

ASSIGNMENT CLASS XII DIFFERENTIATION

1. Find $\frac{dy}{dx}$ for the following:

(a) $y = \frac{1}{\sqrt{a^2 - x^2}}$

(b) $y = \frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x+3)$

(c) $y = \frac{\cos x + \sin x}{\cos x - \sin x}$

(d) $y = \log \sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$

(e) $y = \log(x + \sqrt{1+x^2})$

(f) $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$

2. Show that $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right] = \sqrt{a^2 - x^2}$.

3. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2) \frac{dy}{dx} + y = 0$.

4. If $y = (x + \sqrt{x^2 + a^2})^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$.

5. Find $\frac{dy}{dx}$ for the following:

(a) $\sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

(b) $\tan^{-1} \left(\frac{1 - \cos x}{\sin x} \right)$

(c) $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$

(d) $\tan^{-1} \left(\sqrt{\frac{1+\sin x}{1-\sin x}} \right)$

(e) $\tan^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right)$

6. Find $\frac{dy}{dx}$ for the following:

(a) $\cos^{-1}(4x^3 - 3x)$

(b) $\cot^{-1} \left(\frac{1-x}{1+x} \right)$

(c) $\tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right)$

(d) $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$

(e) $\sin^{-1} \left(\frac{5x + 12\sqrt{1-x^2}}{13} \right)$

7. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

8. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that $2x \frac{dy}{dx} + y = 2\sqrt{x}$.

9. If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1}-x)$, show that $(x^2+1) \frac{dy}{dx} + xy + 1 = 0$.

10. If $y \log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.
11. If $\log(\sqrt{x^2 + y^2}) = \tan^{-1} \frac{y}{x}$, prove that $\frac{dy}{dx} = \frac{x + y}{x - y}$.
12. If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$, prove that $(1 - x^2) \frac{dy}{dx} = xy + 1$.
13. If $y = x^{\cos x} + \cos x^{\sin x}$, find $\frac{dy}{dx}$.
14. If $x^a y^b = (x + y)^{(a+b)}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
15. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.
16. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.
17. If $x = a \sin 2t(1 + \cos 2t)$, $y = b \cos 2t(1 - \cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{at t = \frac{\pi}{4}} = \frac{b}{a}$.
18. If $x = a\left(\frac{1+t^2}{1-t^2}\right)$, $y = \frac{2t}{1-t^2}$, show that $\frac{dy}{dx} = \frac{1+t^2}{2at}$.
19. If $x = 2 \cos \theta - \cos 2\theta$ and $y = 2 \sin \theta - \sin 2\theta$, find $\left(\frac{d^2 y}{dx^2}\right)_{\theta = \frac{\pi}{2}}$.
20. If $y = A \cos nx + B \sin nx$, prove that $\frac{d^2 y}{dx^2} + n^2 y = 0$.
21. If $y = e^x (\sin x + \cos x)$, prove that $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$.
22. If $y = \tan^{-1} x$, show that $(1 + x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = 0$.
23. If $y = \left[\log(x + \sqrt{x^2 + 1})\right]^2$, show that $(1 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 2$.

ANSWERS

- 1.(a) $\frac{x}{(a^2 - x^2)^{3/2}}$ (b) $\frac{15 - 5x^2}{3(1 - x^2)^{4/3}} + 2 \sin(4x + 6)$ (c) $\sec^2(x + \frac{\pi}{4})$ (d) $\frac{-\sin x \cos x}{1 + \cos^2 x} + \frac{e^{2x}}{1 - e^{2x}}$ (e) $\frac{1}{\sqrt{1 + x^2}}$
- (f) $-\sec^2(\frac{\pi}{4} - x)$ 5. (a) -2 (b) $\frac{1}{2}$ (c) -1 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$ 6.(a) $\frac{-3}{\sqrt{1 - x^2}}$ (b) $\frac{1}{1 + x^2}$ (c) $\frac{1}{2(1 + x^2)}$
- (d) $\frac{x}{\sqrt{1 - x^4}}$ (e) $\frac{-1}{\sqrt{1 - x^2}}$ 13. $x^{\cos x} \left\{ \frac{\cos x}{x} - (\log x) \sin x \right\} + \cos x^{\sin x} \left\{ -\sin x \tan x + \cos x \log(\cos x) \right\}$
15. $27 \log 3 - 12$ 16. 1 19. $\frac{-3}{2}$