ATOMIC STRUCTURE

Lecture - 1

Modern Physics

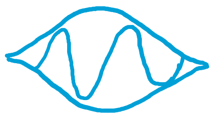
Newton

Particle

Huygen

wave

Einstein



Photon

Energy of Photon

$$E = h\nu = \frac{hc}{\lambda}$$

$$h = 6.6 \times 10^{-34}\ \text{Js}$$

freq: f or $\nu = \frac{1}{T} = \frac{c}{\lambda}$

Momentum of Photon

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

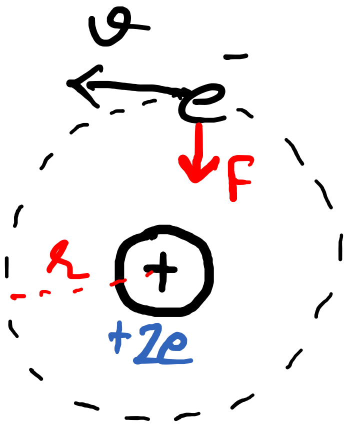
$$hc = 12400\ \text{eV}\text{Å}$$

Properties of photons

- These are particles with wave nature.
- Rest mass of photon is zero.
- In vacuum photon travels with a speed of $3x10^8$ m/s.
- Photons have frequency and wavelength.
- Photos has energy and Momentum.

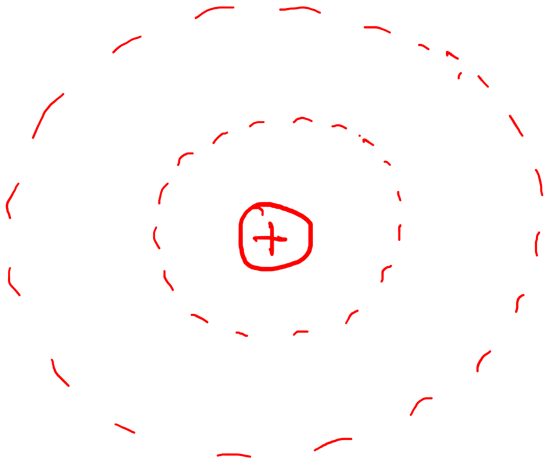
Bohr’s Postulate

1. The electrons revolve around the nucleus in circular orbits. The coulomb force is the essential centripetal force to keep it a revolving in circular orbits.

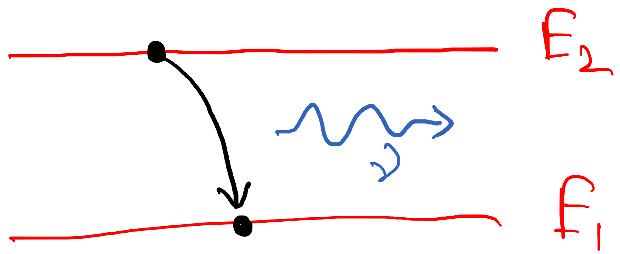


$$F = \frac{(e)(Ze)}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

2. The electron revolves around the nucleus in some permitted orbits only. These orbits are known as stationary orbit or fixed orbits. In these orbits of special radius the electron does not radiate any energy.

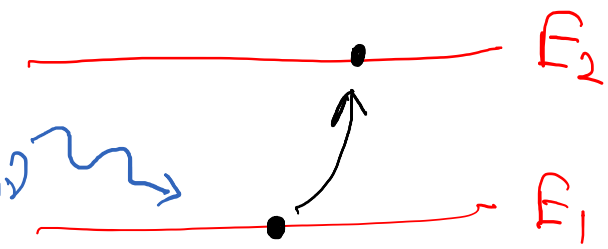


3. If the electron jumps from orbit of higher energy E_2 to an orbit of lower energy E_1 it emits a photon. On the other hand the electron can jump from an orbit of lower energy E_1 to an orbit of higher energy E_2 by absorbing energy from some outside source.



$h\nu = E_2 - E_1$

$E_2 - E_1 = h\nu$



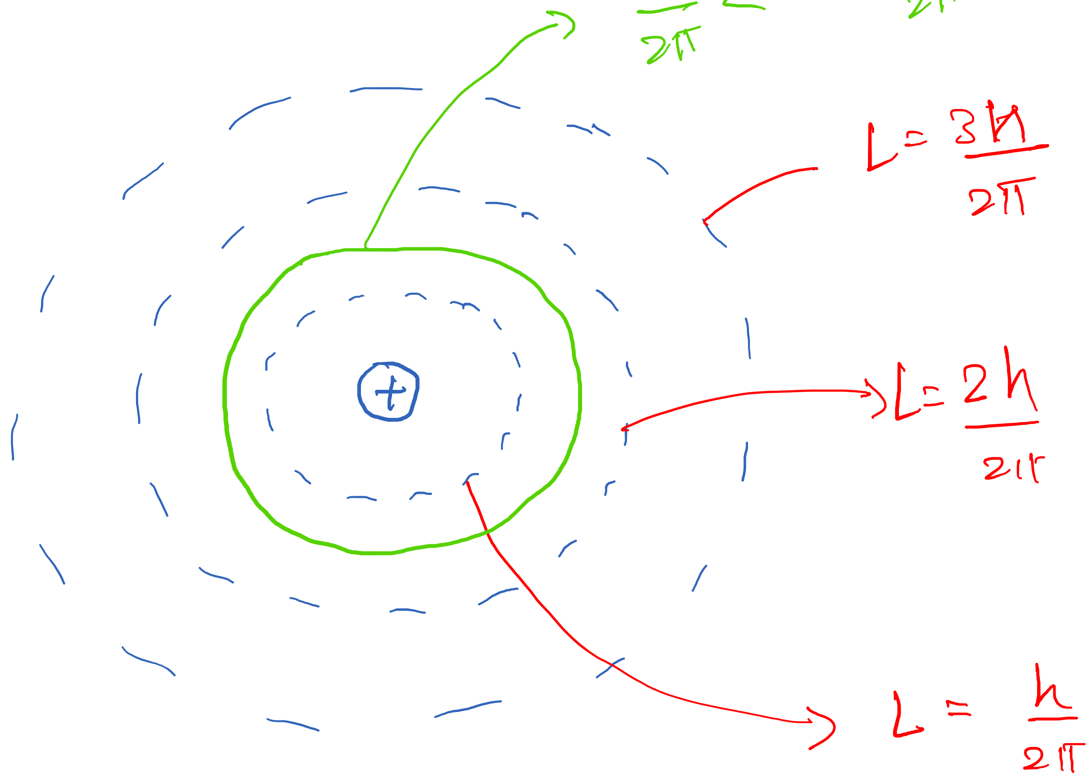
$E_2 > E_1$

4. Electron revolves only in those orbits in which its angular momentum is integral multiple of $\frac{h}{2\pi}$. This is Bohr's quantisation rule.

$L = mvr$

$L = n\frac{h}{2\pi}$

$n = 1, 2, 3, \ldots$



Circular Motion

$$F = \frac{mv^2}{r}$$

Representation of element

$${}^{A}_{Z}X$$

$A \rightarrow$ mass no. = no. of Protons + no. of neutrons

$Z \rightarrow$ Atomic no. = no. of Protons

According to 1st Postulate

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} = \frac{mv^2}{r} \quad \text{--- (1)}$$

According to 4th Postulate

$$mvr = \frac{nh}{2\pi} \quad \text{--- (2)}$$

by eqn (1) & (2)

$$rv = \frac{nh}{2\pi m} \Rightarrow \frac{Ze^2}{4\pi\varepsilon_0 mv} = \frac{nh}{2\pi m}$$

$$\boxed{v = \frac{Ze^2}{2\varepsilon_0 nh}} \quad * \quad v = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{Z}{n}$$

$$\boxed{r = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}} \quad * \quad r = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right)\frac{n^2}{Z}$$

Kinetic Energy

$$K = \frac{1}{2} m v^2$$

$$\boxed{K = \frac{m Z^2 e^4}{8 \varepsilon_0^2 n^2 h^2}}$$

Total Energy = K+U

$$\boxed{TE = \frac{-m Z^2 e^4}{8 \varepsilon_0^2 n^2 h^2}} \Rightarrow E = -\left(\frac{m e^4}{8 \varepsilon_0^2 h^2}\right) \frac{Z^2}{n^2}$$

Potential Energy

$$U = -\frac{1}{4\pi\varepsilon_0} \frac{(Ze)(e)}{r}$$

$$\boxed{U = \frac{-m Z^2 e^4}{4 \varepsilon_0^2 n^2 h^2}}$$

$$E = -K = \frac{U}{2}$$

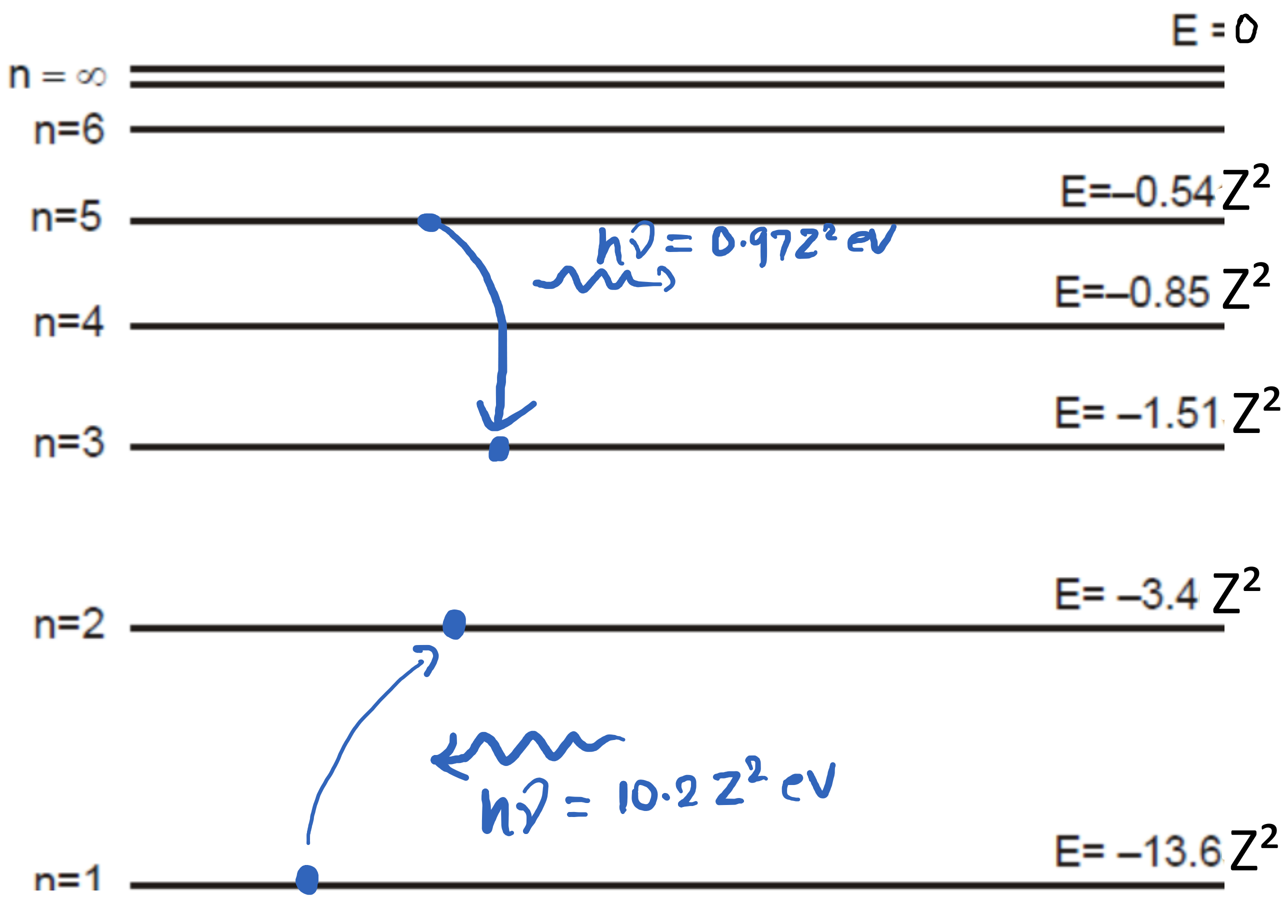
Results to be remembered

* $$\boxed{v = \frac{c}{137}\frac{Z}{n} \ m/s}$$ $\frac{c}{137} = \underline{\underline{2.18 \times 10^{6}}}$

* $$\boxed{r = 0.53 \frac{n^2}{Z} \ \mathring{A}}$$ $a_0 = 0.53 \mathring{A}$

* $$\boxed{E = -13.6 \frac{Z^2}{n^2} \ eV}$$

Energies of Different Energy Level in Hydrogen Like Atoms



$$E_1 = -13.6\ Z^2\ eV$$
$$E_2 = -3.40\ Z^2\ eV$$
$$E_3 = -1.51\ Z^2\ eV$$
$$E_4 = -0.85\ Z^2\ eV$$
$$E_5 = -0.54\ Z^2\ ev$$
$$E_6 = -0.36\ Z^2\ eV$$

frequency $f = \frac{v}{2\pi r}$

Current $i = ef$

Magnetic field $B = \frac{\mu_0 i}{2r}$

Magnetic Moment $|\vec{M}| = iA$

$= i\pi r^2$

Note: Bohr's theory is only applicable to hydrogen like atom (single e^- system)

H, He^{+}, Li^{2+}

$Z=1$ $\quad Z=2$ $\quad Z=3$

Ex. ***What is the angular momentum of an electron in Bohr's Hydrogen atom whose energy is -3.4 eV ?***

Sol.

$$E = -13.6 \frac{Z^2}{n^2} \, eV = -3.4 \, eV \qquad (Z = 1 \text{ for Hydrogen})$$

$$\boxed{n = 2}$$

$$L = \frac{nh}{2\pi} \Rightarrow \boxed{L = \frac{h}{\pi}} \text{ Ans}$$

Binding energy (B.E.)

Binding energy of a system is defined as the energy released when it's constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate it's constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 eV energy will be released.

The binding energy of a hydrogen atom is therefore 13.6 eV.

Ans. [4]

Sol.

Q **In a hydrogen-like atom, if the electron transits from lower energy state to higher energy state, then which of the following statement is correct?**

(1) KE of electron and PE of atom, both increase

(2) KE and PE of atom, both decrease

(3) KE increases while PE decreases

(4) KE decreases while PE increases

Ans. [4]

Sol.

$$L = \frac{nh}{2\pi}$$

Q Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states ?

(1) radius of the orbit ✗

(2) speed of the electron ✗

(3) energy of the atom ✗

(4) orbital angular momentum of the electron ✓

$$r = 0.53\frac{n^2}{Z} \qquad n=1$$

$$v = \frac{c}{137}\frac{Z}{n} \qquad n=1$$

$$E = -13.6\frac{Z^2}{n^2} \qquad n=1$$

$$L = \frac{nh}{2\pi} \qquad n=1$$

Q **The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is -**

(1) 1 **(2) 8**

(3) 4 **(4) 16**

Sol^n

$$a = \frac{v^2}{r}$$

$$v^2 \propto \frac{Z^2}{n^2}$$

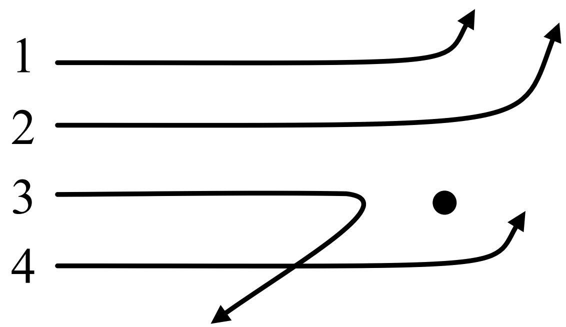
$$\frac{1}{r} \propto \frac{Z}{n^2}$$

$$\Rightarrow a \propto \frac{Z^3}{n^4}$$

$$\boxed{a \propto Z^3}$$

$$\frac{a_{He}}{a_H} = \left(\frac{2}{1}\right)^3 = 8$$

Q The diagram shows the path of four α-particles of the same energy being scattered by the nucleus of an atom simultaneously. Which of these are/is not physically possible –



(1) 3 and 4 (2) 2 and 3
(3) 1 and 4 (4) 4 only

Ans. [1]

Sol. In case (3) particle will retrace its path and in case (4) particle will experience repulsive force.

Q Angular momentum (L) and radius (r) of a hydrogen atom are related as -

(1) Lr = constant (2) Lr^2 = constant

(3) Lr^4 = constant (4) none of these

Sol

Ans. [4]

$$L^2 = \frac{n^2h^2}{2^2\pi^2}$$

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$$

$$L^2 \propto r$$

$$L^2 r^{-1} = Const$$

or

$$L r^{-1/2} = Const$$

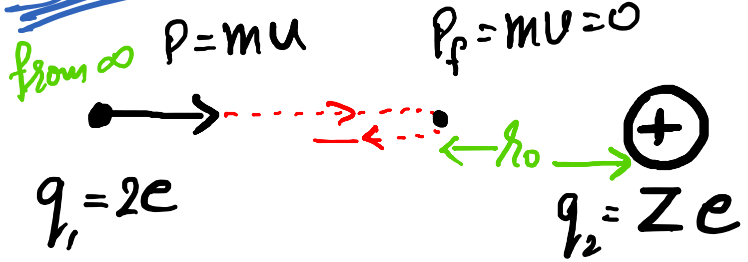
Q The distance of closest approach of an α-particle is fired at a nucleus with momentum p is r_0. When the α-particles are fired at the same nucleus with momentum 5p, the distance of closed approach will be -

(1) $5r_0$ (2) $25r_0$

(3) $\frac{1}{5}r_0$ (4) $\frac{1}{25}r_0$

$$K = \frac{P^2}{2m} = \frac{1}{2}mv^2$$

Sol^n



$$V_i + K_i = V_f + K_f$$

$$0 + \frac{P^2}{2m} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r_0}$$

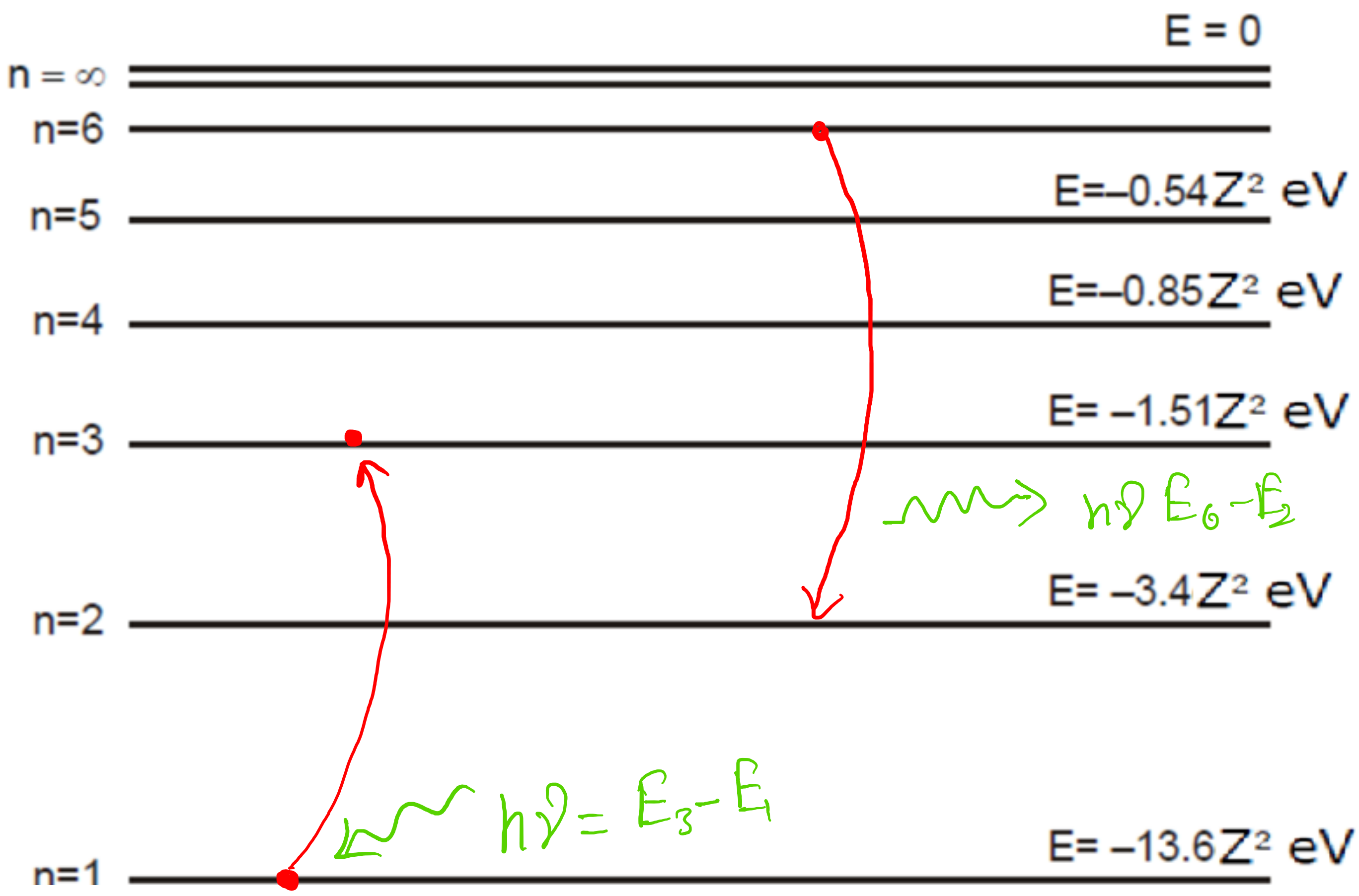
$$P^2 r_0 = \text{Const.}$$

$$r_0 \propto \frac{1}{P^2}$$

$$P \rightarrow 5P$$

$$r_0 \rightarrow \frac{1}{25}r_0$$

Energies of Different Energy Level in Hydrogen Like Atoms



$E_1 = -13.6\ Z^2\ eV$

$E_2 = -3.40\ Z^2\ eV$

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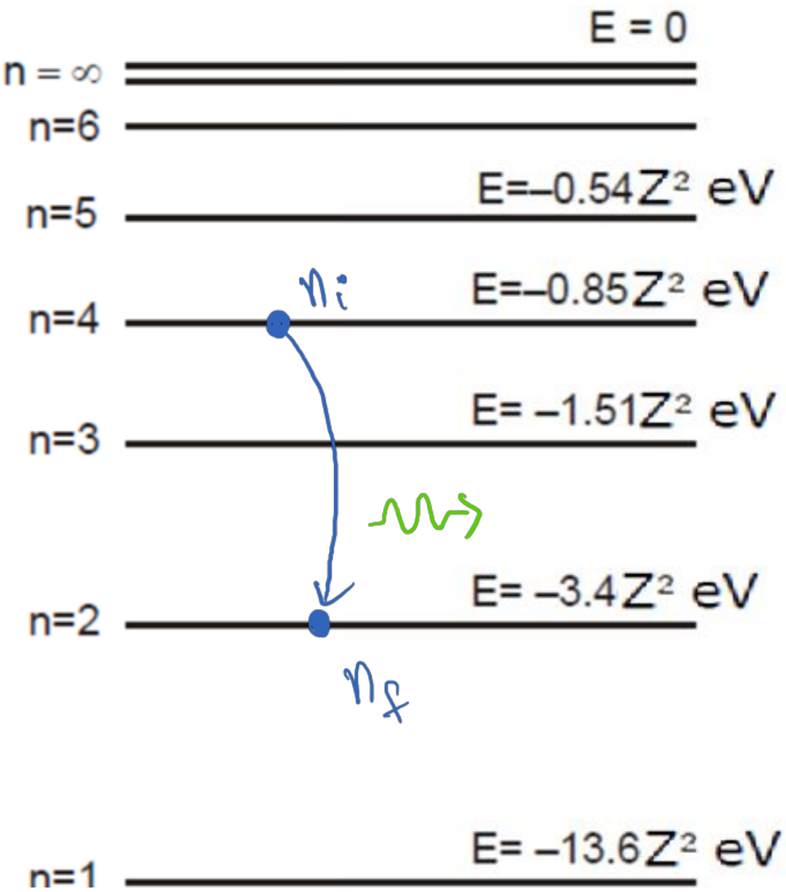


if e^- jumps from n_i to n_f
then $E_i - E_f$ energy will be released

$$h\nu = E_i - E_f$$

$$\frac{hc}{\lambda} = E_i - E_f$$

$$\frac{1}{\lambda} = \frac{E_i - E_f}{hc} = \frac{-13.6\,Z^2}{hc}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

$$\boxed{\frac{1}{\lambda} = R_H Z^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]}$$

where R_H = Rydberg Constant

$$R_H = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.0973 \times 10^7\ m^{-1}$$

ATOMIC STRUCTURE

Lecture - 2

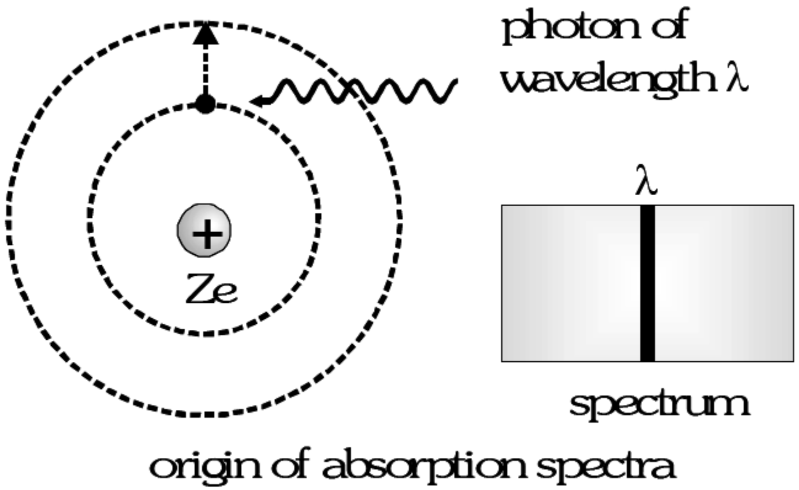
Note:-

➢ **The process of excitation and ionisation both are <u>absorption phenomena</u>.**

Ionization Energy

The minimum energy needed to ionize an atom is called ionization energy.

The potential difference through which an electron should be accelerated to acquire the value of ionization energy is called ionization potential.

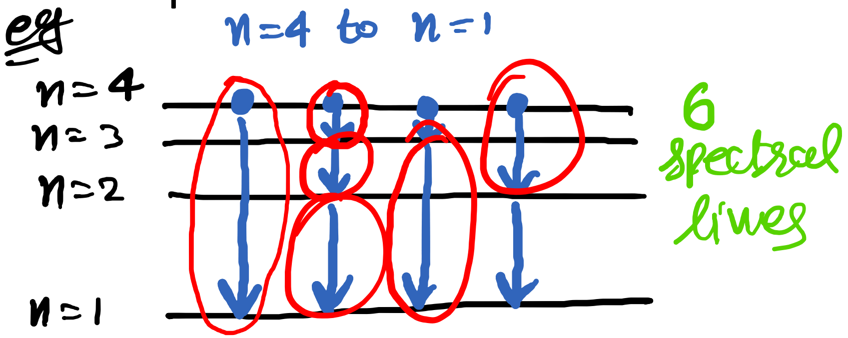


Number of spectral lines

If an electron jumps from higher energy orbit to lower energy orbit it emits radiations with various spectral lines. If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

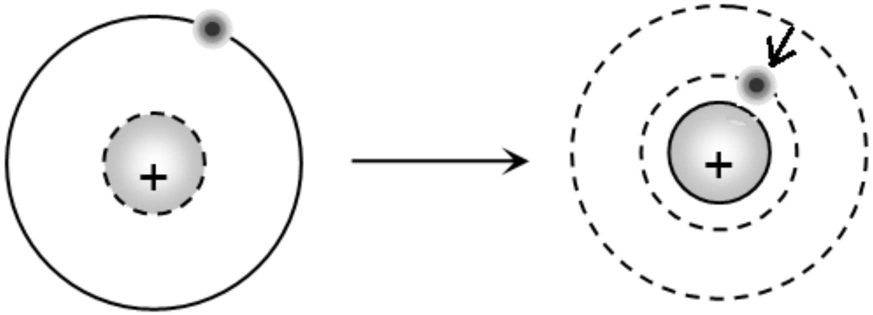
$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

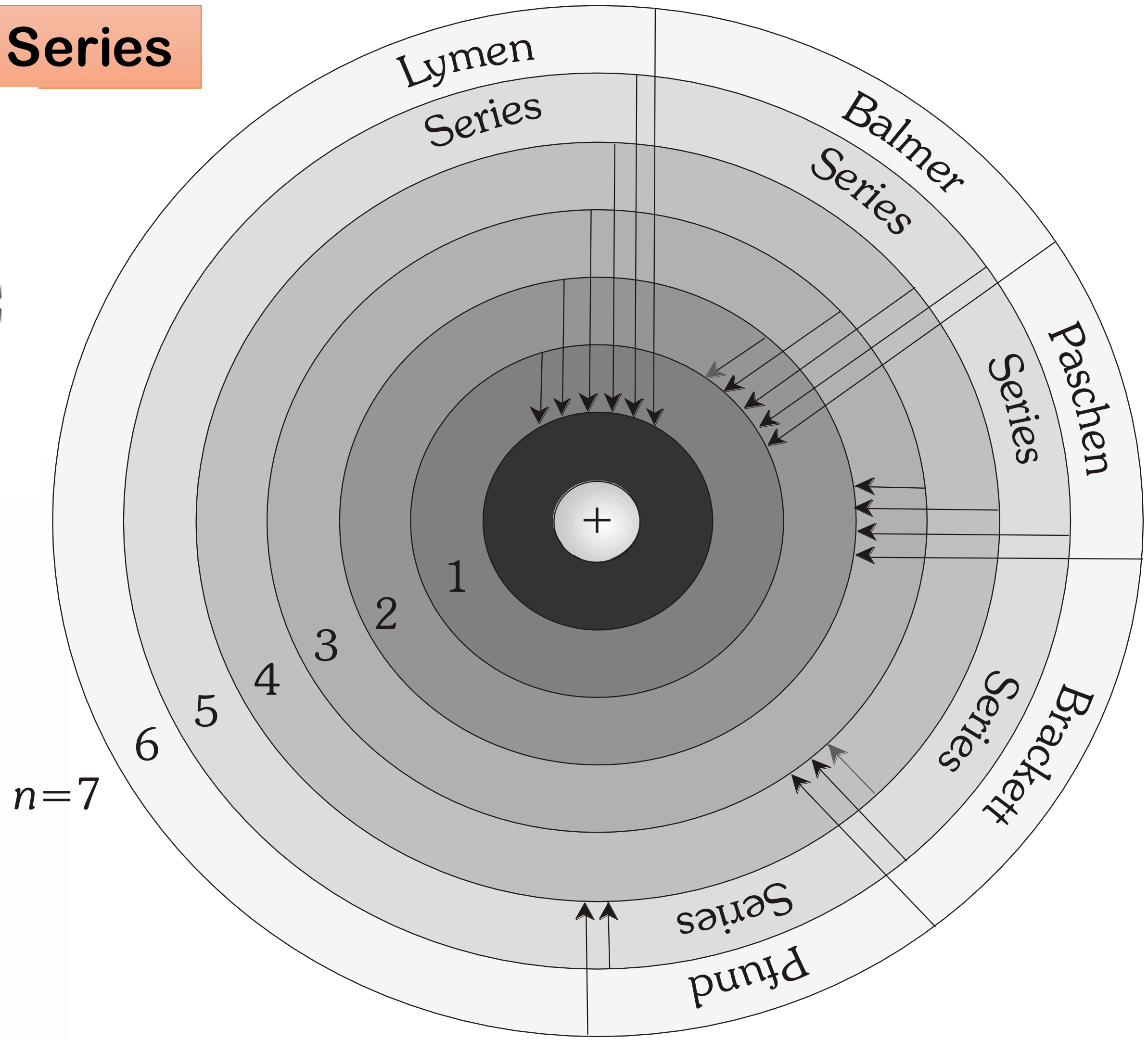
If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted



$$\boxed{N_E = \frac{n(n-1)}{2}}$$

Hydrogen Spectrum and Spectral Series

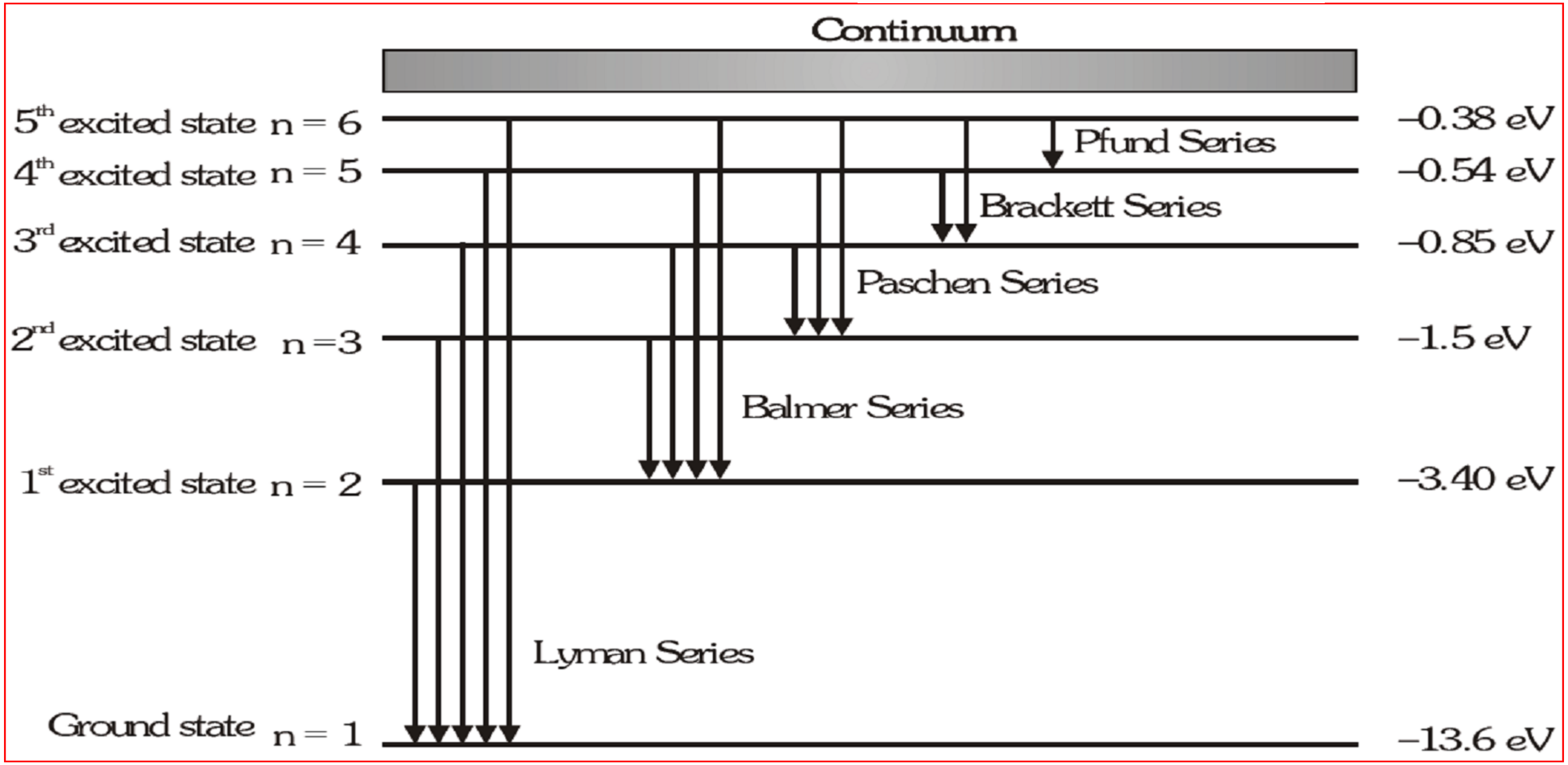




Hydrogen Spectrum and Spectral Series

Z=1

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$



The result are tabulated below :

S. No.	Series Observed	Value of n_1	Value of n_2	Position in the Spectrum
1.	Lyman Series	1	$2,3,4...\infty$	Ultra Violet
2.	Balmer Series	2	$3,4,5...\infty$	Visible
3.	Paschen Series	3	$4,5,6....\infty$	Infra-red
4.	Brackett Series	4	$5,6,7....\infty$	Infra-red
5.	Pfund Series	5	$6,7,8....\infty$	Infra-red

Ex. A single electron orbits around a stationary nucleus of charge +Ze where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third orbit. Find

(i) The value of Z.

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV,

Bohr radius (a_o) = 5.3×10^{-11} m,

Velocity of light = 3×10^8 m/s,

Planck's constant = 6.6×10^{-34} J–s]

Sol^n (i) $E_3 - E_2 = 47.2\,eV$

$$-13.6Z^2\left[\frac{1}{3^2} - \frac{1}{2^2}\right]\cancel{eV} = 47.2\cancel{eV}$$

$$Z^2 \approx 25$$

$\boxed{Z = 5}$ Ans (i)

(ii) $E_4 - E_3 = -13.6Z^2\left[\frac{1}{4^2} - \frac{1}{3^2}\right]eV$

$\boxed{E_4 - E_3 \approx 16.53\,eV}$ Ans (ii)

Q.

$Z = 1$

The time period of revolution of an electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16} s$. The frequency of the electron in its first excited state (in s^{-1}) is :

a. 7.8×10^{14}

b. 7.8×10^{16}

c. 3.7×10^{14}

d. 3.7×10^{16}

Sol^n

$$T = \frac{2\pi r}{v}$$

$$T \propto \frac{r}{v} \propto \frac{n^3}{Z^2}$$

$$r \propto \frac{n^2}{Z}$$

$$v \propto \frac{Z}{n}$$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$$

$$T_1 f_2 = \left(\frac{1}{2}\right)^3$$

$$f_2 = \frac{1}{8 \times 1.6 \times 10^{-16}}$$

$$\Rightarrow \boxed{f_2 = 7.8 \times 10^{14} Hz}$$

Ans

Q. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is

(1) $n = 2 \rightarrow n = 4$

(2) $n = 2 \rightarrow n = 5$

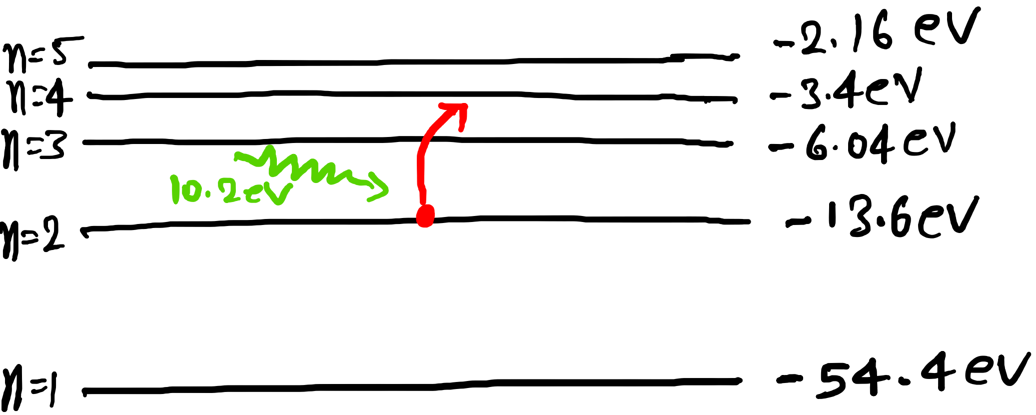
(3) $n = 2 \rightarrow n = 3$

(4) $n = 1 \rightarrow n = 4$

Sol^n for Hydrogen : (Z = 1)

Energy of photon released from Hydrogen is 10.2eV

for Helium (Z=2)



Possible transition for He atom is

n = 2 to n = 4

Q. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm

(2) 488.9 nm

(3) 388.9 nm

(4) 642.7 nm

Sol^n

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

$$\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad \text{—}(1)$$

$$\frac{1}{\lambda_2} = R(1)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \quad \text{—}(2)$$

divide (1) by (2) $(\lambda_1 = 660\,nm)$

$$\boxed{\lambda_2 = 489\,nm}$$ Ans

Q. A He^+ ion is in its first excited state. Its ionization energy is :

(1) 13.60 eV (2) 6.04 eV

(3) 48.36 eV (4) 54.40 eV

Example

The excitation energy of a hydrogen-like ion in its first excited state is 40.8 eV. Find the energy needed to remove the electron from the ion, which is in ground state.

Sol^n

$$40.8\,eV = -13.6\,Z^2\left[\frac{1}{2^2} - \frac{1}{1^2}\right]$$

$$\Rightarrow Z^2 = 4$$

$$\Rightarrow \boxed{Z = 2}$$

Ionisation energy $= -13.6\,Z^2\left[\frac{1}{\infty^2} - \frac{1}{1^2}\right]$

$= 13.6 \times 4$

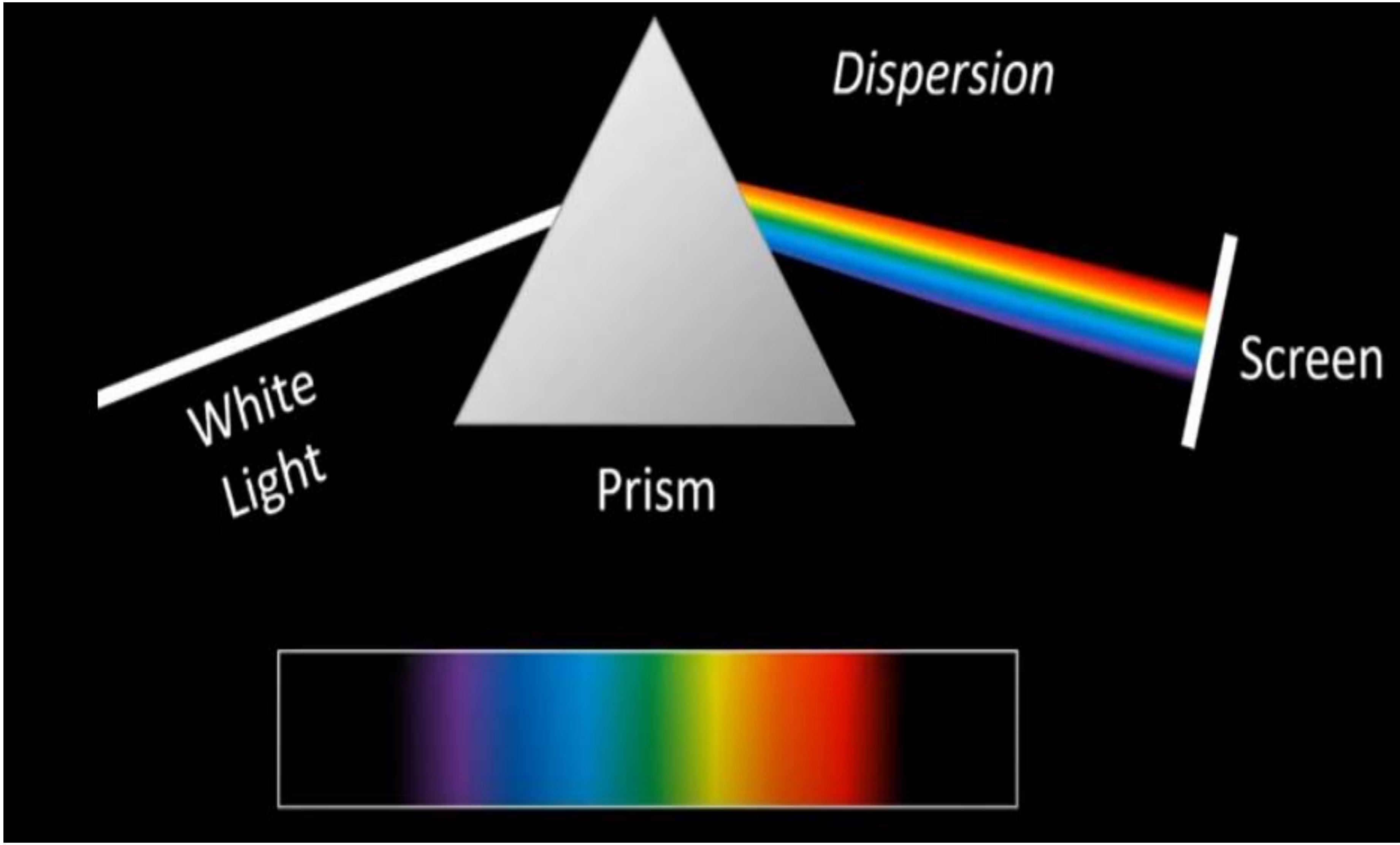
$= 54.4\,eV$

SPECTRUM

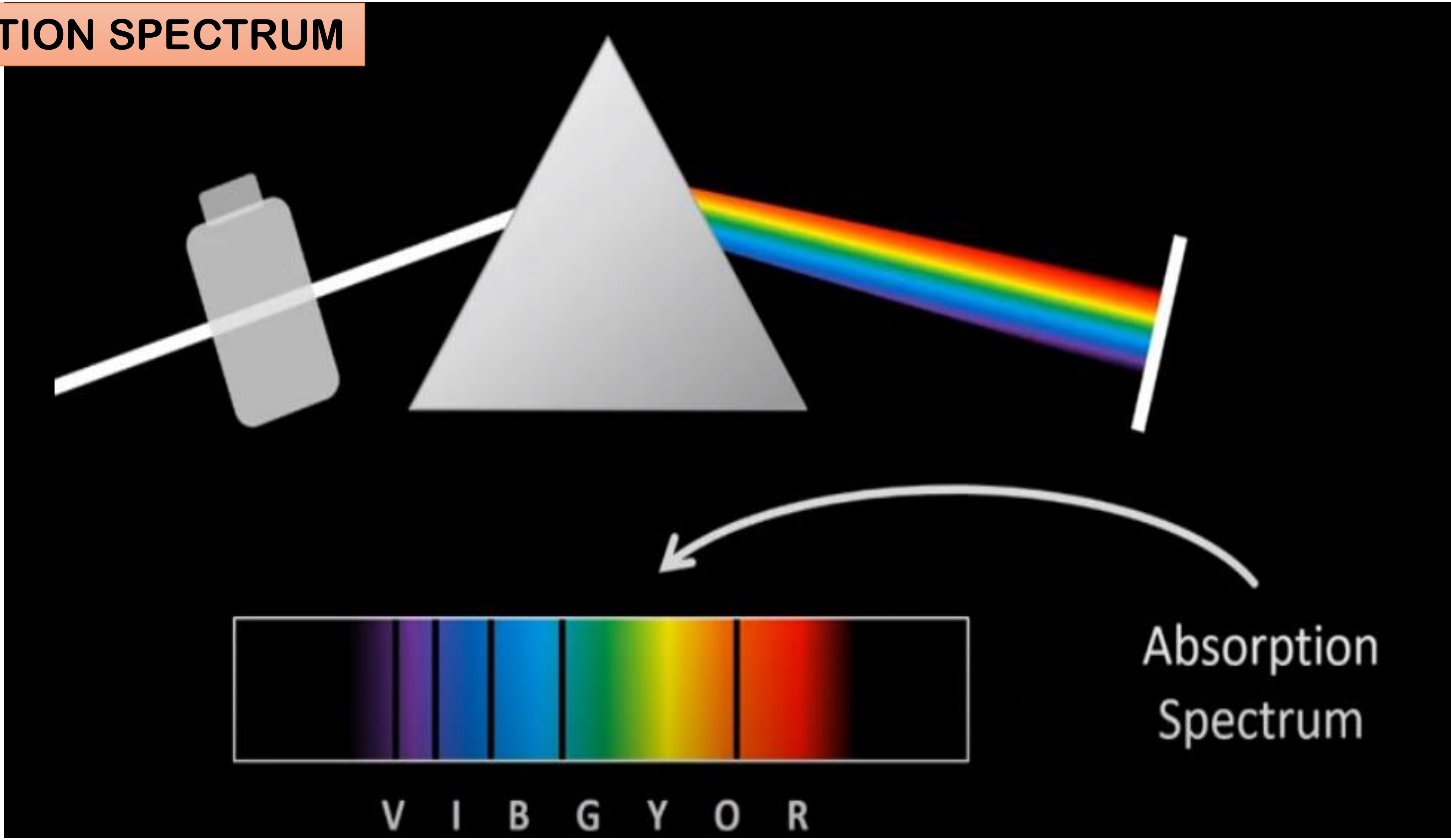
Dispersed light arranging itself in a pattern of different wavelengths is referred to as a spectrum.

- Spectra
  - Emission
    - Continuous
    - Line
    - Band
  - Absorption
    - Line
    - Band

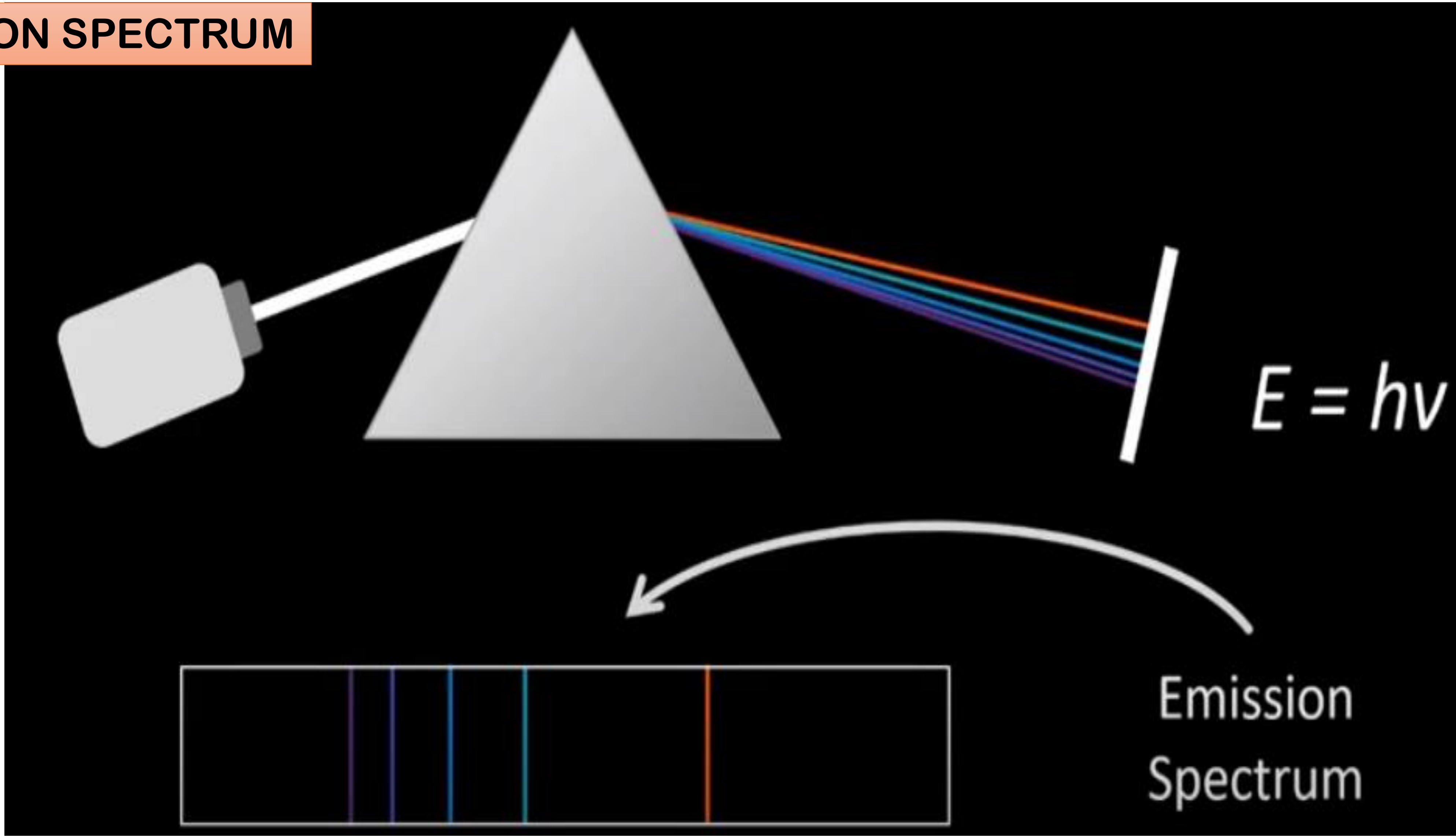
CONTINUOUS SPECTRUM



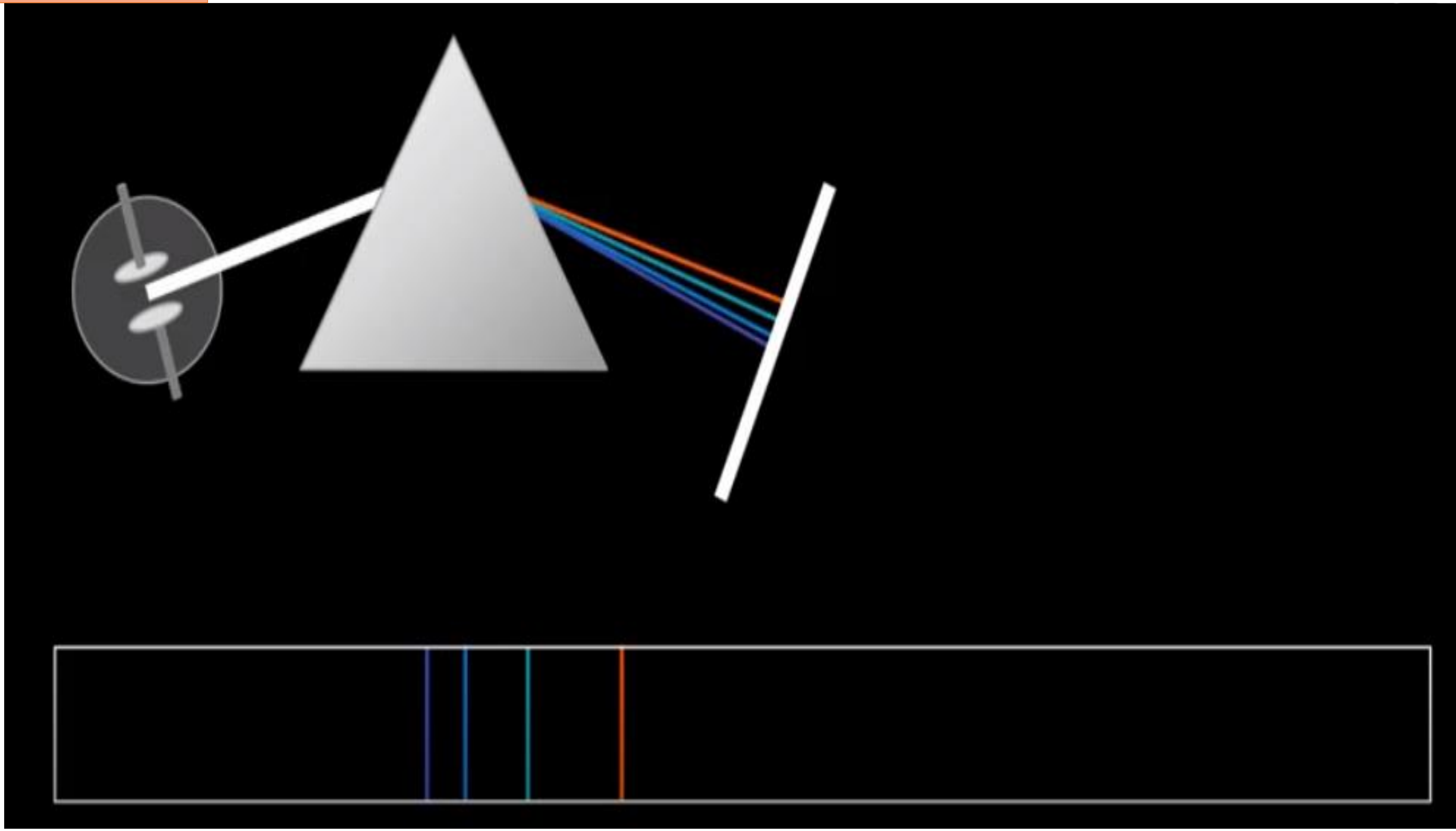
ABSORPTION SPECTRUM

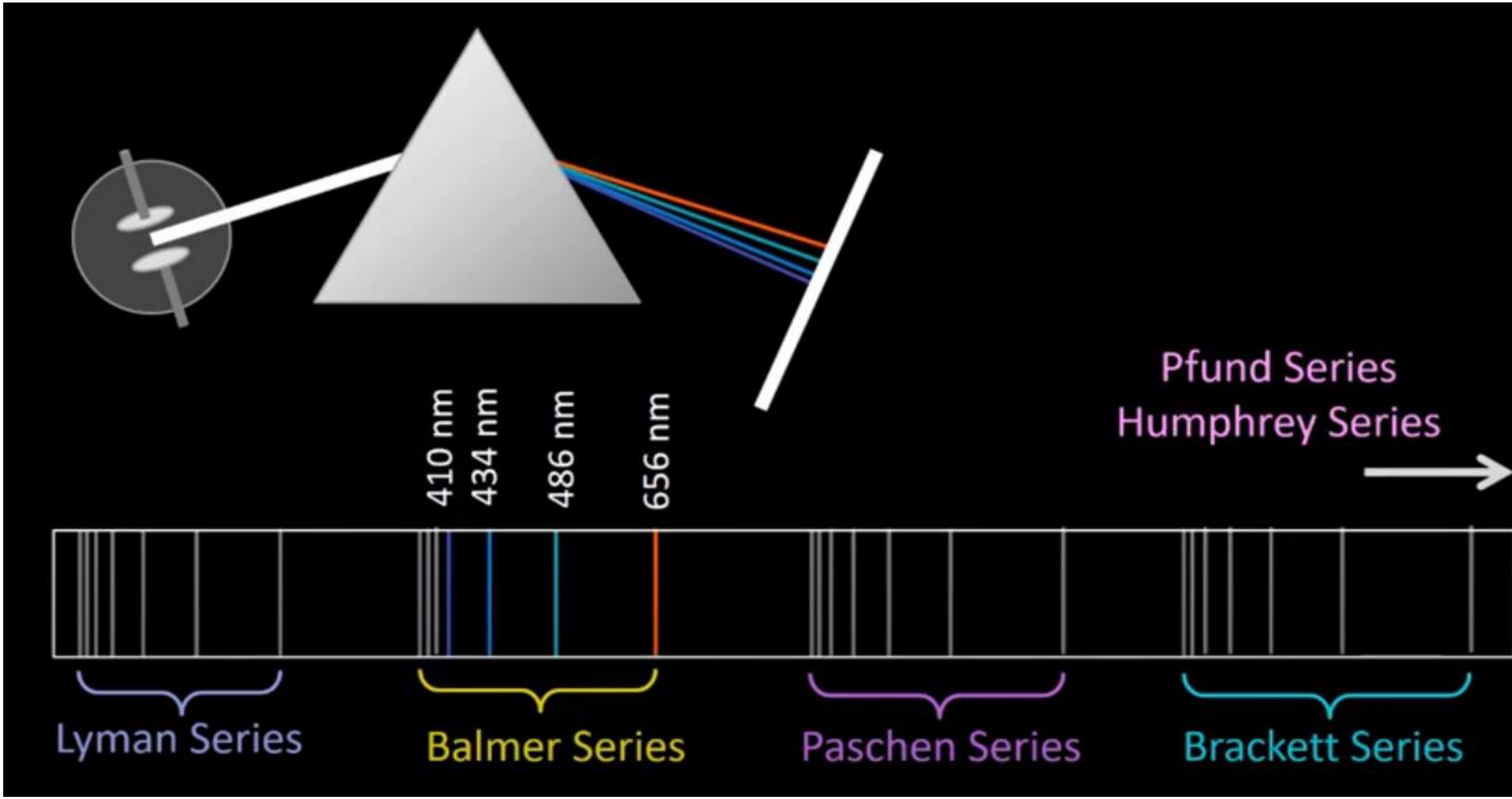


EMISSION SPECTRUM



Hydrogen Spectra

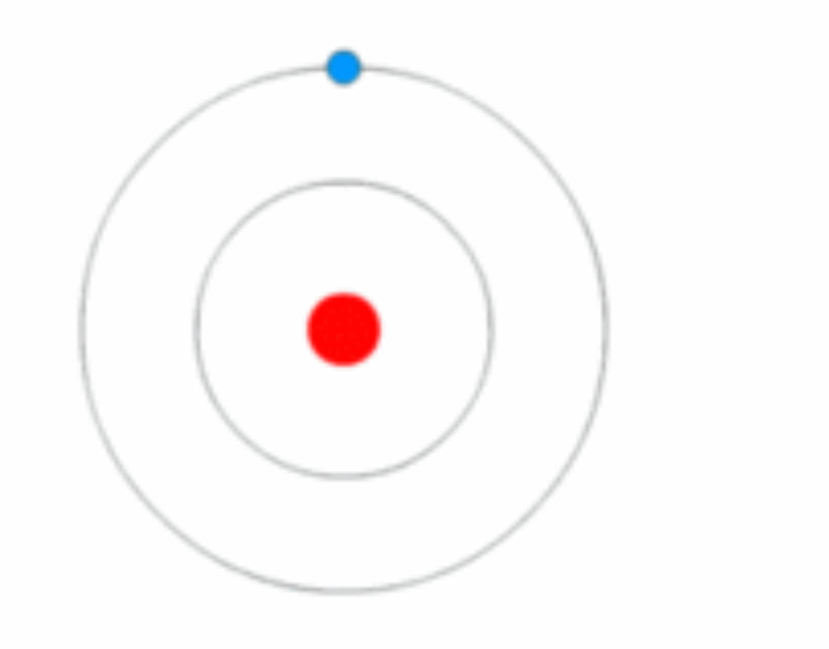



Pfund Series
Humphrey Series
410 nm
434 nm
486 nm
656 nm
Lyman Series
Balmer Series
Paschen Series
Brackett Series

Bohr's theory is unable to explain the following facts :

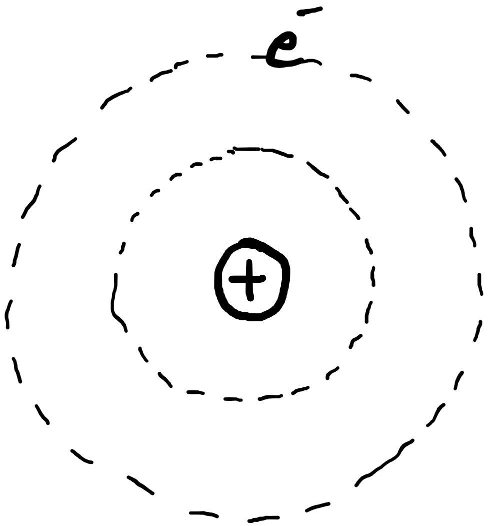
- The spectral lines of hydrogen atom are not single lines but each one is a collection of several closely spaced lines.
- The structure of multi electron atoms is not explained.
- No explanation for using the principles of quantization of angular momentum.
- No explanation for Zeeman effect. If a substance which gives a line emission spectrum is placed in a magnetic field, the lines of the spectrum get splitted up into a number of closely spaced lines. This phenomenon is known as Zeeman effect.

ATOMIC STRUCTURE

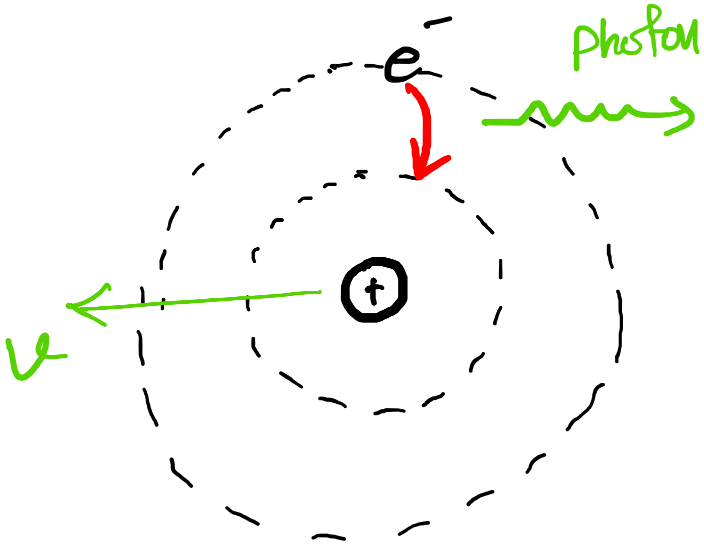


Lecture - 3

Recoil Velocity of Atom



initially at rest



$$\vec{P}_i = \vec{P}_f$$

$$0 = \frac{h}{\lambda}\hat{i} - mv\hat{i}$$

$$\boxed{mv = \frac{h}{\lambda}}$$

Q The recoil speed of a free H-atom at rest in fourth excited state, when it undergoes a transition to ground state is ______ (mass of the H atom = 1.67×10^{-27}) in m/s is [Take $h=6.68\times10^{-34}$ Js and $R_H=1\times10^{7}$ m^{-1}]

A) 2.5 m/s B) 3.8 m/s C) 8.6 m/s D) 12.2 m/s

Ans. [B]

Sol^n

$z = 1$

$$\frac{1}{\lambda} = R_H Z^2\left[\frac{1}{1^2} - \frac{1}{5^2}\right]$$

$$\frac{h}{\lambda} = hR_H\left[\frac{24}{25}\right]$$

also $mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} = \frac{hR_H}{m}\left[\frac{24}{25}\right]$

$$v = \frac{6.68\times\cancel{10^{-34}}\times1\times\cancel{10^{7}}\times24}{1.67\times\cancel{10^{-27}}\times25}$$

$$\boxed{v = 3.84\ m/s}$$ Ans

Atomic Collision:

Q. An neutron with kinetic energy 5eV is incident on a H-atom in its ground state. The collision:

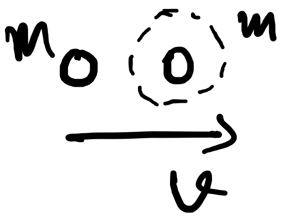
(A) must be elastic ✓ (B) may be partially elastic

(C) must be completely inelastic (D) may be partially inelastic

Sol^{n} Energy loss is maximum when collision is perfectly inelastic

momentum Conservation $mu = (m+m)v$

$$\boxed{v = u/2}$$



after Collision

max. loss is 2.5eV but for excitation, min. energy required is 10.2eV for H-atom

So Collision is elastic

KE loss : $\Delta K = \left(\frac{1}{2}mu^2\right) - \frac{1}{2}(2m)\frac{u^2}{4}$

$\Delta K = \frac{1}{2}\left(\frac{1}{2}mu^2\right) \Rightarrow$ max. energy loss is half of initial energy

Q **In previous question, what should be the minimum energy of neutron to make collision inelastic.**

(1) 10 eV **(2) 20.4 eV**

(3) 10.2 eV **(4) 40.8 eV**

Ans. [2]

As in previous question, max. energy loss is half of initial energy so to make collision inelastic (to excite e^-), the min. energy of neutron required is

2×10.2 eV

$= \underline{\underline{20.4 \text{ eV}}}$

Ans

Q. An electron of energy 10.8 eV undergoes an inelastic collision with a hydrogen atom in its ground state. Then (assuming $m_H >> m_e$, neglecting recoil of atom) -

(A) the outgoing electron has energy 10.8 eV

(B) 10.2 eV of the incident electrons energy is absorbed by H-atom and the electron would come out with 0.6 eV energy

(C) the entire energy is absorbed by H-atom and the electron stops

(D) none of the above

Sol^n (i) $\frac{M}{L} = \frac{q}{2m}$

$$M = \frac{e}{2m}\frac{nh}{2\pi}$$

$$\boxed{M = \frac{eh}{4\pi m}}$$ for $n=1$

(ii) $\tau = MB\sin\theta$

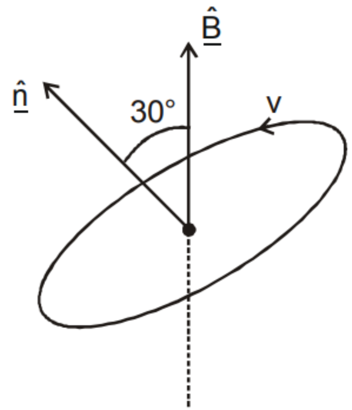
$$\tau = \left(\frac{eh}{4\pi m}\right)B\left(\frac{1}{2}\right)$$

$$\boxed{\tau = \frac{ehB}{8\pi m}}$$

Q. ***An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in the circular orbit ~~of radius R~~ as shown in figure***

(i) Obtain an expression for the orbital magnetic dipole -
moment of the electron.

(ii) The atom is placed in a uniform magnetic induction B such that the plane normal of the electron orbit makes an anlge 30° with the magnetic induction. Find the torque experienced by the orbiting electron.



Q. Suppose that the potential energy of an hypothetical atom consisting of a proton and an electron is given by $U = -ke^2/3r^3$. Then if Bohr's postulates are applied to this atom, then the radius of the n^{th} orbit will be proportional to - ________

Sol^n

$$F = -\frac{dU}{dr}$$

$$F = \frac{ke^2}{r^3\,\cancel{r^4}} = \frac{mv^2}{\cancel{r}} \quad -(1)$$

$$mvr = \frac{nh}{2\pi} \quad -(2)$$

by (1) & (2)

$$\frac{ke^2}{rm} = \frac{n^2h^2}{4m^2\pi^2}$$

$$\Rightarrow \boxed{r \propto \frac{1}{n^2}}$$

Ans

$Z=1$

Q. In a hypothetical atom like that of hydrogen, the mass of the electrons is doubled. Then the energy E_0 and radius r_0 of the first Bohr orbit will be (a_0 = Bohr radius of hydrogen) -

$n=1$

(1) $E_0 = -27.2$ eV ; $r_0 = a_0/2$ ✓
(2) $E_0 = -27.2$ eV ; $r_0 = a_0$
(3) $E_0 = -13.6$ eV ; $r_0 = a_0/2$
(4) $E_0 = -13.6$ eV ; $r_0 = a_0$

Sol^n

$$E = -\frac{m Z^2 e^4}{8\varepsilon_0^2 n^2 h^2} \Rightarrow E = -13.6\,\frac{Z^2}{n^2} \times \left(\frac{2m}{m}\right) \text{eV} \Rightarrow \boxed{E_0 = -27.2\text{eV}}$$

Ans

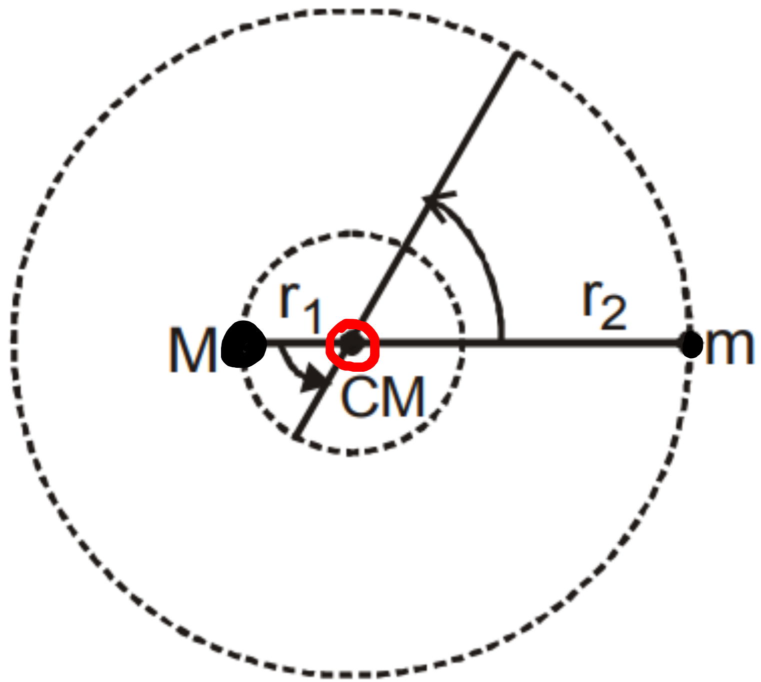
$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2} \Rightarrow r = 0.53\,\frac{n^2}{Z} \times \left(\frac{m}{2m}\right) \text{Å} \Rightarrow \boxed{r = \frac{a_0}{2}\ \text{Å}}$$

$0.53 \rightarrow a_0$

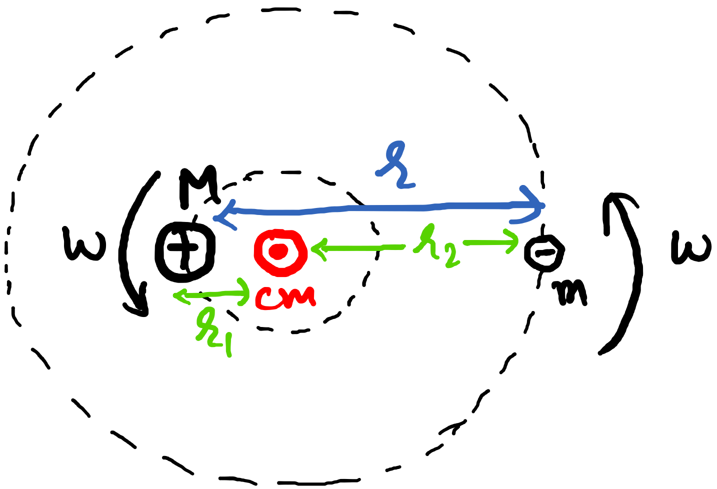
Ans

APPLICATION OF NUCLEUS MOTION ON ENERGY OF ATOM

Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level.



Let r be the distance between the nucleus and the electron. Then



$$r_1 + r_2 = r$$

$$M r_1 = m r_2$$

$$r_1 = \frac{m r}{m+M}$$

$$r_2 = \frac{M r}{M+m}$$

$$F = m\omega^2 r_2$$

$$\frac{(Ze)(e)}{4\pi\varepsilon_0 r^2} = m\omega^2 r_2$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m\omega^2 M r}{M+m}$$

$$\underbrace{\left(\frac{mM}{m+M}\right)}\omega^2 r^3 = \frac{Ze^2}{4\pi\varepsilon_0}$$

$$\mu r^3 \omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$$

where $\mu = \dfrac{mM}{m+M}$

Moment of inertia about C.M.

$$I = m r_2^2 + M r_1^2$$

$$I = \frac{m M^2 r^2}{(M+m)^2} + \frac{M m^2 r^2}{(M+m)^2}$$

$$\boxed{I = \mu r^2}$$

$$\mu = \frac{mM}{m+M}$$

According to 4^{th} Postulate:

$$I\omega = \frac{nh}{2\pi}$$

$$\mu r^2 \omega = \frac{nh}{2\pi}$$

$(\omega = v/r)$

$$\boxed{r = \frac{\varepsilon_0 n^2 h^2}{Z e^2 \pi \mu}}$$

$$\mu = \frac{Mm}{M+m}$$

$$\boxed{r = 0.53 \frac{n^2}{Z} \times \frac{m}{\mu}} \text{ Å}$$

Potential Energy:

$$U = \frac{-\mu Z^2 e^4}{4\varepsilon_0^2 n^2 h^2}$$

Kinetic Energy:

$$K = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\mu = \frac{mM}{m+M}$$

Total energy:

$$E = K + U$$

$$\boxed{E = \frac{-\mu Z^2 e^4}{8\varepsilon_0^2 n^2 h^2}}$$

$$\boxed{E = -13.6 \frac{Z^2}{n^2} \times \frac{\mu}{m}} \text{ eV}$$

$$\mu = \frac{mM}{m+M}$$