

Explanation

1) Can I write every natural number as a sum of consecutive numbers?

Every number can be written as itself

Example:

$$7 = 7$$

$$12 = 12$$

So yes, every number can be written, but that is not interesting.

If you want at least two consecutive numbers, then:

👉 No. Not every number can be written as a sum of two or more consecutive numbers.

Example of numbers that cannot be written:

2, 4, 8, 16, 32 ... (powers of 2)

2) Which numbers can be written in more than one way?

Some numbers can be written in many ways.

Examples:

$$9 = 4 + 5$$

$$9 = 2 + 3 + 4$$

$$9 = 9$$

(so 3 ways)

$$15 = 7 + 8$$

$$15 = 4 + 5 + 6$$

$$15 = 1 + 2 + 3 + 4 + 5$$

$$15 = 15$$

(4 ways)

👉 Numbers that have more than one way usually have more factors (especially odd factors).

👉 Just remember: many odd numbers and many non-power-of-2 numbers have multiple ways.

3) Can we write all odd numbers as a sum of two consecutive numbers?

✓ YES! Every odd number can be written as two consecutive numbers.

Examples:

$$5 = 2 + 3$$

$$7 = 3 + 4$$

$$9 = 4 + 5$$

$$21 = 10 + 11$$

👉 Always works because:

$$\text{Odd number} = (a) + (a+1)$$

4) Can we write all even numbers as a sum of consecutive numbers?

✗ NO. Not all even numbers.

✓ Some even numbers can be written:

$$6 = 1 + 2 + 3$$

$$10 = 1 + 2 + 3 + 4$$

$$14 = 2 + 3 + 4 + 5$$

✗ But some even numbers cannot be written as a sum of two or more consecutive numbers:

2

4

8

16

(these are all powers of 2)

5) Can I write 0 as a sum of consecutive numbers? Maybe using negatives?

✓ YES, if you allow negative numbers.

Examples:

$$-1 + 0 + 1 = 0$$

$$-2 -1 + 0 + 1 + 2 = 0$$

0 (itself)

So 0 can be written in many ways using negative consecutive numbers.