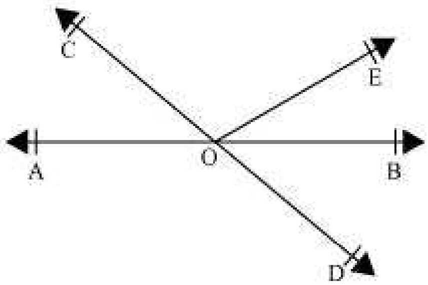
Class IX Chapter 6

Lines and Angles

Mathematics

Exercise 6.1

Question 1: In the given figure, lines AB and CD intersect at O. If ∠ AOC + ∠ BOE = 70°
And ∠ BOD = 40° find ∠BOE and reflex ∠COE.



Answer 1:

Given: ∠ AOC + ∠ BOE = 70°
And ∠ BOD = 40°

To Prove: find ∠BOE and reflex ∠COE

Proof: AB I a straight line, rays OC and OE stand on it

∴∠AOC + ∠COE + ∠BOE = 180^0

∴ (∠AOC + ∠BOE) + ∠COE = 180^0

∴ (70) + ∠COE = 180^0

∴ ∠COE = 180^0 - 70

∴ ∠COE = 110^0

∴ Reflex ∠COE = 360^0 - 110^0

∴ Reflex ∠COE = 250^0

CD is a straight line, ray OE and OB stand on it.

∴∠BOD + ∠COE + ∠BOE = 180^0

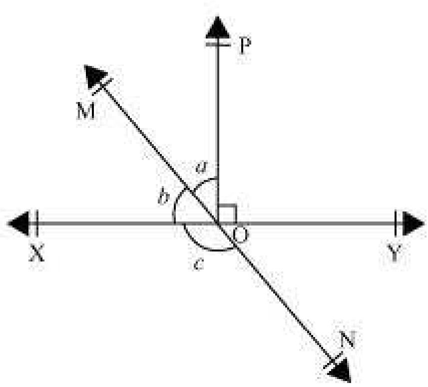
∴ 40 + 110 + ∠BOE = 180^0

∴ ∠BOE = 180^0 -150^0

∴ ∠BOE = 30^0

Question 2:

In the given figure, lines XY and MN intersect at O. If ∠POY = 90°and a:b = 2 : 3, find c.



Answer 2:

Given: ∠POY = 90° and a:b = 2 : 3

To Prove: find c

Proof: Let the common ratio between a and b be x.

∴ a = 2x, and b = 3x

XY is a straight line, rays OM and OP stand on it.

∴ ∠XOM + ∠MOP + ∠POY = 180°

b + a + ∠POY = 180°

3x + 2x + 90° = 180°

5x = 90°

x = 18°

a = 2x = 2 × 18 = 36°

b = 3x= 3 ×18 = 54°

MN is a straight line. Ray OX stands on it.

∴ b + c = 180° (Linear Pair)

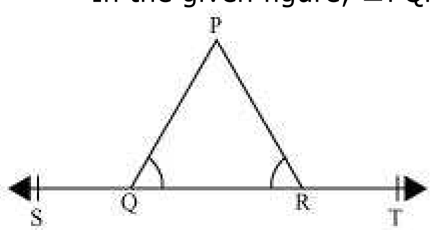
54° + c = 180°

c = 180° − 54° = 126°

∴ c = 126°

Question 3:

In the given figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Answer3:

Given: ∠PQR = ∠PRQ

To Prove: ∠PQS = ∠PRT

Proof: In the given figure, ST is a straight line and ray QP stands on it.

∴ ∠PQS + ∠PQR = 180° (Linear Pair)

∠PQR = 180° − ∠PQS (1)

∠PRT + ∠PRQ = 180° (Linear Pair)

∠PRQ = 180° − ∠PRT (2)

It is given that ∠PQR = ∠PRQ.

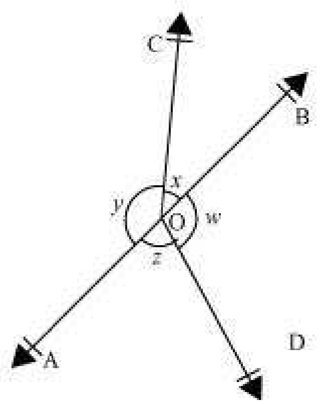
From Equating equations (1) and (2), we obtain

180° − ∠PQS = 180 − ∠PRT

∠PQS = ∠PRT

Question 4:

In the given figure, if x + y = w + z, then prove that AOB is a line.



Answer 4:

Given: $\angle x + \angle y = \angle w + \angle z$

To Prove: AOB is a line

Proof: It can be observed that,

$\angle x + \angle y + \angle z + \angle w = 360°$ (Complete angle) (1)

It is given that,

$\angle x + \angle y = \angle z + \angle w$, replacing this value in equation 1

$\therefore \angle x + \angle y + \angle x + \angle y = 360°$

$2(\angle x + \angle y) = 360°$

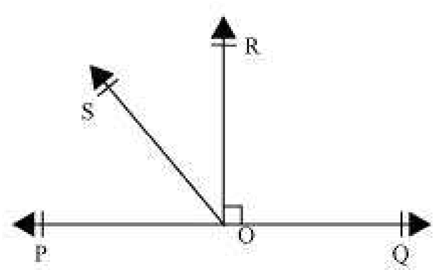
$\angle x + \angle y = 180°$

Since x and y form a linear pair, AOB is a line.

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$\angle ROS = ½(\angle QOS - \angle POS)$



Answer 5:

Given: POQ is a line. Ray OR is perpendicular to line PQ

To Prove: $\angle ROS = ½(\angle QOS - \angle POS)$

Proof: It is given that OR $\perp$ PQ

$\therefore \angle POR = 90°$

$\therefore \angle POS + \angle SOR = 90°$

$\angle ROS = 90° - \angle POS$ … (1)

$\angle QOR = 90°$ (As OR $\angle$ PQ)

$\angle QOS - \angle ROS = 90°$

$\angle ROS = \angle QOS - 90°$ … (2)

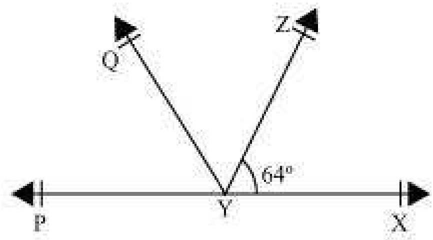
On adding equations (1) and (2), we obtain

$2 \angle ROS = \angle QOS - \angle POS$

$\angle ROS = ½(\angle QOS - \angle POS)$

Question 6:

It is given that $\angle XYZ =$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.



Answer 6:

Given: ∠XYZ = 64^0, ray YQ bisects ∠ZYP

To Prove: find ∠XYQ and reflex ∠QYP

Proof:

It is given that line YQ bisects ∠PYZ.

Hence, ∠QYP = ∠ZYQ

It can be observed that PX is a line. Rays YQ and YZ stand on it.

∴ ∠XYZ + ∠ZYQ + ∠QYP = 180°

∠ 64° + 2∠QYP = 180°

∴ 2∠QYP = 180° − 64° = 116°

∴ ∠QYP = 58°

Also, ∠ZYQ = ∠QYP = 58°

Reflex ∠QYP = 360° − 58° = 302°

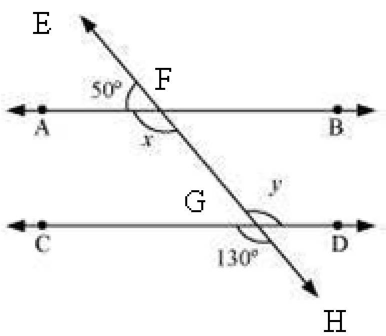
∠XYQ = ∠XYZ + ∠ZYQ

∠XYQ = 64° + 58° = 122°

Exercise 6.2

Question 1:

In the given figure, find the values of x and y and then show that AB || CD.



Answer 1:

Given: ∠ AFE = 50^0, ∠ CGH = 130^0

To Prove: find the values of x and y and then show that AB || CD

Proof: It can be observed that,

50° + x = 180° (Linear pair)

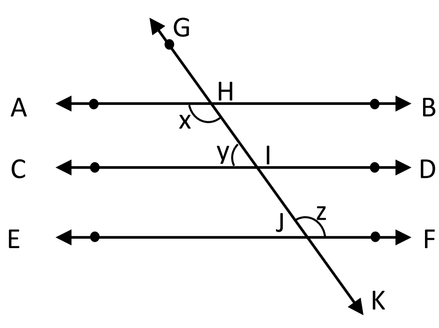
x = 130° … (1)

Also, y = 130° (Vertically opposite angles)

As x and y are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line AB || CD.

Question 2:

In the given figure, if AB || CD, CD || EF and y: z = 3: 7, find x.



Answer 2:

Given: AB || CD, CD || EF and y: z = 3: 7

To Prove: find x

Proof: It is given that AB || CD and CD || EF

∴ AB || CD || EF (Lines parallel to the same line are parallel to each other)

It can be observed that

x = z (Alternate interior angles) … (1)

It is given that y: z = 3: 7

Let the common ratio between y and z be a.

∴ y = 3a and z = 7a

Also, x + y = 180° (Co-interior angles on the same side of the transversal)

z + y = 180° [Using equation (1)]

7a + 3a = 180°

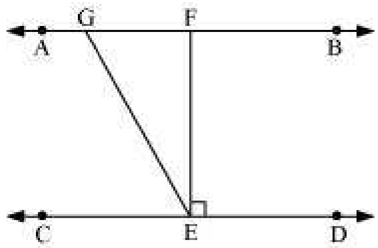
10a = 180°

a = 18°

∴ x = 7a = 7 × 18° = 126°

Question 3:

In the given figure, If AB || CD, EF || CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



Answer:

Given: AB || CD, EF || CD and ∠GED = 126°

To Prove: find ∠AGE, ∠GEF and ∠FGE

Proof: It is given that, AB || CD and EF || CD

∠GED = 126°

∴ ∠GEF + ∠FED = 126° (As given in figure)

∴ ∠GEF + 90° = 126°

∴ ∠GEF = 36°

∠AGE and ∠GED are alternate interior angles.

∴ ∠AGE = ∠GED = 126°

However, ∠AGE + ∠FGE = 180° (Linear pair)

∠ 126° + ∠FGE = 180°

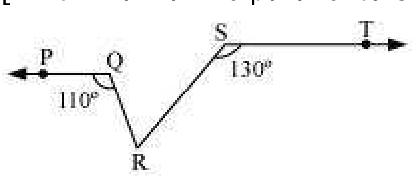
∴ ∠FGE = 180° − 126° = 54°

∴ ∠AGE = 126°, ∠GEF = 36°, ∠FGE = 54°

Question 4:

In the given figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[Hint: Draw a line parallel to ST through point R.]

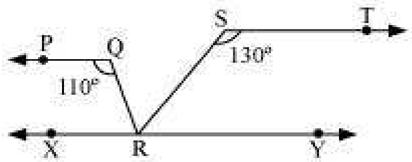


Answer 4:

Given: PQ || ST, ∠PQR = 110° and ∠RST = 130°

To Prove: find ∠QRS

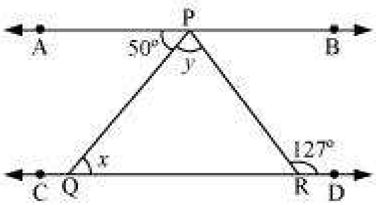
Construction: Let us draw a line XY parallel to ST and passing through point R.



Proof: ∠PQR + ∠QRX = 180° (Co-interior angles on the same side of transversal QR)
∠ 110° + ∠QRX = 180°
∴ ∠QRX = 70°
Also,
∠RST + ∠SRY = 180° (Co-interior angles on the same side of transversal SR)
130° + ∠SRY = 180°
∠SRY = 50°
XY is a straight line. RQ and RS stand on it.
∴ ∠QRX + ∠QRS + ∠SRY = 180°
70° + ∠QRS + 50° = 180°
∠QRS = 180° − 120° = 60°

Question 5:

In the given figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

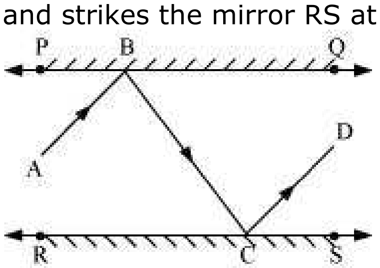


Answer 5:

Given: AB || CD, ∠APQ = 50° and ∠PRD = 127°
To Prove: find x and y
Proof: ∠APR = ∠PRD (Alternate interior angles)
50° + y = 127°
y = 127° − 50°
y = 77°
Also, ∠APQ = ∠PQR (Alternate interior angles)
50° = x
∠ x = 50° and y = 77°

Question 6:

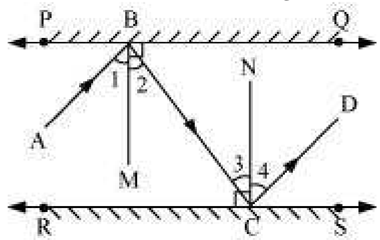
In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Answer 6:

Given: PQ and RS are parallel
An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD.
To Prove: AB || CD

Construction: Let us draw BM ⊥ PQ and CN ⊥ RS.



Proof: As PQ || RS,

Therefore, BM || CN

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

∴∠2 = ∠3 (Alternate interior angles)

However, ∠1 = ∠2 and ∠3 = ∠4 (By laws of reflection)

∴ ∠1 = ∠2 = ∠3 = ∠4

Also, ∠1 + ∠2 = ∠3 + ∠4

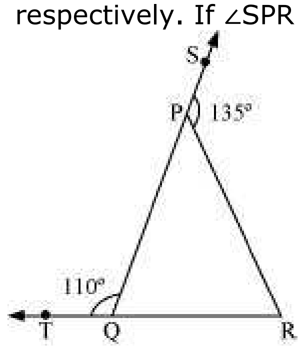
∠ABC = ∠DCB

However, these are alternate interior angles.

∴ AB || CD

Exercise 6.3

Question 1: In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.



Answer 1:

Given: ∠SPR = 135° and ∠PQT = 110°

To Prove: find ∠PRQ

Proof: It is given that,

∠SPR = 135° and ∠PQT = 110°

∠SPR + ∠QPR = 180° (Linear pair angles)

∠ 135° + ∠QPR = 180°

∴ ∠QPR = 45°

Also, ∠PQT + ∠PQR = 180° (Linear pair angles)

∠ 110° + ∠PQR = 180°

∴ ∠PQR = 70°

As the sum of all interior angles of a triangle is 180°, therefore, for MPQR,

∠QPR + ∠PQR + ∠PRQ = 180°

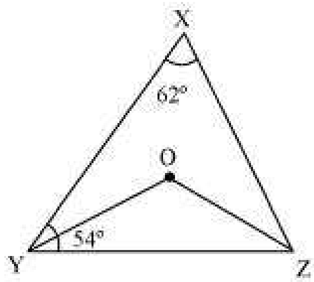
∠ 45° + 70° + ∠PRQ = 180°

∴ ∠PRQ = 180° − 115°

∴ ∠PRQ = 65°

Question 2:

In the given figure, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of MXYZ, find ∠OZY and ∠YOZ.



Answer 2:

Given: ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of MXYZ

To Prove: find ∠OZY and ∠YOZ

Proof: As the sum of all interior angles of a triangle is 180°, therefore, for MXYZ,

∠X + ∠XYZ + ∠XZY = 180°

62° + 54° + ∠XZY = 180°

∠XZY = 180° − 116°

∠XZY = 64°

∠OZY = 32° (OZ is the angle bisector of ∠XZY)

Similarly, ∠OYZ = 27°

Using angle sum property for MOYZ, we obtain

∠OYZ + ∠YOZ + ∠OZY = 180°

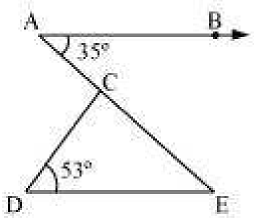
27° + ∠YOZ + 32° = 180°

∠YOZ = 180° − 59°

∠YOZ = 121°

Question 3:

In the given figure, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.



Answer 3:

Given: AB || DE, ∠BAC = 35° and ∠CDE = 53°

To Prove: find ∠DCE

Proof: AB || DE and AE is a transversal.

∠BAC = ∠CED (Alternate interior angles)

∴ ∠CED = 35°

In MCDE,

∠CDE + ∠CED + ∠DCE = 180° (Angle sum property of a triangle)

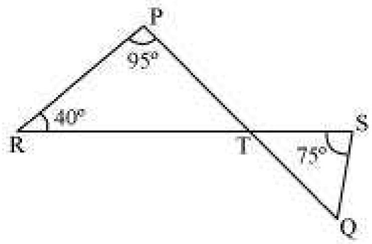
53° + 35° + ∠DCE = 180°

∠DCE = 180° − 88°

∠DCE = 92°

Question 4:

In the given figure, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.



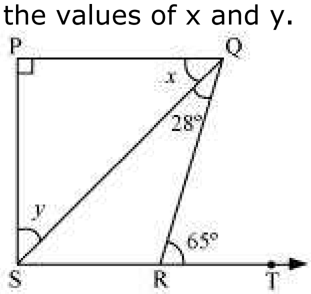
Answer 4:

Given: ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°
To Prove: find ∠SQT
Proof: Using angle sum property for MPRT, we obtain

∠PRT + ∠RPT + ∠PTR = 180°
40° + 95° + ∠PTR = 180°
∠PTR = 180° − 135°
∠PTR = 45°
∠STQ = ∠PTR = 45° (Vertically opposite angles)
∠STQ = 45°
By using angle sum property for MSTQ, we obtain
∠STQ + ∠SQT + ∠QST = 180°
45° + ∠SQT + 75° = 180°
∠SQT = 180° − 120°
∠SQT = 60°

Question 5:

In the given figure, if PQ || PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.



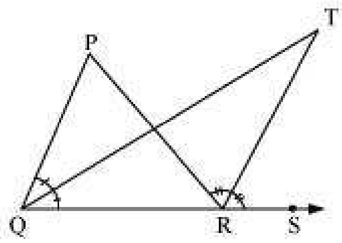
Answer 5:

Given: PQ || PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°
To Prove: find the values of x and y.
Proof: It is given that PQ || SR and QR is a transversal line.

∠PQR = ∠QRT (Alternate interior angles)
x + 28° = 65°
x = 65° − 28°
x = 37°
By using the angle sum property for MSPQ, we obtain
∠SPQ + x + y = 180°
90° + 37° + y = 180°
y = 180° − 127°
y = 53°
x = 37° and y = 53°

Question 6:

In the given figure, the side QR of ∆PQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR= ∠QPR.



Answer 6:

Given: bisectors of ∠PQR and ∠PRS meet at point T

To Prove: ∠QTR= ∠QPR

Proof: In ∆QTR, ∠TRS is an exterior angle.

∠QTR + ∠TQR = ∠TRS

∠QTR = ∠TRS − ∠TQR (1)

For ∆PQR, ∠PRS is an external angle.

∠QPR + ∠PQR = ∠PRS

∠QPR + 2∠TQR = 2∠TRS (As QT and RT are angle bisectors)

∠QPR = 2(∠TRS − ∠TQR)

∠QPR = 2∠QTR [By using equation (1)]

∠QTR = ½ ∠QPR