Motion in a straight line-5

Kinematic Equations for constant acc.

Motion under gravity

29/07/2025

1. (a)	2. (c)	3. (d)	4. (a)	5. (a)	6. (b)

1. (b)	2. (d)	3. (b)	4. (b)	5. (a)

Kinematic Equations

$\boxed{a = \text{Constant}}$

$$\left.\begin{aligned} S &= ut + \frac{1}{2}at^2 \\ v &= u + at \\ v^2 &= u^2 + 2as \end{aligned}\right\} \text{Kinematic equation.}$$

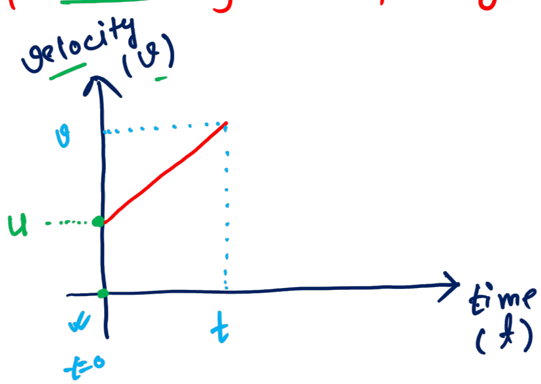
Kinematic → motion

equation of motion

Derivation of kinematic equations for uniformly accelerated motion :

1 : $\boxed{v = u + at}$

- v → final velocity
- u → initial velocity
- a → acceleration constant.
- t → time.



$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t - 0}$$

$$\text{accelerati} = \frac{v - u}{t}$$

$$\Rightarrow a = \frac{v-u}{t}$$

$$\Rightarrow \boxed{v = u + at}$$

proved.

2 : $\boxed{s = ut + \frac{1}{2}at^2}$

displacement

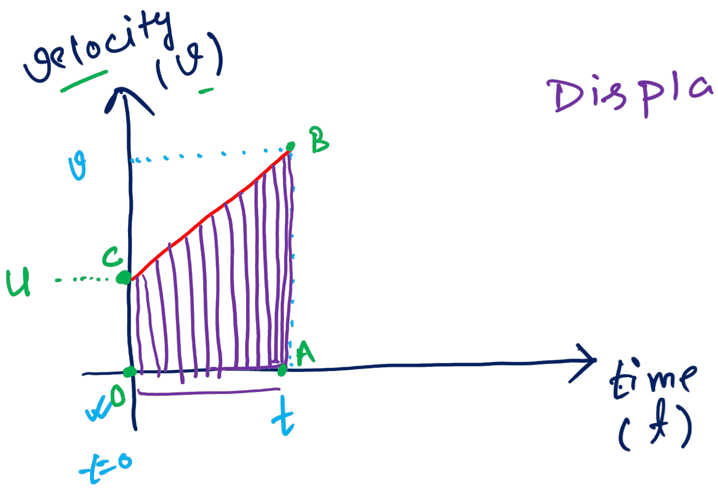
initial velocity

acc = const.

Y X

velocity. time

Displacement.



Displacement, s = area under the curve and time-axis

$= \text{area of } \triangle \text{ OABC}$

$= \frac{1}{2} \times (OC + AB) \times OA$

$= \frac{1}{2}(u + v)\, t$

$= \frac{1}{2}(u + u + at)\, t$

$= \frac{1}{2}(2u + at)\, t$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

3: $\boxed{v^2 = u^2 + 2as}$

$$\left\{\begin{array}{l} v = u + at \quad \text{— (i)} \\ s = ut + \frac{1}{2}at^2 \quad \text{— (ii)} \end{array}\right\} t$$

$$s = ut + \frac{1}{2}t \cdot at$$

$$s = ut + \frac{1}{2}t\,(v - u)$$

$$2s = 2ut + tv - tu$$

$$= 2ut + vt - ut$$

$$= ut + vt$$

$$2s = (u+v)t$$

$$\Rightarrow \frac{2s}{t} = (v+u) \quad \text{— (iii)}$$

$$v = u + at$$

$$\Rightarrow v - u = at$$

$$\Rightarrow at = v - u \quad \text{— (iv)}$$

(iii) × (iv)

$$\frac{2s}{t} \times at = (v+u)(v-u)$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow \boxed{v^2 = u^2 + 2as}$$

4: $\boxed{S_n = u + \frac{1}{2}a(2n-1)}$

Displacement travelled by the particle in n^{th} second.

~~n second~~

$S = \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times t^2$

$= 5t^2$

t	S
0	0
1	5
2	20
3	45
4	80
5	125
6	180
7	245

S_1

S_2: $20-5 = 15$

$S_3 = 45-20 = 25$

$S_4 = 125-80 = 45$ m

t=0 A — 1st Second $\rightarrow S_1$

t=1 B — 2nd Second $\rightarrow S_2$

t=2 C — 3rd Second $\rightarrow S_3$

t=3 D — 4th Second $\rightarrow S_4$

t=4 E

Displacement in 4th second = DE = 45 m.

Displacement in 4 Second = 80 m

$S_n = u + \frac{1}{2}\cdot a\,(2n-1)$

$S_3 = u + \frac{1}{2}a\,(2\times 3 - 1)$

$= u + \frac{1}{2}a\cdot 5$

$S_n = (\text{disp. in } n \text{ sec}) - (\text{disp in } (n-1)\,\text{sec})$

$= \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a\,(n-1)^2\right]$

$= un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 - 2n + 1)\right]$

$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{an^2}{2}} + an - \frac{a}{2}$

$= u + an - \frac{a}{2}$

$$\boxed{S_n = u + \frac{1}{2}a\,(2n-1)}$$

proved

$$\boxed{S = ut + \frac{1}{2}at^2}$$

$t = 3$

$t = 2$

3 sec   2 sec

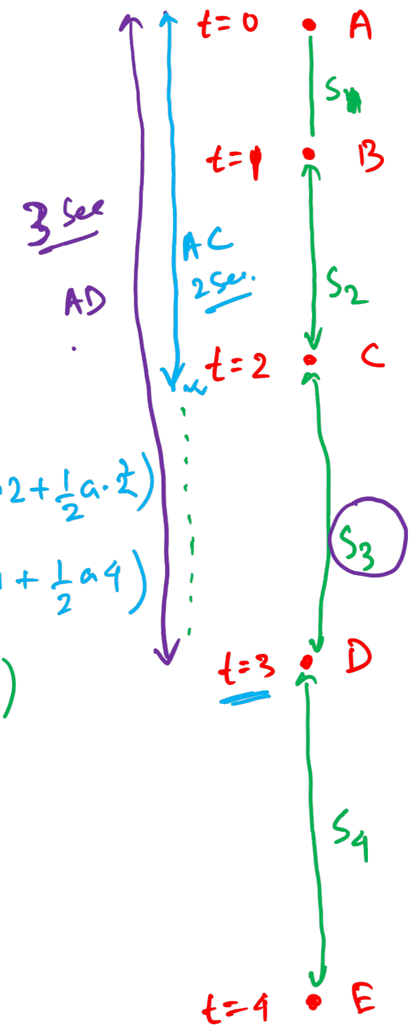
$S_3 = AD - AC$

$= u\cdot 3 + \frac{1}{2}a\cdot 3^2 - \left(u\cdot 2 + \frac{1}{2}a\cdot 2^2\right)$

$= 3u + \frac{1}{2}a\cdot 9 - \left(2u + \frac{1}{2}a\,4\right)$

$= u + \frac{1}{2}a\,(9-4)$

$S_3 = u + \frac{1}{2}a\cdot 5$



Q1 A particle with initial speed 10 m/s moves for 2 minute with constant acceleration $a = 2\ m/s^2$. Find displacement, final velocity.

Sol:

$u = +10\ m/s$

$a = 2\ m/s^2$

$t = 120$ sec.

$s = ?$

$v = ?$

$v = u + at$

$v^2 = u^2 + 2as$

Displa= $s = ut + \frac{1}{2}at^2$

$= (10 \times 120) + \frac{1}{2} \times 2 \times (120)^2$

$= 1200 + 14400$

$= 15600\ m$

Final velocity, $v = u + at$

$= 10 + (2 \times 120)$

$= 10 + 240$

$= 250\ m/s$

Q. What is the velocity of a particle which travelled 20 m when accelerated with 4 m/s^2 from rest.

Sol^n.

$u = 0$

$s = 20$ m.

$a = 4$ m/s^2

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 4 \times 20$$

$$v^2 = 160$$

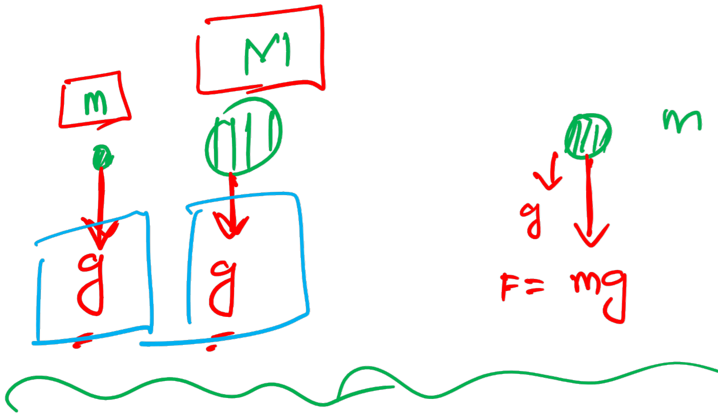
$$v = 4\sqrt{10} \text{ m/s}$$

Motion under Gravity

↳ acc = $a = g$ = **Constant.**

$= 9.8\ m/s^2$

$\approx 10\ m/s^2$ } near earth surface



$F = ma$

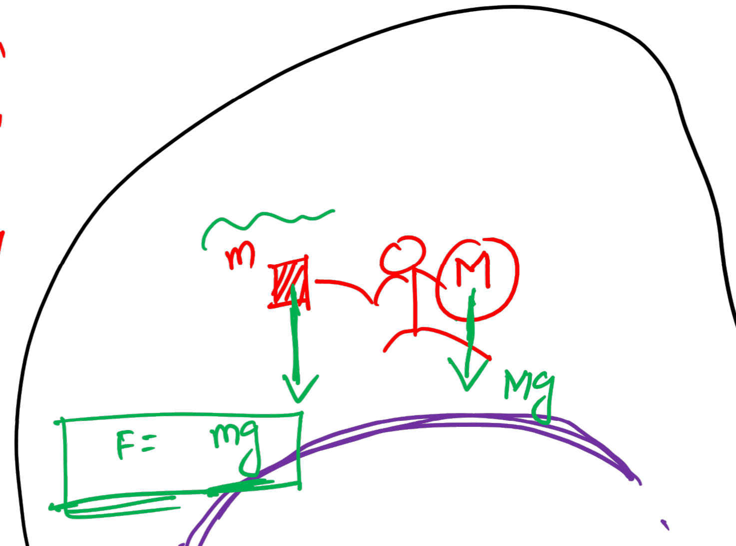
$a = F/m$

$= \frac{mg}{m}$

$\boxed{a = g}$

not depend on mass

Direction of g is always downward.



$F = ma$

$a = \frac{F}{m} = \frac{mg}{m} = g$

$\boxed{a = g}$

Case 1: Downward Motion [Dropped]

$u = 0$

$a = g$

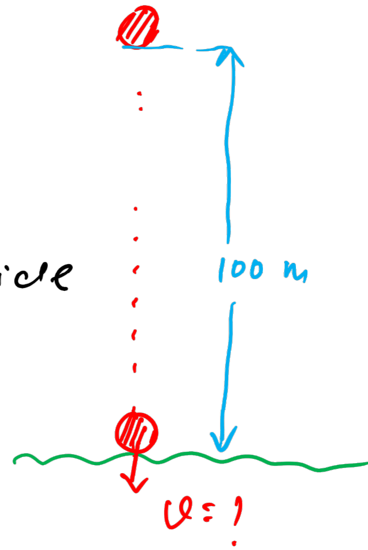
$v = gt$ — (i)

$s = \frac{1}{2} g t^2$ — (ii)

$v^2 = 2as$

$v^2 = 2ad$ — (iii)

Q. A particle is released from height 100 m then (i) what will be the speed when it will hit ground? (ii) How much time the particle will take to reach ground?



$v_f = ?$

$t = ?$

Solⁿ. (i)

$$v^2 = 2a \cdot s$$
$$= 2 \times 10 \times 100$$
$$v^2 = 2000$$
$$v = 10\sqrt{20}$$
$$v = 20\sqrt{5} \text{ m/s}$$

(ii)

$$s = \frac{1}{2} g t^2$$
$$\Rightarrow t^2 = \frac{2s}{g} = \frac{2 \times 100}{10}$$
$$\Rightarrow t = \sqrt{20}$$
$$t = 2\sqrt{5} \text{ sec.}$$

Case 2: Downward Motion [Thrown]

$u = 0$

$a = g$

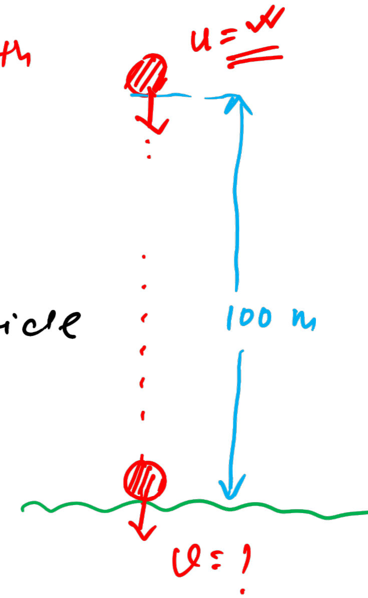
$s = ut + \frac{1}{2}gt^2$ —(i)

$v = u + t$ —(ii)

$v^2 = u^2 + 2gs$ —(iii)

Q. A particle is thrown vertically downward with speed 10 m/s height 100 m then (i) what will be the speed when it will hit ground?

(ii) How much time the particle will take to reach ground?



$v_f = ?$

$t = ?$

Sol^n: (i) $v = u + gt$

$v^2 = u^2 + 2gs$

$= 10^2 + 2 \times 10 \times 100$

$= 100 + 2000$

$= 2100$

$v = \sqrt{2100}$

$= 10\sqrt{21}$ m/s

(ii) $v = u + gt$

$t = \frac{v - u}{g}$

$= \frac{10\sqrt{21} - 10}{10}$

$= (\sqrt{21} - 1)$ sec.

Case-3: Upward Motion [Thrown]

$u = 0$

$a = g$

$\boxed{s = ut - \frac{1}{2}gt^2}$ — (i)

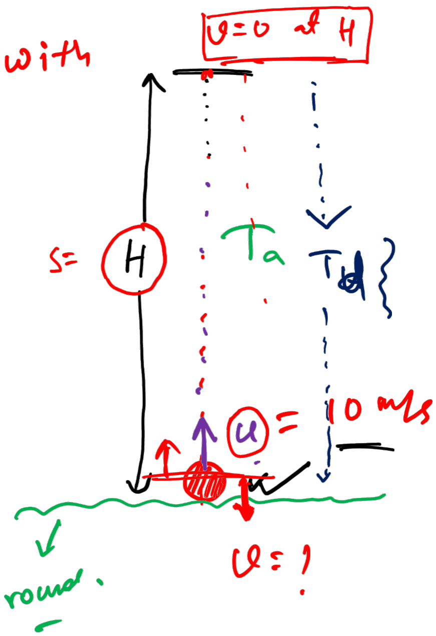
$\boxed{v = u - gt}$ — (ii)

$v^2 = u^2 - 2gs$ — (iii)

Q. A particle is thrown vertically upward with speed 10 m/s.

(i) Maximum height attained by the particle? $H = \frac{u^2}{2g}$ [not depends on mass]

(ii) Time taken to reach maximum height.



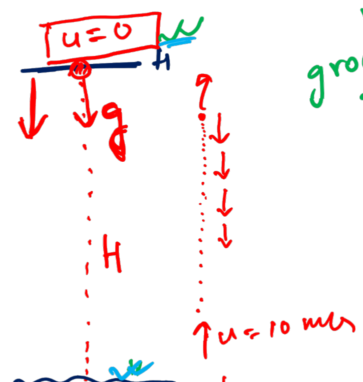
Sol^n

$0^2 = u^2 - 2gH$

$\Rightarrow 2gH = u^2$

$\Rightarrow \boxed{H = \frac{u^2}{2g}}$

$0 = u - gT_a \Rightarrow \boxed{T_a = \frac{u}{g}}$



$v_f = ?$

$t = ?$

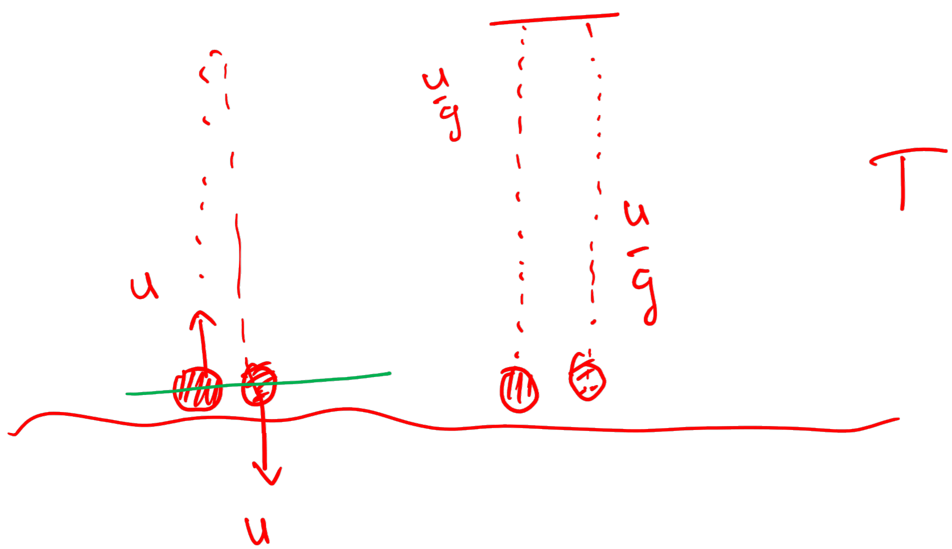
$H = 0 \cdot T_d + \frac{1}{2}gT_d^2$

$H = \frac{1}{2}gT_d^2 \Rightarrow T_d = \sqrt{\frac{2H}{g}}$

$v = 0 + gT_d$

$T_d = \frac{v}{g} = \frac{u}{g}$

$T_d = \frac{u}{g} = \frac{u}{g}$



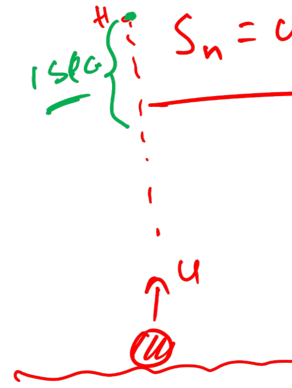
(iii) Time taken by the particle from maximum height to ground?

$\Rightarrow T_d = \frac{u}{g}$

(iv) Time of flight? $\Rightarrow T = T_a + T_d = \frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$

$$\boxed{T = \frac{2u}{g}}$$

(v) Distance travelled by the particle in 1 second before the maximum height ?



$S_n = u + \frac{1}{2}a(2n-1)$

$u = 90$ m/s

$\Rightarrow \quad T_a = \frac{u}{g} = \frac{90}{10} = 9$ sec.

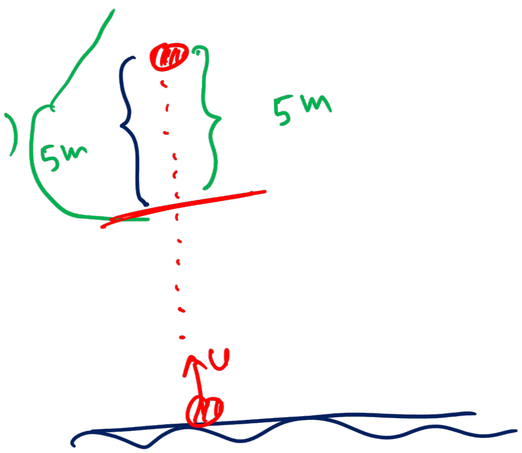
$$S_{T_a} = S_9 = u + \frac{1}{2}a(2\times 9 - 1)$$

$$= 90 + \frac{1}{2}(-10)(18-1)$$

$$= 90 - (5 \times 17)$$

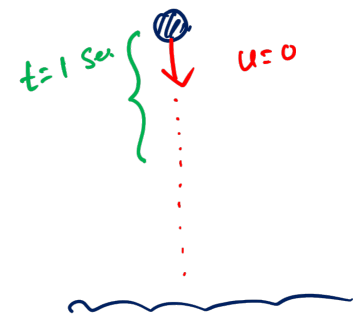
$$= 90 - 85$$

$$= 5\text{ m}$$



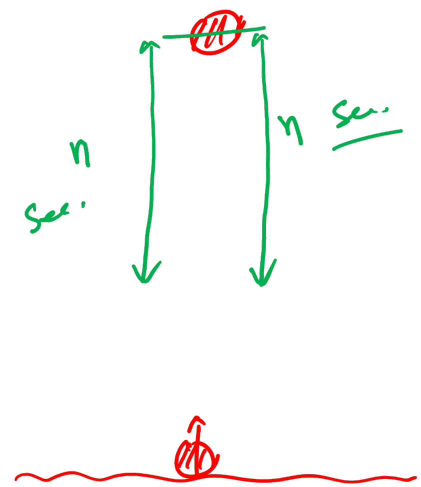
(vi) Distance travelled in first second of desending ?

$\Rightarrow \quad s = ut + \frac{1}{2}gt^2 = \frac{1}{2}\times 10 \times 1^2 = 5\text{m}$



(v) Distance travelled by the particle in ~~1~~ 5 seconds before the maximum height?

$u = 90$ m/s



(vi) Distance travelled in ~~first~~ 5 seconds of desending?

Q. A particle is thrown vertically upward with speed 100 m/s ↓10 Take $g = 10\ m/s^2$.

↳ 10 sec

(i) Distance travelled in 9 sec.

(ii) Distance travelled in 11 sec. → T_a

(iii) distance travelled between $t = 9$ sec. and $t = 11$ sec.

Sol^n.

$x_i = 100 \times 9 - \frac{1}{2} \times 10 \times 9^2$

$= 900 - \frac{810}{2}$

$= 900 - 405$

$= 495$ m

$x_f = 100 \times 11 - \frac{1}{2} \times 10 \times 11^2$

$= 1100 - 5 \times 121$

$1100 - 605 = 495$ m

True for disp.

$\boxed{x_f - x_i = 0}$

not possible.

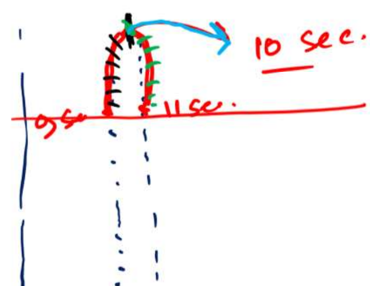
for distance.

$s = ut + \frac{1}{2} g t^2$

$v = u - gt$

$\Rightarrow 0 = 100 - 10 T_a$

$\Rightarrow T_a = \frac{100}{10} = 10$ sec.



$u = 100$ m/s

$g = -10$ m/s^2

↳ in 1 sec

speed ↓ g

$u = 100$ m/s

↳ = 90

↳ = 80

↳ = 70

↳ = 60

12 sec.

$\boxed{u = 120 \text{ m/s}}$, $g = 10 \text{ m/s}^2$

$\boxed{T_a = 12 \text{ sec.}}$

(i) Distance travelled between 9 sec and 11 sec.

$\Rightarrow$

$x = ut - \frac{1}{2} g t^2$

$= 120 \times 9 - \frac{1}{2} \times 10 \times 9^2$

$= 1080 - 405$

$= 1055$ m.

$x = 120 \times 11 - \frac{1}{2} \times 10 \times 11^2$

$= 1331 - 605$

$=$

121
121
1331

121
5
605

Ans $= 1331 - 605 - 1055$

$= 331 -$

1. Velocity of a body moving along a straight line with uniform acceleration (a) reduces by $\frac{3}{4}$ of its initial velocity in time t_0. The total time of motion of the body till its velocity becomes zero is

(a) $\frac{4}{3} t_0$ (b) $\frac{3}{2} t_0$
(c) $\frac{5}{3} t_0$ (d) $\frac{8}{3} t_0$

2. The displacement of a body in 8 s starting from rest with an acceleration of 20 cms^{-2} is

(a) 64 m (b) 64 cm
(c) 640 cm (d) 0.064 m

3. The motion of a particle is described by the equation $v = at$. The distance travelled by the particle in the first 4 s is

(a) $4a$ (b) $12a$
(c) $6a$ (d) $8a$

4. A particle starts with a velocity of 2 ms^{-1} and moves in a straight line with a retardation of 0.1 ms^{-2}. The first time at which the particle is 15 m from the starting point is

(a) 10 s (b) 20 s
(c) 30 s (d) 40 s

5. A particle starts from rest, accelerates at 2 ms^{-2} for 10 s and then moves with constant speed of 20 ms^{-1} for 30 s and then decelerates at 4 ms^{-2} till it stops after next 5 s. What is the distance travelled by it?

(a) 750 m (b) 800 m (c) 700 m (d) 850 m

6. A body is moving with uniform velocity of 8 ms^{-1}. When the body just crossed another body, the second one starts and moves with uniform acceleration of 4 ms^{-2}. The time after which two bodies meet, will be

1. If a stone is thrown up with a velocity of 9.8 ms^{-1}, then how much time will it take to come back?

(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

2. If a ball is thrown vertically upwards with speed u, the distance covered during the last t second of its ascent is

(a) $ut - (gt^2/2)$ (b) $(u + gt)\, t$
(c) ut (d) $gt^2/2$

3. A person throws balls into air after every second. The next ball is thrown when the velocity of the first ball is zero. How high do the ball rise above his hand?

(a) 2 m (b) 5 m (c) 8 m (d) 10 m

4. A particle is thrown vertically upwards. Its velocity at half of the height is 10 ms^{-1}. Then, the maximum height attained by it is (Take, $g = 10\ ms^{-2}$)

(a) 16 m (b) 10 m
(c) 20 m (d) 40 m

5. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity,

(a) $\sqrt{3}\, v_0$ (b) $3\, v_0$ (c) $9\, v_0$ (d) $3/2\, v_0$