(x) = 2x^2 - ax + 3$ is increasing on [2, 4].
    **OR**
    **(b)** If $f(x) = x + \frac{1}{x}, x \geq 1$, show that f is increasing.
12. A cylindrical water container has a leak. Water leaks at 5 cm$^3$/s. If the radius is 15 cm, find the rate at which the height is decreasing when the height is 2 meters.

13. Let A and B be two square matrices of order 3 such that det $(A) = 3$ and det $(B) = -4$. Find the value of det $(-6AB)$.

14. For the curve $y = 5x - 2x^3$, if x increases at 2 units/s, how fast is the slope changing when $x = 2$?

15. Find the local maxima and local minima of $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$.

16. Let the volume of a metallic hollow sphere be constant. If the inner radius increases at 2 cm/s, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm.

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**SECTION C (Short Answer - 3 marks each)**

1. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is increasing in $[0, \frac{\pi}{4}]$.

2. **(a)** The probability a student buys a colouring book is 0.7, and a box of colours is 0.2. The probability she buys the book given she buys the colours is 0.3. Find: (i) P(buys both), (ii) P(buys colours | buys book).
   **OR**
   **(b)** A fruit box has 6 apples and 4 oranges. A fruit is picked three times with replacement. Find: (i) Probability distribution of number of oranges, (ii) Expectation of the number of oranges.

3. Find the particular solution of: $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}(\frac{y}{x}) = 0$; given $y = 0$ when $x = 1$.

4. **(a)** Find $\int \frac{2x}{(x^2+3)(x^2-5)} dx$.
   **OR**
   **(b)** Evaluate $\int_1^4 (|x - 2| + |x - 4|)\, dx$.

5. In an LPP, maximize $Z = 5x + 10y$ subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.

6. **(a)** If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.
   **OR**
   **(b)** If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at angle θ, prove that $\frac{1}{2} |\vec{a} - \vec{b}| = \sin \frac{\theta}{2}$.

7. Find the value of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.

8. Find the particular solution of: $[x\sin^2(\frac{y}{x}) - y]dx + xdy = 0$; given $y = \frac{\pi}{4}$ when $x = 1$.

9. Find the interval in which $f(x) = \sin\ 3x - \cos\ 3x, 0 < x < \frac{\pi}{2}$ is strictly increasing.

10. Solve: $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$.

11. Find $\int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}}dx$.

12. Find the distance of the point (-1, -5, -10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

13. **(a)** If $f: R^+ \to R$ is $f(x) = \log_a x\ (a > 0, a \neq 1)$, prove it is a bijection.

    **OR**

    **(b)** Let $A = \{1,2,3\}, B = \{4,5,6\}$. Relation $R = \{(x,y): x + y = 6\}$. (i) List elements of R, (ii) Is R a function? Justify, (iii) Find domain and range of R.

14. **(a)** Find k so that $f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$.

    **OR**

    **(b)** Check the differentiability of $f(x) = x\ |\ x\ |$ at $x = 0$.

15. Evaluate $\int_{\pi/2}^{\pi} \quad e^x\left(\frac{1-\sin\ x}{1-\cos\ x}\right)dx$.

16. **(a)** Find the probability distribution of the number of boys in families with 3 children (equal probability for boy/girl).

    **OR**

    **(b)** A coin is tossed twice. X = (number of heads) - (number of tails). Find the probability distribution of X and its mean.

17. **(a)** Toss a coin. If head, toss again; if tail, throw a die. Find P(die shows >3 | there is at least one tail).

    **OR**

    **(b)** Probability distribution of X is given. (i) Calculate λ if E(X)=3.2, (ii) Find P(X>1).

18. Solve graphically: Minimise Z = 3x + 5y subject to $x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x, y \geq 0$.

19. For a given LPP graph, write all the constraints satisfying the feasible region.

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**SECTION D (Long Answer - 5 marks each)**

1. Sketch the curve $y = \sqrt{x}$. Using integration, find the area of the region bounded by $y = \sqrt{x}, x = 4$ and the x-axis in the first quadrant.

2. ₹10,000 is invested at 10%, 12%, and 15% per annum. The combined annual income is ₹1,310. The combined income from the first two investments is ₹190 less than the income from the third. Use the matrix method to find the amount in each investment.

3. **(a)** Find the foot of the perpendicular from (1, 1, 4) to the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

   **OR**

   **(b)** Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance $2\sqrt{2}$ from (-1, -1, 2).

4. **(a)** For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $(t + \frac{1}{t})^a, t \neq 0.$

   **OR**

   **(b)** Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$.

5. The height of a plant (y cm) related to sunlight exposure (x days) is $y = 4x - \frac{1}{2}x^2$. (i) Find the rate of growth w.r.t. sunlight. (ii) Find the days to maximum height and the maximum height.

6. Show that the area of a parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2} | \vec{a} \times \vec{b} |$. Find the area for diagonals $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

7. **(a)** Evaluate $\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx.$

   **OR**

   **(b)** Find $\int \sqrt{x^2 + 1}[\log(x^2 + 1) - 2\log x]dx.$

8. If A is an invertible $3 \times 3$ matrix, show that $(kA)^{-1} = \frac{1}{k}A^{-1}$ for scalar $k \neq 0$. Hence calculate $(3A)^{-1}$ for a given matrix A.

9. Sketch the region bounded by $y = 1 + | x + 1 |, x = -2, x = 2, y = 0$. Find its area using integration.

10. Three runners' speeds add to 6 km/h. Double the third's speed plus the first's equals 7 km/h. Thrice the first's speed plus the other two's original speeds equals 12 km/h. Find each runner's original speed using the matrix method.

11. **(a)** Evaluate $\int_0^{3/2} | x\cos \pi x | dx.$

    **OR**

    **(b)** Find $\int \frac{dx}{\sin x + \sin 2x}$.

12. Find the equation of a line through (1, 2, -4) perpendicular to two given lines.

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**SECTION E (Case Study - 4 marks each)**

**Case Study 1: Decimal Misconception**

- **Scenario:** Students compare decimals. A test involves identifying who threw a javelin the farthest given distances (e.g., 47.7, 47.07, 43.09, 43.9, 45.2). 40% have a misconception. 80% with misconception answer "Bijoy". 90% without misconception do not answer "Bijoy".
- **Questions:**
    1. Find P(No misconception but answers Bijoy).
    2. Find P(Randomly selected student answers Bijoy).
    3. **(a)** Find P(Student has misconception | answered Bijoy).
       **OR**
       **(b)** Find P(Student does not have misconception | answered Bijoy).

**Case Study 2: Metro Rail Network**

- **Scenario:** An engineer designs two metro lines with given vector equations.
- **Questions:**
    1. Are the two metro tracks parallel?
    2. Find the equation for solar panel placement on Line A's stations (parallel to Line A, through point (1, -2, -3)).
    3. **(a)** Find the equation of a pedestrian pathway perpendicular to both lines passing through (3, 2, 1).
       **OR**
       **(b)** Find the shortest distance between Line A and Line B.

**Case Study 3: Laptop Cooling**

- **Scenario:** A laptop processor cools after use. Rate of cooling: $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$. Initial temp $T(0) = 85°C$.
- **Questions:**
    1. Find the expression for $T(t)$.
    2. Find the time for the temperature to reach $40°C$ (given $k = 0.03, \log_e 4 = 1.3863$).

**Case Study 4: Wooden Box Optimization**

- **Scenario:** A carpenter makes a closed cuboidal box with a square base and fixed volume, aiming for minimum surface area.

- **Questions:**
    1. Express surface area (S) in terms of base side (x) and volume (V).
    2. Find $\frac{dS}{dx}$.
    3. **(a)** Find relation between x and y for minimum S.
       **OR**
       **(b)** If S is constant and $V = \frac{1}{4}(Sx - 2x^3)$, show V is maximum when $x = \sqrt{\frac{S}{6}}$.

**Case Study 5: Student Roll Number Function**

- **Scenario:** In a class of 30, f(x) assigns a student their roll number (a natural number).
- **Questions:**
    1. Is f a bijective function?
    2. Justify your answer.
    3. **(a)** Relation R = {(x, y) | y = 3x}. List elements of R. Check if R is reflexive, symmetric, transitive.
       **OR**
       **(b)** Relation R = {(x, y) | y = $x^3$}. List elements of R. Is R a function? Justify.

**Case Study 6: Seed Germination**

- **Scenario:** A gardener buys 10 brinjal, 12 cabbage, and 8 radish seeds with germination probabilities 25%, 35%, and 40% respectively. They get mixed.
- **Questions:**
    1. Find the probability that a randomly chosen seed germinates.
    2. Find the probability that a seed is cabbage, given that it germinates.