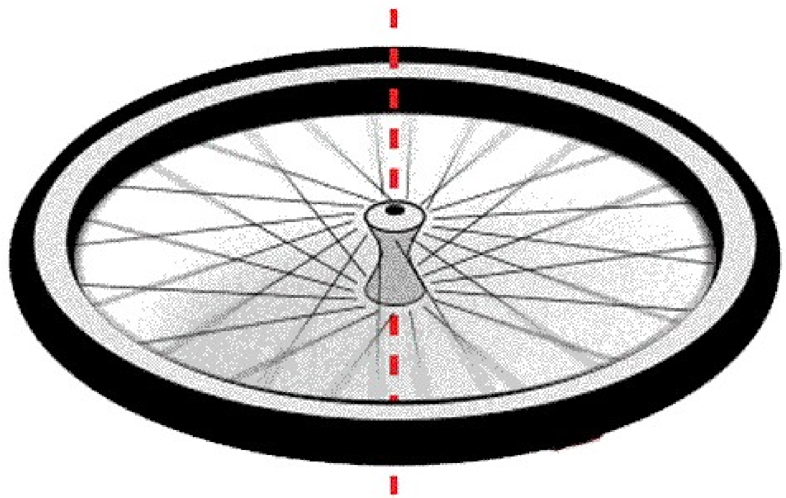
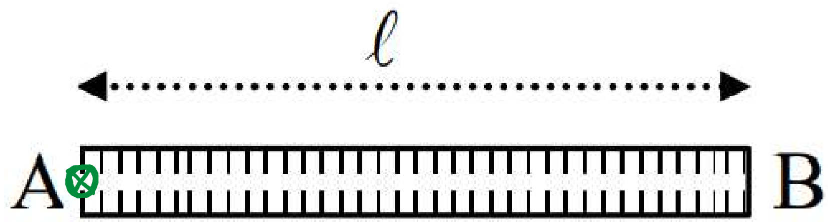
ROTATIONAL MOTION



Lecture - 3

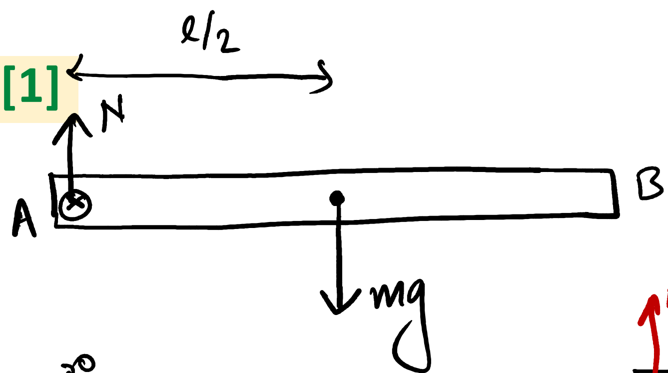
Q. A uniform rod AB of length *l* and mass m is free to rotate about A. The rod is released from rest in the horizontal position, the initial angular acceleration of the rod will be-

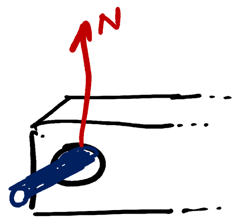


(1) $\frac{3g}{2\ell}$ (2) $\frac{2g}{3\ell}$ (3) $mg\frac{\ell}{2}$ (4) $\frac{3}{2}g\ell$

Ans. [1]

Sol.





$$\tau = \tau_N + \tau_{mg} \quad (\tau_N \to 0)$$

$$\tau = mg\frac{\ell}{2}$$

$$I\alpha = \frac{mg\ell}{2}$$

$$\frac{m\ell^2}{3}\alpha = \frac{mg\ell}{2} \Rightarrow \boxed{\alpha = \frac{3g}{2\ell}}$$

Q. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about the axis. The torque which can stop the wheel's rotation in one minute would be-

(1) $\frac{\pi}{12}$ N-m

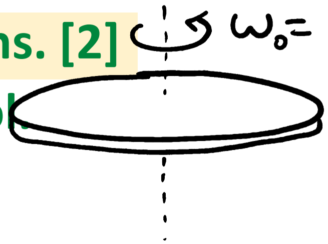
(2) $\frac{\pi}{15}$ N-m

(3) $\frac{\pi}{18}$ N-m

(4) $\frac{2\pi}{15}$ N-m

Ans. [2]

Sol.



$\omega_0 = 60\,rpm = 2\pi\ rad/s$

$\omega_f = 0$

$t = 60\ sec$

$\omega_f = \omega_i + \alpha t$ — (1)

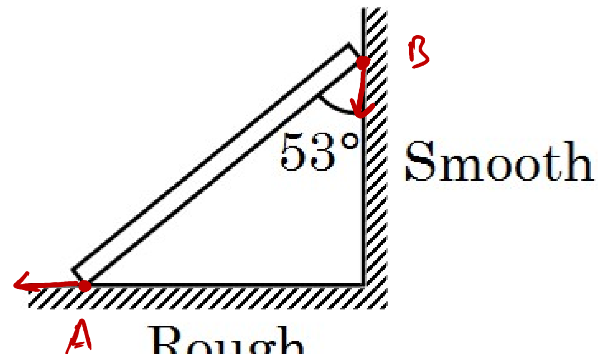
$0 = 2\pi + \alpha\, 60$

$\Rightarrow \alpha = -\frac{\pi}{30}\ rad/s^2$

$\tau = I\alpha$

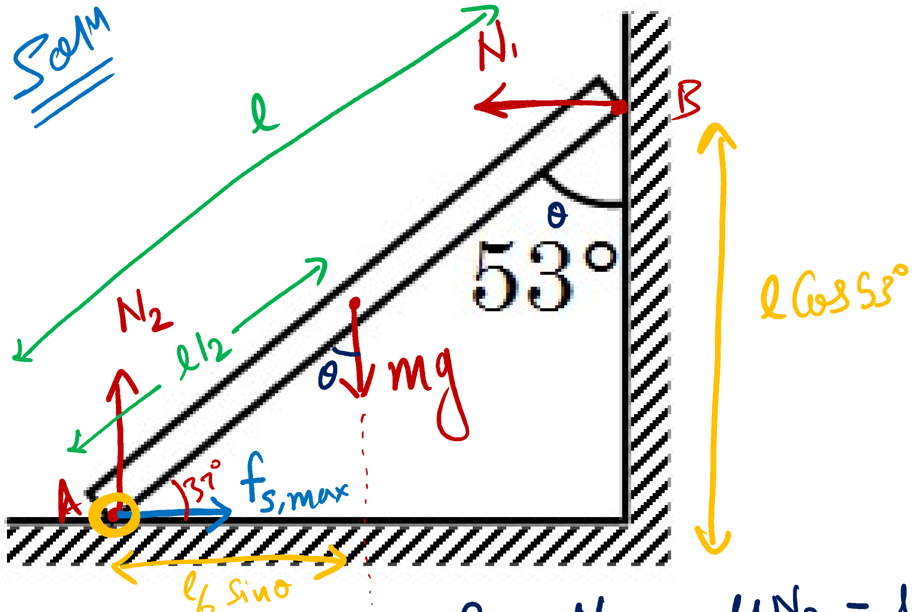
$= (2)\left(\frac{\pi}{30}\right) = \frac{\pi}{15}\ Nm$

Q. A uniform ladder of mass 10 kg leans against smooth vertical wall making an angle 53° with it. The other end rests on rough horizontal floor. The friction coefficient just necessary for ladder to be at rest is approximately.



(A) 0.45 (B) 0.75 (C) 0.55 (D) 0.67

Soln



Eqm (Trans.)

x-dir: $f = N_1 = \mu N_2 = \mu mg$

y-dir: $mg = N_2$

Rot.

$\tau_{net} = 0$ (About point A)

$-mg\frac{l}{2}\sin 53^\circ + N_1 l\cos 53^\circ = 0$

$-mg\frac{l}{2}\cdot\frac{4}{5} + \mu mg\, l\,\frac{3}{5} = 0$

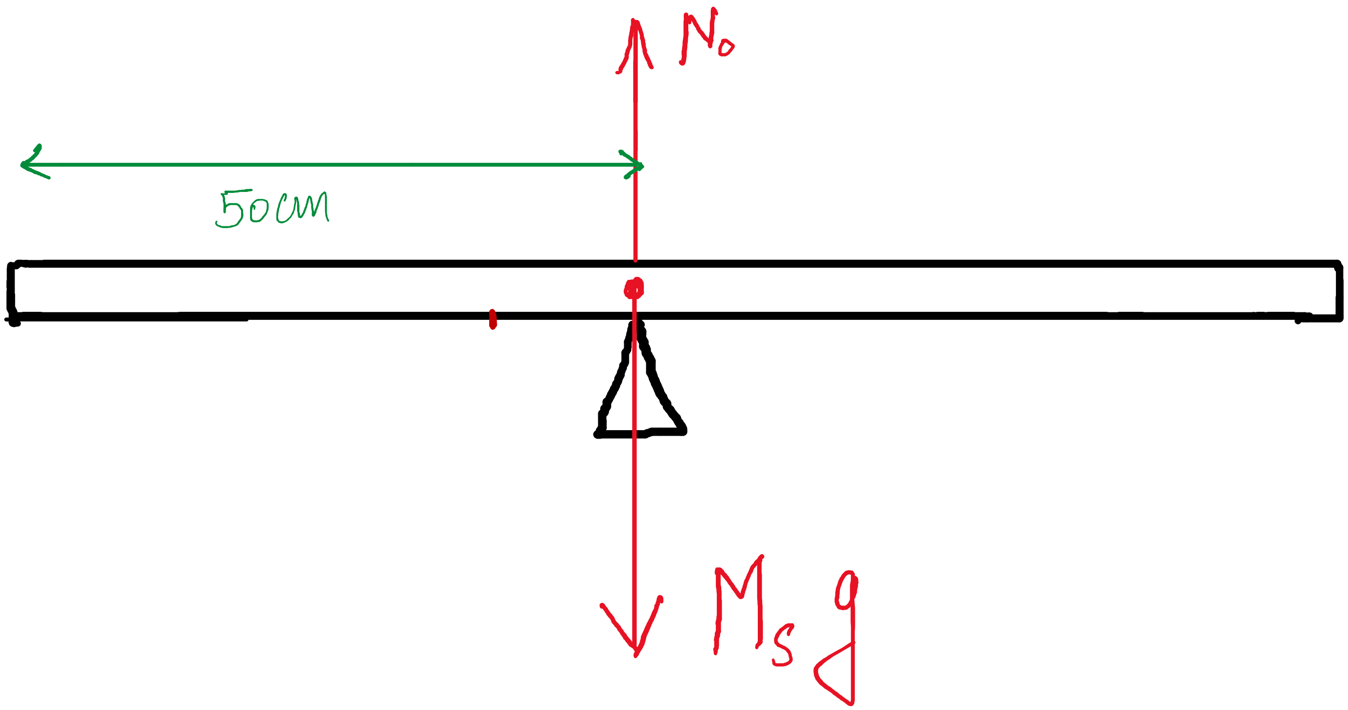
$-2 + 3\mu = 0$

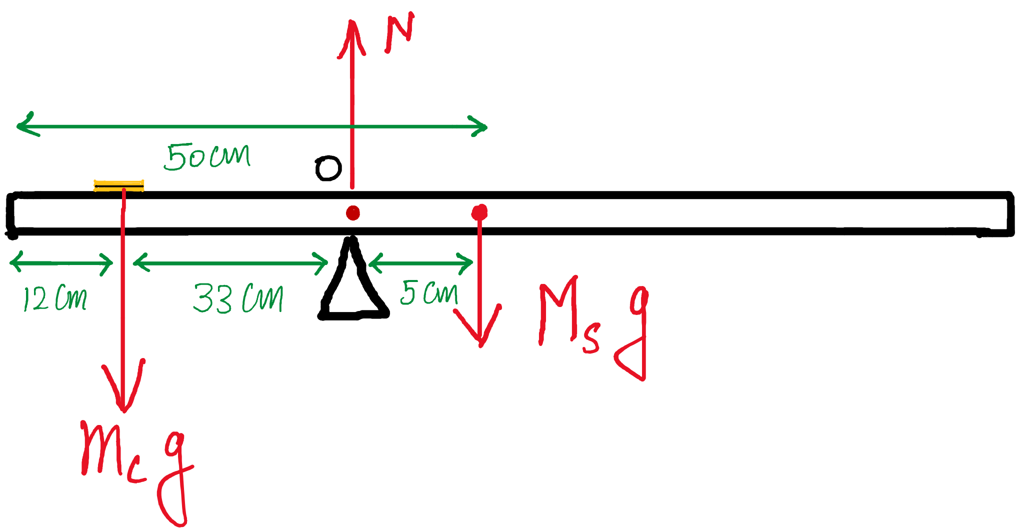
$\mu = \frac{2}{3}$

$\boxed{\mu = 0.67}$

Q. **A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?**

Solⁿ





For Rotational Equilibrium:

$\tau_{net} = 0$ about every point

Putting net torque zero about 'o'

$$m_c g \frac{(33)}{100} - M_s g \frac{(5)}{100} = 0$$

$$M_s = \frac{33(m_c)}{5} = \frac{33 \times (10\,g)}{5}$$

$$\boxed{M_s = 66 \text{ gram}}$$ Ans

For translational Equilibrium:

$$N = m_c g + M_s g$$

$$N = \frac{10}{1000} \times 10 + M_s(10)$$

$$N = 0.1 + M_s$$ ×

Q. Two men support a uniform horizontal beam at its two ends. If one of them suddenly lets go, the force exerted by the beam on the other man will

(1) remain unaffected

(2) increase

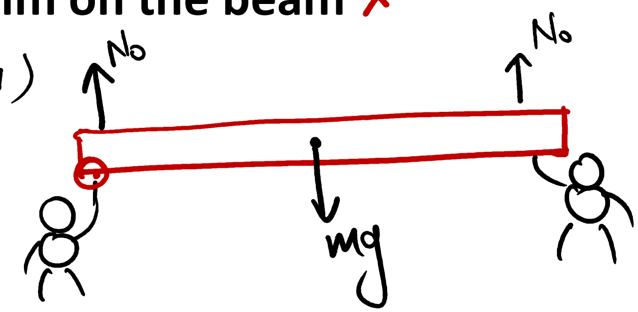
(3) decrease

(4) become unequal to the force exerted by him on the beam

Ans. [3]

Sol. (i) When both support, the force exerted by each = $\frac{mg}{2}$

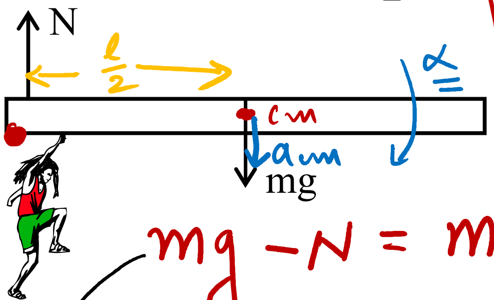
(i)



$2N_0 = mg$

$N_0 = \frac{mg}{2}$

(ii)



~~$N = mg$~~

$N < mg$

$a = \frac{l}{2}\alpha$

$mg - N = m a_{cm}$

$mg\frac{l}{2} = \left(\frac{ml^2}{3}\right)\alpha = \frac{ml^2}{3}\frac{2a_{cm}}{l}$

$a_{cm} = \frac{3g}{4}$

$mg - m a_{cm} = N$

$m\left(g - \frac{3g}{4}\right) = N$

$N = mg/4$

Ans. [4]

Sol.

Net torque about O should be zero, hence

$$mg\left(\frac{\ell}{2}\sin 60^\circ\right) = Mg\left(\frac{\ell}{2}\sin 30^\circ\right)$$

$$\frac{M}{m} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3}$$

HW

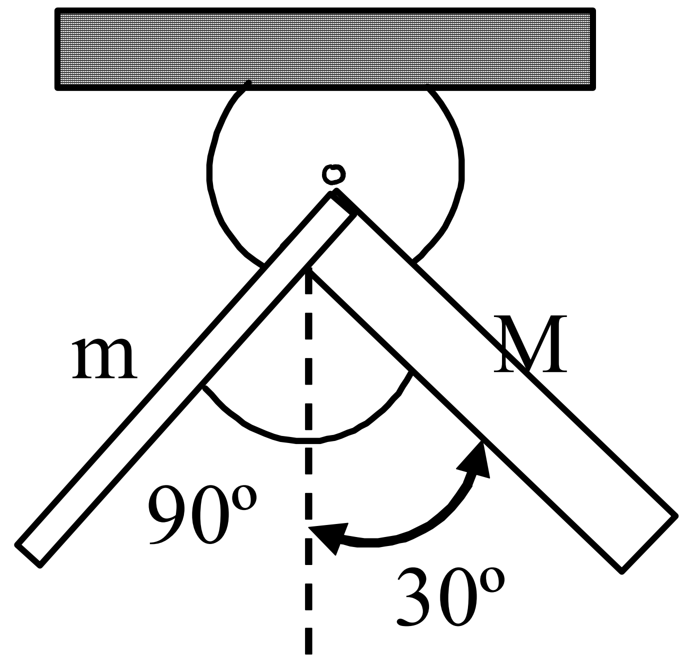
Q. Two uniform rods of equal length but different masses are rigidly joined to form an L–shaped body, which is then pivoted as shown. If in equilibrium the body is in the shown configuration, ratio M/m will be –

(1) 2

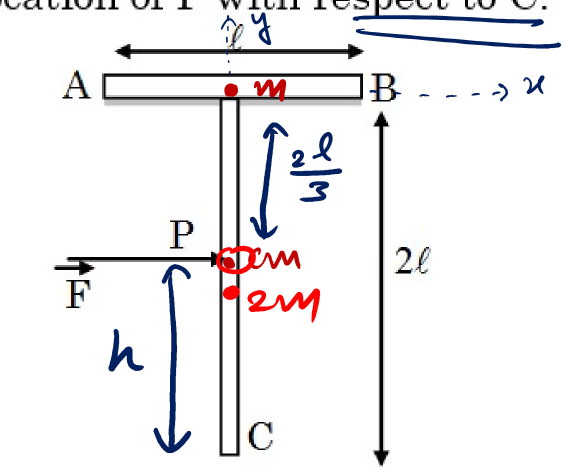
(2) 3

(3) $\sqrt{2}$

(4) $\sqrt{3}$



Q. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(A) $\frac{2}{3}\ell$ (B) $\frac{3}{2}\ell$ (C) $\frac{4}{3}\ell$ (D) ℓ

Sol^n

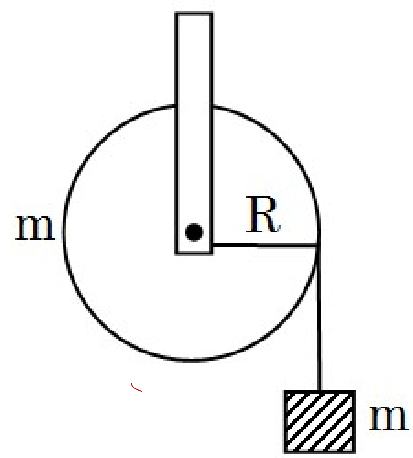
$$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2}{m_1 + m_2} = \frac{m(0) + 2m(-\ell)}{3m}$$

$$Y_{cm} = -\frac{2}{3}\ell$$

$$h = 2\ell - \frac{2\ell}{3}$$

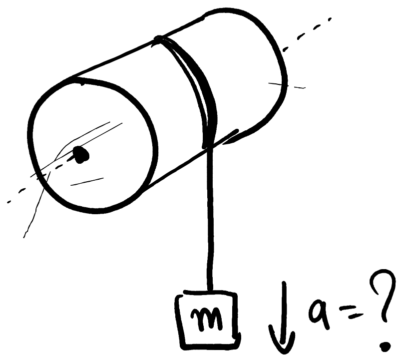
$$h = \frac{4\ell}{3}$$

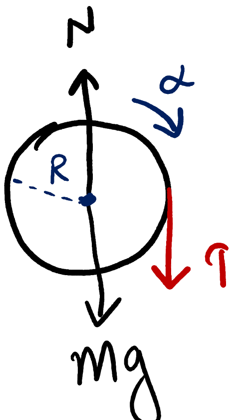
Q. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?

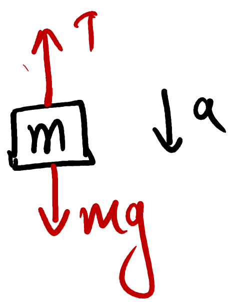


(A) $\frac{g}{2}$ (B) $\frac{5g}{6}$ (C) g (D) $\frac{2g}{3}$

Sol^n







$\tau_{net} = I\alpha$

$TR = (MR^2)\frac{a}{R}$

$T = ma$

$mg - T = ma$

$mg = 2ma$

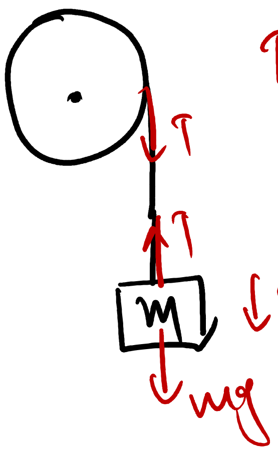
$a = \frac{g}{2}$

Q. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is:

(A) $\frac{3}{2}g$ (B) g (C) $\frac{2}{3}g$ (D) $\frac{g}{3}$

Sol

$I = \frac{MR^2}{2}$



$TR = \left(\frac{MR^2}{2}\right)\frac{a}{R}$

$T = \frac{Ma}{2}$

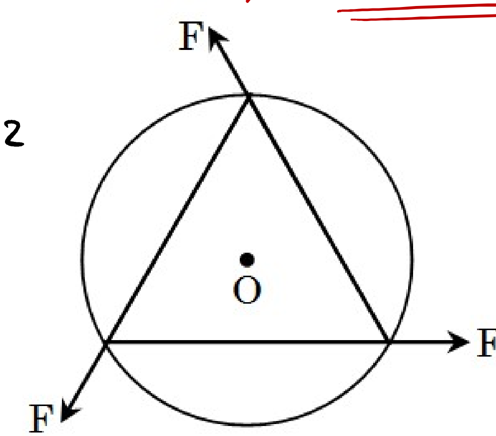
$mg - T = ma$

$mg = \frac{3ma}{2}$

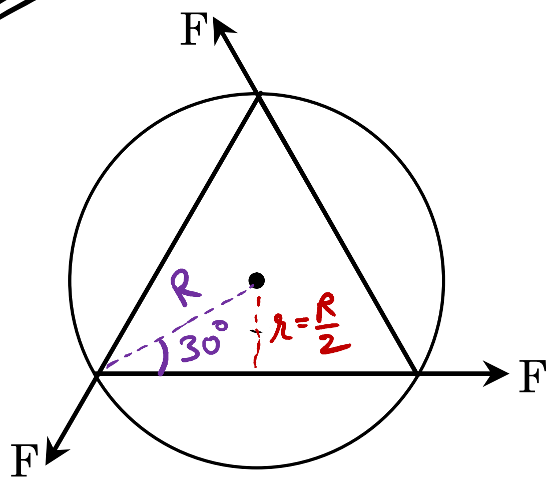
$a = \frac{2g}{3}$

Q. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is.

[JEE-Advance-2014]



Sol^n



$I = \frac{MR^2}{2}$

$\tau = \frac{3FR}{2}$

$I\alpha = \frac{3FR}{2}$

$\alpha = \frac{3FR}{2I} = \frac{3FR}{2\frac{MR^2}{2}}$

$\alpha = \frac{3F}{MR} = \frac{3(\frac{1}{2})}{(\frac{3}{2})\frac{1}{2}} = 2\ rad/s^2$

at t = 1

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 2(1)$

$\omega = 2\ rad/s$ Ans