Physics Problems: Static Electricity and Electrostatic Forces (ISC Curriculum)

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Instructions

Solve the following problems, showing all necessary steps.

Problem 1 (Medium Difficulty):

Description for Diagram: Draw two point charges, $Q_1 = +5\,\mu C$ and $Q_2 = -3\,\mu C$, placed at points A and B respectively. The distance between A and B is $20\,cm$. Point P is located on the line connecting A and B, outside the segment AB, such that it is $10\,cm$ from Q_2 (and $30\,cm$ from Q_1).

Problem Statement: Two point charges, $Q_1 = +5\,\mu C$ and $Q_2 = -3\,\mu C$, are placed $20\,cm$ apart. Find the electric potential at a point P located $10\,cm$ from Q_2 on the line joining the two charges, but outside the region between them and closer to Q_2.

Key Concepts: Electric potential due to a point charge, superposition principle.

Solution Key: The electric potential at point P is the algebraic sum of the potentials due to Q_1 and Q_2. Distance from Q_1 to P $(r_1) = 20\,cm + 10\,cm = 30\,cm = 0.3\,m$. Distance from Q_2 to P $(r_2) = 10\,cm = 0.1\,m$.

$$V_P = V_1 + V_2 = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$V_P = (9 \times 10^9\,\mathrm{Nm^2/C^2})\left(\frac{5 \times 10^{-6}\,\mathrm{C}}{0.3\,\mathrm{m}} + \frac{-3 \times 10^{-6}\,\mathrm{C}}{0.1\,\mathrm{m}}\right)$$

$$V_P = 9 \times 10^9 \left(\frac{5}{0.3} \times 10^{-6} - \frac{3}{0.1} \times 10^{-6}\right)$$

$$V_P = 9 \times 10^9 \times 10^{-6}\left(\frac{50}{3} - 30\right)$$

$$V_P = 9 \times 10^3\,(16.67 - 30)$$

$$V_P = 9 \times 10^3 \times (-13.33)$$

$$V_P \approx -1.2 \times 10^5\,\mathrm{V}$$

Problem 2 (Medium Difficulty):

Description for Diagram: Draw an equilateral triangle ABC with side length a. Place charges $+q$, $+q$, and $-q$ at vertices A, B, and C respectively.

Problem Statement: Three point charges, $+q$, $+q$, and $-q$, are placed at the vertices A, B, and C respectively of an equilateral triangle of side length a. Calculate the magnitude and direction of the net electrostatic force on the charge at vertex C.

Key Concepts: Coulomb's Law, vector addition of forces.

Solution Key: Let F_{AC} be the force on C due to A, and F_{BC} be the force on C due to B.

$$|F_{AC}| = \frac{k|q(-q)|}{a^2} = \frac{kq^2}{a^2}$$

This force is attractive, directed along CA.

$$|F_{BC}| = \frac{k|q(-q)|}{a^2} = \frac{kq^2}{a^2}$$

This force is attractive, directed along CB.

The angle between F_{AC} and F_{BC} is 60°. The resultant force F_{net} can be found using the parallelogram law of vector addition:

$$F_{\text{net}} = \sqrt{F_{AC}^2 + F_{BC}^2 + 2F_{AC}F_{BC}\cos 60^\circ}$$

Since $F_{AC} = F_{BC} = F = \frac{kq^2}{a^2}$:

$$F_{\text{net}} = \sqrt{F^2 + F^2 + 2F^2(1/2)} = \sqrt{3F^2} = F\sqrt{3}$$

$$F_{\text{net}} = \frac{\sqrt{3}kq^2}{a^2}$$

Direction: The resultant force will bisect the angle between F_{AC} and F_{BC}, acting symmetrically. It is directed inwards along the angle bisector of angle C.

Problem 3 (Hard Difficulty):

Description for Diagram: Draw a square ABCD of side length L. Place charges $+q$ at A and C, and charges $-q$ at B and D.

Problem Statement: Four point charges, $+q$, $-q$, $+q$, and $-q$, are placed at the corners A, B, C, and D respectively of a square of side length L. Find the magnitude and direction of the electric field at the center of the square.

Key Concepts: Electric field due to a point charge, vector addition, symmetry.

Solution Key: Let the center of the square be O. The distance from each corner to the center is $r = \frac{L}{\sqrt{2}}$. Magnitude of electric field due to each charge at the center:

$$E_0 = \frac{k|q|}{r^2} = \frac{kq}{(L/\sqrt{2})^2} = \frac{2kq}{L^2}$$

Consider the contributions from each charge at the center O:

- E_A (due to $+q$ at A): Points away from A, i.e., along the diagonal AC, towards C.
- E_B (due to $-q$ at B): Points towards B, i.e., along the diagonal DB, towards B.
- E_C (due to $+q$ at C): Points away from C, i.e., along the diagonal CA, towards A.
- E_D (due to $-q$ at D): Points towards D, i.e., along the diagonal BD, towards D.

Now, let's analyze the vector sum:

- E_A and E_C: These fields are along the same diagonal (AC) but point in opposite directions (E_A towards C, E_C towards A). Since their magnitudes are equal (E_0), their vector sum is zero.
- E_B and E_D: These fields are along the same diagonal (BD) but point in opposite directions (E_B towards B, E_D towards D). Since their magnitudes are equal (E_0), their vector sum is zero.

Therefore, the net electric field at the center of the square is the sum of these zero contributions.

Final Answer: The net electric field at the center of the square is **zero**.

Problem 4 (Medium Difficulty):

Description for Diagram: Draw two parallel conducting plates, labeled P and Q. Plate P is given a charge of $+Q$ and plate Q is given a charge of $-Q$. The area of each plate is A, and the distance between them is d. Show the electric field lines between the plates, pointing from P to Q.

Problem Statement: A parallel plate capacitor has plates of area $A = 100\,cm^2$ and are separated by a distance $d = 2\,mm$. If a charge of $5\,nC$ is transferred from one plate to the other, forming charges of $+5\,nC$ and $-5\,nC$ on the plates, calculate: (a) The electric field strength between the plates. (b) The potential difference between the plates.

Key Concepts: Electric field due to infinite sheet of charge, electric field in a capacitor, relationship between electric field and potential difference.

Solution Key: Given: $A = 100\,cm^2 = 100 \times 10^{-4}\,m^2 = 10^{-2}\,m^2$. $d = 2\,mm = 2 \times 10^{-3}\,m$. $Q = 5\,nC = 5 \times 10^{-9}\,C$.

(a) Surface charge density $\sigma = \frac{Q}{A} = \frac{5\times10^{-9}\,\mathrm{C}}{10^{-2}\,\mathrm{m}^2} = 5 \times 10^{-7}\,\mathrm{C/m}^2$. Electric field strength $E = \frac{\sigma}{\epsilon_0}$ (ignoring fringing effects)

$$E = \frac{5 \times 10^{-7}\,\mathrm{C/m}^2}{8.854 \times 10^{-12}\,\mathrm{F/m}} \approx 5.65 \times 10^4\,\mathrm{N/C}$$

(b) Potential difference $V = E \times d$

$$V = (5.65 \times 10^4\,\mathrm{V/m}) \times (2 \times 10^{-3}\,\mathrm{m})$$

$$V = 113\,\mathrm{V} \quad \text{(approximately)}$$

Problem 5 (Hard Difficulty):

Description for Diagram: Draw two identical small conducting spheres, A and B. Sphere A has an initial charge of $+4Q$ and Sphere B has an initial charge of $-2Q$. Show them initially separated. Then, show them touching, and finally separated again. For part (c), show sphere C, uncharged, touching sphere A (which now has its new charge).

Problem Statement: Two identical small conducting spheres, A and B, carry charges of $+4Q$ and $-2Q$ respectively. (a) If the spheres are brought into contact and then separated, what is the final charge on each sphere? (b) If they are then brought close together (but not touching) with a separation of r, calculate the magnitude of the electrostatic force between them. (c) Now, a third identical uncharged conducting sphere C is brought into contact with sphere A (after the first contact and separation). What is the final charge on A, B, and C after this second contact and separation?

Key Concepts: Conservation of charge, charge distribution on conductors, Coulomb's Law.

Solution Key: (a) When identical conducting spheres are brought into contact, the total charge is redistributed equally between them. Total charge = $+4Q + (-2Q) = +2Q$. Final charge on A = Final charge on B = $\frac{+2Q}{2} = +Q$.

(b) The force between the spheres after contact and separation (each having charge $+Q$):

$$F = \frac{k(+Q)(+Q)}{r^2} = \frac{kQ^2}{r^2}$$

The force is repulsive.

(c) Sphere A now has charge $+Q$. Sphere C is uncharged (0). When A and C are brought into contact, the total charge is redistributed equally. Total charge = $+Q + 0 = +Q$. Final charge on A = $\frac{+Q}{2}$. Final charge on C = $\frac{+Q}{2}$. Sphere B's charge remains unchanged from part (a): $+Q$.

So, final charges: A: $+Q/2$ B: $+Q$ C: $+Q/2$

Problem 6 (Hard Difficulty):

Description for Diagram: Draw an infinitely long straight wire with uniform linear charge density λ. Show an arbitrary point P at a perpendicular distance r from the wire. Indicate the direction of the electric field at P.

Problem Statement: An infinitely long thin straight wire has a uniform linear charge density $\lambda = 2.0 \times 10^{-8}\,C/m$. Calculate the electric field strength at a point P located $5.0\,cm$ away from the wire.

Key Concepts: Gauss's Law, electric field due to an infinitely long charged wire.

Solution Key: For an infinitely long straight charged wire, the electric field strength at a perpendicular distance r from the wire is given by:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Given: $\lambda = 2.0 \times 10^{-8}\,\mathrm{C/m}$ $r = 5.0\,cm = 0.05\,m$ We know that $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\mathrm{Nm^2/C^2}$, so $\frac{1}{2\pi\epsilon_0} = 2 \times 9 \times 10^9\,\mathrm{Nm^2/C^2} = 1.8 \times 10^{10}\,\mathrm{Nm^2/C^2}$.

$$E = (1.8 \times 10^{10}\,\mathrm{Nm^2/C^2}) \times \frac{2.0 \times 10^{-8}\,\mathrm{C/m}}{0.05\,\mathrm{m}}$$

$$E = (1.8 \times 10^{10}) \times (4 \times 10^{-7})$$

$$E = 7.2 \times 10^3\,\mathrm{N/C}$$

The direction of the electric field is radially outwards if λ is positive.

Problem 7 (Hard Difficulty):

Description for Diagram: Draw a thin spherical shell of radius R. Show a positive charge $+Q$ uniformly distributed over its surface. Show two points: P (inside the shell, distance $r_1 < R$ from center) and Q (outside the shell, distance $r_2 > R$ from center).

Problem Statement: A thin spherical shell of radius $R = 10\,cm$ has a total charge of $50\,nC$ uniformly distributed on its surface. Calculate the electric field strength: (a) at a point $5\,cm$ from the center of the shell. (b) at a point $20\,cm$ from the center of the shell.

Key Concepts: Gauss's Law, electric field due to a uniformly charged spherical shell (inside and outside).

Solution Key: Given: $R = 10\,cm = 0.1\,m$ $Q = 50\,nC = 50 \times 10^{-9}\,C$

(a) Point at $r_1 = 5\,cm = 0.05\,m$ from the center. This point is *inside* the spherical shell. For a uniformly charged thin spherical shell, the electric field inside the shell is **zero**.

$$E_{\text{inside}} = 0\,\mathrm{N/C}$$

(b) Point at $r_2 = 20\,cm = 0.2\,m$ from the center. This point is *outside* the spherical shell. For points outside, the spherical shell behaves as if all its charge were concentrated at its center.

$$E_{\text{outside}} = \frac{kQ}{r_2^2}$$

$$E_{\text{outside}} = \frac{(9 \times 10^9\,\mathrm{Nm^2/C^2}) \times (50 \times 10^{-9}\,\mathrm{C})}{(0.2\,\mathrm{m})^2}$$

$$E_{\text{outside}} = \frac{9 \times 10^9 \times 50 \times 10^{-9}}{0.04}$$

$$E_{\text{outside}} = \frac{450}{0.04} = 11250\,\mathrm{N/C} = 1.125 \times 10^4\,\mathrm{N/C}$$

Problem 8 (Hard Difficulty):

Description for Diagram: Draw two fixed point charges, $Q_1 = +4q$ and $Q_2 = -q$, separated by a distance L. Show a point P on the line connecting them where a test charge would experience zero net electrostatic force. This point P should be outside the segment connecting Q_1 and Q_2, closer to $-q$.

Problem Statement: Two point charges, $Q_1 = +4q$ and $Q_2 = -q$, are separated by a distance L. Find the position along the line connecting the two charges where a third charge (a test charge) would experience zero net electrostatic force.

Key Concepts: Coulomb's Law, principle of superposition, condition for zero net force.

Solution Key: Let the third charge be q_0. Since the charges are opposite in sign, the point of zero electric field (and thus zero force on a test charge) must lie *outside* the segment connecting Q_1 and Q_2, and closer to the smaller magnitude charge (Q_2). Let's assume the origin is at Q_1. So Q_1 is at $x = 0$ and Q_2 is at $x = L$. Let the point of zero force be at x. Since it's outside and closer to Q_2, x must be greater than L. The distance from Q_1 to x is x. The distance from Q_2 to x is $x - L$.

For the net force to be zero, the forces due to Q_1 and Q_2 must be equal in magnitude and opposite in direction. Magnitude of force due to Q_1 on q_0: $F_1 = \frac{k(4q)q_0}{x^2}$ Magnitude of force due to Q_2 on q_0: $F_2 = \frac{k|(-q)|q_0}{(x-L)^2}$

For equilibrium, magnitudes must be equal:

$$|F_1| = |F_2|$$

$$\frac{k(4q)q_0}{x^2} = \frac{kqq_0}{(x-L)^2}$$

$$\frac{4}{x^2} = \frac{1}{(x-L)^2}$$

Taking square root on both sides:

$$\sqrt{\frac{4}{x^2}} = \pm\sqrt{\frac{1}{(x-L)^2}}$$

$$\frac{2}{x} = \pm\frac{1}{x-L}$$

Case 1: $\frac{2}{x} = \frac{1}{x-L}$ $2(x-L) = x$ $2x - 2L = x$ $x = 2L$ This position ($x = 2L$) is outside the segment and closer to Q_2, making it a valid solution.

Case 2: $\frac{2}{x} = -\frac{1}{x-L}$ $2(x-L) = -x$ $2x - 2L = -x$ $3x = 2L$ $x = \frac{2L}{3}$ This position ($x = 2L/3$) is between Q_1 and Q_2. If a positive test charge were placed here, Q_1 would repel it to the right, and Q_2 would attract it to the right. Both forces would be in the same direction, so they cannot cancel out. Thus, this is not the correct physical solution.

Final Answer: The point where the net electrostatic force is zero is at a distance $2L$ from Q_1 (on the side of Q_2, away from Q_1). Alternatively, it is at a distance L from Q_2 on the side away from Q_1.

Problem 9 (Additional Problem):

Problem Statement: A small non-metallic object made of glass acquires 3.2×10^{-10} coulomb charge on rubbing against another object. Calculate the mass gained or lost by the glass object. Take mass and charge of an electron as 9.109×10^{-31} kg and -1.602×10^{-19} coulomb respectively.

Key Concepts: Quantization of charge, charge of an electron, mass of an electron.

Solution Key:

1. **Determine the type of charge and particle transferred:** The glass object *acquires* a positive charge ($+3.2 \times 10^{-10}$ C). For an object to become positively charged, it must have **lost electrons** (since electrons are negatively charged).

2. **Calculate the number of electrons lost (n):** Given charge gained by glass (Q) = 3.2×10^{-10} C. Magnitude of charge of one electron (e) = 1.602×10^{-19} C.

$$n = \frac{Q}{e}$$

$$n = \frac{3.2 \times 10^{-10}\,\text{C}}{1.602 \times 10^{-19}\,\text{C/electron}}$$

$$n \approx 1.9975 \times 10^{9} \text{ electrons}$$

Rounding to two significant figures (consistent with the given charge):

$$n \approx 2.0 \times 10^{9} \text{ electrons}$$

3. **Calculate the mass lost:** Mass of one electron (m_e) = 9.109×10^{-31} kg. Total mass lost = Number of electrons lost × Mass of one electron

$$\text{Mass lost} = n \times m_e$$

$$\text{Mass lost} = (2.0 \times 10^9) \times (9.109 \times 10^{-31}\,\text{kg})$$
$$\text{Mass lost} = 18.218 \times 10^{-22}\,\text{kg}$$
$$\text{Mass lost} = 1.8218 \times 10^{-21}\,\text{kg}$$

Conclusion: The glass object **lost** approximately $\mathbf{1.82 \times 10^{-21}}$ kilograms of mass.