Sequence and Series

Que 1: The Maharaja of Mysore was highly fond of chess. Two artisans, Dodda and Chikka, designed a large chessboard for him with king-size chessmen. The Maharaja was pleased and decided to award them with grains of gold. He asked them how many grains they would like to have. Dodda said he would like 100 golden grains on the first square of the board and in every consecutive square, 50 grains more than in the previous. Chikka said that he wanted only one grain in the first square, but in every consecutive square, he wanted twice as many grains as in the previous.

a) How many grains will Dodda get in the 10th square?
b) In how many squares will Chikka get more grains than Dodda?
c) How many grains will Chikka get in the last square?
d) How many grains will Dodda get in all?

Que 2: Consider the series 1, 4, 7, 10, 13...

a) What will be the 100th number (term) in the series?
b) Will the number 172 be there in this series? If yes, what will be its position?
c) What will be the sum of the first 100 terms in the series?
d) What will be the difference between the 100th and 102nd terms?

Que 3: Consider the series 2, 6, 18, 54...

a) What will be the 50th term in the series?
1) 2×3^{49}
2) 2×3^{50}
3) 2×3^{51}
4) 3^{50}

b) Will the number 1008 be there in this series? If yes, what will be its position?
1) No
2) Yes, 336th
3) Yes, 168th
4) Yes, 56th

c) What will be the sum of the first 10 terms of this series?
1) 59048
2) 29524
3) 118098
4) 19682

Que 4: What will be the sum of infinite terms of the series: 1, 1/2, 1/4, 1/8 .....?

Que 5: Find the sum of 3/4 + 15/16 + 63/64 ..... up to 'n' terms

Que 6: Find the next term in the following series:

a) 4, 7, 13, 25, 49, 97, …….
b) 5, 8, 13, 21, 34, 55, …….
c) 3, 6, 10, 15, 21, 28, …….

Que 7: Find the expression for the nth term (Tn) of the following series:
a) 15, 22, 29, 36, 43,...
b) 5, 7, 11, 19, 35,....
c) 3, 6, 10, 15, 21, 28...

Que 8: Find the sum of the first ten terms of the following series:
a) 11, 12, 13, 14, 15....
b) 36, 49, 64, 81, 100...
c) 3, 6, 10, 15, 21, 28...

Que 9: Find the sum of $1/4 + 1/4^2 + 1/4^3 +...$

Que 10: In a bacterial culture, the number of bacteria doubles after every minute. If a dish is full of bacteria exactly after 10 minutes, when was it half full?

Que 11: Marbles are arranged in the shape of a regular tetrahedron such that the marble at the top rests on 3 marbles, which further rest on 6 marbles, and so on.
a) If there are 120 marbles in all, how many layers of marbles are there?
b) If there are 120 marbles in the last row, how many layers of marbles are there?

Sequences, Series, and Progressions: Examples

Que 1: A certain sequence is defined by the recursive relation $T_{n+1} = 2T_n + 3$
a) If $T_1 = 2$ find T_5
b) If $T_5 = 125$ find T_1
c) If $T_1 = 3$ and $T_{11} = 6141$ find $T_2 + T_3 + T_4 +...+T_{10}$

Que 2: If in an Arithmetic Progression with general term a_n, it is known that $a_7 + a_9 = a_3 + a_4 + a_{10}$, then what is the first term of the AP?

Que 3: A group of 630 children is arranged in rows for a group photograph session. Each row contains three fewer children than the row in front of it. What number of rows is not possible?
1) 3
2) 4
3) 5
4) 6
5) 7

Que 4: A tennis ball is initially dropped from a height of 180 m. After striking the ground, it rebounds (3/5)th of the height from which it has fallen. The total distance that the ball travels before it comes to rest is
1) 540 m
2) 600 m

3) 720 m
4) 900 m

Que 5: If the sum 3 + 3/2 + 3/4 + 3/8 + 3/16 + … to n terms is 3069/512, find the value of n.
1) 5
2) 10
3) 15
4) 20

Que 6: Find the last digit of the 200-digit number 122333444455555......

Que 7: Calculate sum of 1/(1×2) + 1/(2×3) + 1/(3×4) + 1/(4×5) +...+ 1/(99×100)

Que 8: 1/(√2 + √1) + 1/(√3 + √2) + 1/(√4 + √3) +...+ 1/(√100 + √99) equals

Que 9: The inverse of the sum of the following series up to n terms, 3/4 + 5/36 + 7/144 +.... can be written as

1) $\dfrac{(n-1)^2}{n^2+2n}$

2) $\dfrac{n^2+2n}{(n-1)^2}$

3) $\dfrac{n^2+2n}{(n+1)^2}$

4) $\dfrac{(n+1)^2}{n^2+2n}$

Que 10: The sum of the series $1^2 - 2^2 + 3^2 - 4^2 +...+2001^2 - 2002^2 + 2003^2$ is
1) 2007006
2) 1005004
3) 200506
4) None of the above

Que 11: A child was asked to add the first few natural numbers (i.e., 1 + 2 + 3 +...) so long as his patience permitted. When he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered that he had missed one number in the sequence during addition. The number he missed was
1) 10
2) 18
3) 20
4) None of the above

Que 12: If the sum of the first 5 terms of an Arithmetic Progression is equal to the sum of the first 10 terms, then which term of the AP must necessarily be zero?
1) 6th
2) 8th
3) 10th

4) 9th

Que 13: A square is drawn by joining the midpoints of the sides of a given square. A third square is drawn inside the second square in the same way, and this process is continued indefinitely. If a side of the first square is 8 cm, the sum of the areas of all the squares formed (in sq. cm) is
1) 128
2) 120
3) 96
4) None of these

Que 14: In an Arithmetic Progression, the 8th, 12th, and 17th terms are in Geometric Progression. What is the ratio of the first and tenth terms?

Que 15: What is the sum of the series 1/3 + 1/8 + 1/15 + 1/24 + 1/35 +.... to 10 terms?
1) 175/132
2) 36/55
3) 175/264
4) 209/312

Que 16: Two men, X and Y, started working for a certain company at similar jobs on January 1, 1950. X asked for an initial monthly salary of Rs. 300 with an annual increment of Rs. 30 on the monthly salary. Y asked for an initial monthly salary of Rs. 200 with a rise of Rs.15 on a monthly salary every 6 months. Assume that the arrangements remained unaltered till December 31, 1959. Salary is paid on the last day of the month. What is the total amount paid to them as salary during the period?
1) Rs. 93,300
2) Rs. 93,200
3) Rs. 93,100
4) None of the above

Que 17: The infinite sum $1 + 4/7 + 9/7^2 + 16/7^3 + 25/7^4 + \ldots$ equals
1) 27/14
2) 21/13
3) 49/27
4) 256/147

Challengers

Que 1: Consider a sequence where the nth term, $t_n = n/(n + 2)$, n = 1, 2, … The value of $t_3 \times t_4 \times t_5 \times \ldots \ldots \times t_{53}$ equals:
1) 2/495
2) 2/477
3) 12/55
4) 1/1485
5) 1/2970

Que 2: If $a_1 = 1$, $a_{n+1} = 2a_n + 5$, n = 1, 2, ... then a_{100} is equal to
1) $5 \times 2^{99} - 6$
2) $5 \times 2^{99} + 6$
3) $6 \times 2^{99} + 5$
4) $6 \times 2^{99} - 5$

Que 3: Given that $1/1^2 + 1/2^2 + 1/3^2 + 1/4^2 + ... = \pi/6$ find the value of $1/1^2 + 1/3^2 + 1/5^2 + 1/7^2 + ...$
1) $\pi/8$
2) $\pi/4$
3) $\pi/10$
4) $\pi/12$

Que 4: There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is
1) 34
2) 38
3) 36
4) 32

Que 5: There is a series of terms such that $T_1 = 1$, $T_2 = 5 + 5^2$, $T_3 = 5^3 + 5^4 + 5^5$, and so on. Find T_{10}
1) $\dfrac{5^{54}\left(5^{10}-1\right)}{4}$
2) $\dfrac{5^{45}\left(5^{9}-1\right)}{4}$
3) $\dfrac{5^{45}\left(5^{10}-1\right)}{4}$
4) $\dfrac{5^{44}\left(5^{10}-1\right)}{4}$

Practice Exercise

Directions for questions 1 to 4: Choose the correct alternative.

Que 1: Five numbers are in geometric progression such that their product is 1024. Find the third number.
1) 1
2) 4
3) 16
4) 2

Que 2: A ball dropped from a height of 24 m rebounds two-thirds of the distance it falls. How much distance will it travel before coming to rest?
1) 240 m
2) 60 m

3) 120 m
4) Cannot be determined

Que 3: The three angles of a triangle are in GP. What type of triangle is it if one of its interior angles is 60°?
1) Equilateral triangle
2) Isosceles triangle
3) Scalene triangle
4) Data insufficient

Que 4: a, b, c are in Arithmetic Progression with a common difference of d. Which of the following is certainly TRUE (select the best option):
1) At least one of a, b, c, d is divisible by 3
2) At most one of a, b, c, d is divisible by 3
3) Either all or none of a, b, c, d is divisible by 3
4) None of the above

Que 5: The sum of the first 10 terms of an AP is equal to the sum of the first 15 terms.
a) If the sum of the first 'k' terms is 0, find the value of 'k'.
b) If the 25th term is 10, find the first term and the common difference of the A.P.

Que 6: Find the sum to infinity of a decreasing GP with the common ratio x such that $|x| < 1$; $x \neq 0$. The ratio of the fourth term to the second term is 1/16, and the ratio of the third term to the square of the second term is 1/9.
1) 6
2) 12
3) 18
4) None of these

Que 7: A series has terms of the form a^b where a is the base and b is the exponent. The bases in the series increase as a GP, while the exponents increase as an AP. Another series is constructed from this series as follows.
If T_1, T_2, T_3,.....,T_r ... are the terms of the given series.
The new series is T_2/T_1, T_3/T_2, T_4/T_3, … etc
Thus, the new series will be a:
1) AP
2) GP
3) HP
4) None of these

Que 8: For a generalized Fibonacci sequence, which of the following statement/s is/are true? Fibonacci sequence is 1, 1, 2, 3, 5, …

I. $\sum_{k=1}^{n} a_k = a_{n+2} - a_2$

II. $a_n^2 = (a_{n-1} \times a_{n+1}) + a_2$ (Where a_n is the nth term of the Fibonacci sequence)

1) Only I

2) Only II
3) Both I and II
4) None of these

Que 9: The sum of the reciprocal of the product of first three natural numbers, the reciprocal of the product of three consecutive natural numbers starting with 2, the reciprocal of the product of next three consecutive natural numbers starting with 3 and so on till the reciprocal of the product of $(n - 2)^{th}$, $(n - 1)^{th}$, and n^{th} natural numbers will be:

1) $\dfrac{n^2-n-2}{4(n-1)n}$

2) $\dfrac{n-1}{(n-2)n}$

3) $\dfrac{4n}{n(n-1)}$

4) $\dfrac{n(n+1)}{2}$

Que 10: A sequence, a_n, is defined recursively as $a_1 = 2$; $a_{n+1} = a_n + 2n$ $(n \geq 1)$. Find a_{100}.
1) 9900
2) 9902
3) 9904
4) 9906

Que 11: Sum of the series: 1 + 2 + 7/4 + 5/4 + 13/16 + 16/32 + 19/64 +..... is:
1) 8
2) 10
3) 9
4) ∞

Que 12: If the product of z positive real numbers is unity, then their sum is necessarily:
1) a multiple of z
2) equal to z + 1/z
3) never less than z
4) a positive integer

Que 13: Consider three numbers in GP such that the middle number is 36. If A = Sum of the three numbers and B = Sum of the products of two numbers taken at a time, then find the relation between A and B.
1) $B = 36^2 A$
2) $B = 6A$
3) $B = 36A$
4) $A = 36B$

Que 14: A body dropped from an airplane falls 10 meters in the first second of its motion, 15 meters in the second, 20 meters in the third, 25 meters in the fourth, and so on. Find the distance travelled by the body if it hits the Earth in 120 seconds.
1) 73.8 km
2) 40 km

3) 36.9 km
4) 1428 km

Que 15: The price of an X-ray machine is Rs.3,00,000. If the machine is bought in monthly installments in a period of 3 years with Rs.500 as the first installment and increasing the value of the installment by Rs.500 every succeeding month, find the extra amount paid by the buyer in the installment scheme.
1) Rs.3000
2) Rs.3300
3) Rs.30000
4) Rs.33000

Que 16: The sum of n terms of a sequence is $4n^2 + 7n$. What is the sum of the 6th, 7th, and 8th terms of the sequence?

Que 17: The kth term of an AP is given by the formula $T_k = 2016 - 23k$. Find the smallest value of n for which S_n, the sum of the first n terms, is negative.

Que 18: The fourth term of an arithmetic progression is 8. What is the sum of the first 7 terms of the arithmetic progression?
1) 7
2) 64
3) 56
4) Indeterminate

Que 19: If $1/(y + z)$, $1/(z + x)$, and $1/(x + y)$ are in A.P., which of the following terms are also in A.P.?
1) x, y, z
2) x^2, y^2, z^2
3) $1/(y - z)$, $1/(z - x)$, $1/(x - y)$
4) None of these

Que 20: What is the sum of the following series:
$1/(1×2) + 1/(2×3) + 1/(3×4) +....+ 1/(100×101)$?
1) 99/100
2) 1/100
3) 100/101
4) 101/102

Answer Key:
1-a-550 b-54 c-2^{63} d-107200, 2-a-298 b-58th c-14950 d-6, 3-a-1 b-1 c-1, 4-2, 5-n-1/3(1-1/4^n),
6-a-193 b-89 c-36, 7-a-7n+8 b-2^n+3 c-½(n+1)(n+2), 8-a-155 b-1185 c-285, 9-1/3, 10-9 min,
11-a-8 b-15

Sequence, Series & Progression Examples: 1-a-77 b-5 c-6105, 2-0, 3-4, 4-3, 5-2, 6-1,
7-99/100, 8-9, 9-4, 10-1, 11-4, 12-2, 13-1, 14-1/2, 15-3, 16-1, 17-3

Challengers: 1-1, 2-4, 3-1, 4-3, 5-3

Practice Exercise: 1-2, 2-3, 3-1, 4-1, 5-25, 6-2, 7-2, 8-1, 9-1, 10-2, 11-1, 12-3, 13-3, 14-3, 15-4, 16-177, 17-175, 18-3, 19-2, 20-3