Mathematics-X

Real Numbers

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“Education is not just about passing exams; it is about shaping the person you are becoming.”

Overview

- Number System – Recap
- Real Number – Diagram
- Prime Factorisation
- Fundamental Theorem of Arithmetic
- HCF & LCM
- Special Property of HCF & LCM of Two Numbers

Number System - Recap

- **Natural Numbers (N):** These are counting numbers starting from 1.
  - 👉 Examples: 1, 2, 3, 4, 5, …
  - 👉 Used for counting objects.

- **Whole Numbers (W):** Natural numbers including 0.
  - 👉 Examples: 0, 1, 2, 3, 4, 5, …
  - 👉 No negative numbers.

Number System - Recap

- **Integers (Z):** Whole numbers including negative numbers.
  - Examples: …, −3, −2, −1, 0, 1, 2, 3, …
  - No fractions or decimals.

$\frac{p}{q}; q \neq 0$

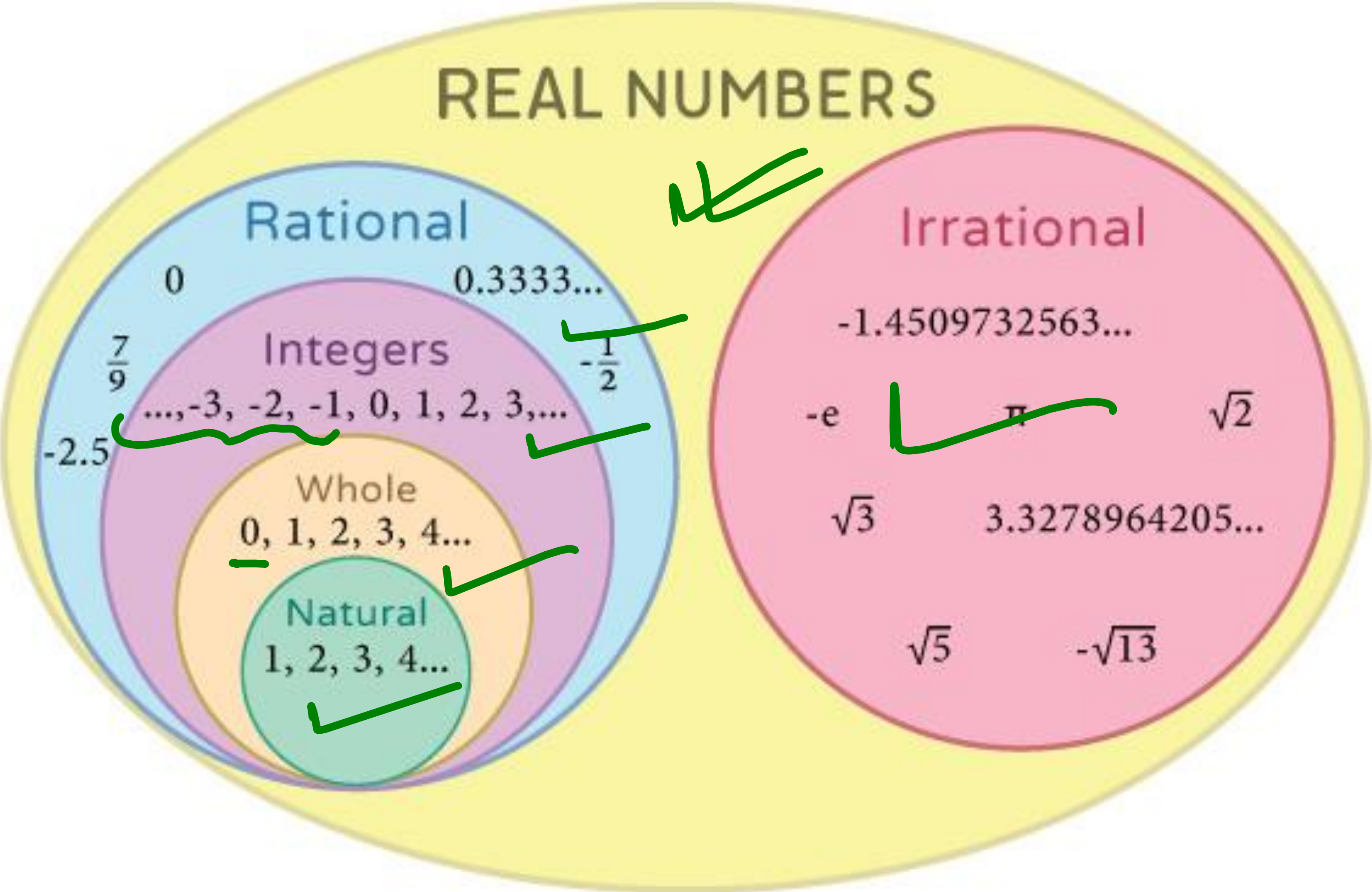
- **Rational Numbers (Q):** Numbers that can be written in the form p/q, where p and q are integers and q ≠ 0.
  - Examples: 1/2, −3/4, 5, 0.75, −2
  - All integers are rational numbers (because 5 = 5/1).

Number System - Recap

- **Irrational Numbers:** Numbers that cannot be written as p/q.
  - 👉 Examples: $\sqrt{2} = 1.414213...$, $\pi = 3.14159...$, $\sqrt{3}$
  - 👉 Decimal expansion never end and never repeat.

- **Real Numbers (R):** All rational + irrational numbers together.
  - 👉 Examples: -5, 0, $3/4$, $\sqrt{2}$, π, 7.8
  - 👉 Any number that can be placed on the number line.

Number System - Diagram



Some More Concepts

- ❑ Even Numbers 2 4 6 8 10 . . . .
- ❑ Odd Numbers 3 7 9 11 . . . .
- ❑ Prime Numbers 3 7 11 13 . . . .
- ❑ Composite Numbers 4 6 8 . . . .

- ❑ Multiples $4 = 4, 8, 12, 16 \ldots$
- ❑ Factors $4 = 1, 2, 4$ only 3
- ❑ Squares $4 = 16$, $5 = 25$
- ❑ Cubes $4 = 64$, $5 = 125$

9

→ odd
→ composite
→ 9, 18, 27 . . .
→ 9 = 1, 3, 9
→ $9^2 = 81$
= 9 = 729

Prime Factorisation

- **Definition:** Prime factorisation is the process of expressing a number as the product of prime numbers only.

- **Division Method:**

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$12 = 2 \times 2 \times 3$$
$$= 2^4 \times 3$$

$$\begin{array}{r|l} 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline & 7 \end{array}$$

$$56 = 2 \times 2 \times 2 \times 7$$
$$= 2^3 \times 7$$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$16 = 2 \times 2 \times 2 \times 2$$
$$= 2^4$$

Fundamental Theorem Of Arithmetic

- Statement: Every composite number can be uniquely expressed as a product of primes, except for the order in which these prime factors occurs.

$12 = 2 \times 2 \times 3$

$69 = 2 \times 23$

$75 = 3 \times 5 \times 5$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

$2 \times 2 \times 3 = 12$

$\cancel{2 \times 3 \times 2}$

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

so,

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$= 2^3 \times 3^2$

unique

HCF & LCM

HCF: Highest Common Factor

Example: HCF of 6 and 8

$6 \Rightarrow 1, 2, 3, 6$

$8 \Rightarrow 1, 2, 4, 8$

$HCF = 2$

LCM: Lowest Common Multiple

Example: LCM of 6 and 8

$6 \Rightarrow 6, 12, 18, 24, 30, 36, 42, 48 \ldots$

$8 \Rightarrow 8, 16, 24, 32, 40, 48, 56 \ldots$

$LCM = 24$

HCF & LCM By Prime Factorisation

Question: Find HCF and LCM of 126 and 156 by prime factorisation.

Ans:

$$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$$

$$156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$$

HCF(126, 156) = Product of common terms with lowest power

$= 2^1 \times 3^1 = 2 \times 3 = 6$

LCM(126, 156) = Product of prime factors with highest power

$= 2^2 \times 3^2 \times 7 \times 13 = 4 \times 9 \times 7 \times 13 = 3276$

Q 01: Find HCF and LCM of 24 and 36 by prime factorisation.

Ans:

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$HCF = 2^2 \times 3 = 4 \times 3 = 12$ (Lowest)

$LCM = 2^3 \times 3^2 = 8 \times 9 = 72$ (highest) ✓

Q 02: Find HCF and LCM of 45 and 75 by prime factorisation.

Ans:

$45 = 3 \times 3 \times 5 = 3^2 \times 5$

$75 = 3 \times 5 \times 5 = 3 \times 5^2$

$$\begin{array}{r|l} 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$HCF = 3 \times 5 = 15$

$LCM = 3^2 \times 5^2 = 9 \times 25 = 225$ Ans

Q 03: Find HCF and LCM of 18 and 30 by prime factorisation.

Ans:

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$30 = 2 \times 3 \times 5 = 2 \times 3 \times 5$

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$HCF = 2 \times 3 = 6$

$LCM = 2 \times 3^2 \times 5 = 2 \times 9 \times 5$

$= 90$ Ans

Q 04: Find HCF and LCM of 96 and 144 by prime factorisation.

Ans:

$96 = 2^5 \times 3$

$144 = 2^4 \times 3^2$

HCF $= 2^4 \times 3 = 48$

LCM $= 2^5 \times 3^2 = 32 \times 9$

$= 288$ Ans

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Q 05: Find HCF and LCM of 108 and 180 by prime factorisation.

Ans:

$108 = 2^2 \times 3^3$

$180 = 2^2 \times 3^2 \times 5$

$HCF = 2^2 \times 3^2 = 4 \times 9 = 36$

$LCM = 2^2 \times 3^3 \times 5$

$= 4 \times 27 \times 5$

$= 4 \times 5 \times 27 = 20 \times 27$

$= 540$

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Special Property of HCF & LCM

Product of two given numbers = product of their HCF and LCM.

Thus,

$$(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{HCF}(\boldsymbol{a}, \boldsymbol{b}) \times \boldsymbol{LCM}(\boldsymbol{a}, \boldsymbol{b})$$

CAUTION: The above result is true for two numbers only.

Q 06: The HCF of two numbers is 6 and their LCM is 180. If one number is 30, find the other number.

Ans:

Let the numbers be x and y.

then, $HCF(x, y) = 6$

$LCM(x, y) = 180$

Now, $x \times y = HCF \times LCM$

$$y = \frac{6 \times \cancel{180}^{6}}{\cancel{30}} = 6 \times 6$$

$= 36$ Ans

Q 07: The HCF of two numbers is 8 and their LCM is 96. If one number is 24, find the other number.

Ans: HCF = 8

LCM = 96

$x \times y = HCF \times LCM$

$24 \times y = 8 \times 96$

$y = \frac{8 \times \cancel{96}^{\cancel{16}^{4}}}{\cancel{24}_{4}} = 8 \times 4 = 32$ Ans

Q 08: The HCF of two numbers is 5 and their LCM is 225.If one number is 25, find the other number.

Ans:

Try it yourself!

Q 09: The product of two numbers is 1728 and their HCF is 12. Find their LCM.

Ans:

$x \times y = HCF \times LCM$

$1728 = 12 \times LCM$

$\frac{1728}{12} = LCM$

$\text{or, } LCM = \frac{\cancel{1728}^{144}}{\cancel{12}_{1}} = 144$ Ans

Q 10: The HCF of two numbers is 21 and their product is 4851.Find their LCM.

Ans:

$x \times y = LCM \times HCF$

$4851 = LCM \times 21$

so, $LCM = \dfrac{\cancel{4851}^{\cancel{693}^{231}}}{\cancel{21}_{\cancel{3}_{1}}} = 231$ Ans

RS EX09: Find the HCF and LCM of 108, 120 and 252 using prime factorisation

Ans:

$108 = 2^2 \times 3^3$

$120 = 2^3 \times 3 \times 5$

$252 = 2^2 \times 3^2 \times 7$

$HCF = 2^2 \times 3 = 4 \times 3 = 12$

$LCM = 2^3 \times 3^3 \times 5 \times 7$

$= 8 \times 27 \times 5 \times 7$

$= 8 \times 5 \times 7 \times 27$

$= 280 \times 27$

$= 7560$ Ans.

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

28×27

560
140
56

756

RS EX10: Find the largest number which divides 245 and 1037, leaving remainder 5 in each case.

Ans:

$245 \rightarrow 245 - 5 = 240$

$1037 \rightarrow 1037 - 5 = 1032$

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 1032 \\ \hline 2 & 516 \\ \hline 2 & 258 \\ \hline 3 & 129 \\ \hline & 43 \end{array}$$

$240 = 2^4 \times 3 \times 5$

$1032 = 2^3 \times 3 \times 43$

$HCF = 2^3 \times 3$

$= 8 \times 3$

$= 24$ Ans

RS EX11: Find the largest number which divides 129 and 545, leaving remainders 3 and 5 respectively.

Ans:

$129 \rightarrow 129 - 3 = 126$ ✓

$545 \rightarrow 545 - 5 = 540$ ✓

$$\begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$126 = 2 \times 3^2 \times 7$

$540 = 2^2 \times 3^3 \times 5$

$HCF = 2 \times 3^2 = 2 \times 9 = 18$ Ans

Key Takeaways

- ❑ Different Types of Numbers
- ❑ Prime Factorisation
- ❑ Fundamental Theorem of Arithmetic
- ❑ HCF & LCM
- ❑ Special Property of HCF & LCM of Two Numbers

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