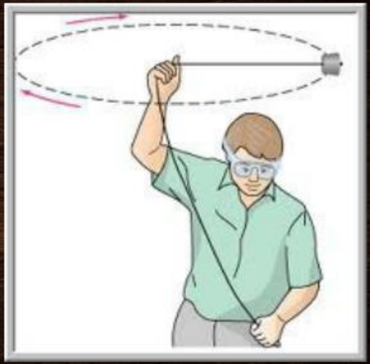


CIRCULAR MOTION

BY NANDHINI HARIKRISHNAN

Circular Motion : (circle)

a, F

Uniform circular motion

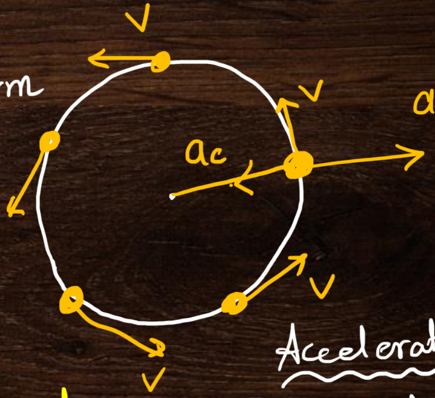
* Speed - constant
Velocity - change
↑
direction → $\Delta v \to a$
↓
centripetal acceleration $a_c = \frac{v^2}{r}$

$F_c = \frac{mv^2}{r}$

Non - uniform circular motion

* $\vec{a_T} + \vec{a_C} = \vec{a}$
* Speed - change
* Variable ang. speed
* $\alpha \neq 0$
* Cent, tang

$F = ma$



Acceleration
* Centripetal acc → (UCM + NUCM)
* Tangential acc.
⇓
(NUCM)

Circular motion - Important terms

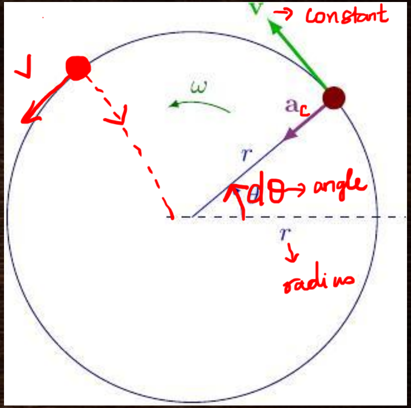
* Angular displacement. $(d\theta)$

↳ angle = $\frac{\text{arc [radian]}}{\text{radius}} = \frac{ds}{r} = d\theta$

↳ dimensionless (degree)

linear disp → dx
↓
metre

Kinemat → UCM
NLOM → NUCM
+ extra
Rotational



* frequency (n) (f)

↳ No. of revolution per second

↳ $T = \frac{1}{f}$

$\therefore$ 1rps = 60rpm

Hz

A.C

cm → m

$\times 10^{-2}$ m

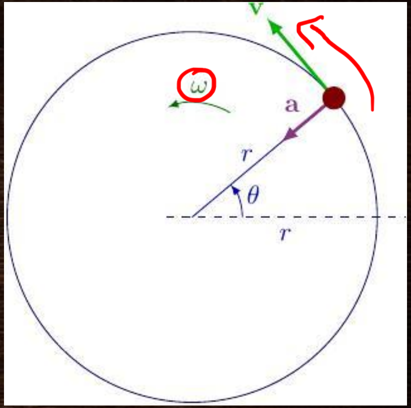
Angular frequency ⇒ $2\pi f = \omega$

ang. freq (ω)

ω

↓

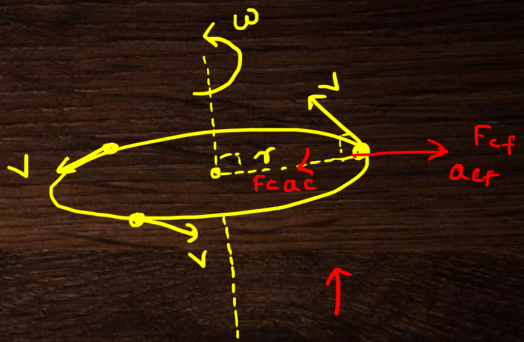
ang. velocity ($\vec{\omega}$)



Angular velocity (ω)

$v = dr/dt$

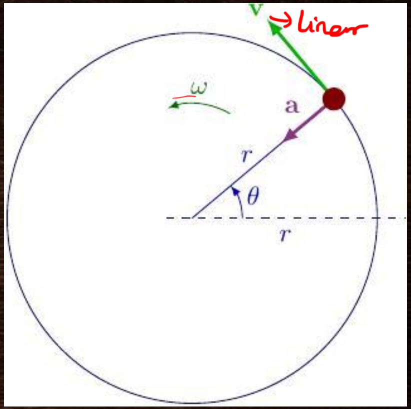
$$\omega = \frac{\text{ang. disp}}{\text{time}} = \frac{d\theta}{dt}$$



Unit: rad/sec **dim:** T^{-1}

Relation b/w angular & Linear velocity (v-ω)

$$v = \frac{\text{disp}}{\text{time}} = \frac{ds}{dt}$$



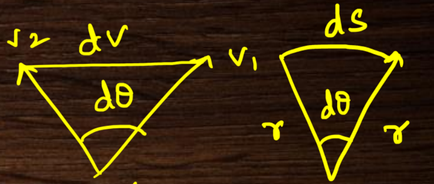
$\omega = v/r$

$v = \omega r$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

Centripetal acceleration & Centripetal force

$|v_1| = |v_2| = v$



(1) (2)

Cent. acc $\longrightarrow$ centri force

$d\theta = \dfrac{ds}{r}$ $\qquad$ $d\theta = \dfrac{dv}{v}$

$\dfrac{ds}{r\,dt} = \dfrac{dv}{v\,dt} \qquad \div \dfrac{1}{dt}$

$\dfrac{v}{r} = \dfrac{a_c}{v} \Rightarrow a_c = \dfrac{v^2}{r}$

$\boxed{a_c = r\omega^2}$

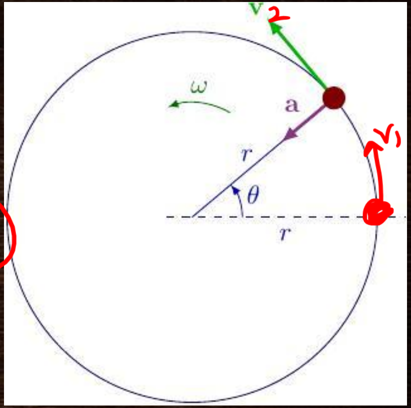
$F_c = m a_c$

$= m \times \dfrac{v^2}{r}$

$F_c = \dfrac{mv^2}{r}$ $\qquad$ $\boxed{v = r\omega}$

$F_c = \dfrac{m r^2 \omega^2}{r}$

$\boxed{F_c = m r \omega^2}$



$\omega = v/r$

$\hat{v} = r\omega$ (diff wrt dt)

direction of a_c → act towards the centre along the radius

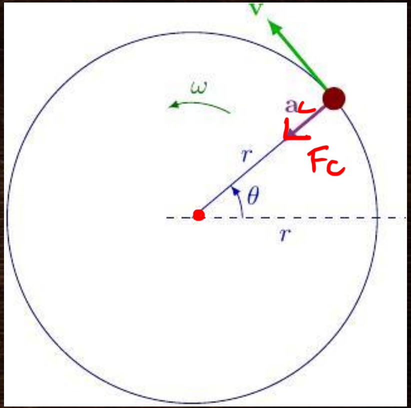
$$\frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

$$a_c = \omega \frac{dr}{dt}$$

$\omega = v/r$

$$\boxed{a_c = \omega v}$$

$$\boxed{a_c = v^2/r}$$



Uniform circular motion

* Spee = |velo| = const
* vel ≠ cons
* $K.E = \frac{1}{2}mv^2 = \text{const}$
* $\omega = \text{const.}$

$$T_f = mv^2/r$$

* $a_T = 0$; $a_{cp} = \vec{a} \neq \text{const.}$
* $\alpha = 0 \quad (\alpha = a_T/r)$
* Total work done = 0 = power

Non-uniform motion:

* a <
  * $a_{cp} \rightarrow$ responsible $\rightarrow \Delta$ direction
  * $a_T \rightarrow$ " $\rightarrow \Delta$ speed
* $|\vec{\omega}| = \vec{v}/r$
* $a_{cp} = \omega v = \omega^2 r = v^2/r$
* $\alpha \neq 0$
* $\vec{a} = \vec{a}_T + \vec{a}_{cp}$
* $a_T \neq 0$

$$|\vec{a}| = \sqrt{a_T^2 + a_{cp}^2}$$

$a_T \rightarrow \alpha r$, $a_{cp} \rightarrow v^2/r$

* $\vec{F} = \sqrt{F_T^2 + F_{cp}^2}$

Circular Motion

- → U.C.M → H.C.M
- → NUCM → V.C.M

Uniform circular motion

Linear motion

* $v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

$S_n = \omega_0 + \frac{a}{2}(2n-1)$

$v \rightarrow \omega$

$u \rightarrow \omega_0$

$s \rightarrow \theta$

$a \rightarrow \alpha$

Rotational Motion

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega = \omega_0 + \alpha t$

$\theta = \left(\frac{\omega + \omega_0}{2}\right)t$

$\alpha = \frac{\omega^2}{2\theta}$

$a = \frac{v^2}{2s}$