EXERCISE 5.11

Short answer questions (1 to 3) :

1. Find the second order derivatives of the following functions :

   (*i*) $x^3 + \tan x$ (NCERT) (*ii*) $\log x$ (NCERT) (*iii*) $\tan^{-1} x$ (NCERT)

2. (*i*) If $y = \log(x-2)$, $x > 2$, find $\frac{d^2y}{dx^2}$. (*ii*) If $y = \cot x$, find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$.

   (*iii*) If $x = at^2$ and $y = 2at$, then find $\frac{d^2y}{dx^2}$.

3. Find the second order derivatives of the following functions :

   (*i*) $\sin^{-1} x$ (NCERT) (*ii*) $x \cos x$ (*iii*) $x \sin 2x$

   (*iv*) $e^x \sin 5x$ (NCERT) (*v*) $e^{2x} \sin 3x$ (*vi*) $\log(\log x)$ (NCERT)

   (*vii*) $\frac{\log x}{x}$ (*viii*) $x^2 \log|\cos x|$ (*ix*) $\frac{2x+1}{2x+3}$.

   Hint. (*ix*) $\frac{2x+1}{2x+3} = 1 - \frac{2}{2x+3}$.

Long answer questions (4 to 26) :

4. Find the second derivatives of the following functions :

   (*i*) $\sec ax$ (*ii*) $\cot(1-2x)$ (NCERT) (*iii*) $\sin 3x \cos 5x$

   (*iv*) $\sin^3 x$ (NCERT) (*v*) $\cos(2x^2 - 1)$ (*vi*) $\sqrt{1-x^2}$. (NCERT)

   Hint. (*iv*) $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$.

5. If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone. (NCERT)

6. (*i*) If $y = \cot x$, prove that $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$.

   (*ii*) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$. (NCERT)

7. (*i*) If $y = x + \tan x$, prove that $\cos^2 x \cdot \frac{d^2y}{dx^2} - 2y + 2x = 0$.

   (*ii*) If $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$. (*Exemplar*)

   Hint. (*ii*) $y = \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{1+\sin x}{\cos x} \times \frac{1-\sin x}{1-\sin x} = \frac{\cos x}{1-\sin x}$.

8. (*i*) If $y = \sec x - \tan x$, prove that $\cos x \cdot \frac{d^2y}{dx^2} = y^2$.

   (*ii*) If $y = x + \cot x$, prove that $\sin^2 x \cdot \frac{d^2y}{dx^2} - 2y + 2x = 0$.

9. (*i*) If $y = ae^{mx} + be^{-mx}$, prove that $y_2 - m^2 y = 0$.

   (*ii*) If $y = ae^{2x} + be^{-x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.