Mechanical Properties Of Fluid

Fluid-

fluid is the name given to a substance which begins to flow when external force is applied on it. The fluids do not have their own shape but take the shape of the containing vessel.

The branch of physics which deals with the study of fluids at rest is called hydrostatics.

The branch of physics which deals with the study of fluids in motion is called hydrodynamics.

Pressure of liquid-

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust.

The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it, is called pressure of liquid or hydrostatic pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \qquad \boxed{P = \frac{F}{A}}$$

SI Unit - N/m^2 or Pascal   Dimensions - $[M^1L^{-1}T^{-2}]$

Atmospheric Pressure -

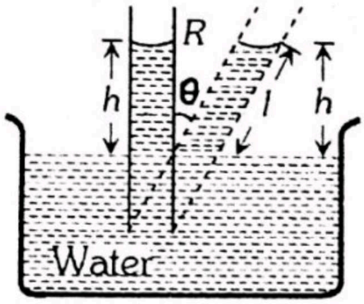
Force exerted by air column on unit cross-section area of sea level is called atmospheric pressure.

$$\boxed{1\ \text{atm} = 1.01 \times 10^5\ \text{Pa} = 1.01\ \text{bar}}$$

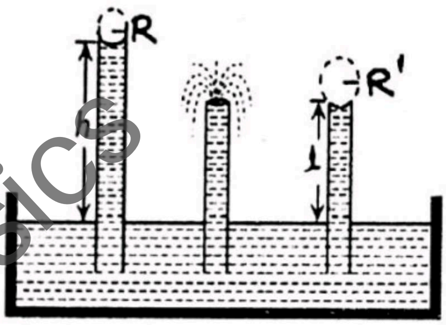
* 'Barometer' is used to measure atmospheric pressure.

$$h = l\cos\theta$$

or $$\boxed{l = \frac{h}{\cos\theta}}$$



2.) In case of capillary of insufficient length i.e. $l < h$, the liquid will not overflow from the upper end. The liquid after reaching the upper end will increase the radius of its maniscus without changing nature such that -

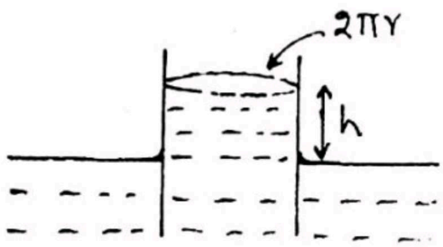


$$hR = lR'$$

If $l < h$ then $R' > R$

3.) Rise of liquid in capillary tube does not violete the law of conservation of energy.

4.) Weight of the liquid Column -



$2\pi r \times T$ = weight of liquid Column

Gauge Pressure -

The pressure difference between hydrostatic pressure (P) and atmospheric pressure P_0 is called gauge pressure

$$P - P_0 = \rho g h$$

* Gauge pressure is always measured with the help of 'manometer.'

Density (ρ) -

mass per unit volume of a substance is defined as density.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \boxed{\rho = \frac{M}{V}}$$

SI Unit - Kg/m^3 Dimensions - $[M^1 L^{-3} T^0]$

Specific weight or weight density (ω)-

It is defined as the ratio of the weight of the fluid to its volume or the weight acting per unit volume of the fluid.

$$\omega = \frac{\text{Weight}}{\text{Volume}} = \frac{mg}{V} = \rho g$$

SI Unit - N/m^2 Dimensions - $[M^1 L^{-2} T^{-2}]$

* Specific weight of pure water at 4°c is 9.8 KN/m^2

Relative density-

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°c.

$$\text{Relative density} = \frac{\text{density of given fluid}}{\text{density of pure water at } 4^\circ C}$$

* Relative density is a unitless scalar quantity.

Note-

1.] If m_1 mass of liquid of density ρ_1 and m_2 mass of liquid of density ρ_2 are mixed, then

$$\text{Density of Mixture, } \rho_{mix} = \frac{M_{mix}}{V_{mix}}$$

or

$$\rho_{mix} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

2.] If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed then

$$\rho_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

Pascal's Law -

If the gravity effect is ignored then the pressure at every point of liquid at rest is same

or

The change in pressure at a particular point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container.

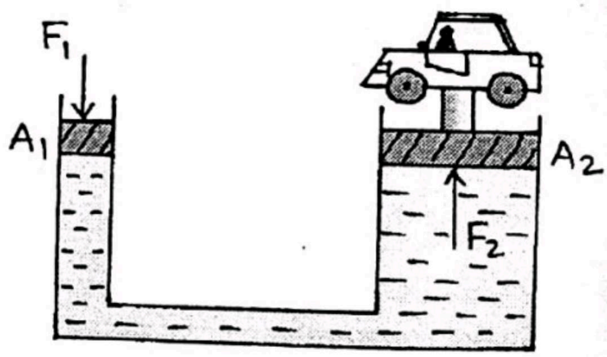
Applications - hydraulic jacks, lifts, breaks etc.

Working of hydraulic Lift -

It is used to lift the heavy loads by applying a small force.

Pressure applied $= \frac{F_1}{A_1}$

Pressure transmitted $= \frac{F_2}{A_2}$



By Pascal's law the pressure is transmitted equally, so that

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$\therefore$ Upward force on A_2 is $F_2 = \frac{F_1}{A_1} \times A_2$

As $A_2 > A_1$, therefore $F_2 > F_1$.

Buoyancy and Buoyant Force -

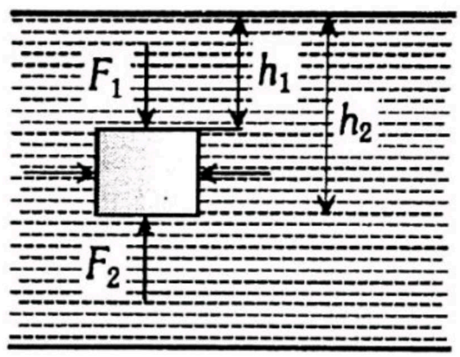
If a body is partially or wholly immersed in a fluid, then it experiences an upward force due to the fluid surrounding it. This phenomenon is called buoyancy and the force exerted by the fluid is called buoyant force.

Buoyancy- जब किसी object को किसी liquid में डुबोया जाता है तो यह object liquid के Molecules को displace कर देता है, इसके कारण liquid के molecules अपनी Initial position में आने के लिए object पर एक force लगाते हैं जो object को liquid से बाहर निकालने की कोशिश करता है। इसे ही Buoyant force या Buoyancy कहते हैं।

Archimede's Principle-

It states that the buoyant force on a body that is partially or totally immersed in a liquid is equal to the weight of the fluid displaced by it.

Let a body is immersed in a liquid of density ρ.



Top surface of the body experiences a downward force F_1

$$F_1 = AP_1 = A[P + \rho g h_1]$$

Lower face of the body will experiences a upward force F_2

$$L = h_2 - h_1$$

$$F_2 = AP_2 = A[P + \rho g h_2]$$

As $h_2 > h_1$ so F_2 is greater than F_1, so that

Net Upward force $F = F_2 - F_1 = A\rho g[h_2 - h_1]$

$\therefore \quad F = A\rho g L = V\rho g \qquad [\because V = AL]$

or $\quad F = mg$ = weight of the fluid displaced

* Loss in the weight of body in liquid = weight of the fluid displaced

Note -

1.] Upthrust or buoyancy is independent of the mass, size, density of the body. It depends upon the volume of the body inside the fluid and nature of displaced fluid (density of the fluid).

i.e. Upthrust $\propto V_{in}$ and Upthrust $\propto \rho$

Eg. Upthrust on a fully submerged body is more in sea water than in fresh water because $\rho_{sea} > \rho_{Pure}$

$$\text{Upthrust} = V_{in}\rho g$$

Downthrust $= mg$ [m = mass of the body]

★ Upthrust और downthrust हमेशा Pair में लगते हैं।

2.] Due to upthrust the weight of the body decreases.

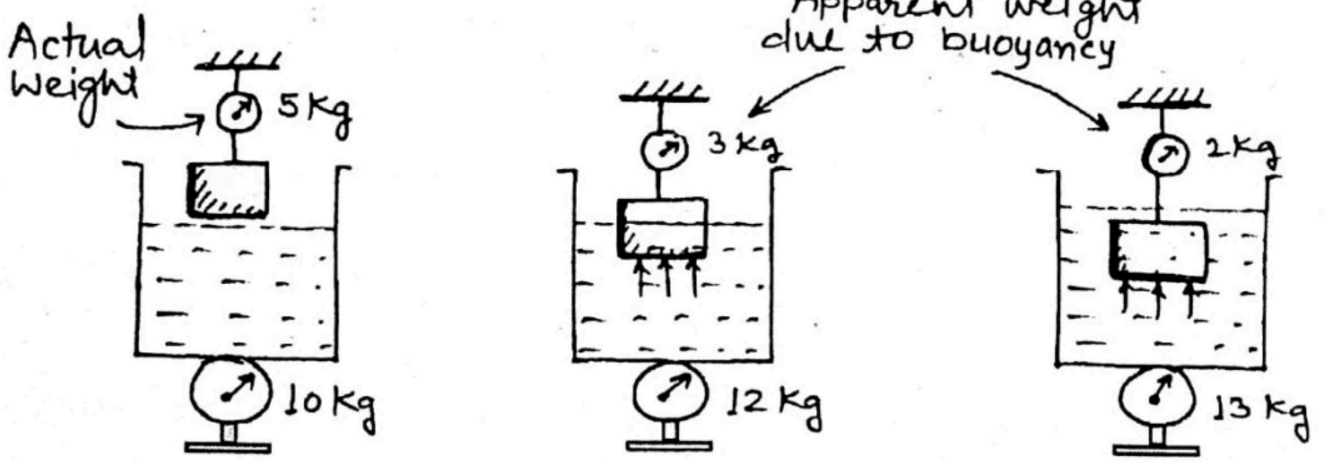
Apparent Weight = Actual Weight − Upthrust

$$W_{app} = W - F_{up}$$

$$W_{app} = V\sigma g - V\rho g \Rightarrow \boxed{W_{app} = W\left[1 - \frac{\rho}{\sigma}\right]}$$

where ρ and σ are density of liquid and body respectively.

Also Loss in Weight $= W_{app} - W = F_{up}$



3.] Using Archimede's principle we can determine relative density of a body as

$$\text{Relative density} = \frac{\text{density of body}}{\text{density of water at } 4°C}$$

$$\text{R.D.} = \frac{\text{Weight of body}}{\text{weight of equal volume of water}}$$

$$\text{R.D.} = \frac{\text{Weight of body}}{\text{Water thrust}} = \frac{\text{Weight of body}}{\text{Loss of weight in water}}$$

$$\text{R.D.} = \frac{\text{weight of body in air}}{\text{weight in air} - \text{weight in water}}$$

or

$$\boxed{\text{R.D.} = \frac{W_1}{W_1 - W_2}}$$

Where W_1 and W_2 are weight of body in air and water respectively.

Floation -

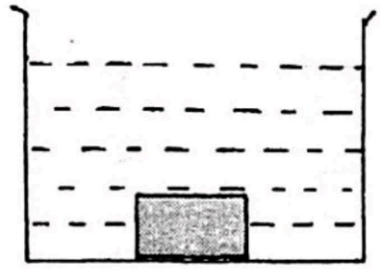
When a body of density σ and volume V is completely immersed in a liquid of density ρ, the forces acting on the body are

(i) weight of the body $W = mg = V\sigma g$ directed vertically downwards through center of gravity of the body.

(ii) Buoyant Force or upthrust $F_{Th} = V\rho g$ directed vertically upwards through centre of gravity of the displaced liquid i.e. centre of buoyancy.

Case 1. If density of body is greater than that of liquid $(\sigma > \rho)$ then $W > F_{Th}$

So the body will sink to the bottom of the liquid

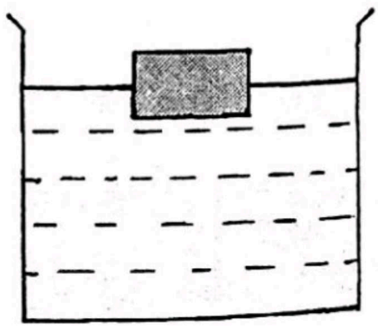


$$W_{app} = W - F_{Th} = V\sigma g - V\rho g$$

$$W_{app} = V\sigma g\left(1 - \frac{\rho}{\sigma}\right) = W\left(1 - \frac{\rho}{\sigma}\right)$$

Case 2. Density of the body is lesser than that of liquid $(\sigma < \rho)$ then $W < F_{Th}$

So the body will float partially submerged in the liquid. In this case the body will move up and the volume of liquid displaced by the body will be less than the volume of body (V) i.e. $V_{in} < V$.



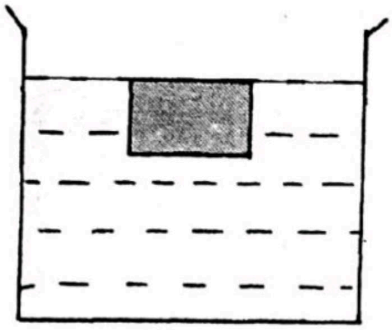
Hence $W_{app} = W - V_{in}\rho g$

or $W_{app} = V\sigma g - V_{in}\rho g$

where V_{in} is the volume of body in the liquid.

Case 3. If density of the body is equal to the density of liquid ($\sigma = \rho$) then $W = F_{Th}$

so the body will float fully submerged in the liquid. It will be in neutral equilibrium.



$\therefore \quad W_{app} = W - F_{Th} = 0$

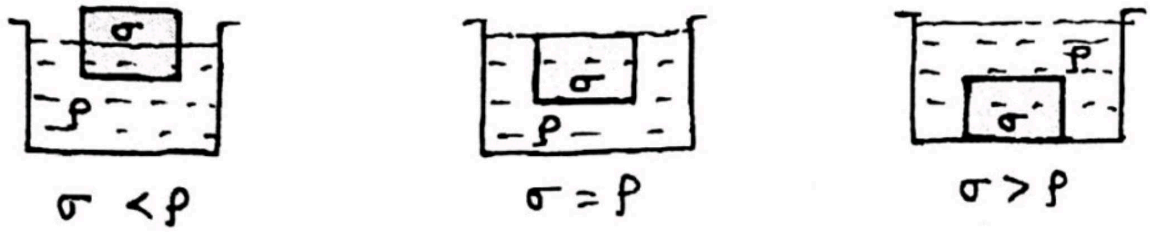
Apparent Weight $W_{app} = 0$

$V\sigma g = V_{in}\rho g \Rightarrow \boxed{V\sigma = V_{in}\rho}$

* The above three cases constitute the law of floation which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

Note - Fraction of volume immersed in liquid is

$$V_{in} = \frac{\sigma}{\rho} V$$



Hydro Dynamics

Types of fluid flow -

1.) Steady and Unsteady flow-

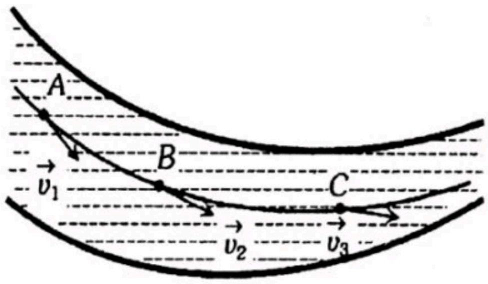
The flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time then it is called steady flow.

If the velocity, pressure and density at a point in the flow varies with time then it is called unsteady flow.

2.) Streamline flow -

The flow in which each particle of the liquid passing through a point travels along the same path and with the same velocity as the preceding particle passes through that point is called streamline flow.

A streamline is the path (straight or curved) such that tangent to it at any point gives the direction of the flow of liquid at that point.

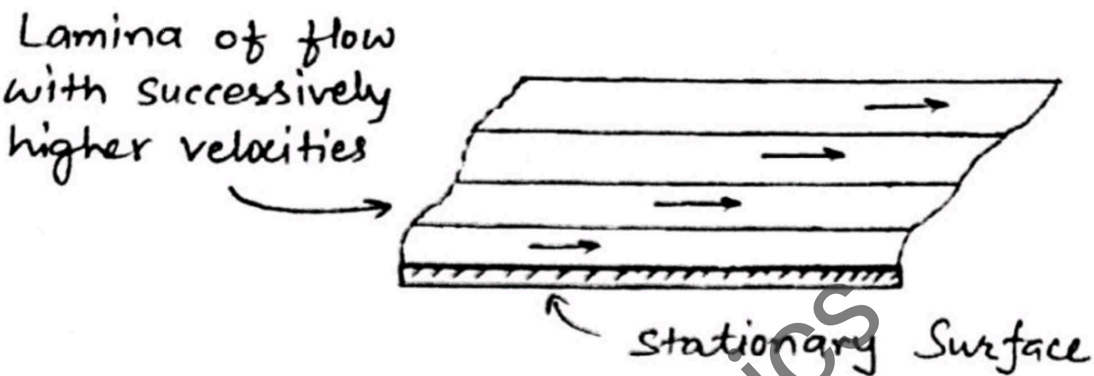


Note -
The two streamlines cannot cross each other and the geater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

3.) Laminar Flow -

It is the flow in which the fluid particles move along well defined streamlines which are straight and parallel.

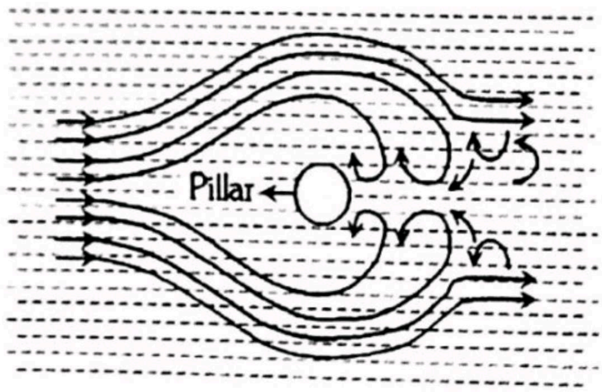
In laminar flow the velocities at different points in the fluid may have different magnitudes, but there directions are parallel. Thus the particles move in the form of layers gliding smoothly over the adjacent layer.



4.) Turbulent Flow -

Turbulent flow is an irregular flow in which the particles can move in zig-zag way due to which eddies formation take place which are responsible for high energy losses.

Eg. eddies are seen by the sides of the pillars of a river bridge.



Critical Velocity (V_c) -

The critical velocity is that velocity of liquid flow upto which its flow is streamlined and above which its flow becomes turbulent.

Reynold's Number (N_R) -

Reynold's number is a pure number which determines the nature of flow of liquid through a pipe.

$$N_R = \frac{\rho V d}{\eta}$$

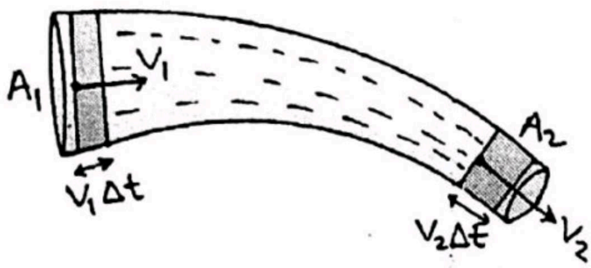
where ρ = density of the fluid
η = viscosity of fluid
V = mean speed of fluid
and d = diameter of the tube

(i) If $0 < N_R < 2000$ then flow is streamline / laminar.
(ii) If $N_R > 3000$ then flow is turbulent.
(iii) If $2000 < N_R < 3000$ then flow may be laminar or turbulent.

Equation of Continuity -

The continuity equation is the mathematical expression of the law of conservation of mass in fluid dynamics.

In the steady flow of mass of fluid entering into a tube in a particular time interval is equal to the mass of fluid leaving the tube.



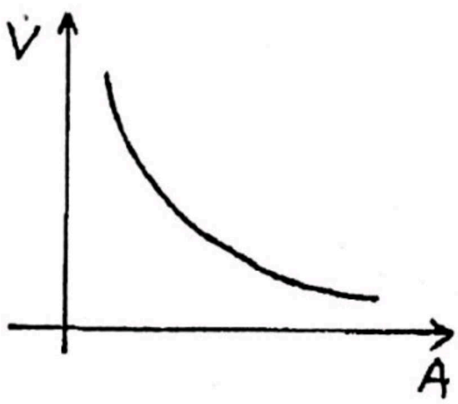
$$\frac{m_1}{\Delta t} = \frac{m_2}{\Delta t} \Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

or $A_1 V_1 = A_2 V_2$ $[\because \rho_1 = \rho_2]$

or $\boxed{AV = \text{Constant}}$

* Volume of water flowing per sec. remains constant.

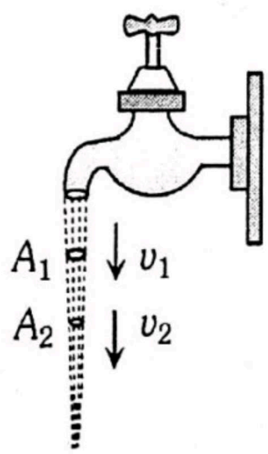
or $\boxed{A \propto \frac{1}{V}}$



Hence the velocity of flow will increase if cross-section decreases and vice-versa.

Eg. When water falls from a tap the velocity of falling water under the action of gravity will increase with distance from the tap (i.e. $V_2 > V_1$).

So in accordance with the continuity equation the cross section of the water stream will decrease (i.e. $A_2 < A_1$) and the falling stream of water becomes narrower.



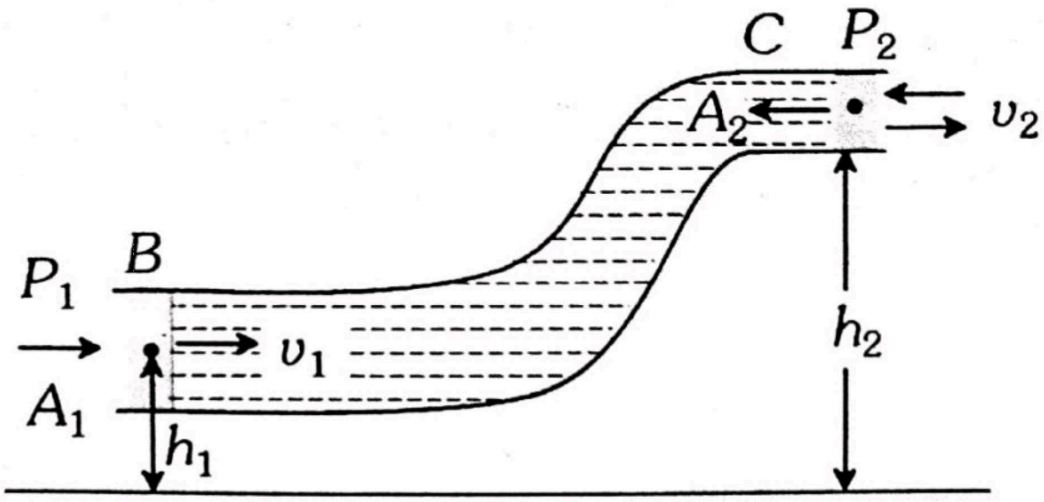
Energy of a flowing Fluid-

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure without imparting any velocity to it.	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.	It is the energy possessed by a liquid by virtue of its motion or velocity.
Pressure energy of the liquid PV	Potential energy of the liquid mgh	Kinetic energy of the liquid $\frac{1}{2}mv^2$
Pressure energy per unit mass of the liquid $\frac{P}{\rho}$	Potential energy per unit mass of the liquid gh	Kinetic energy per unit mass of the liquid $\frac{1}{2}v^2$
Pressure energy per unit volume of the liquid P	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\frac{1}{2}\rho v^2$

Bernoulli's Theorem-

According to this theorem the total energy (pressure energy, potential energy, Kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow.

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$ [Energy per unit volume]



or $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{Constant}$ [Energy per unit mass]

or $\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{Constant}$ [Energy per unit weight]

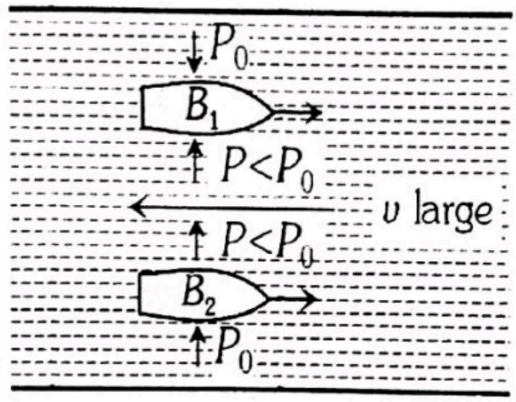
Here $\frac{P}{\rho g}$ is called 'pressure head', h is called the 'gravitational head' and $\frac{v^2}{2g}$ is called velocity head.

Application of Bernoulli's Theorem

1.) Attraction between two closely parallel moving boats

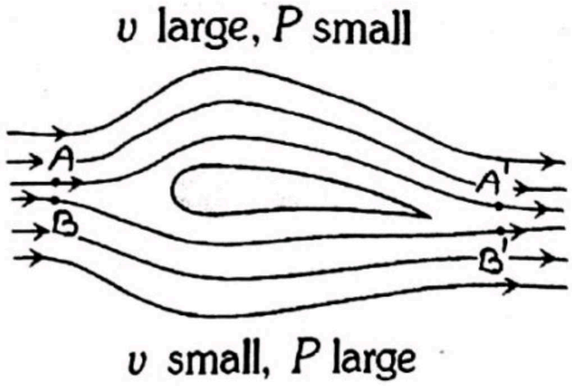
When the two boats move side by side in the same direction then the water in the region between them moves faster than that on the remote sides.

Hence according to Bernoulli's theorem the pressure between them reduces and hence due to pressure difference they are pulled towards each other and may even collide.



2.] Working of an aeroplane -

Due to specific shape of wings of the aeroplane, when the aeroplane runs, air passes at higher speed over it as compared to its lower surface.

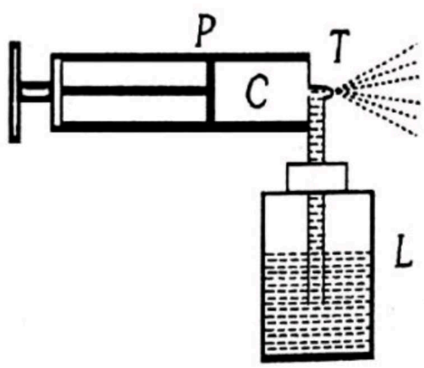


This difference of air speed above and below the wings, creates a pressure difference due to which an upward force called 'dynamic lift' (= pressure difference x area of wing) acts on the plane If this force becomes greater than the weight of the plane, then the plane will rise up.

> ★ Since path of particle A is longer than path of B but the time period to reach at point A' and B' respectively is same, so that $V_A > V_B$ and $P_A < P_B$.

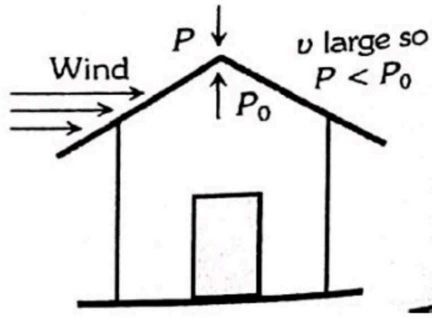
3.] Action of sprayer or atomizer -

In sprayer by means of motion of a piston P in a cylinder C, high speed air is passed over a tube T dipped in liquid to be sprayed. High speed air creates low pressure over the tube due to which liquid rises in it and is then blown off in very small droplets with expelled air.



4.] Blowing off roofs by wind Storm -

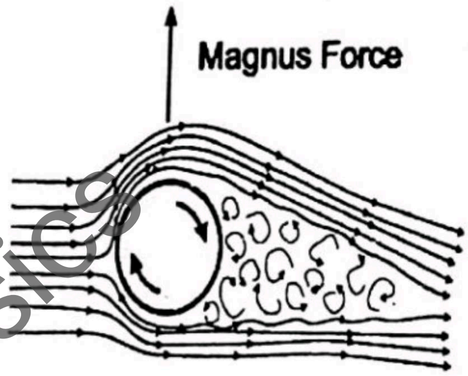
When wind blows with a high velocity above a tin roof, it creates a low pressure in accordance with Bernoulli's principle.



However the pressure below the roof (inside the room) is still atmospheric. So due to this difference of pressure the roof is lifted up and is then blown off by the wind.

5.] Magnus Effect (Spinning Ball)-

Tennis and cricket players usually experience that when a ball is thrown spinning, it moves along a curved path.



This is due to the air which is being dragged round by the spinning ball. When the ball spins the layer of the air around it also moves with the ball. So as shown in fig. the resultant velocity of air increases on the upper side and reduces on the lower side.

Hence the pressure on the upper side becomes lower than that on the lower side. This pressure difference exerts a force on the ball due to which it moves along a curved path. This is known as Magnus effect.

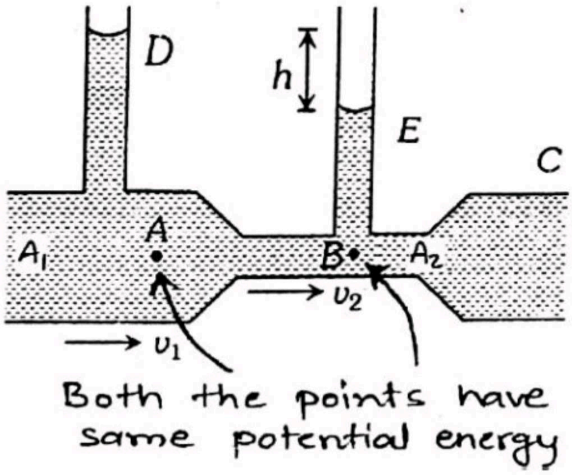
6.] Venturimeter -

It is a device used for measuring the rate of flow of liquid through pipes

Let P_1, P_2 and V_1, V_2 are pressure and velocity at point A and B respectively, then by Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$



If Q be the volume of water flowing per second then

$$Q = A_1 V_1 = A_2 V_2$$

Also $\quad P_1 - P_2 = \rho g h$

Hence $\quad \rho g h = \frac{\rho}{2}(V_2^2 - V_1^2) \Rightarrow \boxed{2gh = V_2^2 - V_1^2}$

or $\quad 2gh = \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}$

$$\Rightarrow Q^2 = 2gh \frac{A_1^2 A_2^2}{A_1^2 - A_2^2}$$

$$\Rightarrow \boxed{Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

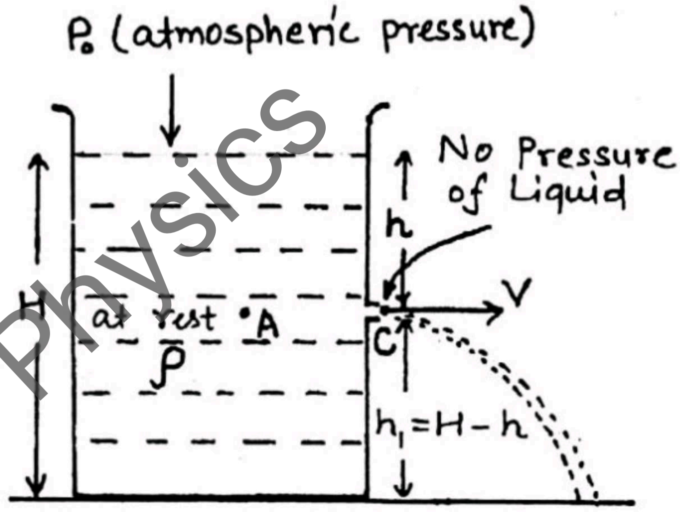
Thus at a point where the cross-section area is smaller velocity is greater and pressure is lower and vice-versa.

Torricelli's Law of Efflux -

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid.

Let V is the velocity with which the liquid comes out from hole at C, Velocity of the liquid inside the tank at point A is zero.

Total energy at point A and C should be same.



Applying Bernoulli's theorem at A and O, we get

$$P_0 + \rho g h = P_0 + \frac{1}{2}\rho V^2$$

Or
$$\boxed{V = \sqrt{2gh}}$$

* Equation is same as that of freely falling body after falling through height h and is known as Toricelli's Law.

Note-

* Now writing equation of uniformly accelerated motion in vertical direction.

$$H - h = 0 + \frac{1}{2}gt^2$$

$$\Rightarrow \boxed{t = \sqrt{\frac{2(H-h)}{g}}} \quad \text{Or} \quad \boxed{t = \sqrt{\frac{2h_1}{g}}}$$

Horizontal Range $R = V \times t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$

$$\boxed{R = 2\sqrt{h(H-h)}} \quad \text{Or} \quad \boxed{R = 2\sqrt{h h_1}}$$

* By using maxima - minima

Range will be maximum when $h = H - h$

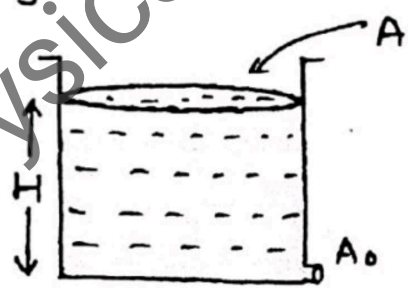
or $\boxed{h = \frac{H}{2}}$

$$\therefore \quad R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} \Rightarrow \boxed{R_{max} = H}$$

* If A and A_0 are the area of cross section of tank and hole and the hole is at the bottom then

Time required to make empty the tank

$$\boxed{t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}}$$



* Time taken by the water to fall from level h to h'

$$\boxed{t = \frac{A}{A_0}\sqrt{\frac{2h}{g}} - \frac{A}{A_0}\sqrt{\frac{2h'}{g}}}$$

Viscosity -

Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its adjacent layers. It is the fluid friction or internal friction.

The internal tangential force which try to retard the relative motion between the layers is called viscous force.

* Viscous force is due to intermolecular forces.

Newton's Law of Viscosity -

It states that the viscous force F acting between two layers of a liquid flowing in stream lined motion depends upon following two factors

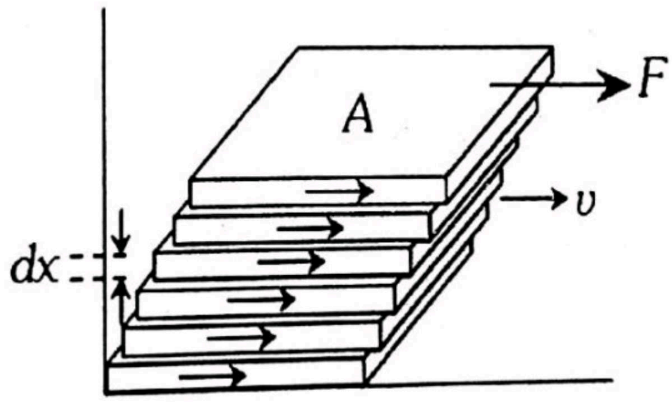
(i) $F \propto A$ — A = Contact - area of the layers

(ii) $F \propto \frac{dv}{dx}$ — $\frac{dv}{dx}$ = velocity gradient between layers

or $F \propto A\frac{dv}{dx}$

or $$\boxed{F = \eta A\frac{dv}{dx}}$$

where η is called the coefficient of viscosity of the liquid.



If $A = 1$ and $\frac{dv}{dx} = 1$ then $F = \eta$

The coefficient of viscosity of a liquid is defined as the viscous force per unit area of contact between two layers having a unit velocity gradient.

SI Unit – N-s/m^2 **Dimensions** - $[M^1L^{-1}T^{-1}]$

Dependency of Viscosity of fluids-

(1.) On Temperature-

Since cohesive forces decreases with increase in temperature. Therefore with the rise in temperature the viscosity of liquids decreases.

(2.) On Pressure -

with increase in pressure the viscosity of liquids (except water) increases. The viscosity of water decreases with increase in pressure.

(3.) On density -

The viscosity of thick liquids like honey, coltar, glycerin etc is more than that of thin liquids like water etc.

Poiseuille's Formula -

In case of steady flow of liquid of viscosity η in a capillary tube of length L and radius r under a pressure difference P across it, the volume of liquid flowing per second is given by

$$Q = \frac{dV}{dt} = \frac{\pi P r^4}{8 \eta L}$$

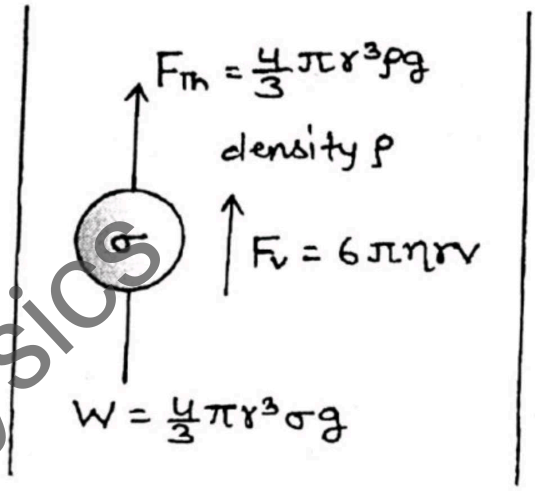
With the help of poiseuille's formula, the coefficient of viscosity of a liquid can be determined.

Stoke's Law -

When a body moves through a fluid, then the fluid exerts viscous force on the body to oppose its motion.

According to stoke's law if a sphere of radius r moves with velocity v through a fluid of viscosity η then the viscous force opposing the motion of the sphere is

$$F_v = 6\pi\eta r v$$



Difference between Viscous force & Friction -

(i) Friction force depends on mass of the body but viscous force does not depends on mass.

(ii) Viscous force depends on velocity of body but friction force does not depends on velocity.

Terminal Velocity – (V_T)

As the body falls through a medium, its velocity goes on increasing due to gravity. Therefore the opposing viscous force which acts upwards also goes on increasing. A stage reaches when the true weight of the body is just equal to the sum of the upward thrust due to buoyancy and the upward viscous force. At this stage there is no net force to accelerate the body.

Hence the body starts falling freely with a constant velocity which is called terminal velocity.

$$F_{Th} + F_v = W$$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_T = \frac{4}{3}\pi r^3 \sigma g$$

or

$$\boxed{V_T = \frac{2}{9} r^2 \frac{(\sigma - \rho)}{\eta} g}$$