Trigonometry Identities & Telescoping Tricks

1. Basic Trigonometric Identities

sin^2theta + cos^2theta = 1

1 + tan^2theta = sec^2theta

1 + cot^2theta = cosec^2theta

sin(-theta) = -sintheta, cos(-theta) = costheta, tan(-theta) = -tantheta

2. Angle Transformation Identities

sin(90° - theta) = costheta

cos(90° - theta) = sintheta

tan(90° - theta) = cottheta

sin(180° - theta) = sintheta

cos(180° - theta) = -costheta

tan(180° - theta) = -tantheta

3. Trigonometric Ratios of Standard Angles

theta | 0° | 30° | 45° | 60° | 90°

---

sintheta | 0 | 1/2 | 1/sqrt2 | sqrt3/2 | 1

costheta | 1 | sqrt3/2 | 1/sqrt2 | 1/2 | 0

tantheta | 0 | 1/sqrt3 | 1 | sqrt3 | undefined

4. Multiple Angle Identity (Used in Your Question)

costheta/cos3theta + cos3theta/cos9theta + cos9theta/cos27theta

= 1/2 (tan27theta - tantheta)

This is a telescoping identity where each term cancels internally.

Trigonometry Identities & Telescoping Tricks

5. Special Telescoping Series in Trigonometry

tantheta + tan2theta + tan4theta + ... + tan(2^ntheta)

Can sometimes be simplified using identities or induction.

Key idea: look for cancelation patterns.

Another example:

cotA - tanA = 2cot2A