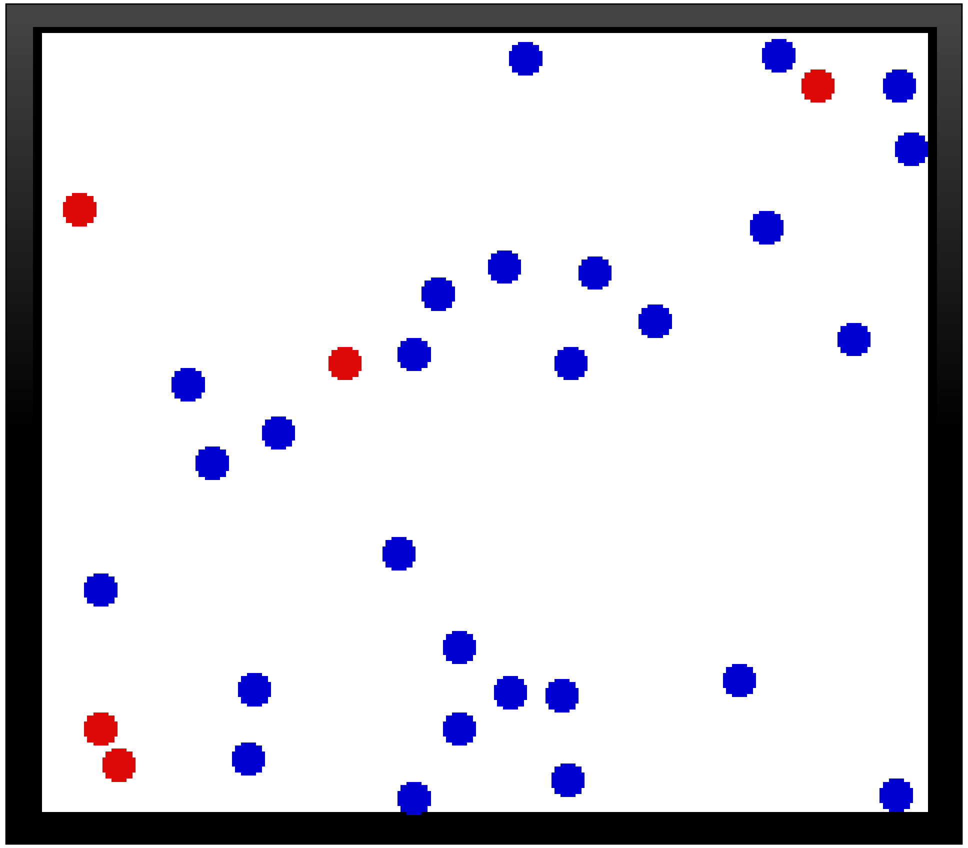
KINETIC THEORY OF GASES



Revision

GAS LAWS

Ideal Gas Law :

$$\frac{PV}{nT} = R$$ → universal gas constant.

$$\boxed{PV = nRT}$$

$R = 0.0821$ atm-L/mol-K

or

$R = 8.314$ J/mol-K

or

$R = 2$ Cal/mol-K

n = no. of moles

P = Pressure of Gas

V = Volume of gas

$n = \frac{m}{M}$ m → given mass, M → molar mass

$n = \frac{N}{N_A}$ N → no. of molecules, N_A → Avogadrono.

- if n & T are Const ⇒ $P \propto \frac{1}{V}$ (Boyle's law)
- if n & P are Const ⇒ $V \propto T$ (Charle's law)
- if n & V are Const ⇒ $P \propto T$ (Lussac's Law)
- if P & T are Const ⇒ $n \propto V$ (Avogadro's Principle)

Boltzmann Constant : $K = \frac{R}{N_A}$

Example.

A sample of oxygen with volume 1000 cc at a pressure of 1 atm is compressed to a volume of 900 cc. What pressure is needed to do this if the temperature is kept constant ?

$$PV = \underline{\underline{n}}\,\underline{\underline{R}}\,\underline{\underline{T}}$$

$$P_1 V_1 = P_2 V_2$$

$$(1\,atm)(1000\,cc) = P_2\,(900\,cc)$$

$$\underline{\underline{P_2 = \frac{10}{9} = 1.11\,atm}}$$

Example.

V_1 $T_1 = 296K$

1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person. $T_2 = 310K$ $V_2 = ?$

Solⁿ

$$PV = nRT$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{T_2}{T_1} \times V_1 = \frac{310}{296}(1500 ml)$$

$$\boxed{V_2 = 1570.95 ml}$$

A) 1570.95 ml ✓

B) 1520.27 ml

C) 2413.04 ml

D) 1830.25 ml

Example.

$T_1 = 300K$

A sample of helium gas is at a pressure of 1 atm (P_1) when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm (P_2) and volume remains 100 ml.

$T_2 = ?$

A) 54°C

B) 600°C

C) 327°C ✓

D) 540°C

Sol^n

$$PV = nRT$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = \frac{P_2}{P_1} \times T_1 = \frac{2}{1} \times 300$$

$$T_2 = 600K \Rightarrow \boxed{T_2 = 327°C} \text{ Ans}$$

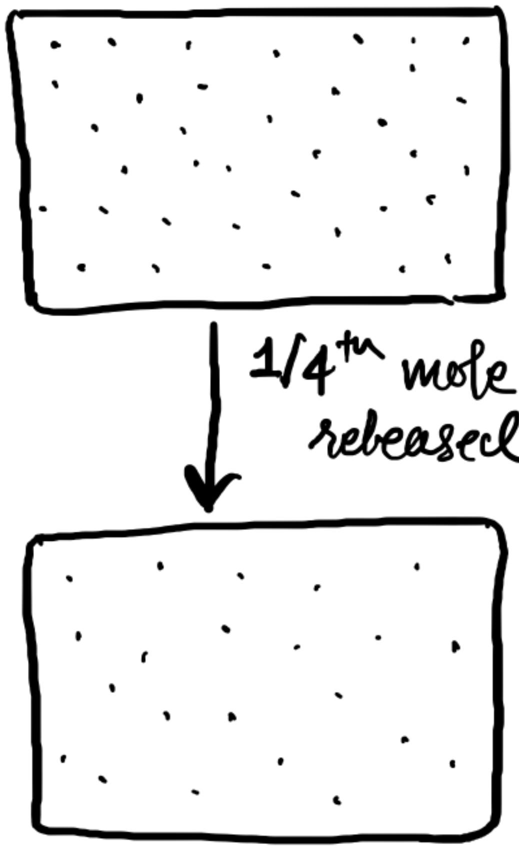
Example.

An ideal gas has been placed in a tank at 37°C. The gauge pressure is initially 609 kPa. If one fourth of the gas is released from the tank and after thermal equilibrium is established temperature becomes 317°C, then the gauge pressure will be – (atmospheric Pressure = 101 kPa) P_0

(A) 101 kPa (B) 710 kPa (C) 912 kPa (D) 1013 kPa

Sol^n

initial



$P_1 = 710\,kPa$
$V_1 = V$
$T_1 = 310\,K$
$n_1 = n$

final

$P_2 = ?$
$V_2 = V$
$T_2 = 590\,K$
$n_2 = \frac{3}{4}n$

$(P_1, V_1, T_1, n_1) \to (P_2, V_2, T_2, n_2)$

$\frac{PV}{nT} = \text{Const.}$

$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$

$P_2 = \frac{n_2}{n_1} \frac{T_2}{T_1} \frac{\cancel{V_1}}{\cancel{V_2}} P_1$

$P_2 = \left(\frac{3}{4}\right) \frac{590}{310} (710\,kPa)$

$P_2 = 1013\,kPa$

final gauge Pressure is

$\Delta P_2 = P_2 - P_0$

$\boxed{\Delta P_2 = 912\,kPa}$ Ans

$n = \frac{m}{M}$

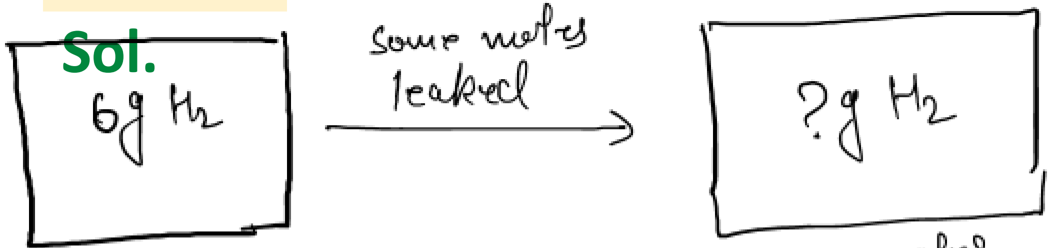
Q A vessel has 6 g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{2}$ and temperature falls to 300 K ?

(1) 2 g (2) 3 g

(3) 4 g (4) 1 g

Ans. [4]

Sol.



n_1 = moles
$P_1 = P$
$T_1 = 500K$
$V_1 = V$

n_2 = moles
$P_2 = P/2$
$T_2 = 300K$
$V_2 = V$

$$\frac{P_1 \cancel{V_1}}{n_1 T_1} = \frac{P_2 \cancel{V_2}}{n_2 T_2}$$

$$n_2 = \frac{P_2}{P_1} \frac{T_1}{T_2} n_1$$

$$n_2 = \left(\frac{1}{2}\right)\left(\frac{5}{3}\right) n_1$$

$$n_2 = \frac{5}{6} n_1$$

no. of moles leaked $= n_1 - n_2 = \frac{1}{6} n_1 = \frac{1}{6}\left(\frac{6}{2}\right) = \frac{1}{2}$

mass of H_2 leaked = (no. of moles leaked) × M_{H_2}

$= \frac{1}{2} \times 2 = 1g$

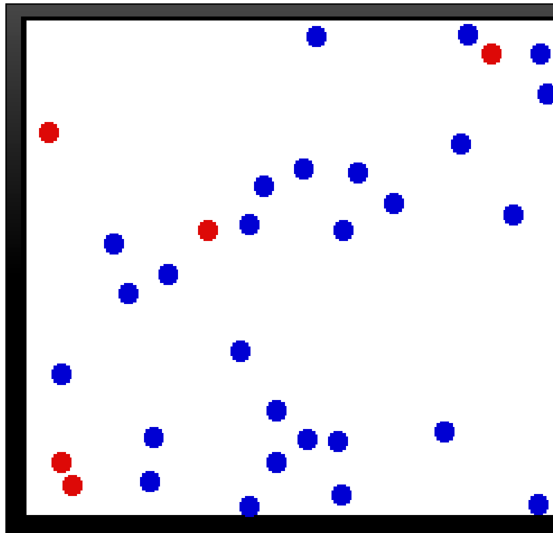
Ans

Assumptions Of KTG

1. A gas consists of a large number of molecules which move randomly in all directions.

2. The size of a molecule is much smaller than average separation between the molecules.

3. There is no force of attraction between molecules i.e. gas molecules do not interact with each other so total internal energy is total kinetic energy of molecules.

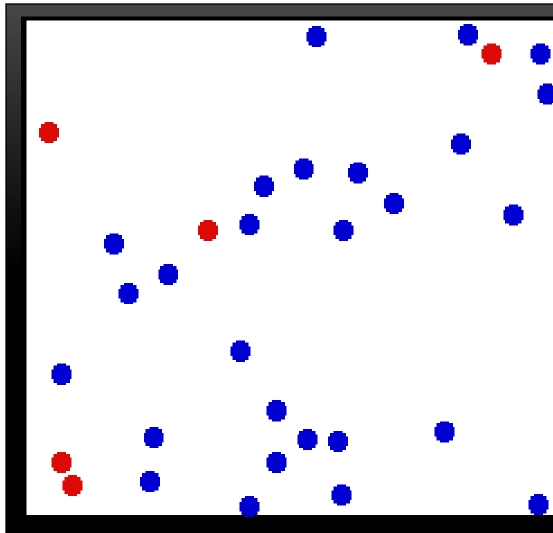


4. This theory is based on translation motion of gas molecules and it obey Newton’s Laws of motion.

5. The collision between two molecules or between a molecule and a wall are perfectly elastic.

6. Volume occupied by gas molecule is very small or negligible as compared to volume of vessel.

7. Gas molecule move with very high velocity, therefore effect of gravity on them is negligible due to which gas density remains same throughout the vessel.



Instantaneous Velocity: The Velocity of molecules at any instant.

most probable Speed:

$$\boxed{v_{mps} = \sqrt{\frac{2RT}{M}}}$$

M = molar mass

(*) put every variable in SI unit then we get v in m/s

eg O_2 $M = 32\,g/mol = \underline{\underline{32 \times 10^{-3}\,kg/mol}}$
SI

average Velocity

$$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2 + \dots \vec{v}_N}{N}$$

$$\boxed{\vec{v}_{av} = 0}$$

average speed

$$v_{av} = \frac{|\vec{v}_1| + |\vec{v}_2| + \dots |\vec{v}_N|}{N}$$

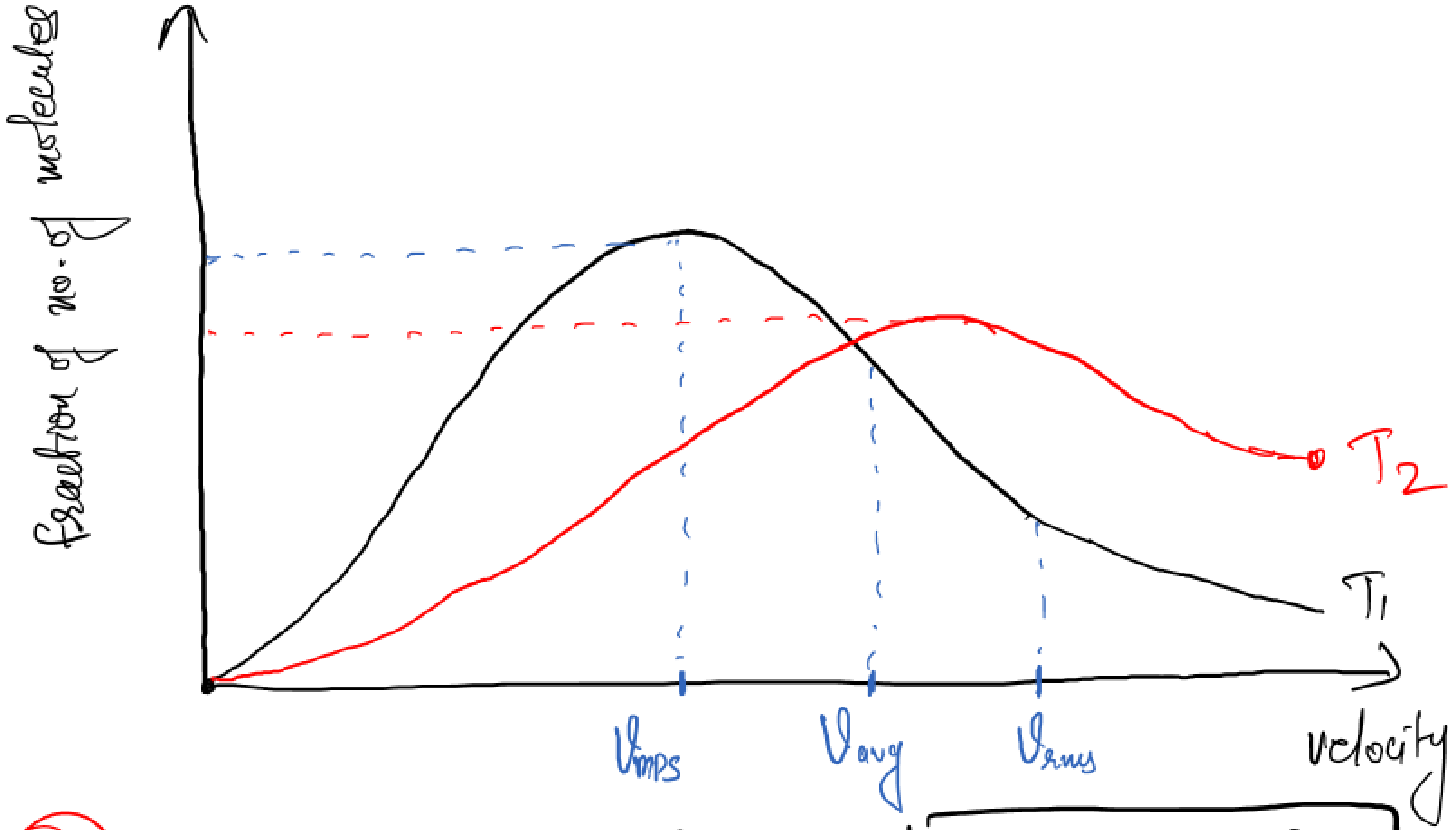
$$\boxed{v_{av} = \sqrt{\frac{8RT}{\pi M}}}$$

RMS Speed:

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$$

$$\boxed{v_{rms} = \sqrt{\frac{3RT}{M}}}$$

Maxwell - Boltzmann Distributive Curve



$T_2 > T_1$

Pressure of ideal gas : $\boxed{P = \frac{1}{3} \rho V_{rms}^2}$ $\rho = \frac{m}{V}$

Example.

A flask of $10^{-3}\ m^3$ volume contains 3×10^{22} molecules of oxygen at a certain temperature. The mass of one molecule of oxygen is 5.3×10^{-26} Kg and rms velocity of its molecules at the same temperature is 400 m/s. Calculate the pressure of the gas.

V   N m_o v_{rms}

Sol^n

$$P = \frac{1}{3}\rho v_{rms}^2$$

$$P = \frac{1}{3}\frac{M}{V} v_{rms}^2$$

$$P = \frac{1}{3}\frac{m_o N}{V} v_{rms}^2$$

$$P = \frac{1}{3}\frac{(5.3\times10^{-26})(3\times10^{22})}{10^{-3}}(400)^2$$

$$\boxed{P = 8.48 \times 10^4\ N/m^2}$$

Ans

Example.

What is the rms speed of Helium gas molecules at STP.

Sol^n

$T = 273 K$

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314)(273)}{(4\times10^{-3})}}$$

all in SI unit

$$\boxed{V_{rms} = 1.3\times10^{3}\ m/s}$$

Ans

Example.

The rms speed of oxygen at room temperature is about 500 m/s. The rms speed of Hydrogen at the same temperature is about

(A) 125 m/s (B) 2000 m/s (C) 8000 m/s (D) 31 m/s

Sol

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{v_{O_2}}{v_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$v_{O_2}\sqrt{\frac{M_{O_2}}{M_{H_2}}} = v_{H_2}$$

$$v_{H_2} = 500\sqrt{\frac{32\times\cancel{10^{-3}}}{2\times\cancel{10^{-3}}}}$$

$$v_{H_2} = 500 \times 4$$

$$\boxed{v_{H_2} = 2000\ m/s}$$

Five molecules of a gas have speeds, 1, 2, 3, 4 and 5 kms^{-1}. The value of the root mean square speed of the gas molecules is:

(1) 3 kms^{-1} (2) $\sqrt{11}$ kms^{-1}

(3) $\frac{1}{2}\sqrt{\pi}$ kms^{-1} (4) 3.5 kms^{-1}

Sol^n

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2}{5}} = \sqrt{\frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2}{5}}$$

$$= \sqrt{\frac{1 + 4 + 9 + 16 + 25}{5}} = \sqrt{\frac{55}{5}}$$

$$\boxed{V_{rms} = \sqrt{11} \text{ km/s}}$$ Ans

The average velocity of an ideal gas molecule is:

(1) proportional to $\sqrt{T}$

(2) proportional to T^2

(3) proportional to T^3

(4) zero

At what temperature, is the root mean square speed of molecules of hydrogen is twice of that at STP?

(A) 273 K

(B) 546 K

(C) 819 K

(D) 1092 K

Sol^n

at STP $T_1 = 273K$

$v_{rms} \propto \sqrt{T}$ if M is same

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{2v_1}{v_1}\right)^2 T_1$$

$$\boxed{T_2 = 1092K}$$ Ans

DEGREE OF FREEDOM (f)

Number of modes in which a gas molecule can store energy is called degree of freedom. It can also be referred as the number of modes in which motion of gas molecule takes place.

eg

₹ 2500000

5 lakh insurance

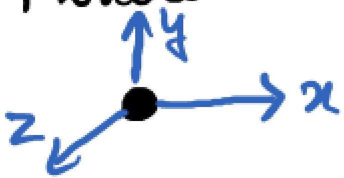
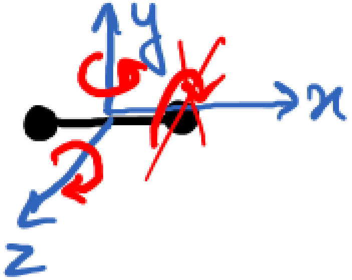
5 lakh Papa

5 lakh Property

5 lakh FD

5 lakh Share

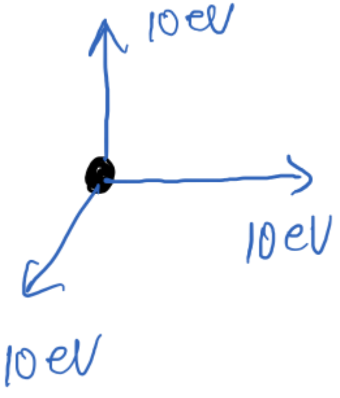
degree of freedom = 5

	Translational	Rotational	Total
(i) Monoatomic 	3	0	3
(ii) Diatomic 	3	2	5
(iii) Triatomic (linear) 	3	2	5
(iv) Triatomic (non linear) or Polyatomic (non linear) 	3	3	6

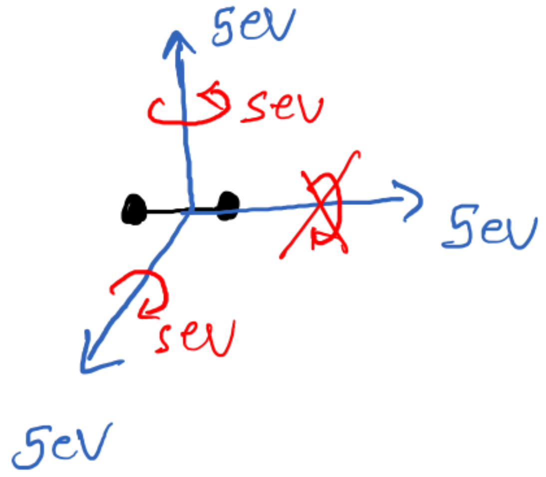
(*) At very high temperature, we add 2 vibrational degree of freedom for diatomic, polyatomic, and triatomic

(If nothing specified, don't assume vibrational D.O.f)

eg if one atom of He has 30 eV energy



eg if one H_2 molecule has 25 eV energy



Maxwell's Law of Equipartition of Energy

I. According to this law, total energy of each gas molecule is equally divided among its degree of freedom.

II. At 0 Kelvin temperature, energy of each degree of freedom is zero

III. At T Kelvin, energy of each degree of freedom is $\frac{1}{2}kT$

Avogadro no.

$$R = k N_A$$

Universal Gas Constant

Boltzmann Constant

IV. If ‘f’ is the degree of freedom of a molecule of gas

Total average energy of each molecule $= \frac{fkT}{2}$

Total energy per mole of gas $= \frac{fkT}{2} N_A$

Total energy of n moles of gas $= n\frac{fkT}{2} N_A = \boxed{n\frac{fRT}{2}}$ *

V. According to KTG, the molecules are not interacting with each other. So potential energy is zero and internal energy of gas molecule is only their kinetic energy.

VI. For n moles of gas, internal energy at temperature T Kelvin is given by:

$$\boxed{U = n\frac{fRT}{2}}$$

VII. Change in Internal energy is given by:

$$dU = n\frac{fR}{2}\mathrm{dT}$$

or

$$\Delta U = n\frac{fR}{2}\Delta T$$

The mean translational kinetic energy of a perfect gas molecule at the temperature T Kelvin is:

(1) $\frac{1}{2}kT$ (2) kT (3) $\frac{3}{2}kT$ (4) 2 kT

Q. The average kinetic energy of a gas molecule at 27ºC is 6.21×10^{-21} J. Its average kinetic energy at 227ºC will be

(1) 52.2×10^{-21} J (2) 5.22×10^{-21} J

(3) 10.35×10^{-21} J (4) 11.35×10^{-21} J

Sol^n

$$E = \frac{f KT}{2}$$

$$\frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{500}{300}$$

$$E_2 = \frac{5}{3} \times 6.21 \times 10^{-21}$$

$$\boxed{E_2 = 10.35 \times 10^{-21} J}$$

Ans

Mean Free Path Of Gas Molecules

It is the average distance moved by a gas molecule between two consecutive collisions.

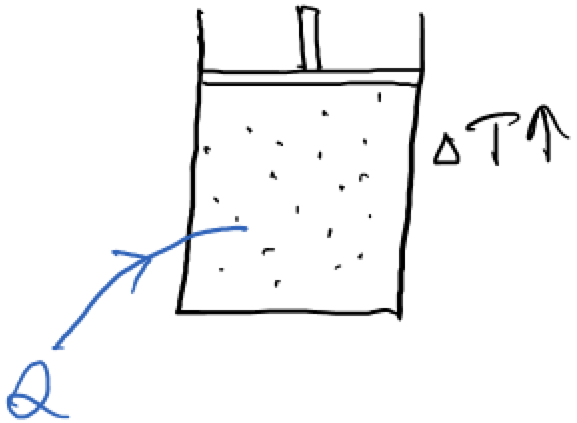
$$\lambda_m \text{ or } \langle l \rangle = \frac{1}{\sqrt{2}\,\pi\, n\, d^2}$$

$d \rightarrow$ diameter

$n \rightarrow$ no. density $= \dfrac{\text{no. of molecules}}{\text{Volume}}$

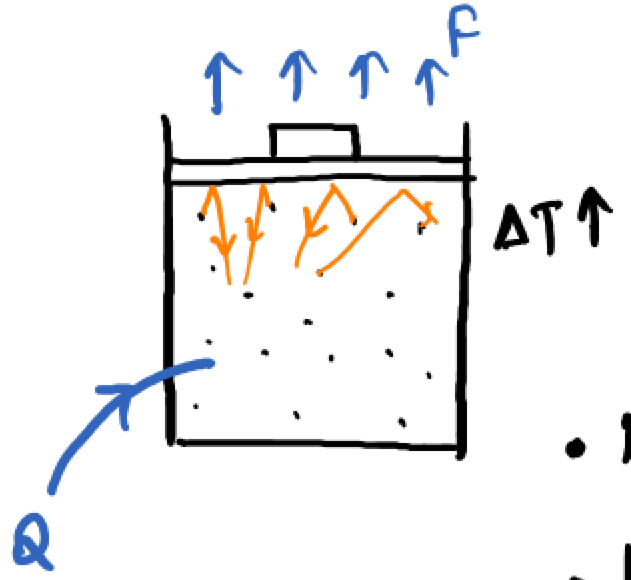
Molar Specific Heat of Gases (c)

- There are infinite ways by which we can supply heat to a gaseous system. So we can define infinite type of specific heat
- Molar specific heat of a substance is the amount of heat required to raise the temperature of 1 mole substance by 1°C



$$\boxed{Q = nC\Delta T}$$

$$C = \frac{Q}{n\Delta T}$$



$$\boxed{Q = nC\Delta T} \qquad C = \frac{Q}{n\Delta T}$$

- Molar specific heat at Const. Pressure : $C_p = \frac{(f+2)R}{2}$
- Molar specific heat at Const. Volume : $C_v = \frac{fR}{2}$
- $C_p - C_v = R$
- Ratio of specific heat : $\gamma = \frac{C_p}{C_v}$

or

adiabatic Constant.

or

Poission's ratio

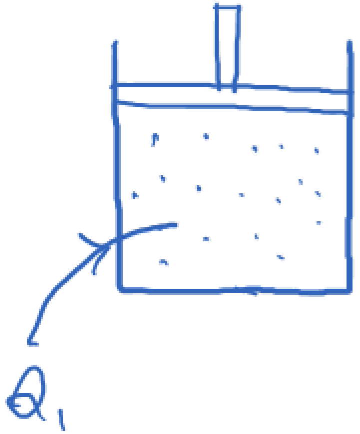
	Monoatomic $f = 3$	Diatomic $f = 5$	Polyatomic $f = 6$
$C_p = \frac{(f+2)R}{2}$	$\frac{5R}{2}$	$\frac{7R}{2}$	$4R$
$C_v = \frac{fR}{2}$	$\frac{3R}{2}$	$\frac{5R}{2}$	$3R$
$\gamma = \frac{f+2}{f}$	$\frac{5}{3} = 1.67$	$\frac{7}{5} = 1.4$	$\frac{4}{3} = 1.33$

Example.

70 calories of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 30ºC to 35ºC. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is –

(A) 30 calories (B) 50 calories (C) 70 calories (D) 90 calories

Sol^n Case(i) Constant Pressure :



$Q_1 = n C_p \Delta T$

$70 \text{ Cal} = 2\, C_p (5°C)$

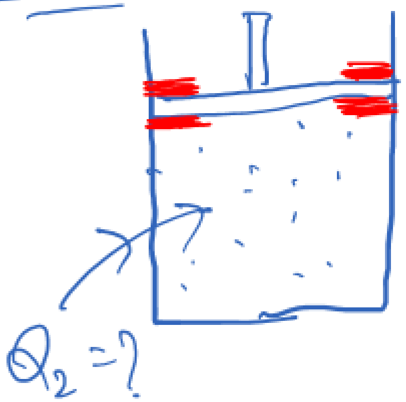
$C_p = 7 \text{ Cal/mol K}$

& $C_p - C_v = R \Rightarrow C_v = C_p - R$

$C_v = 7 \text{ Cal/mol-K} - 2 \text{ Cal/mol-K}$

$\boxed{C_v = 5 \text{ Cal/mol-K}}$

Case(ii) Constant Volume



$Q_2 = n C_v \Delta T$

$Q_2 = (2)(5)(5)$

$\boxed{Q_2 = 50 \text{ Cal}}$

Ans

C_p & C_v for a mixture:

$$C_{P_{mix}} = \frac{n_1 C_{P_1} + n_2 C_{P_2} + \cdots}{n_1 + n_2 + \cdots}$$

$$C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2} + \cdots}{n_1 + n_2 + \cdots}$$

$$\gamma_{mix} = \cancel{\frac{n_1 \gamma_1 + n_2 \gamma_2 + \cdots}{n_1 + n_2}}$$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{n_1 C_{P_1} + n_2 C_{P_2} + \cdots}{n_1 C_{V_1} + n_2 C_{V_2} + \cdots}$$

Example.

1 mole of $\mathbf{H_2}$ is mixed with 1 mole of **He**. Determine γ for the mixture

Sol^n

H_2

$n_1 = 1$

$f = 5$

$C_{v_1} = \frac{5R}{2}$

$C_{p_1} = \frac{7R}{2}$

He

$n_2 = 1$

$f = 3$

$C_{v_2} = \frac{3R}{2}$

$C_{p_2} = \frac{5R}{2}$

$$\gamma_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$$

$$\gamma_{mix} = \frac{(1)\left(\frac{7\cancel{R}}{\cancel{2}}\right) + (1)\left(\frac{5\cancel{R}}{\cancel{2}}\right)}{(1)\left(\frac{5\cancel{R}}{\cancel{2}}\right) + (1)\left(\frac{3\cancel{R}}{\cancel{2}}\right)} = \frac{12}{8}$$

$$\boxed{\gamma_{mix} = \frac{3}{2}}$$ Ans

Q. A gaseous mixture consists of 16g of helium and 16g of oxygen. Then ratio Cp/Cv of mixture is:

1)1.4

2)1.54

3)1.59

4)1.62

Sol^n

$$\gamma_{mix} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

He

$n_1 = \frac{16}{4} = 4$

$f_1 = 3$

$C_{V_1} = \frac{3R}{2}$

$C_{P_1} = \frac{5R}{2}$

O_2

$n_2 = \frac{16}{32} = 0.5$

$f_2 = 5$

$C_{V_2} = \frac{5R}{2}$

$C_{P_2} = \frac{7R}{2}$

$$\gamma_{mix} = \frac{4\left(\frac{5\cancel{R}}{\cancel{2}}\right) + \frac{1}{2}\left(\frac{7\cancel{R}}{\cancel{2}}\right)}{4\left(\frac{3\cancel{R}}{\cancel{2}}\right) + \frac{1}{2}\left(\frac{5\cancel{R}}{\cancel{2}}\right)} = \frac{23.5}{14.5}$$

$$\boxed{\gamma_{mix} = 1.62}$$

Ans