

03. Arithmetic Progressions. (A.P.)

- (A) Key Concepts (Part 1):
- ① An A.P. is a list of nos. in which each term is obtained by adding a fixed no. to the preceding term except the first term. eg:- 1, 2, 3, 4, 5, 6, ...
 - ② This fixed no. is called the common difference of the A.P.
 - ③ $a, a+d, a+2d, a+3d, \dots$ n terms.
 - ④ n^{th} term of an A.P. is denoted by ' a_n ' & $a_n = a + (n-1)d$.
eg:- $a-d, a, a+d$. 3 terms in A.P.
 - ⑤ Sum of the first ' n ' Natural Number = $\frac{n(n+1)}{2}$
 - ⑥ $S_n = \frac{n}{2} [2a + (n-1)d]$ (or) $\frac{n}{2} [a + l]$ } if first, last term given
 - ⑦ $a_n = S_n - S_{n-1}$ where a_n the n^{th} term of an A.P.
 - ⑧ $d = t_2 - t_1$ or $a_n - a_{n-1}$.

⑨ Geometric progression (G.P.) A G.P. is a list of nos. in which each term is obtained by multiplying with a fixed no. to the preceding number. eg:- 3, 3², 3³, ...

⑩ Common Ratio, $r = \frac{t_2}{t_1}$; $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = r$.

⑪ $a, ar, ar^2, ar^3, \dots$

⑫ $a_n = a \cdot r^{n-1}$.

- (B) ① 4, 10, 16, 22, ... Next 3 terms of A.P. are 28, 34, 40.
 $(a = 4 = t_1 ; d = t_2 - t_1 = 10 - 4 = 6)$

② The common difference of A.P. 3, $3+\sqrt{2}$, $3+2\sqrt{2}$, $3+3\sqrt{2}$, ... is $\sqrt{2}$
 $(\because d = t_2 - t_1 = 3 + \sqrt{2} - 3 = \sqrt{2})$

③ The 10th term of the A.P. 5, 1, -3, -7, ... is _____
 $(\because a_{10} = a + 9d = 5 + 9(-4) ; d = t_2 - t_1 = 1 - 5 = -4 ; 5 - 36 = -31)$

④ The first term of the nos. whose n^{th} term is $a_n = 3 + 2n$ is 5
 $(\because a_1 = 3 + 2(1) = 3 + 2 = 5)$

⑤ If $p-1, p+3, 3p-1$ are in A.P. then $p = \underline{4 \text{ or } 1}$ $(\because d = \frac{t_2}{t_1} = \frac{t_3}{t_2} \Rightarrow \frac{p+3}{p-1} = \frac{3p-1}{p+3})$
 $(3p^2 - 4p + 1 = p^2 + 6p + 9 \Rightarrow 2p^2 - 10p - 8 = 0 \Rightarrow p^2 - 5p - 4 = 0)$

⑥ The no. of terms in the A.P. 7, 10, 13, ... 151 is 49.
 $a = t_1 = 7 ; d = t_2 - t_1 = 10 - 7 = 3 \Rightarrow a_n = 7 + (n-1)3 = 7 + 3n - 3$
 $3n + 4 = 151 \Rightarrow 3n = 151 - 4 = 147 \Rightarrow n = 147/3 = 49$

⑦ The next term of the A.P. $\sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$ is $5\sqrt{2}$
 $a_n = a + (n-1)d$ or $a + 3d$
 $a_4 = \sqrt{8} + (4-1)(3\sqrt{2})$
 $= 2\sqrt{2} + 3\sqrt{2}$ (Contd.)