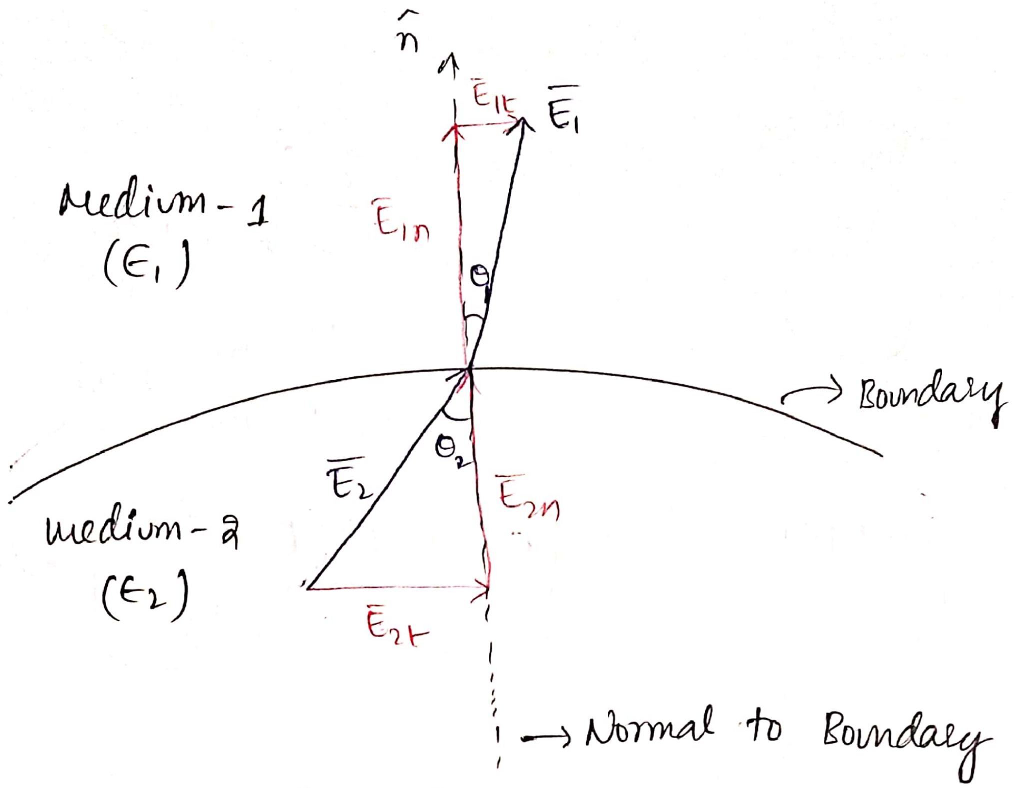
Electrostatic Boundary Conditions



Direction of $\bar{E}$: from medium 2 to medium 1

Electric field intensity in medium 1 $= \bar{E}_1$

" " " " medium 2 $= \bar{E}_2$

Component of $\bar{E}_1$ along the normal to Boundary $= \bar{E}_{1n}$

" " $\bar{E}_2$ " " " to Boundary $= \bar{E}_{2n}$

" " $\bar{E}_1$ " the Boundary $= \bar{E}_{1t}$

" " $\bar{E}_2$ " " " $= \bar{E}_{2t}$

$$\boxed{\bar{E}_1 = \bar{E}_{1n} + \bar{E}_{1t}}$$

$$\boxed{\bar{E}_2 = \bar{E}_{2n} + \bar{E}_{2t}}$$

θ_1 = angle b/w $\bar{E}_1$ & normal to the Boundary

θ_2 = " " $\bar{E}_2$ " " " " "

$$\bar{D}_1 = \epsilon_1 \bar{E}_1$$

↑ flux density in medium 1

$$\bar{D}_1 = \bar{D}_{1t} + \bar{D}_{1n}$$

$\bar{D}_{1n}$ ← Normal Component

$\bar{D}_{1t}$ ↑ tangential Component

$$\boxed{\begin{aligned} \bar{D}_{1t} &= \epsilon_1 \bar{E}_{1t} \\ \bar{D}_{1n} &= \epsilon_1 \bar{E}_{1n} \end{aligned}}$$

$$\bar{D}_2 = \epsilon_2 \bar{E}_2$$

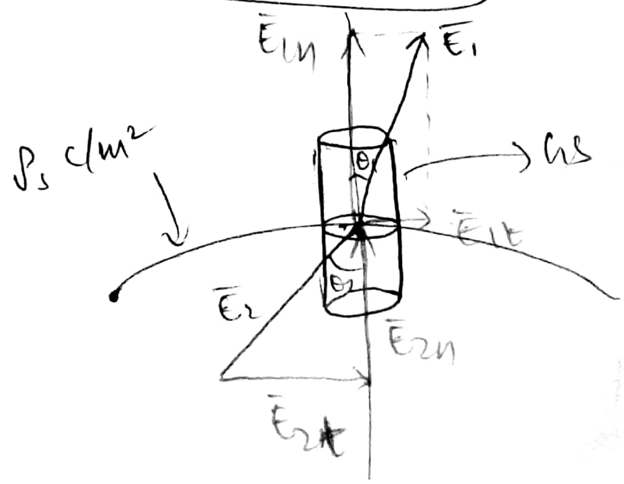
↑ Electric flux density in medium 2

$$\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2n}$$

$\bar{D}_{2n}$ ← normal component

$\bar{D}_{2t}$ ↑ tangential Component

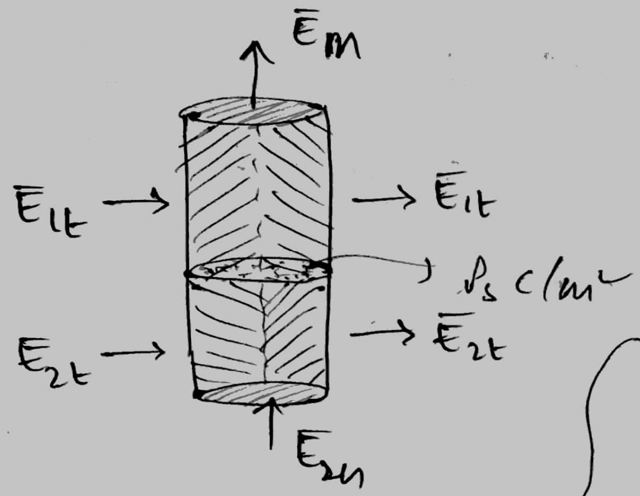
$$\boxed{\begin{aligned} \bar{D}_{2t} &= \epsilon_2 \bar{E}_{2t} \\ \bar{D}_{2n} &= \epsilon_2 \bar{E}_{2n} \end{aligned}}$$



At the Boundary

Charge density $= \rho_s \ C/m^2$

By applying Gauss law on Gaussian Surface



$$\phi_{elec} = Q_{enc}$$

$$\oint \bar{D}\cdot\bar{ds} = \rho_s \cdot A$$

$$\underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{top}} + \underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{Bottom}} + \underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{topleft}} + \underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{Bottom left}} + \underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{top Right}} + \underbrace{\int \bar{D}\cdot\bar{ds}}_{\text{Bottom right}} = \rho_s A$$

top: $D_{1n}A$; Bottom: $-D_{2n}A$; topleft: $-\epsilon_1 \bar{E}_{1t} A_1$

Bottom left: $-\epsilon_2 E_{2t} A_1$; top Right: $\epsilon_1 E_{1t} A_1$; Bottom right: $\epsilon_2 E_{2t} A_1$

$$\boxed{D_{1n} - D_{2n} = \rho_s}$$

$$\boxed{\bar{D}_{1n} - \bar{D}_{2n} = \rho_s \hat{n}}$$

$$\boxed{\bar{D}_{1n} - \bar{D}_{2n} = \rho_s \hat{n}}$$

Source free Boundary: $\boxed{\rho_s = 0}$

$\boxed{D_{1n} = D_{2n}}$ → Electric flux density is continous (normal components)

$$\boxed{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

$\boxed{E_{1n} \neq E_{2n}}$ → Electric flux ~~density~~ intensity are not continous (normal components)

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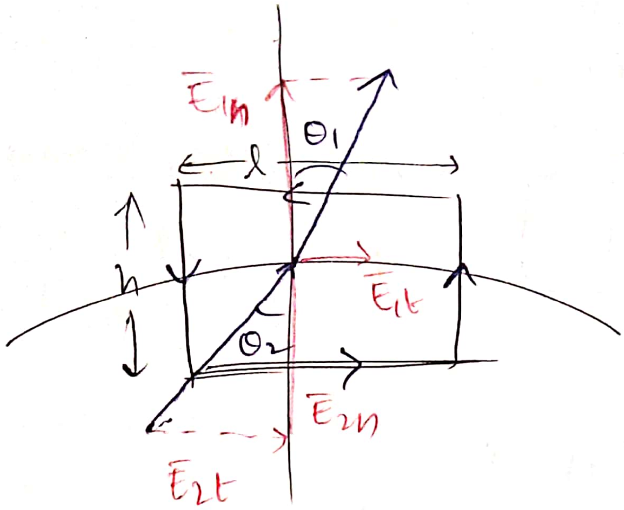

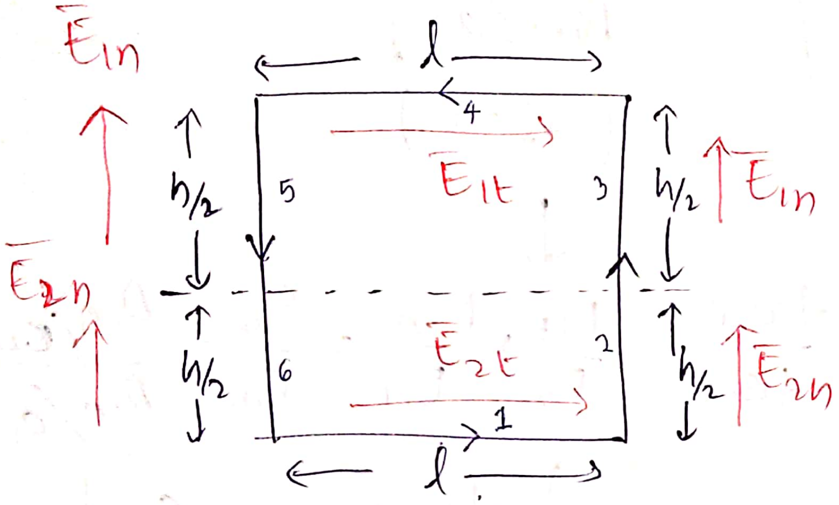

$$\oint_L \bar{E}\cdot d\bar{l} = 0$$ Maxwell Equation 2

$$\int_1 \bar{E}\, d\bar{l} + \int_2 \bar{E}\cdot d\bar{l} + \int_3 \bar{E}\cdot d\bar{l} + \int_4 \bar{E}\cdot d\bar{l} + \int_5 \bar{E}\cdot d\bar{l}$$

$$+ \int_6 \bar{E}\cdot d\bar{l} = 0$$

$$\Rightarrow (E_{2t})\, l + \cancel{(E_{2n})\, h/2} + \cancel{(E_{1n})\, h/2} + (-E_{1t}\, l)$$

$$+ \cancel{(-E_{1n})\, h/2} + \cancel{(-E_{2n})\, h/2} = 0$$

$$\Rightarrow \boxed{E_{2t} = E_{1t}}$$ $\Rightarrow$ Tangential Components of $\bar{E}$ are continuous

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$$\boxed{\frac{D_{2t}}{\epsilon_2} = \frac{D_{1t}}{\epsilon_1}}$$

$$\boxed{D_{1t} \neq D_{2t}}$$ ⇒ Tangential Components of D is not Continuous

(3)

$$\left.\begin{aligned} \tan\theta_1 &= \frac{E_{1t}}{E_{1n}} \\ \tan\theta_2 &= \frac{E_{2t}}{E_{2n}} \end{aligned}\right\}$$ Source free Boundary $(\rho_s = 0)$

↓

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\cancel{E_{1t}}}{E_{1n}} \times \frac{E_{2n}}{\cancel{E_{2t}}} = \frac{E_{2n}}{E_{1n}} = \frac{D_{2n}/\epsilon_2}{D_{1n}/\epsilon_1}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

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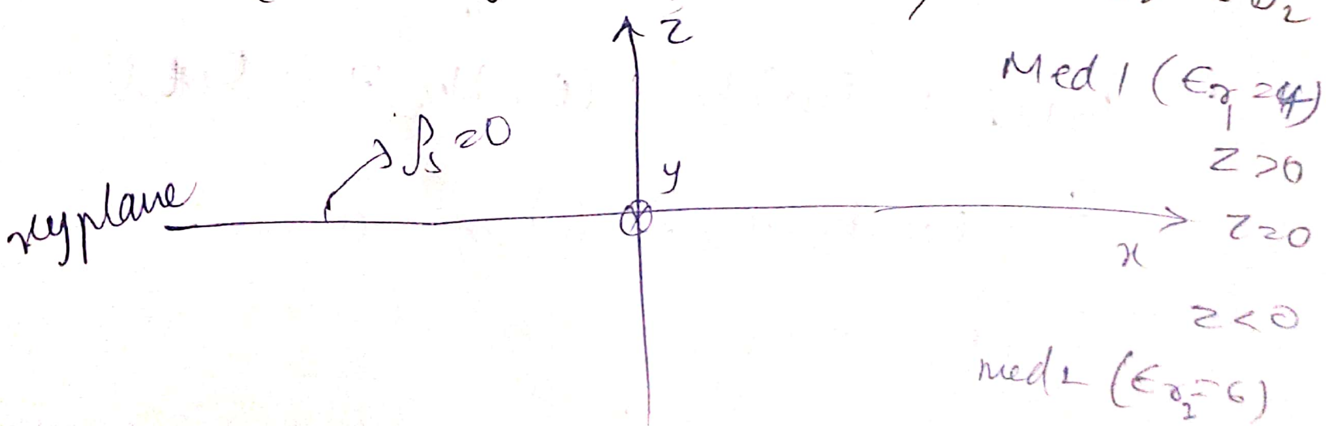
(1) A plane $z=0$ seperates medium 1 $(z>0)$ and medium 2 $(z<0)$ $\epsilon_{r1} = 4$ & $\epsilon_{r2} = 6$. If $D_1 = (16\hat{x} + 30\hat{y} - 20\hat{z})\ nC/m^2$, find $\bar{E}_2$ & $\bar{D}_2$



$$\bar{D}_1 = \underbrace{16\hat{x} + 30\hat{y}}_{\text{tangential}} \underbrace{-20\hat{z}}_{\text{normal}}$$

Component of $\bar{D}_1$ along $\hat{z} = D_1 \cdot \hat{z}$

$$= -20$$

$$\bar{D}_{1n} = -20\hat{z}$$

$$\bar{D}_{1t} = \bar{D}_1 - \bar{D}_{1n}$$

$$\bar{D}_{1t} = 16\hat{x} + 30\hat{y}$$

$$D_{1n} = D_{2n} = -20\hat{z}$$

$$E_{1t} = E_{2t}$$

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

$$\frac{16\hat{x} + 30\hat{y}}{4} = \frac{D_{2t}}{6}$$

$$\boxed{D_{2t} = 24\hat{x} + 45\hat{y}}$$

$$\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2n}$$

$$\bar{D}_2 = 24\hat{x} + 45\hat{y} - 20\hat{z}$$

$$\bar{E}_2 = \frac{\bar{D}_2}{\epsilon_2} = \frac{24\hat{x} + 45\hat{y} - 20\hat{z} \ \text{n C/m}^2}{\varepsilon_{r2}\varepsilon_0}$$

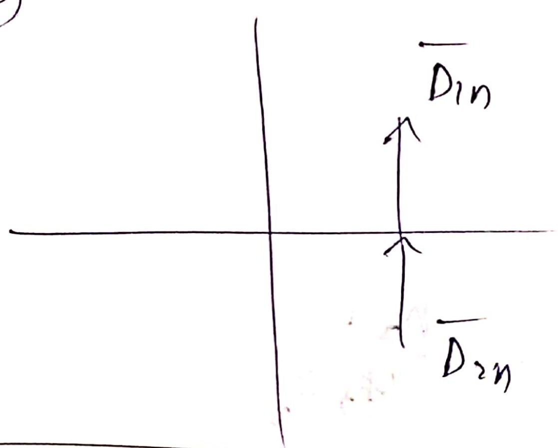
$$\bar{E}_2 = \frac{1}{\varepsilon_0}(4\hat{x} + 7.5\hat{y} - 3.33\hat{z})\, 10^{-9}\ \text{V/m}$$

(2) If $D = (50\hat{x} + 80\hat{y} - 30\hat{z})$ nC/m^2 for $x > 0$ & $\varepsilon = 2\varepsilon_0$ find E in the region $x < 0$ for which $\varepsilon = 8\varepsilon_0$.

(3) Repeat Q(1) if surface charge density @ boundary $= 100 nC/m^2$

(4) Repeat Q(2) if surface charge density @ boundary $= 10 nC/m^2$

(4)



$\bar{D}_2 = 40\hat{x} + 320\hat{y} - 120\hat{z}$

$\bar{E}_2 = \dfrac{5\hat{x} + 40\hat{y} - 15\hat{z}}{\varepsilon_0} \times 10^{-9}$ V/m

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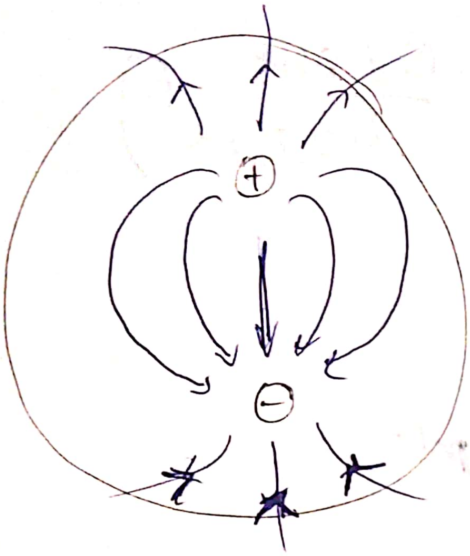
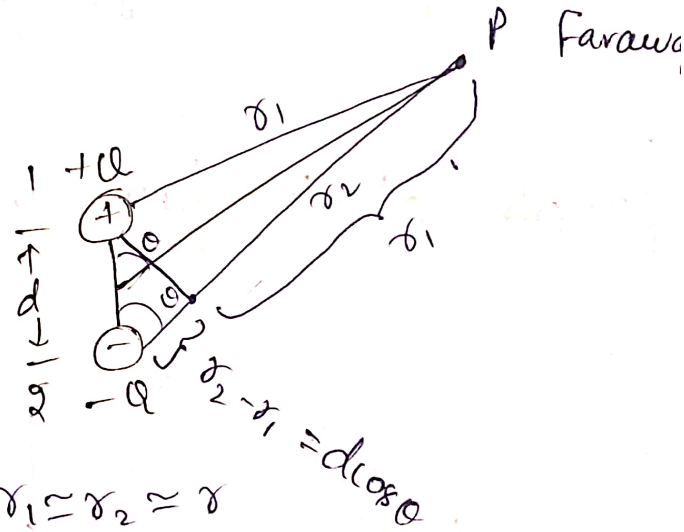
(5) A conducting sphere of radius a is half immersed in liquid of permittivity $\varepsilon = 4\varepsilon_0$. The medium above the liquid is a gas of permittivity $\varepsilon = 2\varepsilon_0$ (1) Find the Electric field intensity in the liquid & gas media (2) Q_1 = charge on top hemisphere & Q_2 = charge on bottom hemisphere then $\frac{Q_1}{Q_2}$ —— total charge on sphere = 10nC

Miscellaneous topics

(1) Electric Dipole :



we can't get $\bar{E}$ from Gauss law



Potential @ P', V

$(r >> d)$

$r_1 \simeq r_2 \simeq r$

$$V = V_1 + V_2$$

$$= \frac{Q_1}{4\pi\varepsilon_0 r_1} + \frac{Q_2}{4\pi\varepsilon_0 r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{(r_2 - r_1)}{\cancel{r_1 r_2} \to r^2}$$

$$\boxed{V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}} \longrightarrow \boxed{V \propto \frac{1}{r^2}} \text{ for dipole}$$

~~For moti~~

For monopole $V \propto \frac{1}{r}$

For quadpole $V \propto \frac{1}{r^3}$

$$\bar{E} = -\bar{\nabla} \cdot V$$

$$= -\left(\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}\right)$$

$$\bar{E} = \left(\frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{r} + \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{\theta}\right)$$

for dipole

$$\boxed{|E| \propto \frac{1}{r^3}} \rightarrow \text{in dipole}$$

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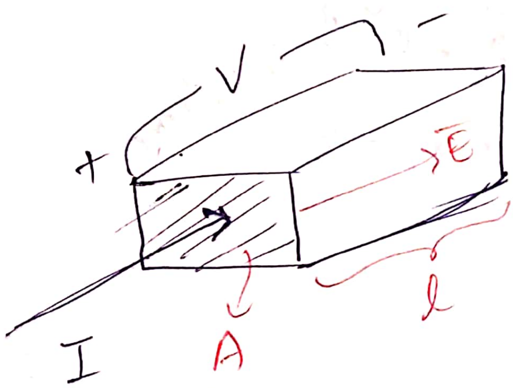
② ~~ohm~~ <u>ohm's law in point form</u>



$$V = IR$$

$$\downarrow$$

$$E \times l = J \cdot A \left(\frac{\rho l}{A}\right)$$

$$J = \frac{1}{\rho}E$$

$$\boxed{\bar{J} = \sigma E}$$

$\downarrow$

ohm's law in point form

<u>Note:</u> when current flow is not uniform, total current through some surface $=(S)$

$$(I) = \int_S \bar{J} \cdot d\bar{s}$$

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③ Electric field in Conducting medium :-

For perfect Conductor, $\sigma = \infty$

$$J = \sigma E$$

$$\downarrow$$

$$E = \frac{J}{\cancel{\sigma}}, \infty$$

$$\boxed{\bar{E} = 0}$$

$\downarrow$

Electric field vanishes inside a conductor

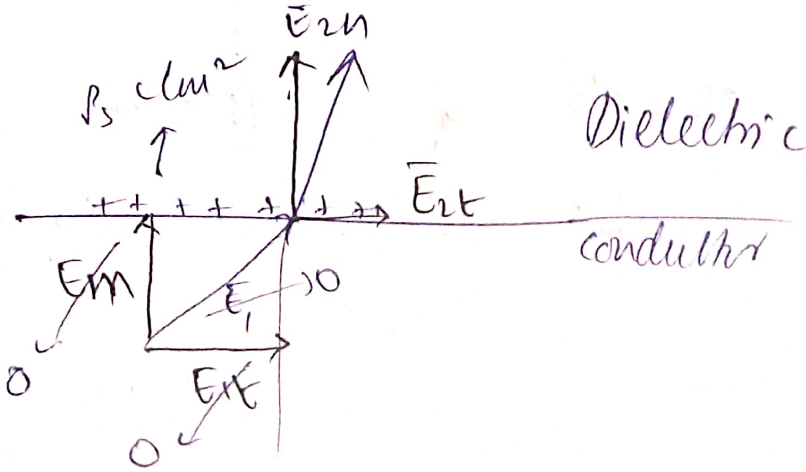
$\downarrow$

Source of $\bar{E}$ = static charge = 0

* Inside a conductor, $\underbrace{\text{static charges}}_{\text{producing } \bar{E} \text{ field}} = 0$

* charge exists on the surface of conductor (static)

④ Boundary Conditions :- (Conductor - Dielectric)



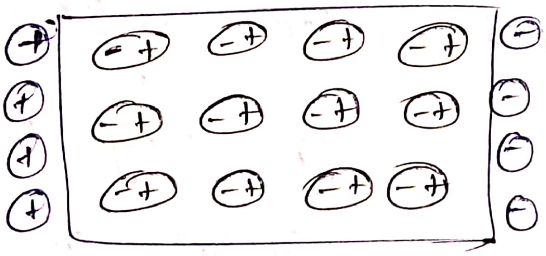
$$\underbrace{\cancel{E_{1t}}}_{0} = E_{2t} \Rightarrow E_{2t} = 0$$

$$\overbrace{D_{2n}}^{\text{outgoing}} - \overbrace{\underbrace{\cancel{D_{1n}}}_{0}}^{\text{incoming}} = \rho_s$$

$$\boxed{D_{2n} = \rho_s}$$

(5) Polarization

Dielectric medium: A material which has dipoles or which can create dipoles in the application of electric field



$$\bar{D} = \epsilon_0 \bar{E} + \underbrace{\bar{P}}_{\text{polarization}} = \epsilon_0 \epsilon_r \bar{E}$$

$$\Rightarrow \bar{P} = \epsilon_0 (\epsilon_r - 1) \bar{E}$$

$$\boxed{\bar{P} = \underbrace{\chi_e}_{\text{Electric Suseptability}} \epsilon_0 \bar{E}}$$

$$\boxed{\chi_e = \epsilon_r - 1}$$

Dielectric strength of a dielectric material:

Maximum dielectric stresses a material can withstand