Class 11th Mathematics

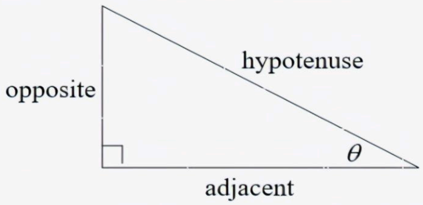
Chapter 3

Trigonometric functions

Revision notes

Right triangle definition

For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



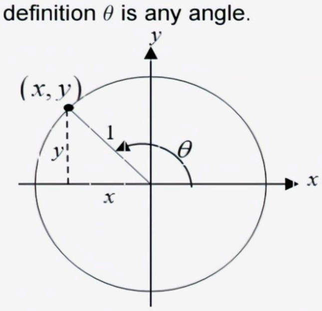
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$$

Unit Circle Definition

For this definition θ is any angle.



$$\sin(\theta) = \frac{y}{1} = y \qquad \csc(\theta) = \frac{1}{y}$$

$$\cos(\theta) = \frac{x}{1} = x \qquad \sec(\theta) = \frac{1}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin(\theta)$, θ can be any angle

$\cos(\theta)$, θ can be any angle

$\tan(\theta),\ \theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm 1, \pm 2, \ldots$

$\csc(\theta),\ \theta \neq n\pi,\ n = 0,\ \pm 1,\ \pm 2, \ldots$

$\sec(\theta),\ \theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm 1, \pm 2, \ldots$

$\cot(\theta),\ \theta \neq n\pi,\ n = 0, \pm 1, \pm 2, \ldots$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cos(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\tan(\omega\,\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

$$\csc(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\sec(\omega\,\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cot(\omega\,\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

Range

The range is all possible values to get out of the function.

$$-1 \leq \sin(\theta) \leq 1 \qquad -1 \leq \cos(\theta) \leq 1$$

$$-\infty < \tan(\theta) < \infty \qquad -\infty < \cot(\theta) < \infty$$

$$\sec(\theta) \geq 1 \text{ and } \sec(\theta) \leq -1 \qquad \csc(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1$$

Formulas and Identities

Tangent and Cotangent Identities

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Reciprocal Identities

$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sin(\theta) = \frac{1}{\csc(\theta)}$

$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cos(\theta) = \frac{1}{\sec(\theta)}$

$\cot(\theta) = \frac{1}{\tan(\theta)} \qquad \tan(\theta) = \frac{1}{\cot(\theta)}$

Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

Even/Odd Formulas

$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\cot(\theta)$

Periodic Formulas

If n is an integer then,

$\sin(\theta + 2\pi n) = \sin(\theta) \qquad \csc(\theta + 2\pi n) = \csc(\theta)$

$\cos(\theta + 2\pi n) = \cos(\theta) \qquad \sec(\theta + 2\pi n) = \sec(\theta)$

$\tan(\theta + \pi n) = \tan(\theta) \qquad \cot(\theta + \pi n) = \cot(\theta)$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180}$ and $x = \frac{180t}{\pi}$

Double Angle Formulas

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= 2\cos^2(\theta) - 1$$
$$= 1 - 2\sin^2(\theta)$$

$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$

Half Angle Formulas

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$

Half Angle Formulas (alternate form)

$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)) \qquad \tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

$\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

Product to Sum Formulas

$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

Sum to Product Formulas

$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

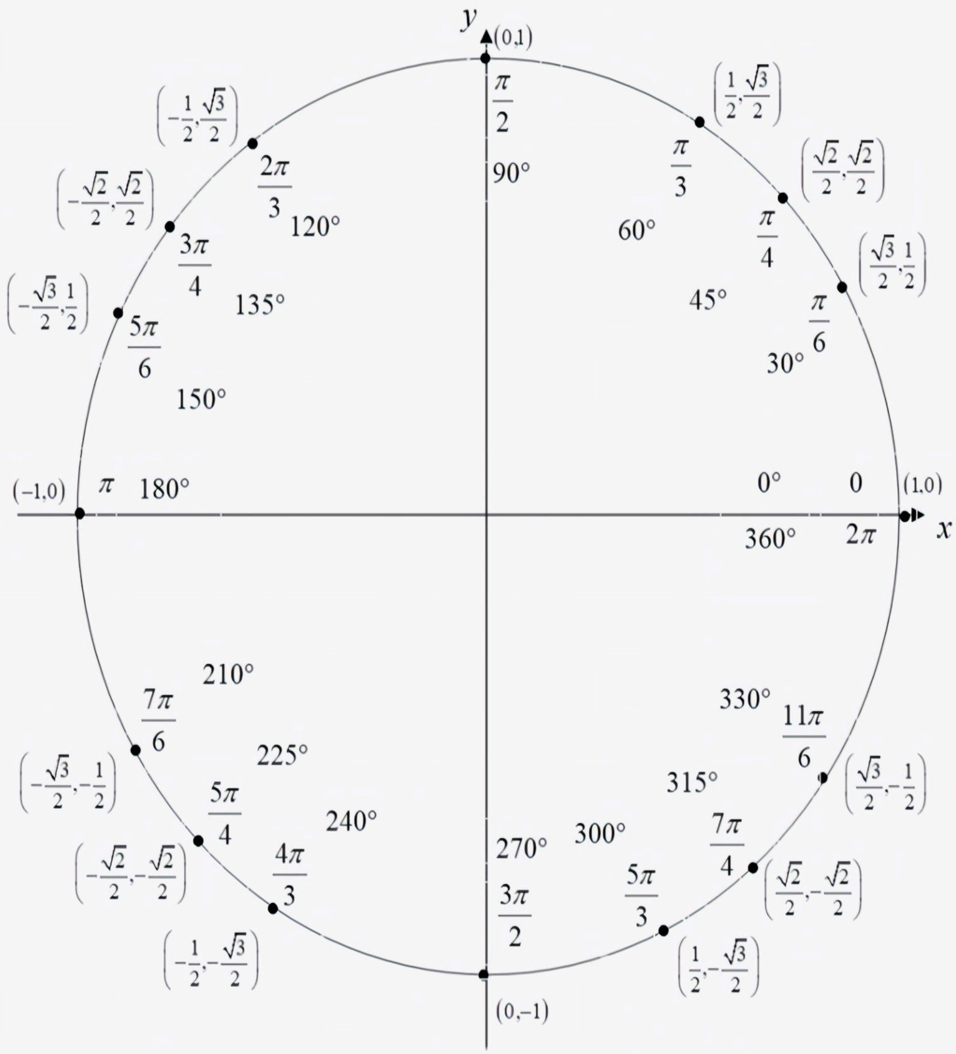
$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

Cofunction Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta) \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$



For any ordered pair on the unit circle (x, y) : cos$(\theta) = x$ and sin$(\theta) = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$

$y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$

$y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \quad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \quad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \quad \tan^{-1}\left(\tan(\theta)\right) = \theta$

Domain and Range

Function	Domain	Range
$y = \sin^{-1}(x)$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1}(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

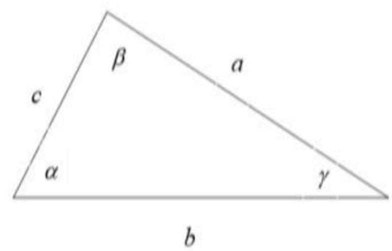
Alternate Notation

$\sin^{-1}(x) = \arcsin(x)$

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\left(\frac{1}{2}(\alpha-\beta)\right)}{\sin\left(\frac{1}{2}\gamma\right)}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(\beta-\gamma)\right)}{\tan\left(\frac{1}{2}(\beta+\gamma)\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$