

Given:

$$\text{If } y_1(x) = e^{-x^2}$$

$$x y'' + \alpha y' + \beta x^3 y = 0.$$

$$\therefore y_1'(x) = e^{-x^2} \times (-2x) = -2x e^{-x^2}$$

$$y_1''(x) = -2x(e^{-x^2} \cdot (-2x)) + (e^{-x^2})(-2).$$

$$e^{-x^2} [4x^3 - 2x - 2x\alpha + \beta x^3] = 0.$$

$$\Rightarrow e^{-x^2} [(\beta + 4)x^3 - 2x(\alpha + 1)] = 0.$$

OR

$\alpha = -1$ & $\beta = -4$ Satisfy the eqn.

$$\therefore \boxed{\alpha\beta = +4}.$$