CBSE CLASS 12 MATHEMATICS - FORMULAE AND CONCEPTS

Chapter 1: Relations and Functions

I. Prerequisites (Class 11)

- **Cartesian Product:** $A \times B = \{(a, b): a \in A, b \in B\}$.
- **Relation:** A subset of $A \times B$.
- **Domain:** Set of all first elements of the ordered pairs in a relation.
- **Range:** Set of all second elements of the ordered pairs in a relation.

II. Types of Relations For a relation R in a set A:

1. **Reflexive:** $(a, a) \in R, \forall a \in A$.
2. **Symmetric:** If $(a, b) \in R$, then $(b, a) \in R$.
3. **Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.
4. **Equivalence Relation:** A relation that is Reflexive, Symmetric, and Transitive.

III. Types of Functions

1. **One-One (Injective):** A function $f: X \to Y$ is one-one if $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$.
2. **Onto (Surjective):** A function $f: X \to Y$ is onto if for every $y \in Y$, there exists $x \in X$ such that $f(x) = y$. (Range = Codomain).
3. **Bijective:** A function that is both One-One and Onto.

IV. Composition and Invertible Functions

- **Composition:** $(f \circ g)(x) = f(g(x))$.
- **Invertible Function:** A function $f: X \to Y$ is invertible if there exists $g: Y \to X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$.
- **Condition:** A function is invertible if and only if it is **Bijective**.

Chapter 2: Inverse Trigonometric Functions

I. Prerequisites (Class 11 Trigonometry)

- $\sin(2\pi \pm \theta) = \pm\sin\theta$, etc.
- $\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$

II. Domain and Range (Principal Value Branch)

Function	Domain	Range (Principal Branch)
$y = \sin^{-1} x$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Function	Domain	Range (Principal Branch)
$y = \cos^{-1}x$	$[-1,1]$	$[0, \pi]$
$y = \mathrm{cosec}^{-1}x$	$R - (-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$R - (-1,1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1}x$	R	$(0, \pi)$

III. Properties of Inverse Trigonometric Functions

1. $\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2. $\sin^{-1}(-x) = -\sin^{-1}x$
   - $\tan^{-1}(-x) = -\tan^{-1}x$
   - $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}x$
3. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
   - $\sec^{-1}(-x) = \pi - \sec^{-1}x$
   - $\cot^{-1}(-x) = \pi - \cot^{-1}x$
4. **Complementary Identity:** $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1,1]$
   - $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in R$
   - $\sec^{-1}x + \mathrm{cosec}^{-1}x = \frac{\pi}{2}$, $|x| \geq 1$
5. **Sum and Difference (Formulae for** $\tan^{-1}x$**):**
   - $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$
   - $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$
6. **Conversion:**
   - $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x| \leq 1$
   - $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $x \geq 0$
   - $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $-1 < x < 1$

Chapter 3: Matrices

I. Matrix Operations

- **Addition:** Defined only for matrices of the same order. $A + B = [a_{ij} + b_{ij}]$.
- **Scalar Multiplication:** $kA = [ka_{ij}]$.
- **Multiplication:** If $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$, then $C = AB$ is of order $m \times p$.
   - $c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$
   - **Note:** $AB \neq BA$ usually.

II. Transpose of a Matrix

- $(A^T)_{ij} = A_{ji}$
- $(A^T)^T = A$

- $(kA)^T = kA^T$
- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$ (Reversal Law)

III. Special Matrices

1. **Symmetric:** $A^T = A$
2. **Skew-Symmetric:** $A^T = -A$ (Diagonal elements are zero).
3. **Decomposition:** Any square matrix A can be expressed as sum of symmetric and skew-symmetric matrices:

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

Chapter 4: Determinants

I. Properties and Expansion

- For order 2: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.
- $\det(AB) = \det(A) \cdot \det(B)$.
- $\det(kA) = k^n \det(A)$ where n is the order of matrix A.
- $\det(A^T) = \det(A)$.

II. Area of Triangle Area $= \frac{1}{2}\left|\det\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}\right|$. If Area = 0, points are collinear.

III. Adjoint and Inverse

1. **Minor (M_{ij}):** Determinant obtained by deleting i-th row and j-th column.
2. **Cofactor (A_{ij}):** $A_{ij} = (-1)^{i+j} M_{ij}$.
3. **Adjoint:** Transpose of Cofactor Matrix. adj $A = \left[A_{ij}\right]^T$.
4. **Properties of Adjoint:**
   - $A(\text{adj } A) = (\text{adj } A)A = |A|I$
   - $|\text{adj } A| = |A|^{n-1}$
5. **Inverse:** $A^{-1} = \frac{1}{|A|}(\text{adj } A)$ (Exists only if $|A| \neq 0$, i.e., Non-singular).

IV. Solving System of Linear Equations

- System: $AX = B$
- Solution: $X = A^{-1}B$
- **Consistency Check:**
   - $|A| \neq 0$: Unique solution (Consistent).
   - $|A| = 0$ and $(\text{adj } A)B \neq O$: No solution (Inconsistent).
   - $|A| = 0$ and $(\text{adj } A)B = O$: Infinite solutions or no solution.

Chapter 5: Continuity and Differentiability

I. Prerequisites (Class 11 Limits)

- $\lim_{x\to 0}\frac{\sin x}{x} = 1$
- $\lim_{x\to 0}\frac{e^x-1}{x} = 1$
- $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

II. Continuity A function $f(x)$ is continuous at $x = c$ if:

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

III. Differentiability $f(x)$ is differentiable at c if Left Hand Derivative (LHD) = Right Hand Derivative (RHD):

$$\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h} = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

IV. Derivative Formulae

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(\sin x) = \cos x$
3. $\frac{d}{dx}(\cos x) = -\sin x$
4. $\frac{d}{dx}(\tan x) = \sec^2 x$
5. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$
6. $\frac{d}{dx}(\sec x) = \sec x \tan x$
7. $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$
8. $\frac{d}{dx}(e^x) = e^x$
9. $\frac{d}{dx}(\log x) = \frac{1}{x}$
10. $\frac{d}{dx}(a^x) = a^x \log a$
11. **Inverse Trig Derivatives:**
    - $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
    - $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$
    - $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

V. Rules of Differentiation

1. **Product Rule:** $(uv)' = u'v + uv'$
2. **Quotient Rule:** $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$
3. **Chain Rule:** $\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$
4. **Parametric Form:** If $x = f(t), y = g(t)$, then $\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$.
5. **Logarithmic Differentiation:** Used for $y = [u(x)]^{v(x)}$. Take log on both sides: $\log y = v(x)\log u(x)$.

Chapter 6: Applications of Derivatives (AOD)

[Image of increasing and decreasing functions graph]

I. Rate of Change Rate of change of y w.r.t x is $\frac{dy}{dx}|_{x=x_0}$.

II. Increasing and Decreasing Functions

1. **Strictly Increasing:** $f'(x) > 0$ for all $x \in (a, b)$.
2. **Strictly Decreasing:** $f'(x) < 0$ for all $x \in (a, b)$.
3. **Constant:** $f'(x) = 0$.

III. Maxima and Minima

1. **Critical Point:** Points where $f'(x) = 0$ or $f'(x)$ is undefined.
2. **First Derivative Test:**
    - $f'(x)$ changes sign from $+$ to $- \rightarrow$ Local Maxima.
    - $f'(x)$ changes sign from $-$ to $+ \rightarrow$ Local Minima.
3. **Second Derivative Test (Standard Method):**
    - Find c such that $f'(c) = 0$.
    - If $f''(c) < 0 \rightarrow$ Local Maxima.
    - If $f''(c) > 0 \rightarrow$ Local Minima.
    - If $f''(c) = 0 \rightarrow$ Test fails (use First Derivative Test).

Chapter 7: Integrals

I. Standard Integrals

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
2. $\int \frac{1}{x} dx = \log|x| + C$
3. $\int e^x dx = e^x + C$
4. $\int a^x dx = \frac{a^x}{\log a} + C$
5. $\int \sin x dx = -\cos x + C$
6. $\int \cos x dx = \sin x + C$
7. $\int \sec^2 x dx = \tan x + C$
8. $\int \operatorname{cosec}^2 x dx = -\cot x + C$
9. $\int \sec x \tan x dx = \sec x + C$
10. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
11. $\int \tan x dx = \log|\sec x| + C$
12. $\int \cot x dx = \log|\sin x| + C$
13. $\int \sec x dx = \log|\sec x + \tan x| + C$
14. $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + C$

II. Special Integrals

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

3. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
1. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
2. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
3. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

III. Integration by Parts $\int uv\,dx = u\int v\,dx - \int \left(\frac{du}{dx}\int v\,dx\right)dx$

- **ILATE Rule** for priority of u: **I**nverse, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential.
- **Special Form:** $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$

IV. Properties of Definite Integrals

1. P_0: $\int_a^b f(x)dx = \int_a^b f(t)dt$
2. P_1: $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. P_2: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
4. P_3: $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
5. P_4 **(King's Prop):** $\int_0^a f(x)dx = \int_0^a f(a - x)dx$
6. P_5: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - x)dx$
7. P_6: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a - x) = f(x)$, else 0.
8. P_7: $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if Even function ($f(-x) = f(x)$); 0 if Odd function ($f(-x) = -f(x)$).

Chapter 8: Applications of Integrals

[Image of area under curve integration]

I. Area under Simple Curves

1. Area bounded by curve $y = f(x)$, x-axis, $x = a$ and $x = b$:

$$A = \int_a^b |y|\,dx$$

2. Area bounded by curve $x = g(y)$, y-axis, $y = c$ and $y = d$:

$$A = \int_c^d |x|\,dy$$

II. Area Between Two Curves Area between $y = f(x)$ and $y = g(x)$:

$$A = \int_a^b |f(x) - g(x)|\,dx$$

Chapter 9: Differential Equations

I. Order and Degree

- **Order:** Highest order derivative appearing in the equation.

- **Degree:** Power of the highest order derivative (defined only when polynomial in derivatives).

II. Variable Separable Method If $f(x)dx + g(y)dy = 0$, integrate both sides: $\int f(x)dx + \int g(y)dy = C$.

III. Homogeneous Differential Equations Form: $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$.

- Step 1: Put $y = vx \Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.
- Step 2: Solve using Variable Separable.

IV. Linear Differential Equations

1. **Form 1:** $\frac{dy}{dx} + Py = Q$ (where P, Q are functions of x)
    - **Integrating Factor (I.F.):** $e^{\int P dx}$
    - **Solution:** $y(\text{I.F.}) = \int Q(\text{I.F.})dx + C$
2. **Form 2:** $\frac{dx}{dy} + Px = Q$ (where P, Q are functions of y)
    - **Integrating Factor (I.F.):** $e^{\int P dy}$
    - **Solution:** $x(\text{I.F.}) = \int Q(\text{I.F.})dy + C$

Chapter 10: Vector Algebra

I. Basic Formulae

- **Unit Vector:** $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- **Position Vector:** $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$; Magnitude $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.
- **Direction Cosines:** $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.
    - $l = \frac{x}{|\vec{r}|}, m = \frac{y}{|\vec{r}|}, n = \frac{z}{|\vec{r}|}$
    - $l^2 + m^2 + n^2 = 1$

II. Dot Product (Scalar Product) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

- Properties: $\hat{\imath} \cdot \hat{\imath} = 1$, $\hat{\imath} \cdot \hat{\jmath} = 0$.
- **Projection of $\vec{a}$ on $\vec{b}$:** $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

[Image of vector projection diagram]

III. Cross Product (Vector Product) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$

- Properties: $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.
- **Area of Triangle:** $\frac{1}{2}|\vec{a} \times \vec{b}|$ (adjacent sides $\vec{a}, \vec{b}$).
- **Area of Parallelogram:** $|\vec{a} \times \vec{b}|$.

Chapter 11: Three Dimensional Geometry

I. Direction Cosines and Line Equations

- **Line through point $\vec{a}$ parallel to $\vec{b}$:**
    - Vector Form: $\vec{r} = \vec{a} + \lambda\vec{b}$

- Cartesian Form: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
- **Line through two points** $A(x_1, y_1, z_1)$ **and** $B(x_2, y_2, z_2)$**:**
  - $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

II. Shortest Distance between Two Lines

1. **Skew Lines:** $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

2. **Parallel Lines:** $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

(Note: Plane equations have been removed from the rationalized CBSE 2023-24 syllabus).

Chapter 12: Linear Programming

I. Concept

- **Objective Function:** $Z = ax + by$ (to be maximized or minimized).
- **Constraints:** Inequalities like $Ax + By \leq C$.

II. Corner Point Method

1. Plot the graph of constraints and identify the **Feasible Region**.
2. Determine the vertices (Corner Points) of the region.
3. Evaluate Z at each corner point.
4. Max/Min value of Z occurs at these points.
5. *Note:* If the region is unbounded, verification is required (plot $ax + by > M$ for max or $ax + by < m$ for min).

Chapter 13: Probability

I. Prerequisites

- $P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Mutually Exclusive:** $A \cap B = \phi \Longrightarrow P(A \cap B) = 0$.

II. Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

III. Multiplication Theorem $P(A \cap B) = P(A) \cdot P(B|A)$

IV. Independent Events Two events are independent if $P(A \cap B) = P(A) \cdot P(B)$.

[Image of Bayes theorem tree diagram]

V. Total Probability and Bayes' Theorem Let $E_1, E_2, \dots, E_n$ be a partition of sample space S.

1. **Total Probability:**

$$P(A) = \sum_{j=1}^{n} P\left(E_j\right) P\left(A|E_j\right)$$

2. **Bayes' Theorem:**

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P\left(E_j\right) P\left(A|E_j\right)}$$