3. Algebra

Ex-3.1

1) (i) $x+y+z=5$ — ①

$2x-y+z=9$ — ②

$x-2y+3z=16$ — ③

Add ①+② ⇒ $3x+2z=14$ — ④

② × 2 ⇒ $4x-\cancel{2y}+2z=18$

$x-\cancel{2y}+3z=16$ — ③

$(-)\quad(+)\quad(-)\quad(-)$

$3x-z=2$ — ⑤

④ − ⑤ ⇒ $3z=12$

$z=\frac{12}{3}=4$

Sub. in ⑤

$3x-4=2$

$3x=2+4=6$

$x=\frac{6}{3}=2$

Sub. in ①, $2+y+4=5$

$y+6=5$

$y=5-6=-1$

$$\boxed{\begin{aligned}x&=2\\y&=-1\\z&=4\end{aligned}}$$

(ii) $\frac{1}{x}-\frac{2}{y}+4=0$ — ①

$\frac{1}{y}-\frac{1}{z}+1=0$ — ②

$\frac{2}{z}+\frac{3}{x}=14$ — ③

Put $\frac{1}{x}=a$, $\frac{1}{y}=b$, $\frac{1}{z}=c$

$a-2b=-4$ — ④

$b-c=-1$ — ⑤

$3a+2c=14$ — ⑥

⑤ × 2 ⇒ $\cancel{2b}-2c=-2$

$a-\cancel{2b}=-4$ — ④

Add

$a-\cancel{2c}=-6$ — ⑦

$3a+\cancel{2c}=14$ — ⑥

Add

$4a=8$

$a=\frac{8}{4}=2$

Sub. in ④, $2-2b=-4$

$-2b=-4-2=-6$

$b=\frac{-6}{-2}=3$

Sub. in ⑤, $3-c=-1$

$-c=-1-3=-4$

$c=4$

$a=2$, $x=\frac{1}{a}=\frac{1}{2}$

$b=3$, $y=\frac{1}{b}=\frac{1}{3}$

$c=4$, $z=\frac{1}{c}=\frac{1}{4}$

(iii) $\overset{I}{x+20}=\overset{II}{\frac{3y}{2}+10}=\overset{III}{2z+5}=\overset{IV}{110-(y+z)}$

From I & II, $x+20=\frac{3y}{2}+10$

$x=\frac{3y}{2}+10-20$

$x=\frac{3y}{2}-10$

multiply by 2, $2x=3y-20$

$2x-3y=-20$ — ①

From I + III, $x+20=2z+5$

$x-2z=5-20$

$x-2z=-15$ — ②

From I & IV, $x+20=110-(y+z)$

$x+y+z=110-20$

$x+y+z=90$ — ③

③ × 2 ⇒ $2x+2y+2z=180$

$x\quad-2z=-15$ — ②

add $3x+2y=165$ — ④

④ × 2 ⇒ $6x+4y=330$

① × 3 ⇒ $6x-9y=-60$

$(-)\quad(+)\quad(+)$

Subtract $13y=390$

$y=\frac{390}{13}$

$\boxed{y=30}$

Sub. in ①

$2x-3(30)=-20$

$2x-90=-20$

$2x=-20+90=70$

$x=\frac{70}{2}$

$\boxed{x=35}$

Sub. in ②

$35-2z=-15$

$-2z=-15-35$

$-2z=-50$

$z=\frac{-50}{-2}$

$\boxed{z=25}$

Solution set is $\{35, 30, 25\}$

Hence, the system has unique solution.

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2) (i) $x + 2y - z = 6$ — ①

$-3x - 2y + 5z = -12$ — ②

$x - 2z = 3$ — ③

$-2x + 4z = -6$ — ④

① + ② ⇒

③ × −2 $-2x + 4z = -6$

(+) (−) (+)

$0 = 0$

Subtract

∴ The system has an infinite no of solutions

(ii) $2y + z = 3(-x + 1)$

$2y + z = -3x + 3$

$3x + 2y + z = 3$ — ①

$-x + 3y - z = -4$ — ②

$3x + 2y + z = -\frac{1}{2}$

$6x + 4y + 2z = -1$ — ③

① × 2 $6x + 4y + 2z = 6$ — ④

(−) (−) (−) (−)

$0 \neq -7$

③ − ④ ⇒

The system is inconsistent and It has no solution.

(iii) $\overset{I}{\frac{y+z}{4}} = \overset{II}{\frac{z+x}{3}} = \overset{III}{\frac{x+y}{2}}$

$x + y + z = 27$ — ①

From I & II, $\frac{y+z}{4} = \frac{z+x}{3}$

$3(y+z) = 4(z+x)$

$3y + 3z = 4z + 4x$

$-4x + 3y + 3z - 4z = 0$

$-4x + 3y - z = 0$ — ②

From II & III, $\frac{z+x}{3} = \frac{x+y}{2}$

$2(z+x) = 3(x+y)$

$2z + 2x = 3x + 3y$

$3x + 3y - 2x - 2z = 0$

$x + 3y - 2z = 0$ — ③

$x + y + z = 27$ — ①

(−) (−) (−) (−)

③ − ① ⇒ $2y - 3z = -27$ — ④

① × 4 ⇒ $4x + 4y + 4z = 108$

$-4x + 3y - z = 0$ — ②

Add $7y + 3z = 108$ — ⑤

Add ④ & ⑤, $9y = 81$

$y = \frac{81}{9}$

$\boxed{y = 9}$

Sub. in ④

$2(9) - 3z = -27$

$18 - 3z = -27$

$-3z = -27 - 18 = -45$

$z = \frac{-45}{-3}$

$\boxed{z = 15}$

Sub. in ①

$x + 9 + 15 = 27$

$x + 24 = 27$

$x = 27 - 24$

$\boxed{x = 3}$

Solution set is $\{3, 9, 15\}$

It has unique solution.

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3) Vani's age $= x$

Her father's age $= y$

Her grand father's age $= z$

Given $\frac{x+y+z}{3} = 53$

$x + y + z = 159$ — ①

$\frac{z}{2} + \frac{y}{3} + \frac{x}{4} = 65$

$\frac{6z + 4y + 3x}{12} = 65$

$3x + 4y + 6z = 65 \times 12 = 780$ — ②

$z - 4 = 4(x - 4)$

$z - 4 = 4x - 16$

$4x - z = 16 - 4 = 12$ — ③

① × 4 ⇒ $4x + 4y + 4z = 636$

$3x + 4y + 6z = 780$ — ②

(−) (−) (−) (−)

$x - 2z = -144$ — ④

④ × 4 ⇒ $4x - 8z = -576$

$4x - z = 12$ — ③

(−) (+) (−)

$-7z = -588$

$z = \frac{-588}{-7}$

$\boxed{z = 84}$

Sub. in ④,

$x - 2(84) = -144$

$x - 168 = -144$

$x = 168 - 144$

$\boxed{x = 24}$

Sub. in ①

$24 + y + 84 = 159$

$y + 108 = 159$

$y = 159 - 108$

$\boxed{y = 51}$

Ans Vani's age = 24

Her father's age = 51

Her grand father's age = 84

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4) Let the number be $100x+10y+z$

Reversed no. be $100z+10y+x$

$x+y+z=11$ — ①

$100z+10y+x = 5(100x+10y+z)+46$

$= 500x+50y+5z+46$

$499x+40y-95z = -46$ — ②

$x+2y = z$

$x+2y-z=0$ — ③

$x+y+z=11$ — ①

Add $2x+3y = 11$ — ④

③ ×95 $95x+190y-95z=0$

$499x+40y-95z=-46$ — ②

(−) (−) (+) (+)

$-404x+150y = 46$ — ⑤

∴ ④ ×50 ⇒ $100x+150y = 550$

(−) (−) (−)

$-504x = -504$

$x = \frac{-504}{-504}$

$\boxed{x=1}$

Sub. in ④,

$2(1)+3y = 11$

$2+3y = 11$

$3y = 11-2 = 9$

$y = \frac{9}{3}$

$\boxed{y=3}$

Sub. in ①

$1+3+z = 11$

$4+z = 11$

$z = 11-4$

$\boxed{z=7}$

∴ The number is xyz = 137

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5) Let x, y and z be no. of currency pieces of ₹ 5, 10, 20 rupees respectively.

$x+y+z = 12$ — ①

$5x+10y+20z = 105$ — ②

(x and y are inter changed)

$10x+5y+20z = 105+20 = 125$

① ×5 ⇒ $5x+5y+5z = 60$

$5x+10y+20z = 105$ — ②

(−) (−) (−) (−)

$-5y-15z = -45$ — ④

② ×2 ⇒ $10x+20y+40z = 210$

$10x+5y+20z = 125$ — ③

(−) (−) (−) (−)

$15y+20z = 85$ — ⑤

④ ×3 ⇒ $-15y-45z = -135$ — ⑥

Add ⑤+⑥, $-25z = -50$

$z = \frac{-50}{-25}$

$\boxed{z=2}$

Sub. in ④,

$-5y-15(2) = -45$

$-5y-30 = -45$

$-5y = -45+30 = -15$

$y = \frac{-15}{-5}$

$\boxed{y=3}$

Sub. in ①

$x+3+2 = 12$

$x+5 = 12$

$x = 12-5$

$\boxed{x=7}$

∴ No. of 5 rupees currency = 7

No. of 10 rupees currency = 3

No. of 20 rupees currency = 2

Ex - 3.2

(i) $f(x) = x^4+3x^3-x-3$

$g(x) = x^3+x^2-5x+3$

$$\begin{array}{r|l} & x+2 \\ \hline x^3+x^2-5x+3 & x^4+3x^3+0x^2-x-3 \\ & x^4+x^3-5x^2+3x \downarrow \\ & (-)\ (-)\ (+)\ (-) \\ \hline & 2x^3+5x^2-4x-3 \\ & 2x^3+2x^2-10x+6 \\ & (-)\ (-)\ (+)\ (-) \\ \hline & 3x^2+6x-9 \\ \hline \end{array}$$

$3(x^2+2x-3) \neq 0$

$$\begin{array}{r|l} & x-1 \\ \hline x^2+2x-3 & x^3+x^2-5x+3 \\ & x^3+2x^2-3x \downarrow \\ & (-)\ (-)\ (+) \\ \hline & -x^2-2x+3 \\ & -x^2-2x+3 \\ & (+)\ (+)\ (-) \\ \hline & 0 \\ \hline \end{array}$$

$\therefore$ GCD $= x^2+2x-3$

(ii) $f(x) = x^4-1$

$g(x) = x^3-11x^2+x-11$

$$\begin{array}{r|l} & x+11 \\ \hline x^3-11x^2+x-11 & x^4+0x^3+0x^2+0x-1 \\ & x^4-11x^3+x^2-11x \downarrow \\ & (-)\ (+)\ (-)\ (+) \\ \hline & 11x^3-x^2+11x-1 \\ & 11x^3-121x^2+11x-121 \\ & (-)\ (+)\ (-)\ (+) \\ \hline & 120x^2+120 \\ \hline \end{array}$$

$120(x^2+1) \neq 0$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3-11x^2+x-11 \\ & x^3-0x^2+x \downarrow \\ & (-)\ (+)\ (-) \\ \hline & -11x^2-11 \\ \hline \end{array}$$

$-11(x^2+1) \neq 0$

$$\begin{array}{r|l} & 1 \\ \hline x^2+1 & x^2+0x+1 \\ & x^2+0x+1 \\ & (-)\ (-)\ (-) \\ \hline & 0 \\ \hline \end{array}$$

$\therefore$ GCD $= x^2+1$

(iii) $3x^4+6x^3-12x^2+24x$

$= 3(x^4+2x^3-4x^2+8x)$

$4x^4+14x^3+8x^2-8x$

$= 2(2x^4+7x^3+4x^2-4x)$

$$\begin{array}{r|l} & 2 \\ \hline x^4+2x^3-4x^2+8x & 2x^4+7x^3+4x^2-4x \\ & 2x^4+4x^3-8x^2+16x \\ & (-)\ (-)\ (+)\ (+) \\ \hline & 3x^3+12x^2+12x \\ \hline \end{array}$$

$3(x^3+4x^2+4x) \neq 0$

$$\begin{array}{r|l} & x-2 \\ \hline x^3+4x^2+4x & x^4+2x^3-4x^2-8x \\ & x^4+4x^2+4x^2 \downarrow \\ & (-)\ (-)\ (-) \\ \hline & -2x^3-8x^2-8x \\ & -2x^3-8x^2-8x \\ & (+)\ (+)\ (+) \\ \hline & 0 \\ \hline \end{array}$$

GCD $= x^3+4x^2+4x = x(x^2+4x+4)$

(iv) $f(x) = 3x^3+3x^2+3x+3$

$g(x) = 6x^3+12x^2+6x+12$

$$\begin{array}{r|l} & 2 \\ \hline 3x^3+3x^2+3x+3 & 6x^3+12x^2+6x+12 \\ & 6x^3+6x^2+6x+6 \\ & (-)\ (-)\ (-)\ (-) \\ \hline & 6x^2+6 \\ \hline \end{array}$$

$6(x^2+1) \neq 0$

$$\begin{array}{r|l} & 3x+3 \\ \hline x^2+1 & 3x^3+3x^2+3x+3 \\ & 3x^3+0.+3x \downarrow \\ & (-)\ (-) \\ \hline & 3x^2+3 \\ & 3x^2+3 \\ & (-)\ (-) \\ \hline & 0 \\ \hline \end{array}$$

G.C.D $= 3(x^2+1)$

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2) (i) $4x^2y = \boxed{2} \times \boxed{2} \times x^2 \times y$

$8x^3y^2 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{x^3} \times \boxed{y^2}$

LCM $= 2 \times 2 \times 2 \times x^3 \times y^2$

$= 8x^3y^2$

(ii) $9a^3b^2 = 3\times3\times a^3\times b^2$

$12a^2b^2c = 3\times2\times2\times a^2\times b^2\times c$

$LCM = 3\times3\times2\times2\times a^3\times b^2\times c$

$= 36a^3b^2c$

(iii) $16m, 12m^2n^2, 8n^2$

$$\begin{array}{r|l} 2 & 16, 12, 8 \\ 2 & 8, 6, 4 \\ 2 & 4, 3, 2 \\ 2 & 2, 3, 1 \\ 3 & 1, 3, 1 \\ & 1, 1, 1 \end{array}$$

$LCM = 2\times2\times2\times2\times3\times m^2\times n^2$

$= 48m^2n^2$

(iv) $p^2-3p+2 = (p-2)(p-1)$ $\quad 2,\ -3,\ -2\ \ -1$

$p^2-4 = (p^2-2^2) = (p-2)(p+2)$

$LCM = (p-2)(p-1)(p+2)$

(v) $2x^2-5x-3 = (x-3)(2x+1)$ $\quad -6,\ -5,\ \frac{-6}{2}\ \ \frac{1}{2}$

$4x^2-36 = (2x)^2-6^2 = (2x+6)(2x-6)$

$= 4(x+3)(x-3)$ $\quad = -3$

$LCM = 4(x-3)(x+3)(2x+1)$

(vi) $(2x^2-3xy)^2 = x^2(2x-3y)^2$

$(4x-6y)^3 = 2^3(2x-3y)^3$

$= 8(2x-3y)^3$

$8x^3-27y^3 = (2x)^3-(3y)^3$

$= (2x-3y)(4x^2+6xy+9y^2)$

$LCM = 8x^2(2x-3y)^3(4x^2+6xy+9y^2)$

Ex-3.3

1)(i) $f(x) = 21x^2y$

$g(x) = 35xy^2$

$$\begin{array}{r|l} 3 & 21, 35 \\ 5 & 7, 35 \\ 7 & 7, 7 \\ & 1, 1 \end{array}$$

$LCM = 105x^2y^2$

$GCD = 7xy$

$LCM = 3\times5\times7$

$= 105$

$GCD = 7$

$f(x)\times g(x) = (21x^2y)(35xy^2)$

$= 735x^3y^3$

$LCM\times GCD = (105x^2y^2)(7xy)$

$= 735x^3y^3$

$\therefore LCM\times GCD = f(x)\times g(x)$

(ii) $f(x) = (x^3-1)(x+1)$

$= (x-1)(x^2+x+1)(x+1)$

$g(x) = x^3+1$

$= (x+1)(x^2-x+1)$

$LCM = (x-1)(x+1)(x^2+x+1)(x^2-x+1)$

$GCD = (x+1)$

$LCM\times GCD = (x-1)(x+1)^2(x^2+x+1)(x^2$

$= (x+1)(x^3+1)(x^3-1)$

$= (x+1)(x^6-1)$

$f(x)\cdot g(x) = (x^3-1)(x+1)\cdot(x^3+1)$

$= (x^6-1)(x+1)$

$\therefore LCM\times GCD = f(x)\times g(x)$

(iii) $f(x) = x^2y+xy^2 = xy(x+y)$

$g(x) = x^2+xy = x(x+y)$

$LCM = xy(x+y)$

$GCD = x(x+y)$

$LCM\times GCD = x^2y(x+y)^2$

$f(x).g(x) = x^2y(x+y)^2$

$\therefore LCM\times GCD = f(x)\cdot g(x)$

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2)(i) $f(a) = a^2+4a-12 = (a+6)(a-$

$g(a) = a^2-5a+6 = (a-3)(a-$

$GCD = (a-2)$

$LCM = \frac{f(a)\times g(a)}{GCD}$

$= \frac{(a+6)(a-\cancel{2})(a-3)(a-2)}{(a-\cancel{2})}$

$= (a+6)(a-3)(a-2)$

(ii) $f(x) = x^4-27a^3x$

$= x(x^3-(3a)^3)$

$= x(x-3a)(x^2+3ax+9)$

$g(x) = (x-3a)^2$

$GCD = (x-3a)$

$LCM = \frac{f(x)\times g(x)}{GCD}$

$= \frac{x(x-\cancel{3a})(x^2+3ax+9a^2)(x-3}{(x-\cancel{3a})}$

$= x(x-3a)^2(x^2+3ax+9a^2)$

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3)(i) $f(x) = 12(x^4-x^3) = 12x^3(x-1)$

$g(x) = 8(x^4-3x^3+2x^2)$

$= 8x^2(x^2-3x+2)$

$= 8x^2(x-1)(x-2)$

$LCM = 24x^3(x-1)(x-2)$

$GCD = \frac{f(x)\times g(x)}{LCM}$

$= \frac{\cancel{12}\,x^{\cancel{3}}(x-\cancel{1})(\cancel{8}x^2)(x-}{\cancel{24}x^3(\cancel{x-1})(x-\cancel{2})}$ $\quad 4$

$= 4x^2(x-1)$

(ii) $f(x) = x^3+y^3$

$g(x) = x^4+x^2y^2+y^4$

$LCM = (x^3+y^3)(x^2+xy+y^2)$

$$h(x) = \frac{f(x).g(x)}{LCM}$$

$$= \frac{(\cancel{x^3+y^3})(x^4+x^2y^2+y^4)}{(\cancel{x^3+y^3})(x^2+xy+y^2)}$$

$$= x^2-xy+y^2$$

$$\begin{array}{r|l}
 & x^2-xy+y^2 \\
\hline
x^2+xy+y^2 & x^4+\cancel{x^2y^2}+y^4 \\
 & x^4+x^3y+\cancel{x^2y^2} \\
 & (-)\quad(-)\quad(-) \\
\hline
 & -\cancel{x^3y}+y^4 \\
 & -\cancel{x^3y}-x^2y^2-xy^3 \\
 & (+)\quad(+)\quad(+) \\
\hline
 & y^4+x^2y^2+xy^3 \\
 & y^4+x^2y^2+xy^3 \\
 & (-)\quad(-)\quad(-) \\
\hline
 & 0 \\
\hline
\end{array}$$

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4)(i) $LCM = a^3-10a^2+11a+70$

$= (a-7)(a^2-3a-10)$

$= (a-7)(a-5)(a+2)$

$$\begin{array}{r|rrrr}
7 & 1 & -10 & 11 & 70 \\
 & 0 & 7 & -21 & 70 \\
\hline
 & 1 & -3 & -10 & \boxed{0}
\end{array}$$

$GCD = (a-7)$

$p(x) = a^2-12a+35$

$= (a^2-5)(a-7)$

$$q(x) = \frac{LCM \times GCD}{p(x)}$$

$$= \frac{(\cancel{a-7})(\cancel{a-5})(a+2)(a-7)}{(\cancel{a-5})(\cancel{a-7})}$$

$$= (a+2)(a-7)$$

(ii) $LCM = (x^4-y^4)(x^4+x^2y^2+y^4)$

$= (x^4-y^4)(x^2+xy+y^2)(x^2-xy+y^2)$

$GCD = x^2-y^2$

$q(x) = (x^4-y^4)(x^2+y^2-xy)$

$$p(x) = \frac{LCM \times GCD}{q(x)}$$

$$= \frac{(\cancel{x^4-y^4})(x^2+xy+y^2)(\cancel{x^2-xy+y^2})(x^2-y^2)}{(\cancel{x^4-y^4})(\cancel{x^2+y^2-xy})}$$

$$= (x^2+xy+y^2)(x^2-y^2)$$

Ex-3.4

1)(i) $$\frac{x^2-1}{x^2+x} = \frac{(\cancel{x+1})(x-1)}{x(\cancel{x+1})} = \frac{x-1}{x}$$

(ii) $$\frac{x^2-11x+18}{x^2-4x+4} = \frac{(\cancel{x-2})(x-9)}{(\cancel{x-2})(x-2)} = \frac{x-9}{x-2}$$

(iii) $$\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9x(x+9)}{x(x^2+8x-9)}$$

$$= \frac{9\cancel{x}(\cancel{x+9})}{\cancel{x}(\cancel{x+9})(x-1)} = \frac{9}{x-1}$$

(iv) $$\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{(p-8)(p+5)}{2p(p^2-12p+32)}$$

$$= \frac{(\cancel{p-8})(p+5)}{2p(\cancel{p-8})(p-4)}$$

$$= \frac{(p+5)}{2p(p-4)}$$

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2)(i) $$\frac{y}{y^2-25}$$

$y^2-25 = 0$

$y^2 = 25$

$y = \pm 5$

Excluded values are +5, -5

(ii) $$\frac{t}{t^2-5t+6}$$

$t^2-5t+6 = 0$

$(t-3)(t-2) = 0$

$t-3 = 0 \quad | \quad t-2 = 0$

$t = 3 \quad | \quad t = 2$

Excluded values are 2, 3

(iii) $$\frac{x^2+6x+8}{x^2+x-2} = \frac{(\cancel{x+2})(x+4)}{(x-1)(\cancel{x+2})} = \frac{x+4}{x-1}$$

$x-1 = 0$

$x = 1$

Excluded value is 1

(iv) $$\frac{x^3-27}{x^3+x^2-6x} = \frac{x^3-3^3}{x(x^2+x-6)}$$

$$= \frac{x^3-27}{x(x+3)(x-2)}$$

$x(x+3)(x-2) = 0$

$x = 0, \quad x = -3, \quad x = 2$

The excluded values are 0, 2, -3

Ex - 3.5

1) (i) $\frac{\cancel{6}x^2 y}{\cancel{2}z^2} \times \frac{\overset{3}{\cancel{6}}xz^5}{\underset{5}{\cancel{20}}y^4} = \frac{3x^3z}{5y^3}$

(ii) $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$

$= \frac{(p-\cancel{7})(p-3)}{(p-\cancel{7})} \times \frac{(p+4)(p-\cancel{3})}{(p-\cancel{3})(p-\cancel{3})}$

$= (p+4)$

(iii) $\frac{\cancel{3}t^3}{4t-8} \times \frac{6t-12}{\cancel{16}t}$

$= \frac{t^{\overset{2}{\cancel{3}}} \times \overset{3}{\cancel{6}}(t-\cancel{2})}{4(t-\cancel{2})\cancel{4t}}$

$= \frac{3t^2}{4}$

2) (i) $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

−40

3

$\frac{8}{2}$ $-\frac{5}{2}$

$=4$

$= \frac{\cancel{x+4}}{(3x+4y)} \times \frac{(3x)^2-(4y)^2}{(\cancel{x+4})(2x-5)}$

$= \frac{(3x+\cancel{4y})(3x-4y)}{(3x+\cancel{4y})(2x-5)} = \frac{3x-4y}{2x-5}$

(ii) $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$

$= \frac{(\cancel{x-y})(x^2+xy+y^2)}{3(x^2+3xy+2y^2)} \times \frac{x^2+2xy+y^2}{(\cancel{x-y})(x+y)}$

$= \frac{(x^2+xy+y^2)}{3(x+2y)(x+y)} \times \frac{x^2+2xy+y^2}{(x+y)}$

$= \frac{(x^2+xy+y^2) \times (\cancel{x+y})(\cancel{x+y})}{3(x+2y)(\cancel{x+y})\ (\cancel{x+y})}$

$= \frac{x^2+xy+y^2}{3(x+2y)}$

3) (i) $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$

$= \frac{(\cancel{2a+3})(a+1)}{(\cancel{2a+3})(a+2)} \div \frac{(a+5)(a+1)}{-5(a^2+7a+10)}$

$= \frac{\cancel{(a+1)}}{\cancel{(a+2)}} \times \frac{-5(\cancel{a+2})(\cancel{a+5})}{(\cancel{a+5})(\cancel{a+1})}$

$= -5$

(ii) $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$

$= \frac{(b+7)(b-4)}{(b+2)(b+2)} \div \frac{(b+7)(b-\cancel{7})}{(b-\cancel{7})(b+2)}$

$= \frac{(\cancel{b+7})(b+4)}{(b+2)(\cancel{b+2})} \times \frac{\cancel{b+2}}{\cancel{b+7}}$

$= \frac{b+4}{b+2}$

(iii) $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$

$= \frac{\cancel{x+2}}{\cancel{4y}} \times \frac{\overset{3}{\cancel{12}}y^{\cancel{2}}}{(x-3)(\cancel{x+2})}$

$= \frac{3y}{x-3}$

(iv) $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

$= \frac{2(6t^2-11t+4)}{3t} \div \frac{(3t-4)(t+\cancel{2})}{2t(t+\cancel{2})}$

$= \frac{2(\cancel{3t-4})(2t-1)}{3\cancel{t}} \times \frac{2\cancel{t}}{(\cancel{3t-4})}$

$= \frac{4(2t-1)}{3}$

4) $x = \frac{a^2+3a-4}{3a^2-3} = \frac{(a+4)(a-1)}{3(a^2-1)}$

$= \frac{(a+4)(\cancel{a-1})}{3(a+1)(\cancel{a-1})}$

$x = \frac{a+4}{3(a+1)}$

$y = \frac{a^2+2a-8}{2a^2-2a-4} = \frac{(a+4)(a-2)}{2(a^2-a-2)}$

$= \frac{(a+4)(\cancel{a-2})}{2(\cancel{a-2})(a+1)}$

$y = \frac{(a+4)}{2(a+1)}$

$x^2y^{-2} = \frac{x^2}{y^2} = \frac{\left(\frac{a+4}{3(a+1)}\right)^2}{\left(\frac{(a+4)}{2(a+1)}\right)^2}$

$= \frac{\cancel{(a+4)^2}}{9\cancel{(a+1)^2}} \times \frac{4\cancel{(a+1)^2}}{\cancel{(a+4)^2}}$

$= \frac{4}{9}$

a) $p(x) = x^2 - 5x - 14$

$= (x-7)(x+2)$

Given $\frac{p(x)}{q(x)} = \frac{x-7}{x+2}$

$$\frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2}$$

$$\cancel{(x-7)}(x+2) \times \frac{(x+2)}{\cancel{(x-7)}} = q(x)$$

$$q(x) = (x+2)^2$$

$$= x^2 + 4x + 4$$

Ex-3.6

Unit Exercise - 3

1) $\overset{A}{\frac{1}{3}(x+y-5)} = \overset{B}{y-z} = \overset{C}{2x-11} = \overset{D}{9-(x+2z)}$

From A and B, $\frac{1}{3}(x+y-5) = y-z$

$x+y-5 = 3y-3z$

$x+y-5-3y+3z = 0$

$x-2y+3z = 5$ —— ①

From B and C, $y-z = 2x-11$

$2x-y+z = 11$ —— ②

From C and D, $2x-11 = 9-(x+2z)$

$2x+x+2z = 9+11$

$3x+2z = 20$ —— ③

② × 2 ⇒ $4x - \cancel{2y} + 2z = 22$

① ⇒ $x - \cancel{2y} + 3z = 5$

(−) (+) (−) (−)

$3x - z = 17$ —— ④

③ ⇒ $\cancel{3x} + 2z = 20$

(−) (−) (−)

$-3z = -3$

$z = \frac{-3}{-3} = 1$

Sub. in ④, $3x - 1 = 17$

$3x = 17+1 = 18$

$x = \frac{18}{3} = 6$

Sub. in ②

$2(6) - y + 1 = 11$

$12 - y + 1 = 11$

$13 - y = 11$

$13 - 11 = y$

$y = 2$

∴ solution set is $\{6, 2, 1\}$.

Let the students in section A, B, C be a, b, c respectively.

$a+b+c = 150$ —— ①

$a-b = c+6$ —— ②

$4c = a+b$ —— ③

Sub. in ①

$4c + c = 150$

$5c = 150$

$c = \frac{150}{5} = 30.$

Sub. in ②

$a-b = 30+6$

$a = 30+6+b$

$a = 42$

Sub. in ①

$42 + b + 30 = 150$

$b + 72 = 150$

$b = 150 - 72$

$b = 78$

Ans $\boxed{\begin{array}{l} a = 42 \\ b = 78 \\ c = 30 \end{array}}$

---

3) Let the 3 digits nos. be $100a + 10b + c$.

Given

$100b + 10a + c = 3(100a+10b+c)+54$

$100b+10a+c-300a-30b-3c = 54$

$-290a + 70b - 2c = 54$ —— ①

$100a + 10b + c + 198 = 100c + 10b + a$

$100a + \cancel{10b} + c - 100c - \cancel{10b} - a = -198$

$99a - 99c = -198$

÷ 99, $a - c = -2$

$a = c - 2$ —— ②

$b - a = 2(b-c)$

$-a = 2b - 2c - b$

$-a = b - 2c$

$a + b = 2c$ —— ③

$(c-2) + b = 2c$ (From ②)

$b = 2c - c + 2$

$b = c + 2$

Sub. in ①

$-290(c-2) + 70(c+2) - 2c = 54$

$-290c + 580 + 70c + 140 - 2c - 54 = 0$

$-222c + 666 = 0$

$-222c = 666$

$c = \frac{666}{222} = 3$

$b = 3 + 2 = 5$

$a = 3 - 2 = 1$

The number is 153.

4) $xy(k^2+1) + k(x^2+y^2)$

$= xyk^2 + xy + kx^2 + ky^2$

$= yk(xk+y) + x(y+kx)$

$= (xk+y)(x+yk)$ —— ①

$xy(k^2-1) + k(x^2-y^2)$

$= xyk^2 - xy + kx^2 - ky^2$

$= yk(xk-y) + x(kx-y)$

$= (xk-y)(x+yk)$ —— ②

$\therefore$ LCM $= (x+yk)(xk+y)(xk-y)$

$= (x+yk)(x^2k^2-y^2)$

---

5) Given

$2x^4+13x^3+27x^2+23x+7,$

$x^3+3x^2+3x+1,$

x^2+2x+1

$$\begin{array}{r|l}
 & 2x+7 \\
x^3+3x^2+3x+1 & 2x^4+13x^3+27x^2+23x+7 \\
 & 2x^4+6x^3+6x^2+2x \\
 & (-)\quad(-)\quad(-)\quad(-) \\
 & 7x^3+21x^2+21x+7 \\
 & 7x^3+21x^2+21x+7 \\
 & (-)\quad(-)\quad(-)\quad(-) \\
 & 0
\end{array}$$

$$\begin{array}{r|l}
 & x+1 \\
x^2+2x+1 & x^3+3x^2+3x+1 \\
 & x^3+2x^2+x \\
 & (-)\quad(-)\quad(-) \\
 & x^2+2x+1 \\
 & x^2+2x+1 \\
 & (-)\quad(-)\quad(-) \\
 & 0
\end{array}$$

GCD $= x^2+2x+1$

---

6) (i) $\dfrac{x^{3a}-8}{x^{2a}+2x^a+4} = \dfrac{(x^a)^3-2^3}{x^{2a}+2x^a+4}$

$= \dfrac{(x^a-2)\cancel{(x^{2a}+2x^a+4)}}{\cancel{x^{2a}+2x^a+4}}$

$= x^a-2$

(ii) $\dfrac{10x^3-25x^2+4x-10}{-4-10x^2}$

$= \dfrac{5x^2(2x-5)+2(2x-5)}{-2(5x^2+2)}$

$= \dfrac{\cancel{(5x^2+2)}(2x-5)}{-2\cancel{(5x^2+2)}}$

$= \dfrac{2x-5}{-2}$

$= -x+\dfrac{5}{2}$

---

7) $\dfrac{\dfrac{1}{p}+\dfrac{1}{q+r}}{\dfrac{1}{p}-\dfrac{1}{q+r}} \times \left[1+\dfrac{q^2+r^2-p^2}{2qr}\right]$

$= \dfrac{\dfrac{q+r+p}{p(q+r)}}{\dfrac{q+r-p}{p(q+r)}} \times \dfrac{2qr+q^2+r^2-p^2}{2qr}$

$= \dfrac{(q+r)+p}{(q+r)-p} \times \dfrac{(q+r)+p}{(q+r)+p} \times \dfrac{2qr+q^2+r^2-p^2}{2qr}$

$= \dfrac{(q+r+p)^2}{(q+r)^2-p^2} \times \dfrac{2qr+q^2+r^2-p^2}{2qr}$

$= \dfrac{(q+r+p)^2}{\cancel{(q^2+r^2+2qr)-p^2}} \times \dfrac{\cancel{2qr+q^2+r^2-p^2}}{2qr}$

$= \dfrac{(q+r+p)^2}{2qr}$

Ex-3.7

1) (i) $\sqrt{\dfrac{4\cancel{00}x^4y^{12}z^{16}}{1\cancel{00}x^8y^4z^4}} = 2\sqrt{\dfrac{y^8z^{12}}{x^4}}$

$= \dfrac{2|y^4z^6|}{|x^2|}$

(ii) $\sqrt{\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}}$

$= \sqrt{\dfrac{(\sqrt{7}x+\sqrt{2})(\sqrt{7}x+\sqrt{2})}{(x-\frac{1}{4})(x-\frac{1}{4})}}$

$= \left|\dfrac{\sqrt{7}x+\sqrt{2}}{(x-\frac{1}{4})}\right|$

$= \dfrac{|\sqrt{7}x+\sqrt{2}|}{\left|\frac{4x-1}{4}\right|}$

$= 4\left|\dfrac{\sqrt{7}x+\sqrt{2}}{4x-1}\right|$