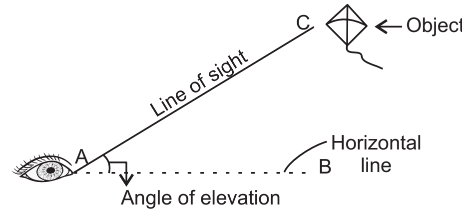
Heights and Distances 22

STUDY NOTES

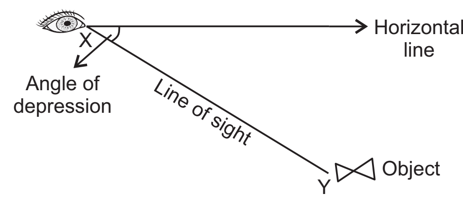
- **Line of Sight:** It is the line drawn from the eye of the observer to the point in the object viewed by the observer.

  In the given figure AC is the line of sight.



- **Angle of Elevation:** It is the angle formed by the line of sight with the horizontal when object is above the point of observation.

  In the given figure, ∠A is the angle of elevation as A is the observer and C is the object.

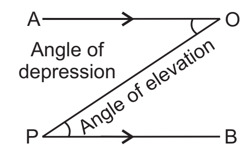


- **Angle of Depression :** It is the angle formed by the line of sight with the horizontal when object is below the point of observation.

  In the given figure, ∠X is angle of depression, as X is the observer and Y is the object.

- Angle of depression of P as seen from O = angle of elevation of O as seen from P.

  ∴ ∠AOP = ∠BPO



QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

1. If the length of the shadow of a pole is equal to its height, then the angle of elevation of Sun is:
(a) 30° (b) 45° (c) 60° (d) 90°

2. If the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is:
(a) increasing (b) first increasing, then decreasing
(c) decreasing (d) not changed

3. If the height of a tower and the distance of the point of observation from its foot both are increased by 10%, then the angle of elevation of its top:
(a) gets doubled (b) remains unchanged (c) gets tripled (d) becomes half

4. If the angle of elevation of the sun is 60° and the length of the shadow of a tower is 30 m, then the height of tower is :
(a) $3\sqrt{3}$ m (b) $\sqrt{3}$ m (c) $30\sqrt{3}$ m (d) $2\sqrt{3}$ m

5. The angles of depression of two objects from the top of a 100 m hill lying to its east are found to be 45° and 30°. The distance between the two objects is ($\sqrt{3}$ = 1.73) :
(a) 73.2 m (b) 107.5 m (c) 150 m (d) 200 m

6. A kite is attached to a string. The length of the string, when the height of the kite is 60 m and the string makes an angle 30°with the ground is:
(a) 120 m (b) 30 m (c) 50 cm (d) 60 m

7. The angle of elevation a plane $2x$ metres above the ground from a point x metres above the ground is θ. At this moment the angle of depression of a point just below the plane will be:

(a) $\sin(45°-\theta)$ (b) $\cot(45°-\theta)$ (c) 2θ (d) θ

8. If the angles of elevation of the top of a vertical tower from two points A and B on the ground are respectively 30° and 60°, then the ratio of the distances of A and B from the upper end of the tower is:

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$ (c) $\sqrt{3} + 1 : 1$ (d) $1 : \sqrt{3} - 1$

9. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is:

(a) 6.5 m (b) 7.5 m (c) 8.5 m (d) $\frac{15}{\sqrt{3}}$ m

10. A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°, then the height of the tower is:

(a) $6\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) 18 m (d) $\frac{25}{\sqrt{3}}$ m

11. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on ground, then the Sun's elevation is:

(a) 30º (b) 45º (c) 60º (d) 90º

12. An observer 1.5 metres tall is 18.5 metres away from the tower. If the angle of elevation of the top of the tower from his eye is 45°, the height of the tower is:

(a) 15 m (b) 20 m (c) 8.5 m (d) 25.6 m

13. The shadow of a tower, standing on a level ground, is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Then height of the tower is:

(a) 20 m (b) $10\sqrt{3}$ m (c) 10 m (d) $20\sqrt{3}$ m

14. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If bridge is at the height of 30 m from the bank, then the width of the river is:

(a) 90 m (b) $30\sqrt{3}$ m (c) $30(\sqrt{3}+1)$ m (d) $25(\sqrt{3} - 1)$ m

15. The angles of depression of two ships from the top of a lighthouse are 45° and 30° towards east. If the ships are 100 m apart, the height of the lighthouse is:

(a) $\frac{20}{(\sqrt{3}+1)}$ m (b) $\frac{20}{(\sqrt{3}-1)}$ m (c) $50(\sqrt{3}-1)$ m (d) $50(\sqrt{3} + 1)$ m

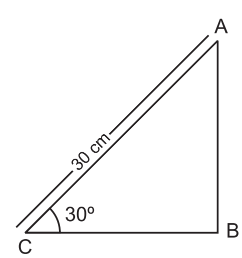
A. Answers

1. (b) **2.** (c) **3.** (b) **4.** (c) **5.** (a) **6.** (a) **7.** (d) **8.** (a) **9.** (b) **10.** (b)
11. (c) **12.** (b) **13.** (d) **14.** (c) **15.** (c)

B. Short Answer Type Questions [3 Marks]

1. A circus artist is climbing a 30 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the distance of the pole to the peg in the ground, if the angle made by the rope with the ground level is 30°.

Sol. AB be the pole and AC be the rope which is 30 m long.
Then, in ΔABC,



$$\cos 30° = \frac{BC}{AC}$$

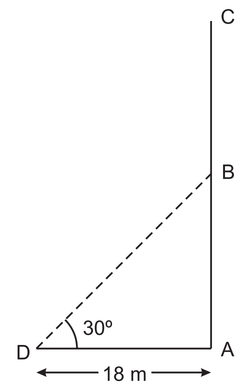
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{30} \Rightarrow BC = 15\sqrt{3} \text{ m.}$$

So, required distance is $15\sqrt{3}$ m.

2. A tree is broken by the wind. Find the total height of the tree if the top struck the ground at an angle of 30° and at a distance of 18 m from the foot of the pole.

Sol. Let AB be the tree which is broken at point B which struck the ground level at point D.

Now, in ΔDAB, $\cos 30° = \frac{AD}{BD}$



$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{BD} \Rightarrow \sqrt{3}BD = 2 \times 18 = 36$ m

$\Rightarrow BD = \frac{36}{\sqrt{3}}$ m $= 12\sqrt{3}$ m

$\Rightarrow \tan 30° = \frac{AB}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{18} \Rightarrow AB = \frac{18}{\sqrt{3}}$ m $\Rightarrow AB = 6\sqrt{3}$ m

Total length of the tree = AB + BD = $6\sqrt{3}+12\sqrt{3} = 18\sqrt{3}$ m

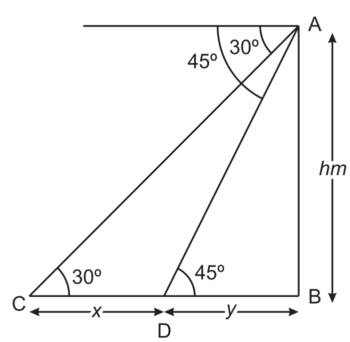
3. A man standing on the top of a vertical tower observes a car moving towards the tower at a uniform speed. If it takes 10 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the tower?

Sol. Let AB be the tower C is the position of car moving towords the pole at an uniform speed. After 10 minutes the position of car is at D.

$AB = h$

$CD = x$

$BD = y$



In ΔABD, $\tan 45° = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{h}{y} \quad \Rightarrow \quad h = y$ ...(i)

In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y} \quad \Rightarrow \quad x + y = \sqrt{3}h$ ...(ii)

Let the speed of car be s m/min and it take t min to cover BD.

Then, $x = 10s$ ...(iii)

And $y = st$ ...(iv)

From (i), (ii), (iii) and (iv), we have

$$x + y = \sqrt{3}h$$

$$10s + st = \sqrt{3} \times y$$

$$10s + st = \sqrt{3} \times st$$

$\Rightarrow 10s = \sqrt{3}st - st$

$10s = st\left(\sqrt{3}-1\right)$

$\Rightarrow t = \frac{10s}{s\left(\sqrt{3}-1\right)} = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{10(1.732+1)}{3-1} = 5 \times 2.732 = 13.66$ minutes.

4. The angles of elevation of the top of a vertical tower from two points, at a distance a and b $(a > b)$ from the base and in the same straight line with it are complementary. Find the height of the tower.

Sol. Let AB be the vertical tower C and D are two points on the straight line.

Let ∠ACB = θ, then ∠ADB = 90° – θ

$AB = h$

Now in ΔABC, $\tan \theta = \frac{AB}{BC}$

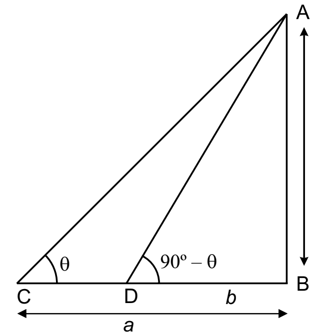
$\Rightarrow \qquad \tan\theta = \frac{h}{a}$ ...(i)

And in ΔADB, $\tan(90° - \theta) = \frac{AB}{BD}$

$\cot\theta = \frac{h}{b}$ ...(ii)

Multiplying (i) and (ii), we have

$\Rightarrow \tan\theta \times \cot\theta = \frac{h}{a}\times\frac{h}{b} \Rightarrow 1 = \frac{h^2}{ab} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$.



5. If the angle of depression of the top and the bottom of a tower as observed from the top of a h metres high cliff are 30° and 60° respectively, prove that the height of the tower is $\frac{2h}{3}$.

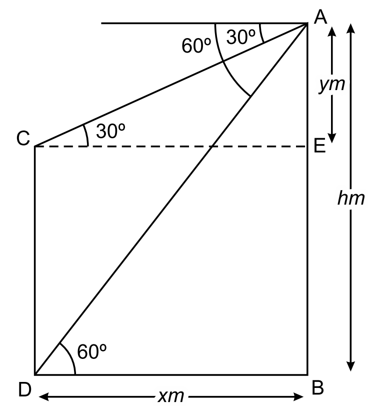
Sol. Let AB be the cliff and CD be the tower.
Let AB = h m and CD = H
∠ACE = 30° and ∠ADB = 60°
BD = CE = x m

Now in ΔADB, $\tan 60° = \frac{AB}{BD} = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$ ...(i)

In ΔACE, $\tan 30° = \frac{AE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{x}$

From (i) $\frac{1}{\sqrt{3}} = \frac{y}{\frac{h}{\sqrt{3}}} \Rightarrow y\sqrt{3} = \frac{h}{\sqrt{3}} \Rightarrow y = \frac{h}{3}$

∴ Height of tower = $h - y = h - \frac{h}{3} = \frac{2h}{3}$ **Proved**



6. The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of the tower is 45°. Find the height of the tower PQ and distance PX. (Use $\sqrt{3} = 1.73$).

Sol. Let PX = a, PE = b = 40 m and QE = c m

In ΔQEY, $\tan 45° = \frac{QE}{EY} \Rightarrow 1 = \frac{c}{a}$

$\Rightarrow \qquad a = c$ ...(i)

In ΔQPX, $\tan 60° = \frac{QP}{XP}$

$\Rightarrow \sqrt{3} = \frac{b+c}{a} \Rightarrow \sqrt{3} = \frac{b+a}{a}$ [From (i)]

$\Rightarrow b + a = \sqrt{3}a \Rightarrow \sqrt{3}a - a = b \Rightarrow a\left(\sqrt{3}-1\right) = b \Rightarrow a = \frac{b}{\sqrt{3}-1}$

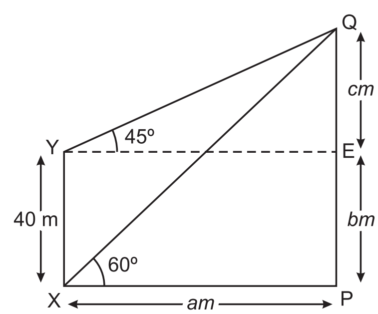
$\Rightarrow a = \frac{40}{\sqrt{3}-1} = \frac{40(\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = 20(\sqrt{3}+1)$

$\Rightarrow a = 54.64$ m

So, XP = 54.64 m

From (i), $c = a = 54.64$

∴ Height of tower = PQ = $b + c$ = 40 + 54.64 = 94.64 m



7. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in horizontal line through its base. On covering 60 m, the angle of elevation changed to 60°. Find the height of the building.

Sol. Let AB = h m be the height of the building.

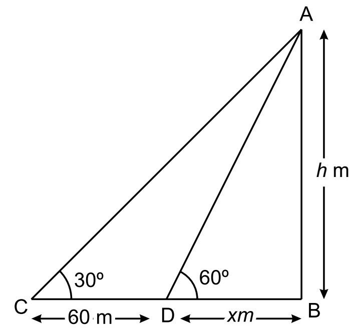
CD = 60 m and BD = x m

Now in ΔABD,

$\tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ ...(i)

In ΔABC, $\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60+\frac{h}{\sqrt{3}}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{60\sqrt{3}+h} \Rightarrow 3h = 60\sqrt{3}+h \Rightarrow 2h = 60\sqrt{3} \Rightarrow h = 30\sqrt{3}$



h = 51.96 = 52 m.

So, height of the building is 52 m.

C. Long Answer Type Questions [4 Marks]

1. An observer measures angles of elevation of two towers of equal height from a point between the towers. The angles of elevation of the tops of the two towers from this point are 60° and 30°. If this point is at a distance of 120 m from the first tower, find the distance between the towers.

Sol. Let AB and CD are two towers of equal height h m.

E is the point between them.

BE = 120 m and DE = x m

Now in ΔABE

$\tan 60° = \frac{AB}{BE} \Rightarrow \sqrt{3} = \frac{h}{120}$

$\Rightarrow h = 120\sqrt{3}$ m

In ΔCDE,

$\tan 30° = \frac{CD}{DE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{x} \Rightarrow x = 360$ m

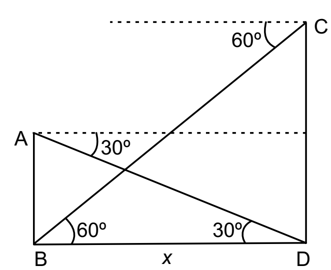
The distance between two towers = DE + BE = 360 m + 120 m = 480 m.

2. Two towers AB and CD are standing at some distance apart. From the top of tower AB, the angle of depression of the foot of tower CD is 30°. From the top of tower CD, the angle of depression of the foot of tower AB is 60°. If the height of tower CD is 'h' m, then prove that the height of tower AB is $\frac{h}{3}$ m.

Sol. In ΔABD,

$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{x} \Rightarrow x = \sqrt{3}AB$ ...(i)

In ΔCDB, $\tan 60° = \frac{CD}{BD} \Rightarrow \sqrt{3} = \frac{h}{x}$ ...(ii)



From (i) and (ii), we have

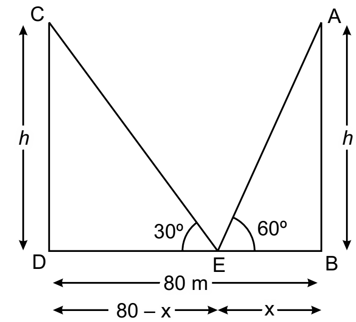
$\sqrt{3} = \frac{h}{\sqrt{3}AB} \Rightarrow 3\,AB = h \Rightarrow AB = \frac{h}{3}$ m

∴ Height of tower AB = $\frac{h}{3}$ m. **Proved.**

3. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of poles are 60° and 30° respectively. Find the height of poles and the distances of the point from the poles.

Sol. Let AB and CD are two poles of equal height h.

$\tan 60° = \frac{AB}{BE} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ ...(i)



In ΔCDE, $\tan 30° = \frac{CD}{DE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80-x}$

From (i), $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{80-x} \Rightarrow 3x = 80 - x$

$\Rightarrow x = 20$ m $\Rightarrow$ BE = 20 m

From (i), $h = \sqrt{3}x = 20\sqrt{3}$

$\therefore$ Height of each pole = $20\sqrt{3}$ m.

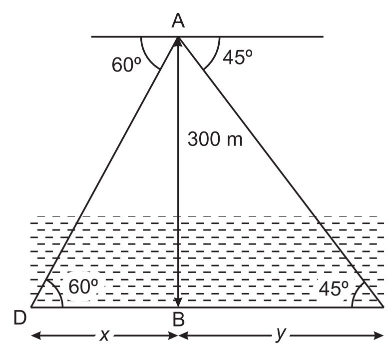
Also, the point is 20 m from first pole and (80 – 20) m = 60 m from the second pole.

4. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the plane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river (Use $\sqrt{3} = 1.73$)

Sol. Let A be the position of aeroplane. C and D are the two points on bank of river in opposite sides.

Let BD = x and BC = y

$\angle ADB = 60°$ and $\angle ACB = 45°$



Now in ΔABC, $\tan 45° = \frac{AB}{BC} \Rightarrow 1 = \frac{300}{y} \Rightarrow y = 300$ m

In ΔABD, $\tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{300}{x} \Rightarrow x = \frac{300}{\sqrt{3}}$

Width of river = BC + BD = $300 + \frac{300}{\sqrt{3}} = \frac{300\sqrt{3}+300}{\sqrt{3}}$

$= \frac{300(\sqrt{3}+1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 100(3+\sqrt{3})$ m.

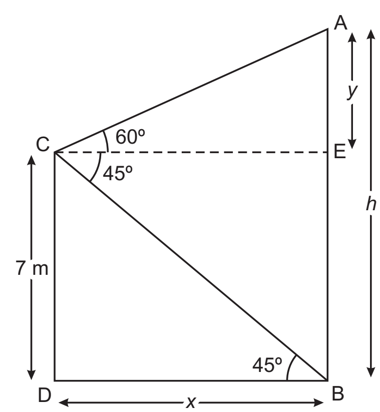
5. There is a building of height 7 m next to a cable tower of unknown height. From the top of the building, the angle of elevation of the top of the tower is 60° and the angle of depression to the foot of the tower is 45°. Find the height of the cable tower.

Sol. Let AB be the cable tower and CD be the building.

Let AB = h m and BD = x

CD = 7 m and AE = y m

$\therefore$ EB = CD = 7 m



Now in ΔAEB, $\tan 60° = \frac{AE}{CE}$

$\Rightarrow \sqrt{3} = \frac{y}{x} \Rightarrow y = \sqrt{3}x$ ...(i)

In ΔCDB, $\tan 45° = \frac{CD}{BD} \Rightarrow 1 = \frac{7}{x} \Rightarrow x = 7$m ...(ii)

From (i) and (ii), we have

$y = \sqrt{3} \times 7 \Rightarrow y = 7\sqrt{3}$ m

$\therefore$ Height of cable tower = BE + AE = $7 + 7\sqrt{3} = 7(1+\sqrt{3})$ m.

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