$\overline{XI}$ - ③  Trigonometric Proofs

$\cos(x+y) = \cos x \cos y - \sin x \sin y.$

$\Delta P_1 O P_3 = \Delta P_2 O P_4$

$P_1 P_3 = P_2 P_4$

P₁ (cos x, sin x)

$P_1 P_3^2 = \left(\cos x - \cos(-y)\right)^2 + \left(\sin x - \sin(-y)\right)^2$

$= \cos^2 x + \cos^2(-y).$

$- 2\cos x \cos(-y) +$

$\sin^2 x + \sin^2(-y) - 2\sin x \sin(-y)$

$P_2\ (\cos(x+y),$
$\sin(x+y))$
$P_3\ (\cos(-y),$
$\sin(-y))$

$= 2 - 2\cos x \cos y + 2 \sin x \sin y.$

$= 2 - 2\left(\cos x \cos y - \sin x \sin y\right)$

P₄ (1, 0)

$P_2 P_4^2 = \left(\ (\cos(x+y) - 1)^2 + (\sin(x+y) - 0)^2\ \right)$

$= \cos^2(x+y) + 1 - 2\cos(x+y) + \sin^2(x+y)$

$= 2 - 2\cos(x+y).$

$P_1 P_3 = P_2 P_4 \implies P_1 P_3^2 = P_2 P_4^2$

$\boxed{\therefore \cos(x+y) = \cos x \cos y - \sin x \sin y.}$ —①

$\cos(x-y) = \cos x \cos y + \sin x \sin y.$

Replacing $y$ by $-y$ in $\cos(x+y).$ eq.①

$\cos(x+y) = \cos x \cos y - \sin x \sin y.$

$\cos(x-y) = \cos x \cos(-y) - \sin x \sin(-y)$

$\boxed{\cos(x-y) = \cos x \cos y + \sin x \sin y.}$ —②

$\cos\left(\frac{\pi}{2} - x\right) = \sin x.$

Replace $x$ by $\frac{\pi}{2}$ ; $y$ by $x$ in ②

$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$\boxed{\cos\left(\frac{\pi}{2} - x\right) = \sin x}$ —③