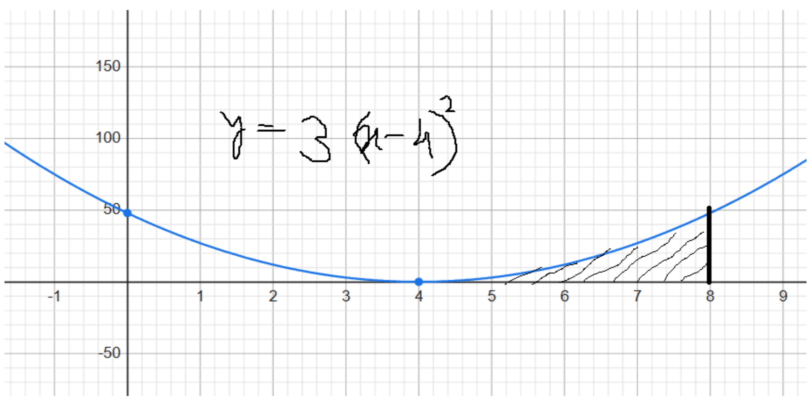
Application of Integrals - part1

Area Under Curve

1. As done in the class,
   a. use the **Σ** summation of the differential-slice-method (D/N, with N → ∞) approach to find the area under the curve:

      y = 3(x – 4)^2 from x = 4 to x = 8



   b. Use the Integral form $\int_{4}^{8} 3\,(x-4)^2\,dx$ to validate your answer above.

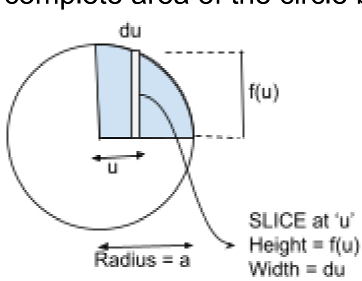
   c. Can we write D/N (with N → ∞) as 'dx'?

      HINT: In that case, the area of the uth slice (or $A_i(u)$) can be referred to as the differential-area: 'dA'.

      In that case, $\sum_{i=0}^{n} A_i(u) = \lim_{n\to\infty} \sum_{i=0}^{n} f(i \times \frac{D}{N}) \times \frac{D}{N}$ becomes,

      $$Area = \int_{a}^{a+D} dA = \int_{a}^{a+D} f(x)\,dx$$

2. Find the area of the circle using the Integral-form of Area Under Curve (or AUC), as outlined in two approaches below:
   a. Find the area of a quarter-circle as shown below, choosing the vertical differential-slices. And then find the complete area of the circle by multiplying the result by 4.



Hint1: Find what is f(u)? Revise conic-sections.

Hint2: What's the span of 'u' ? I.e. 'u' varies from __?___ to __?___

Hint3: Express as definite integral as shown in 1.c

b. Imagine the circle area as the addition of little-little (differential) circular rings (concentric rings) of size. You may use the fact that the circumference of a ring of radius r is $2\pi r$, and that each ring has a uniform thickness (that's infinitesimally small).

   Hint1: What varies with each ring?

   Hint2: What is the area of the u^{th} differential-ring? If we straighten a ring into a nearly rectangle thin-wire, what is the width and height of the same?