NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions

Question 1:

Determine whether each of the following relations are reflexive, symmetric and transitive.

(i) Relation R in the set $A=\{1,2,3,\ldots,13,14\}$ defined as

$R=\{(x,y):3x-y=0\}$

(ii) Relation R in the set of N natural numbers defined as

$R=\{(x,y):y=x+5 \text{ and } x<4\}$

(iii) Relation R in the set $A=\{1,2,3,4,5,6\}$ defined as

$R=\{(x,y):y \text{ is divisible by } x\}$

(iv) Relation R in the set of Z integers defined as

$R=\{(x,y):x-y \text{ is an integer}\}$

(v) Relation R in the set of human beings in a town at a particular time given by

(a) $R=\{(x,y):x \text{ and } y \text{ work at the same place}\}$

(b) $R=\{(x,y):x \text{ and } y \text{ live in the same locality}\}$

(c) $R=\{(x,y):x \text{ is exactly 7cm taller than } y\}$

(d) $R=\{(x,y):x \text{ is wife of } y\}$

(e) $R=\{(x,y):x \text{ is father of } y\}$

Solution:

(i) $R=\{(1,3),(2,6),(3,9),(4,12)\}$

R is not reflexive because $(1,1),(2,2)\ldots$ and $(14,14)\notin R$.

R is not symmetric because $(1,3)\in R$, but $(3,1)\notin R.[\text{since } 3(3)\neq 0]$.

R is not transitive because $(1,3),(3,9)\in R$, but $(1,9)\notin R.[3(1)-9\neq 0]$.

Hence, R is neither reflexive nor symmetric nor transitive.

(ii) $R=\{(1,6),(2,7),(3,8)\}$

R is not reflexive because $(1,1)\notin R$.

R is not symmetric because $(1,6)\in R$ but $(6,1)\notin R$.

R is not transitive because there isn't any ordered pair in R such that $(x,y),(y,z)\in R$, so $(x,z)\notin R$.

Hence, R is neither reflexive nor symmetric nor transitive.

(iii) $R=\{(x,y):y \text{ is divisible by } x\}$

We know that any number other than 0 is divisible by itself.

Thus, $(x,x)\in R$

So, R is reflexive.

$(2,4)\in R$ [because 4 is divisible by 2]

But $(4,2)\notin R$ [since 2 is not divisible by 4]

So, R is not symmetric.

Let (x,y) and $(y,z)\in R$. So, y is divisible by x and z is divisible by y.

So, z is divisible by x $\Rightarrow (x,z)\in R$

So, R is transitive.

So, R is reflexive and transitive but not symmetric.

(iv) $R=\{(x,y):x-y \text{ is an integer}\}$

For $x\in Z$, $(x,x)\notin R$ because $x-x=0$ is an integer.

So, R is reflexive.

For, $x,y\in Z$, if $x,y\in R$, then $x-y$ is an integer $\Rightarrow (y-x)$ is an integer.

So, $(y,x)\in R$

So, R is symmetric.

Let (x,y) and $(y,z)\in R$, where $x,y,z\in Z$.

$\Rightarrow (x-y)$ and $(y-z)$ are integers.

$\Rightarrow x-z=(x-y)+(y-z)$ is an integer.

So, R is transitive.

So, R is reflexive, symmetric and transitive.

(v)

a) $R=\{(x,y):x \text{ and } y \text{ work at the same place}\}$

R is reflexive because $(x,x)\in R$

R is symmetric because ,

If $(x,y)\in R$, then x and y work at the same place and y and x also work at the same place. $(y,x)\in R$.

R is transitive because,

Let $(x,y),(y,z)\in R$

x and y work at the same place and y and z work at the same place.

Then, x and z also works at the same place. $(x,z)\in R$.

Hence, R is reflexive, symmetric and transitive.

b) $R=\{(x,y):x \text{ and } y \text{ live in the same locality}\}$

R is reflexive because $(x,x)\in R$

R is symmetric because,

If $(x,y)\in R$, then x and y live in the same locality and y and x also live in the same locality $(y,x)\in R$.

R is transitive because,

Let $(x,y),(y,z)\in R$

x and y live in the same locality and y and z live in the same locality.

Then x and z also live in the same locality. $(x,z)\in R$.

Hence, R is reflexive, symmetric and transitive.

c) $R=\{(x,y): x \text{ is exactly 7cm taller than } y\}$

R is not reflexive because $(x,x)\notin R$.

R is not symmetric because,

If $(x,y)\in R$, then x is exactly $7cm$ taller than y and y is clearly not taller than x. $(y,x)\notin R$.

R is not transitive because,

Let $(x,y),(y,z)\in R$

x is exactly $7cm$ taller than y and y is exactly $7cm$ taller than z.

Then x is exactly $14cm$ taller than z. $(x,z)\notin R$

Hence, R is neither reflexive nor symmetric nor transitive.

d) $R=\{(x,y): x \text{ is wife of } y\}$

R is not reflexive because $(x,x)\notin R$.

R is not symmetric because,

Let $(x,y)\in R$, x is the wife of y and y is not the wife of x. $(y,x)\notin R$.

R is not transitive because,

Let $(x,y),(y,z)\in R$

x is wife of y and y is wife of z, which is not possible.

$(x,z)\notin R$.

Hence, R is neither reflexive nor symmetric nor transitive.

e) $R=\{(x,y): x \text{ is father of } y\}$

R is not reflexive because $(x,x)\notin R$.

R is not symmetric because,

Let $(x,y)\in R$, x is the father of y and y is not the father of x. $(y,x)\notin R$.

R is not transitive because,

Let $(x,y),(y,z)\in R$

x is father of y and y is father of z, x is not father of z. $(x,z)\notin R$.

Hence, R is neither reflexive nor symmetric nor transitive.

Question 2:

Show that the relation R in the set R of real numbers, defined as $R=\{(a,b): a\le b^2\}$ is neither reflexive nor symmetric nor transitive.

Solution:

$R=\{(a,b): a\le b^2\}$

$\left(\frac{1}{2},\frac{1}{2}\right)\notin R$ because $\frac{1}{2}>\left(\frac{1}{2}\right)^2$

$\therefore$ R is not reflexive.

$(1,4)\in R$ as $1<4$. But 4 is not less than 1^2.

$(4,1)\notin R$

$\therefore$ R is not symmetric.

$(3,2)(2,1.5)\in R$ [Because $3<2^2=4$ and $2<(1.5)^2=2.25$]

$3>(1.5)^2=2.25$

$\therefore (3,1.5)\notin R$

$\therefore$ R is not transitive.

R is neither reflective nor symmetric nor transitive.

Question 3:

Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R=\{(a,b): b=a+1\}$ is reflexive, symmetric or transitive.

Solution:

$A=\{1,2,3,4,5,6\}$

$R=\{(a,b): b=a+1\}$

$R=\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

$(a,a)\notin R, a\in A$

$(1,1),(2,2),(3,3),(4,4),(5,5)\notin R$

$\therefore$ R is not reflexive.

$(1,2)\in R$, but $(2,1)\notin R$

∴ R is not symmetric.

$(1,2),(2,3) \in R$
$(1,3) \notin R$
∴ R is not transitive.

R is neither reflective nor symmetric nor transitive.

Question 4:

Show that the relation R in R defined as $R=\{(a,b):a \le b\}$ is reflexive and transitive, but not symmetric.

Solution:

$R=\{(a,b):a \le b\}$
$(a,a) \in R$
∴ R is reflexive.

$(2,4) \in R$ (as $2<4$)
$(4,2) \notin R$ (as $4>2$)
∴ R is not symmetric.

$(a,b),(b,c) \in R$
$a \le b$ and $b \le c$
$\Rightarrow a \le c$
$\Rightarrow (a,c) \in R$
∴ R is transitive.
R is reflexive and transitive but not symmetric.

Question 5:

Check whether the relation R in R defined as $R=\{(a,b):a \le b^3\}$ is reflexive, symmetric or transitive.

Solution:

$R=\{(a,b):a \le b^3\}$
$\left(\frac{1}{2},\frac{1}{2}\right) \notin R$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^3$
∴ R is not reflexive.

$(1,2) \in R\left(\text{as } 1<2^3=8\right)$
$(2,1) \notin R\left(\text{as } 2^3>1=8\right)$
∴ R is not symmetric.

$\left(3,\frac{3}{2}\right),\left(\frac{3}{2},\frac{6}{5}\right) \in R$, since $3<\left(\frac{3}{2}\right)^3$ and $\frac{2}{3}<\left(\frac{6}{2}\right)^3$
$\left(3,\frac{6}{5}\right) \notin R\, 3>\left(\frac{6}{5}\right)^3$
∴ R is not transitive.

R is neither reflexive nor symmetric nor transitive.

Question 6:

Show that the relation R in the set $\{1,2,3\}$ given by $R=\{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

Solution:

$A=\{1,2,3\}$
$R=\{(1,2),(2,1)\}$
$(1,1),(2,2),(3,3) \notin R$
∴ R is not reflexive.
$(1,2) \in R$ and $(2,1) \in R$
∴ R is symmetric.

$(1,2) \in R$ and $(2,1) \in R$
$(1,1) \in R$
∴ R is not transitive.

R is symmetric, but not reflexive or transitive.

Question 7:

Show that the relation R in the set A of all books in a library of a college, given by $R=\{(x,y):x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.

Solution:

$R=\{(x,y):x \text{ and } y \text{ have same number of pages}\}$

R is reflexive since $(x,x) \in R$ as x and x have same number of pages.

∴ R is reflexive.

$(x,y)\in R$

x and y have same number of pages and y and x have same number of pages $(y,x)\in R$
∴ R is symmetric.

$(x,y)\in R,(y,z)\in R$
x and y have same number of pages, y and z have same number of pages.
Then x and z have same number of pages.
$(x,z)\in R$
∴ R is transitive.

R is an equivalence relation.

Question 8:

Show that the relation R in the set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b):|a-b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other. But no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

Solution:

$a\in A$
$|a-a|=0(\text{which is even})$

∴ R is reflective.

$(a,b)\in R$
$\Rightarrow |a-b|$ [is even]
$\Rightarrow |-(a-b)|=|b-a|$ [is even]
$(b,a)\in R$
∴ R is symmetric.

$(a,b)\in R$ and $(b,c)\in$ R
$\Rightarrow |a-b|$ is even and $|b-c|$ is even
$\Rightarrow (a-b)$ is even and $(b-c)$ is even
$\Rightarrow (a-c)=(a+b)+(b-c)$ is even
$\Rightarrow |a-b|$ is even
$\Rightarrow (a,c)\in R$
∴ R is transitive.

R is an equivalence relation.

All elements of $\{1,3,5\}$ are related to each other because they are all odd. So, the modulus of the difference between any two elements is even.

Similarly, all elements $\{2,4\}$ are related to each other because they are all even.

No element of $\{1,3,5\}$ is related to any elements of $\{2,4\}$ as all elements of $\{1,3,5\}$ are odd and all elements of $\{2,4\}$ are even. So, the modulus of the difference between the two elements will not be even.

Question 9:

Show that each of the relation R in the set $A=\{x\in Z:0\le x\le 12\}$, given by

i. $R=\{(a,b):|a-b| \text{ is a mutiple of } 4\}$
ii. $R=\{(a,b):a=b\}$

Is an equivalence relation. Find the set of all elements related to 1 in each case.

Solution:

$A=\{x\in Z:0\le x\le 12\}=\{0,1,2,3,4,5,6,7,8,9,10,11,12\}$

i. $R=\{(a,b):|a-b| \text{ is a mutiple of } 4\}$

$a\in A,(a,a)\in R \quad [|a-a|=0 \text{ is a multiple of } 4]$
∴ R is reflexive.

$(a,b)\in R\Rightarrow |a-b|$ [is a multiple of 4]
$\Rightarrow |-(a-b)|=|b-a|$ [is a multiple of 4]
$(b,a)\in R$
∴ R is symmetric.

$(a,b)\in R$ and $(b,c)\in$ R
$\Rightarrow |a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4
$\Rightarrow (a-b)$ is a multiple of 4 and $(b-c)$ is a multiple of 4
$\Rightarrow (a-c)=(a-b)+(b-c)$ is a multiple of 4
$\Rightarrow |a-c|$ is a multiple of 4

$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive.
R is an equivalence relation.

The set of elements related to 1 is $\{1,5,9\}$ as
$|1-1| = 0$ is a multiple of 4.
$|5-1| = 4$ is a multiple of 4.
$|9-1| = 8$ is a multiple of 4.

ii. $R = \{(a,b) : a = b\}$
$a \in A, (a,a) \in R$ [since a=a]
$\therefore$ R is reflective.

$(a,b) \in R$
$\Rightarrow a = b$
$\Rightarrow b = a$
$\Rightarrow (b,a) \in R$
$\therefore$ R is symmetric.

$(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow a = b$ and $b = c$
$\Rightarrow a = c$
$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive.

R is an equivalence relation.

The set of elements related to 1 is $\{1\}$.

Question 10:
Give an example of a relation, which is
i. Symmetric but neither reflexive nor transitive.
ii. Transitive but neither reflexive nor symmetric.
iii. Reflexive and symmetric but not transitive.
iv. Reflexive and transitive but not symmetric.
v. Symmetric and transitive but not reflexive.

Solution:
i.
$A = \{5,6,7\}$
$R = \{(5,6),(6,5)\}$
$(5,5),(6,6),(7,7) \notin R$
R is not reflexive as $(5,5),(6,6),(7,7) \notin R$
$(5,6),(6,5) \in R$ and $(6,5) \in R$, R is symmetric.
$\Rightarrow (5,6),(6,5) \in R$, but $(5,5) \notin R$
$\therefore$ R is not transitive.
Relation R is symmetric but not reflexive or transitive.

ii. $R = \{(a,b) : a < b\}$
$a \in R, (a,a) \notin R$ [since a cannot be less than itself]
R is not reflexive.
$(1,2) \in R(\text{as } 1 < 2)$
But 2 is not less than 1
$\therefore (2,1) \notin R$
R is not symmetric.
$(a,b),(b,c) \in R$
$\Rightarrow a < b$ and $b < c$
$\Rightarrow a < c$
$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive.
Relation R is transitive but not reflexive and symmetric.

iii. $A = \{4,6,8\}$
$A = \{(4,4),(6,6),(8,8),(4,6),(6,8),(8,6)\}$
R is reflexive since $a \in A, (a,a) \in R$
R is symmetric since $(a,b) \in R$
$\Rightarrow (b,a) \in R \quad for\, a,b \in R$
R is not transitive since $(4,6),(6,8) \in R, but\, (4,8) \notin R$
R is reflexive and symmetric but not transitive.

iv. $R = \{(a,b) : a^3 > b^3\}$
$(a,a) \in R$
$\therefore$ R is reflexive.
$(2,1) \in R$
$But\, (1,2) \notin R$

$\therefore$ R is not symmetric.

$(a,b),(b,c) \in R$
$\Rightarrow a^3 \geq b^3$ and $b^3 < c^3$
$\Rightarrow a^3 < c^3$
$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive.
R is reflexive and transitive but not symmetric

v. Let $A = \{-5,-6\}$
$R = \{(-5,-6),(-6,-5),(-5,-5)\}$
R is not reflexive as $(-6,-6) \notin R$
$(-5,-6),(-6,-5) \in R$
R is symmetric.
$(-5,-6),(-6,-5) \in R$
$(-5,-5) \in R$
R is transitive.
$\therefore$ R is symmetric and transitive but not reflexive.

Question 11:

Show that the relation R in the set A of points in a plane given by
$R = \{(P,Q)$: Distance of the point P from the origin is same as the distance of the point Q from the origin$\}$
, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0,0)$ is the circle passing through P with origin as centre.

Solution:

$R = \{(P,Q)$: Distance of the point P from the origin is same as the distance of the point Q from the origin$\}$

Clearly, $(P,P) \in R$
$\therefore$ R is reflexive.
$(P,Q) \in R$
Clearly R is symmetric.
$(P,Q),(Q,S) \in R$
$\Rightarrow$ The distance of P and Q from the origin is the same and also, the distance of Q and S from the origin is the same.
$\Rightarrow$ The distance of P and S from the origin is the same.
$(P,S) \in R$
$\therefore$ R is transitive.
R is an equivalence relation.

The set of points related to $P \neq (0,0)$ will be those points whose distance from origin is same as distance of P from the origin.

Set of points forms a circle with the centre as origin and this circle passes through P.

Question 12:

Show that the relation R in the set A of all triangles as $R = \{(T_1,T_2) : T_1 \text{ is similar to } T_2\}$, is an equivalence relation. Consider three right angle triangles T_1 with sides $3,4,5$, T_2 with sides $5,12,13$ and T_3 with sides $6,8,10$. Which triangle among T_1,T_2,T_3 are related?

Solution:

$R = \{(T_1,T_2) : T_1 \text{ is similar to } T_2\}$
R is reflexive since every triangle is similar to itself.

If $(T_1,T_2) \in R$, then T_1 is similar to T_2.
T_2 is similar to T_1.
$\Rightarrow (T_2,T_1) \in R$
$\therefore$ R is symmetric.

$(T_1,T_2),(T_2,T_3) \in R$
T_1 is similar to T_2 and T_2 is similar to T_3.
$\therefore$ T_1 is similar to T_3.
$\Rightarrow (T_1,T_3) \in R$
$\therefore$ R is transitive.
$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \left(\frac{1}{2}\right)$$

$\therefore$ Corresponding sides of triangles T_1 and T_3 are in the same ratio.
Triangle T_1 is similar to triangle T_3.
Hence, T_1 is related to T_3.

Question 13:

Show that the relation R in the set A of all polygons as $R = \{(P_1,P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides $3, 4 \, and \, 5$?

Solution:

$R = \{(P_1, P_2) : P_1 \text{and } P_2 \text{ have same number of sides}\}$

$(P_1, P_2) \in R$ as same polygon has same number of sides.

$\therefore$ R is reflexive.

$(P_1, P_2) \in R$

$\Rightarrow P_1$ and P_2 have same number of sides.

$\Rightarrow P_2$ and P_1 have same number of sides.

$\Rightarrow (P_2, P_1) \in R$

$\therefore$ R is symmetric.

$(P_1, P_2), (P_2, P_3) \in R$

$\Rightarrow P_1$ and P_2 have same number of sides.

P_2 and P_3 have same number of sides.

$\Rightarrow P_1$ and P_3 have same number of sides.

$\Rightarrow (P_1, P_3) \in R$

$\therefore$ R is transitive.

R is an equivalence relation.

The elements in A related to right-angled triangle (T) with sides $3, 4, 5$ are those polygons which have three sides.

Set of all elements in a related to triangle T is the set of all triangles.

Question 14:

Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Solution:

$R = \{(L_1, L_2) : L_1 \text{is parallel to } L_2\}$

R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in R$

If $(L_1, L_2) \in R$, then

$\Rightarrow L_1$ is parallel to L_2.

$\Rightarrow L_2$ is parallel to L_1.

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is symmetric.

$(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow L_1$ is parallel to L_2

$\Rightarrow L_2$ is parallel to L_3

$\therefore L_1$ is parallel to L_3.

$\Rightarrow (L_1, L_3) \in R$

$\therefore$ R is transitive.

R is an equivalence relation.

Set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of the line $y = 2x + 4$ is $m = 2$.

Line parallel to the given line is in the form $y = 2x + c$, where $c \in R$.

Set of all lines related to the given line is given by $y = 2x + c$, where $c \in R$.

Question 15:

Let R be the relation in the set $\{1, 2, 3.4\}$ given by $R = \{(1,2)(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$.

Choose the correct answer.

A. R is reflexive and symmetric but not transitive.
B. R is reflexive and transitive but not symmetric.
C. R is symmetric and transitive but not reflexive.
D. R is an equivalence relation.

Solution:

$R = \{(1,2)(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$

$(a, a) \in R$ for every $a \in \{1, 2, 3.4\}$

$\therefore$ R is reflexive.

$(1,2) \in R$ but $(2,1) \notin R$

$\therefore$ R is not symmetric.

$(a,b),(b,c) \in R$ for all $a,b,c \in \{1,2,3,4\}$

$\therefore$ R is not transitive.

R is reflexive and transitive but not symmetric.

The correct answer is B.

Question 16:

Let R be the relation in the set N given by $R=\{(a,b): a=b-2, b>6\}$. Choose the correct answer.

A. $(2,4)\in R$

B. $(3,8)\in R$

C. $(6,8)\in R$

D. $(8,7)\in R$

Solution:

$R=\{(a,b): a=b-2, b>6\}$

Now,

$b>6,(2,4)\notin R$

$3\neq 8-2$

$\therefore (3,8)\notin R$ and as $8\neq$ 7-2

$\therefore (8,7)\notin R$

Consider $(6,8)$

$8>6$ and $6=8-2$

$\therefore (6,8)\in R$

The correct answer is C.