1. ELECTRIC CHARGE AND FIELD

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1. ELECTRIC CHARGE AND FIELD

Electric Charge -

The property of material by which it exerts or experience electric and magnetic effects is called the electric charge.

Examples -

(i) Experience of seeing a spark or hearing a crackle, when we take of our synthetic cloths.

(ii) Lightning in the sky during thunderstorms due to electric discharge.

Electrostatics -

The branch of Physics, which deals with the study of charges at rest (static charges), the forces, fields and potential due to these charges is called Electrostatics or static Electricity.

Important Facts about charge -

(1.) There are only two types of electric charge.

(2.) Like charges repel and unlike charges attracts each other.

(3.) The unlike charges nullify (Cancel) each other's effect when they come in contact. Therefore the charges are named as positive and negative by the American scientist 'Benjamin Frenklin'.

(4.) Normally the materials are electrically neutral, they do not contain charge because their charges (Protons and electrons) are exactly balanced.

(5.) The electron of the outermost orbit of an atom are far from the nucleus so these electrons are Loosely bounded with the nucleus and can be separated easily from the orbit by giving some energy. These electron are then called free electrons.

(6.) The body or atom can be charged positively by loasing some of its electrons and can be charged negatively by gaining electrons.

(7.) The apparatus used to detect charge on a body is "Gold leaf Electroscope".

(8.) An electric charge can creates electric field (E), magnetic field ($\vec{B}$) and electromagnetic radiations

$V = 0$

→ Only Electric field ($\vec{E}$)

$V = \text{Const.}$

→ Electric Field ($\vec{E}$) & Magnetic Field ($\vec{B}$)

$V \neq \text{Const.}$

→ $\vec{E}$, $\vec{B}$ and Electromagnetic Radiations

(9.) Electric charge uniformly distribute on a uniform surface but the charge density on a non-uniform surface is maximum on that points where the radius of curve is minimum.

Charge density $\boxed{\sigma \propto \frac{1}{R}}$

Point Charge -

It is a point where all the charge of a body is concentrated.

Conductor, Insulator & Semiconductor -

Conductor -

The material through which the electric charge and current can flow easily are called Conductors.

Examples - All Metals, Human body, earth etc.

Insulator -

The materials which resist the flow of electric charge and current are called insulators.

Examples - Wood, Porcelene, plastic etc.

Semiconductors -

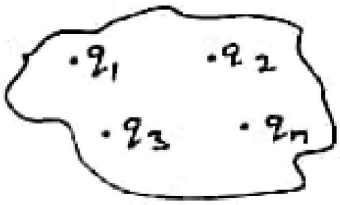
The materials which resist the flow of electric charge and current but the value of their resistance is in between the conductor and insulator are called semiconductors.

Examples - Silicon (Si), Germenium (Ge)
Carbon (C) → Excellent thermal Conductivity.

Properties of Electric Charge -

(1) Additivity of Charges:-

Electric charge is a scalar quantity so the total charge of the system is obtained simply by adding the different charges algebrically with proper sign according to their charge.



$$q = q_1 + q_2 + \dots q_n$$

(2) Charge is conserved:-

Charge can not be either created or distroyed. It can only be transferred from one body to another. Hence the total charge of the isolated system is always conserved.

Total charge = 0+0=0

Total charge = +q−q = 0

(3) Quantisation of Charge :-

The electric charge on a body is always an integral multiple of e (charge on 1 electron). This is called quantisation of charge.

$$Q = \pm ne$$

$n = 0, 1, 2 \dots \infty$

$e = 1.602 \times 10^{-19}$ C

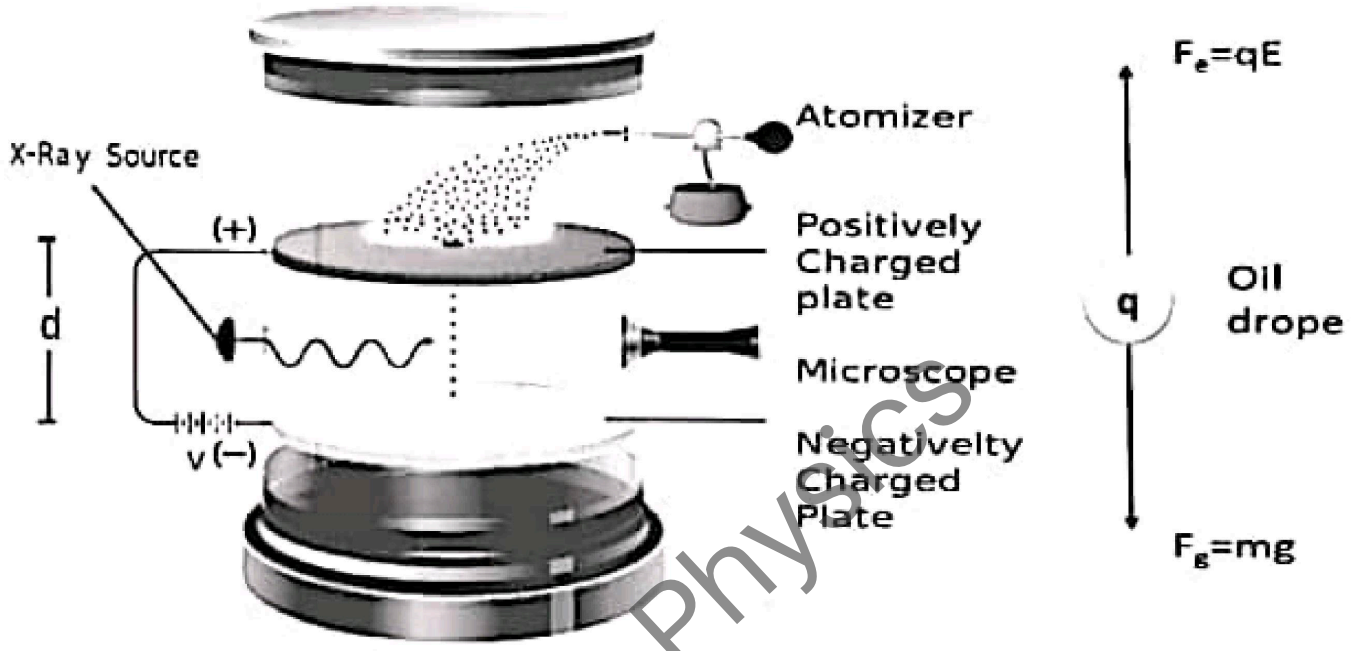
* The unit of electric charge is coulomb.

1 Coulomb -

The total charge on 6.24×10^{18} electrons is equal to 1 coulomb.

Millikan's Oil drop Experiment –

The charge on one electron was found by this oil drop experiment by Millikan in 1912.



When the oil drop remain stationary in the experiment then the forces on oil drop

Electric Force (F_e) = Gravitational Force (F_g)

$$qE = mg$$

or $$\boxed{q = \frac{mg}{E} = \frac{mgd}{V}}$$ where $E = \frac{V}{d}$

We Know that – mass = density x volume

$$\boxed{m = d \times \frac{4}{3}\pi r^3}$$

where $r \rightarrow$ radius of oil drop (0.5 mm)

* Mass of Electron (m_e)	= 9.11×10^{-31} kg.
* Mass of Proton (m_p)	= 1.67×10^{-27} kg.

Q. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/C in Millikan's oil drop experiment. The density of the oil is 1.26 gm/cm³. Then estimate the radius of the oil drop.

Sol. Given that -

Charge $q = ne = 12 \times 1.6 \times 10^{-19}$ C

Density $d = 1.26 \times \frac{10^{-3}}{10^{-6}} = 1.26 \times 10^3$ kg/m³

When the oil drop is held stationary then

$$q = \frac{mg}{E}$$

We know $m = d \times \frac{4}{3}\pi r^3$

so that $q = \frac{d \times \frac{4}{3}\pi r^3 \times g}{E}$

or $r^3 = \frac{qE}{d \times \frac{4}{3}\pi \times g}$

$$r^3 = \frac{12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{1.26 \times 10^3 \times \frac{4}{3} \times 3.14 \times 9.8} = 9.47 \times 10^{-18} \text{ m}^3$$

$$r = (9.47 \times 10^{-18})^{1/3} = 9.81 \times 10^{-7} \text{ m.}$$

Q. What is the total charge on 75 kg of electrons?

Sol. Number of electrons in 75 kg.

$$n = \frac{\text{total mass}}{\text{mass of 1 electron}} = \frac{75}{9 \times 10^{-31}}$$

$$n = 8.3 \times 10^{31}$$

so Total charge $Q = ne$

$$Q = 8.3 \times 10^{31} \times (-1.6 \times 10^{-19})$$

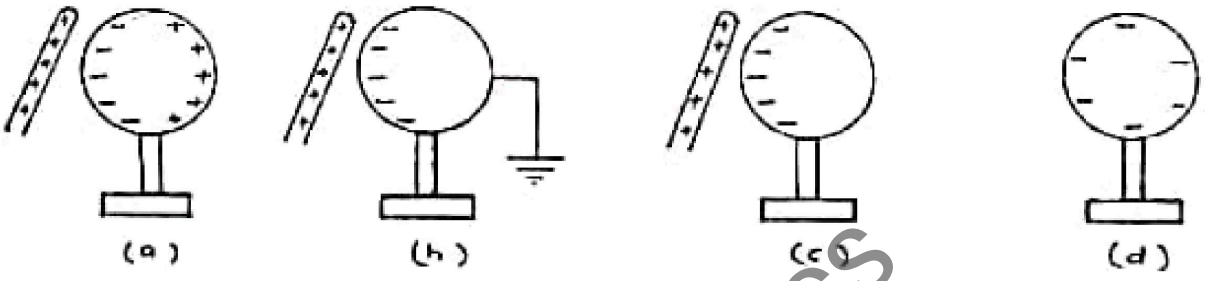
$$Q = -1.33 \times 10^{13} \text{ C}$$

Charging by Induction -

Method 1. (Using a ground Connection)

(i) A positively charged rod is brought near a neutral metal sphere and polarising it.

(ii) The sphere is grounded allowing electrons to be attracted from the earth.



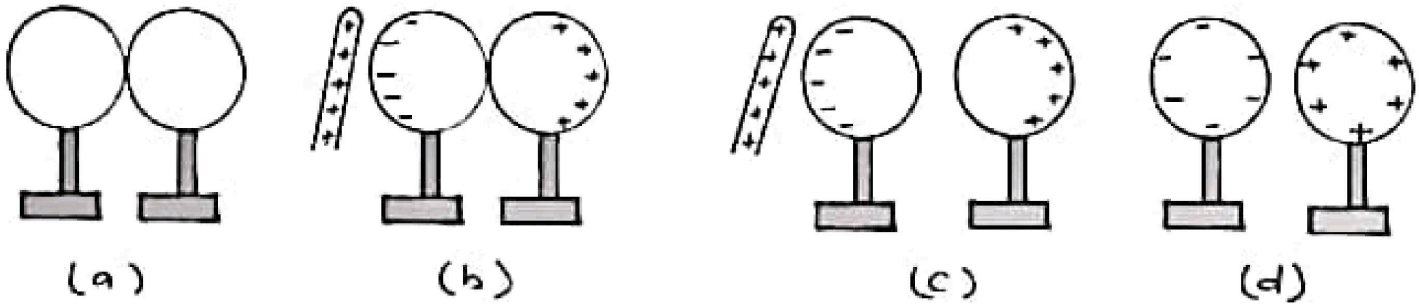
(iii) The ground Connection is broken.

(iv) The positive rod is removed leaving the sphere with an induced negative charge.

Method 2.

(i) Two uncharged metal spheres are in contact with each other but insulated from rest of the world.

(ii) A positively charged rod is brought near the sphere, attracting negative charge and leaving the other sphere positively charged.



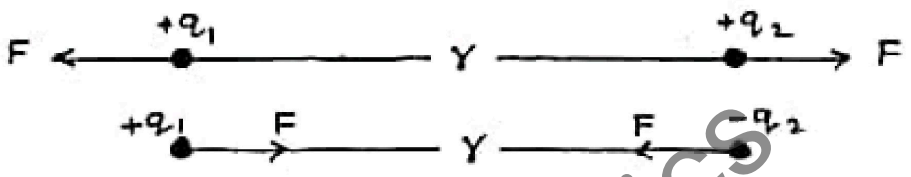
(iii) Now the spheres are separated before the rod is removed and thus separating positive & negative charge.

(iv) Remove the rod. The charges on spheres will rearrange themselves and uniformly distributed over the spheres.

Coulomb's Law -

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, and acts along the line joining the two charges.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r.



According to Coulomb's Law,

$$F \propto q_1 q_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

or $$\boxed{F = \frac{K q_1 q_2}{r^2}}$$ K - Electrostatic Constant

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In cgs system, $K = 1$

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2$

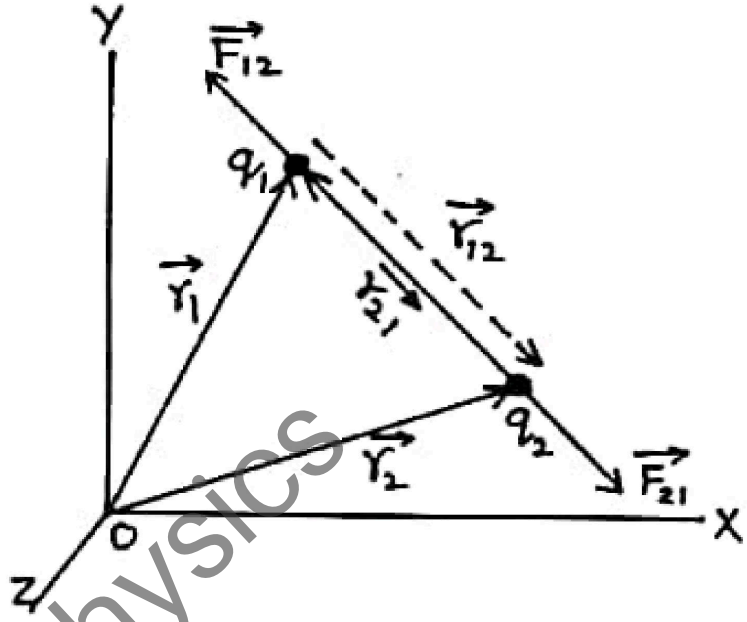
* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \ C^2/Nm^2$$

Coulomb's Law in vector form -

Let the position vector of charges q_1 and q_2 be $\vec{r_1}$ and $\vec{r_2}$ respectively.

We denote the force on q_1 due to q_2 by $\vec{F_{12}}$ and the force on q_2 due to q_1 by $\vec{F_{21}}$.



$$\boxed{F_{12} = F_{21}}$$

Here $\vec{r_1} + \vec{r_{12}} = \vec{r_2}$

$$\boxed{\vec{r_{12}} = \vec{r_2} - \vec{r_1}}$$

also $\vec{r_2} + \vec{r_{21}} = \vec{r_1}$

or $\boxed{\vec{r_{21}} = \vec{r_1} - \vec{r_2}}$ and $\boxed{\vec{r_{12}} = -\vec{r_{21}}}$

The magnitude of vectors $\vec{r_{12}}$ and $\vec{r_{21}}$ is denoted by $|\vec{r_{12}}|$ and $|\vec{r_{21}}|$ and their unit vectors are given as -

$$\hat{r}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \quad \text{and} \quad \hat{r}_{21} = \frac{\vec{r_{21}}}{|\vec{r_{21}}|}$$

So the force $\vec{F_{12}}$ can be given as -

$$\boxed{\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_{21}}|^2} \hat{r}_{21}}$$

(i) If q_1 and q_2 are of same sign ; $q_1 q_2 > 0$ then $\vec{F_{12}}$ is along $\vec{r_{21}}$, which denotes repulsion

(ii) If q_1 and q_2 are of opposite sign ; $q_1 q_2 < 0$ then $\vec{F_{12}}$ is along $-\vec{r_{21}}$ or $\vec{r_{12}}$, which denotes attraction

Q. Two point charges of $+2\,\mu C$ and $+6\,\mu C$ repel each other with a force of 12 N. Now if each is given an additional charge of $-4\,\mu C$, what will be the new force?

Sol. Here $q_1 = +2\,\mu C$, $q_2 = +6\,\mu C$, $F = 12\,N$

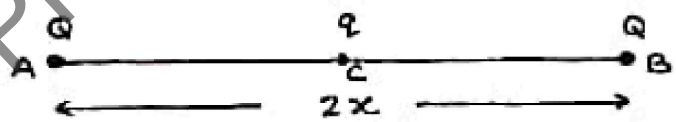
and $q_1' = +2-4 = -2\,\mu C$, $q_2' = +6-4 = 2\,\mu C$

So $$\frac{F'}{F} = \frac{q_1' q_2'}{q_1 q_2} = \frac{(-2)(2)}{(2)(6)} = -\frac{1}{3}$$

$$F' = -\frac{F}{3} = -\frac{12}{3} = -4\,N \text{ (Attractive)}$$

Q. A charge q is placed at the centre of the line joining two equal charges, Q. Show that the system of three charges will be in equillibrium if $q = -Q/4$.

Sol.



Let two equal charges Q each, be held at A and B, where $AB = 2x$.

Here C is the centre of AB, where charge q is held. Net force on q is zero. So q is already in equilibrium.

For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is -

$$\frac{KQq}{x^2} + \frac{K\,Q\cdot Q}{(2x)^2} = 0$$

$$\frac{KQq}{x^2} = -\frac{KQ^2}{4x^2}$$

So $$\boxed{q = -\frac{Q}{4}}$$ Hence Proved.

Dielectric Constant (K) -

force in a medium $(F_m) = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}$ — (1)

where ϵ - electrical permittivity of the medium.

force in vacuum $(F_o) = \frac{1}{4\pi\epsilon_o} \times \frac{q_1 q_2}{r^2}$ — (2)

Dividing eq. (2) by (1), we get -

$$\boxed{\frac{F_o}{F_m} = \frac{\epsilon}{\epsilon_o} = \epsilon_r \text{ or } K}$$

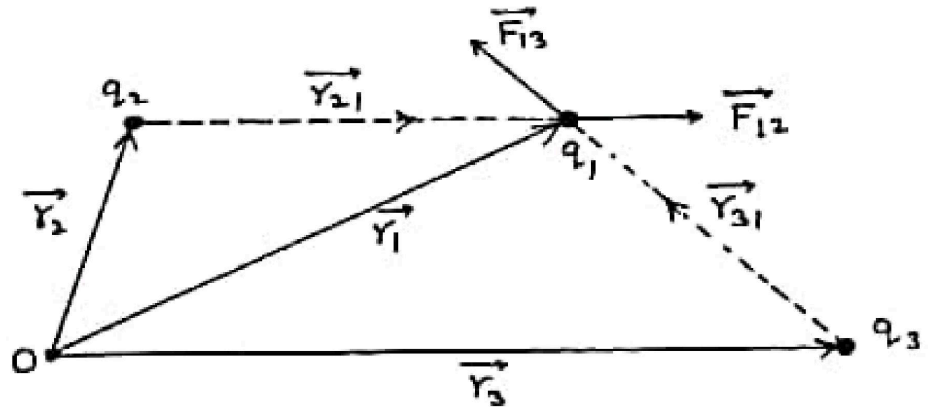
where ϵ_r or K is called dielectric constant or relative electrical permittivity of the medium.

* Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the electrical permittivity of free space.
* Thus force between two given charges in a medium (K) is only $\frac{1}{K}$ times of the force between them in air/ vacuum.

$$\boxed{F_m = \frac{F_o}{K}}$$

Force between Multiple Charges -

Total force on any charge due to a number of charges at rest is the vector sum of all the forces on that charge due to other charges.



Let the force on charge q_1 due to two other charges q_2 and q_3 are $\vec{F_{12}}$ and $\vec{F_{13}}$ respectively.

The position vectors of charges q_1, q_2 and q_3 are $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$.

Here $\boxed{\vec{F_{12}} = \frac{Kq_1q_2}{|\vec{r_{21}}|^2}\hat{r}_{21}}$ and $\boxed{\vec{F_{13}} = \frac{Kq_1q_2}{|\vec{r_{31}}|^2}\hat{r}_{31}}$

Thus the resultant force $\vec{F_1}$ on charge q_1 is obtained by the vector sum of the forces $\vec{F_{12}}$ and $\vec{F_{13}}$ as-

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}}$$

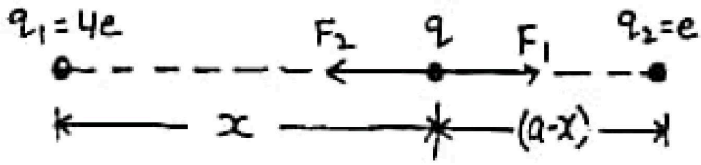
Similarly if a system have n charges $q_1, q_2, \ldots q_n$ then the force on q_1 due to $q_2, q_3, \ldots q_n$ charges can be calculated by the vector sum of forces $\vec{F_{12}}$, $\vec{F_{13}}$, ... $\vec{F_{1n}}$ as-

$$\boxed{\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \ldots \vec{F_{1n}}} \quad \text{or} \quad \boxed{\vec{F_1} = Kq_1\sum_{i=2}^{n}\frac{q_i}{|\vec{r_{i1}}|^2}\hat{r}_{i1}}$$

Q. Two fixed point charges $+4e$ and $+e$ units are separated by a distance 'a'. where should the third point charge be placed for it to be in equilibrium.

Sol: For the charge $+q$ to be in equilibrium

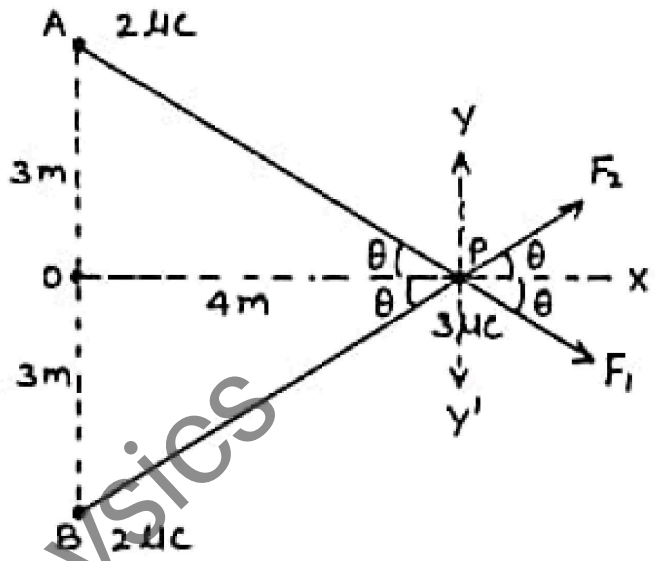
$$F_1 = F_2$$



$$\frac{Kq \cdot 4e}{x^2} = \frac{Kq \cdot e}{(a-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(a-x)^2}$$

on solving - $x = 2a/3$ — Ans.

Q. Two equal positive charges each of $2\,\mu C$ interact with a third positive charge of $3\,\mu C$ situated as shown in fig. Calculate the magnitude and direction of the force on the $3\,\mu C$ charge.



Sol:

Here $OA = OB = 3\,m$

and $OP = 4\,m$

So $AP = BP = \sqrt{3^2 + 4^2}$

$= 5\,m.$

According to Coulomb's law Force on charge at P due to charge at A and B are equal in magnitude so-

$$F_1 = F_2 = \frac{Kq_1q_2}{r^2} = \frac{9\times10^9 \times 2\times10^{-6} \times 3\times10^{-6}}{5^2}$$

So $F_1 = F_2 = 2.16\times10^{-3}\,N$

Here the components of F_1 and F_2 along Y and Y' axis are equal in magnitude ($F_1 \sin\theta = F_2 \sin\theta$) but opposite in direction so they cancel each other

The Components along PX add up. So total force on charge $3\,\mu C$ at P is -

$$F = 2F_1\cos\theta = 2\times2.16\times10^{-3}\times\frac{4}{5}$$

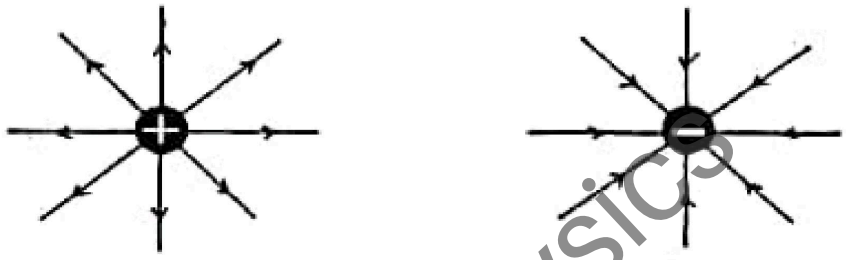
So $F = 3.5\times10^{-3}\,N$ — Ans.

Electric Field -

An electric field is the region of space surrounding electrically charged particles where a force is exerted on other electrically charged objects.

* The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

* The electric field is radially outward from a positive charge and radially inward towards a negative charge.



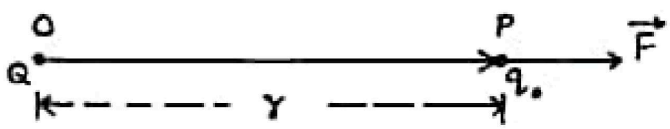
* Concept of Field was introduced first of all by "Faraday".

Electric Field Intensity ($\vec{E}$)

The electric field intensity at any point is the strength of electric field at that point. It is defined as the force experienced by unit Positive charge placed at that Point.

According to Coulomb's Law force on test charge q_0 due to a point charge Q is

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q \cdot q_0}{r^2}$$



by defintion $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ so $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\hat{r}}$

where $\hat{r}$ is unit vector directed from Q towards q_0.

SI Unit of $E = \frac{N}{C}$

Important Points about Electric field -

(1) The magnitude of electric field $\vec{E}$ is same at equal distances from the charge Q, thus it has a spherical symmetry.

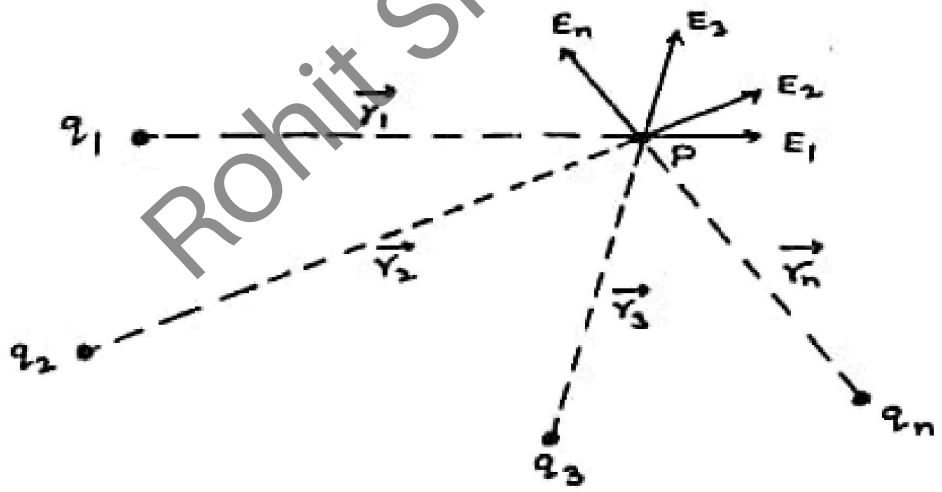
(2) The magnitude of the electric field $\vec{E}$ depends only on the charge Q and the distance r.

(3) The test charge q_0 may have its own electric field, it may modify the electric field of the source charge. Therefore to minimize this effect we rewrite electric field intensity at $\vec{r}$ as -

$$\boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}}$$

Electric Field due to a system of charges -

Consider a system of charges $q_1, q_2, \ldots q_n$ with position vectors $\vec{r_1}, \vec{r_2}, \ldots \vec{r_n}$.



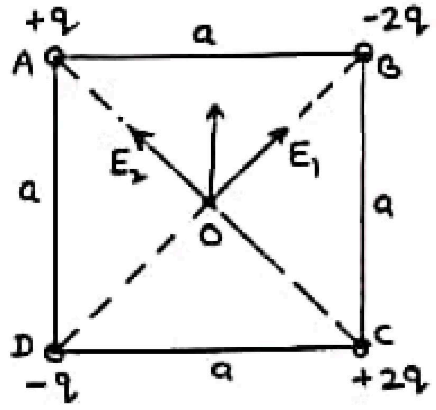
The electric field intensity at a point P due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

$$\boxed{\vec{E} = \vec{E_1} + \vec{E_2} + \ldots \vec{E_n}} \qquad \vec{E} = \frac{Kq_1}{|\vec{r_1}|^2}\hat{r}_1 + \frac{Kq_2}{|\vec{r_2}|^2}\hat{r}_2 + \ldots \frac{Kq_n}{|\vec{r_n}|^2}\hat{r}_n$$

or

$$\boxed{\vec{E} = K\sum_{i=1}^{n} \frac{q_i}{|r_i|^2}\hat{r}_i}$$

Q. What are the magnitude and direction of the electric field at centre of the square in figure. If $q = 10^{-8}C$ and $a = 5$ cm.



Sol. Here

$$OA = OB = OC = OD = r$$

$$r = \frac{5 \times 10^{-2}}{\sqrt{2}} \text{ m.}$$

also here $E_1 = E_2 = \dfrac{q}{4\pi\epsilon_0 r^2}$

or $E_1 = E_2 = \dfrac{10^{-8} \times 9 \times 10^{9}}{(5\times10^{-2}/\sqrt{2})^2} = 7.2 \times 10^{4}$ N/C

In square ABCD the $\angle AOB = 90°$

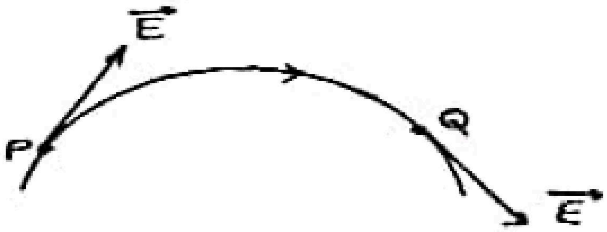
So the Net Electric field at O $= \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 90°}$

$$E = \sqrt{E_1^2 + E_2^2} = E_1\sqrt{2}$$

$E = 7.2\sqrt{2} \times 10^{4}$ N/C at 45° to OA.

Electric Field Lines -

An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

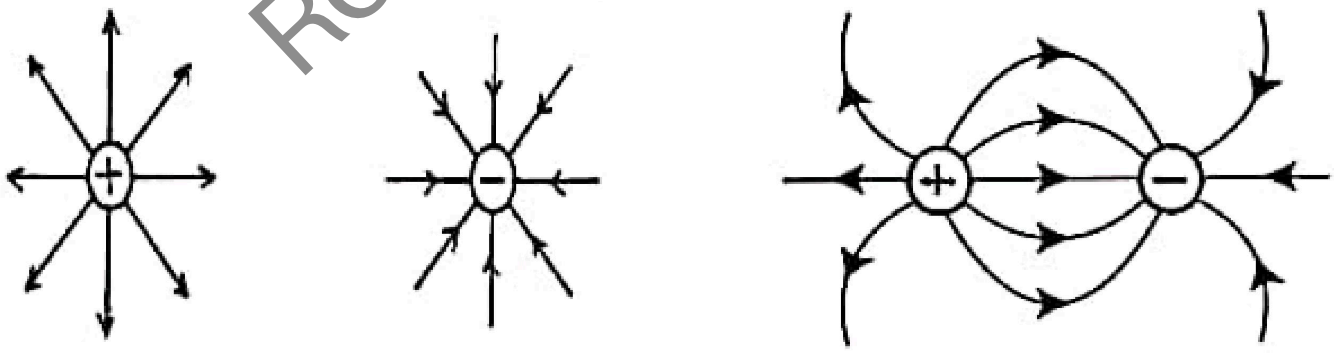


The magnitude of the field is represented by the density of field lines.

Hence Electric field intensity at a point is equal to number of field lines crossing normally a unit area around that point.

Properties of Electric Field Lines -

(1) Electric field lines are continuous curves. They start from a positively charged body and end at a negatively charged body.
If there is a single charge then the field lines may start or end at infinity.

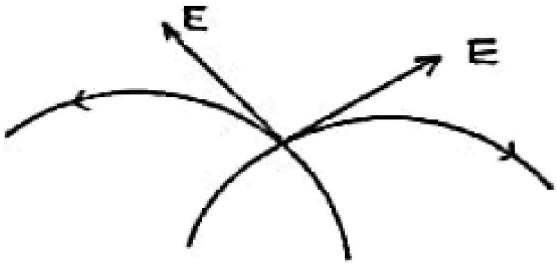


(2) No electric field lines exist inside the charged body. Thus electrostatic field lines do not form continuous closed loops.

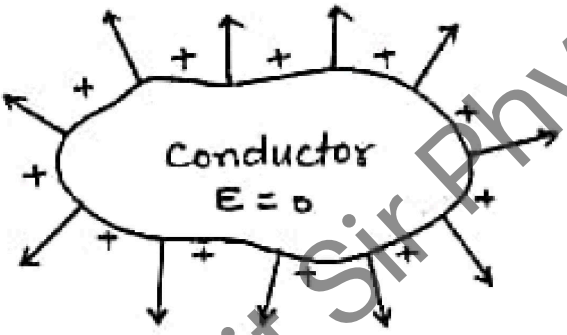
* The magnetic field lines are continuous (or endless) closed loops as against the electric field lines.

(3) Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

(4) No two electric field lines of force can intersect each other. This is because at the point of intersection P, we can draw two tangents PA and PB. This would mean two directions of electric field intensity at the same point, which is not possible.

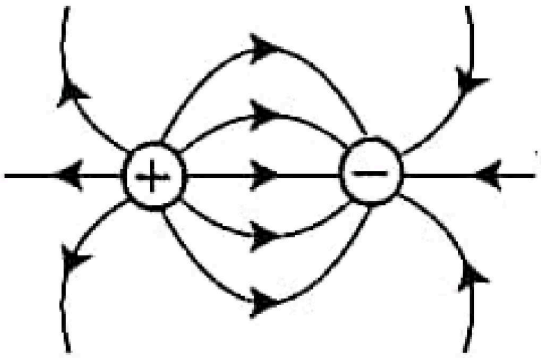


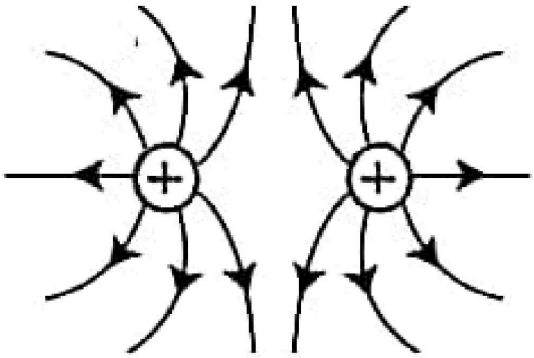
(5) The electric field lines are always normal to the surface of a conductor. So there is no component of electric field intensity parallel to the surface of the conductor.



(6) The field lines around a system of two positive charges (q, q) gives a pictorial representation of their mutual repulsion.

While around the configuration of two equal and opposite charges (q, -q), a dipole, shows the mutual attraction between the charges.

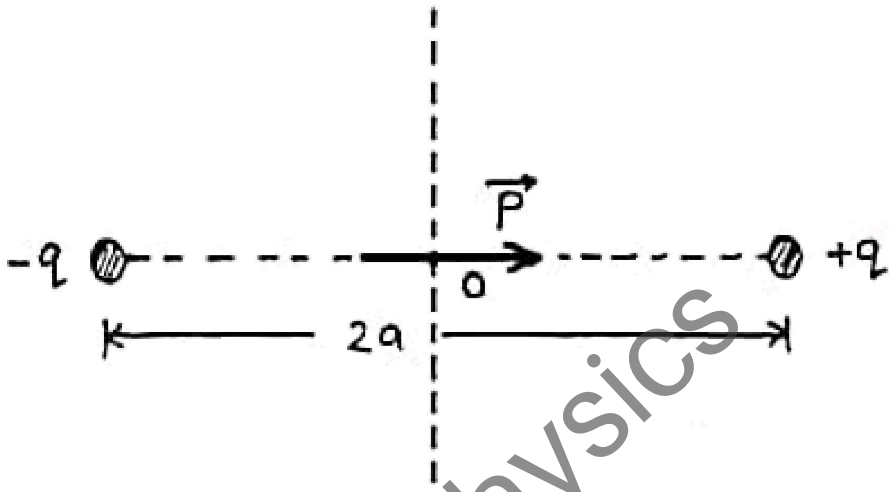




Electric Dipole -

An electric dipole is a pair of equal and opposite point charges separated by a short distance.

Let q and $-q$ are two equal and opposite point charges separated by a small distance '2a'.



* The total charge of the electric dipole is zero, but this does not mean that the field of the electric dipole is zero.
* But at distances much larger than the separation of the two charges forming a dipole ($r >> 2a$) then the field due to q and $-q$ nearly cancel out.

Dipole Moment ($\vec{P}$) -

Dipole moment is a measure of the strength of electric dipole. It is a vector quantity whose direction is from negative charge to positive charge.

Dipole moment is equal to the product of the magnitude of either charge and the distance between them.

$$\boxed{\vec{P} = q \times 2a \, \hat{p}}$$

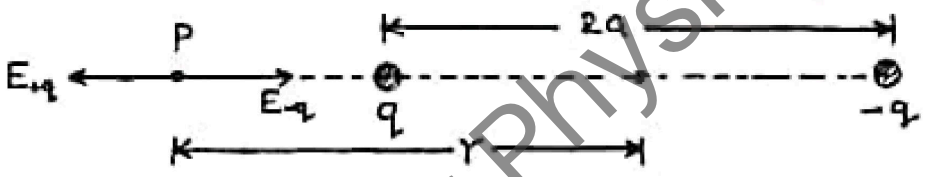
Dipole Field -

It is the space around the dipole in which the electric effect of the dipole can be experienced.

The electric field at any point P is obtained by adding the electric fields E_{-q} due to charge $-q$ and E_{+q} due to the charge q by the parallelogram law of vectors.

(i) Field Intensity on Axial Line of Electric Dipole -

Let the point P be at distance r from the centre of the dipole on the side of the charge q, then



Here $E_{-q} = \dfrac{kq}{(r+a)^2}$ and $E_{+q} = \dfrac{kq}{(r-a)^2}$

So the total Electric field at point P is -

$$E = E_{+q} - E_{-q} \qquad (E_{+q} > E_{-q})$$

$$E = kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = kq \cdot \frac{4ar}{(r^2-a^2)^2}$$

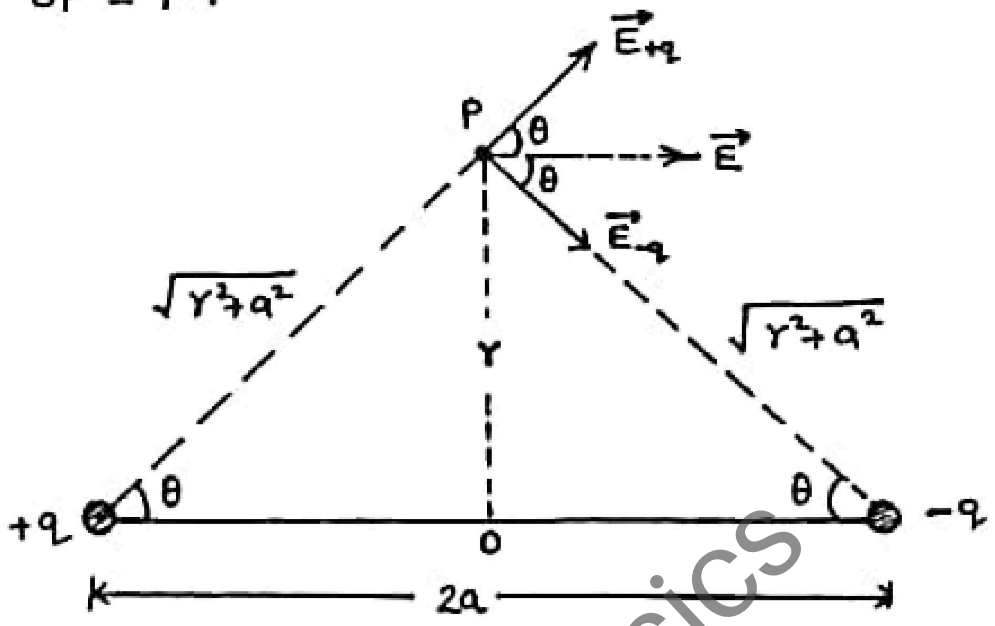
For $(r >> a)$ $\quad E = \dfrac{4kqa}{r^3}$ $\qquad \boxed{\vec{E} = \dfrac{2k\vec{P}}{r^3}}$

The direction of $\vec{E}$ is along $\vec{P}$ i.e from $-q$ to $+q$.

$$* \quad \boxed{E \propto \frac{1}{r^3}}$$

(ii) Field Intensity on Equatorial Line of Electric Dipole -

Let the point P is on the equatorial line, where $OP = r$.



So the magnitudes of the electric fields due to the two charges q and -q are given as -

$$E_{+q} = \frac{Kq}{(r^2+a^2)} \quad \text{and} \quad E_{-q} = \frac{Kq}{(r^2+a^2)}$$

Here the components of E_{+q} and E_{-q} normal to the dipole axis cancel each other and the components along the dipole axis add up.

So the total electric field is in opposite direction to $\hat{P}$.

$$E = E_{+q}\cos\theta + E_{-q}\cos\theta$$

$$\text{or} \quad E = 2E_{+q}\cos\theta \quad \Rightarrow \quad E = \frac{K \cdot 2qa}{(r^2+a^2)^{3/2}}$$

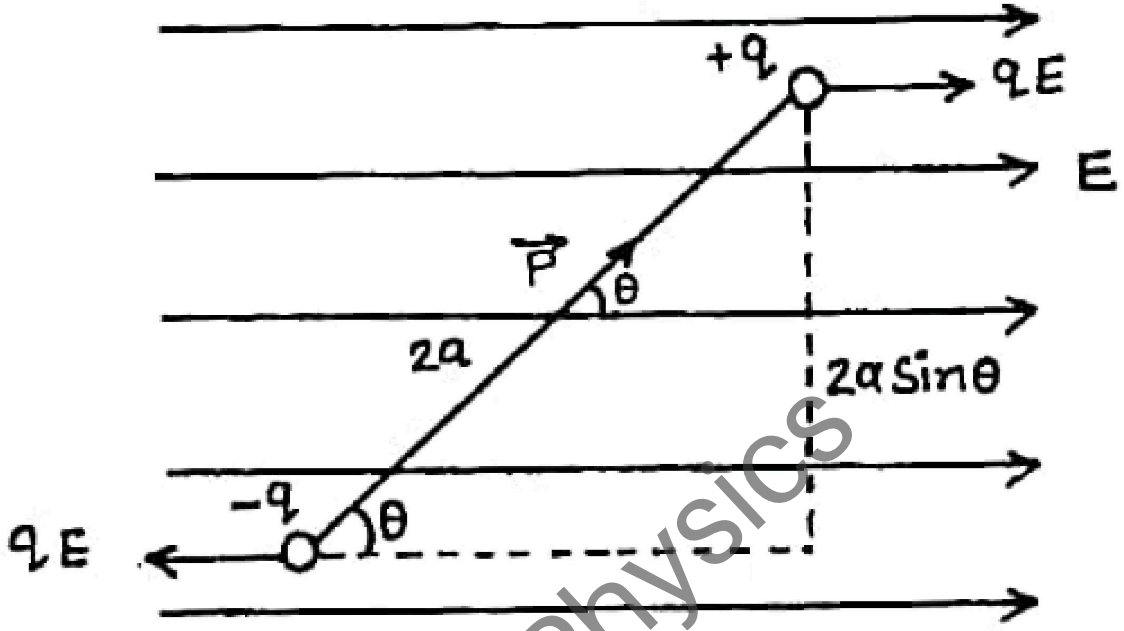
For $(r >> a)$ $\quad E = \dfrac{K \cdot 2qa}{r^3}$ $\qquad \boxed{\vec{E} = \dfrac{-K\vec{P}}{r^3}}$

* Hence the dipole field at large distances falls of not as $1/r^2$ but as $1/r^3$.

* Due to an electric dipole $\boxed{E_{axial} = 2\,E_{equatorial}}$

Dipole in a uniform Electric Field -

Let a dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.



Net force on the dipole = zero

The forces on charge +q and -q resulting in a torque on the dipole which rotates the dipole.

Thus the torque tend to align the dipole axis along the direction of field $\vec{E}$.

The magnitude of torque is equals the magnitude of each force (qE) multiplied by the perpendicular distance between the two antiparallel forces.

Torque $\tau = qE \times 2a\sin\theta$

$$\tau = PE\sin\theta$$

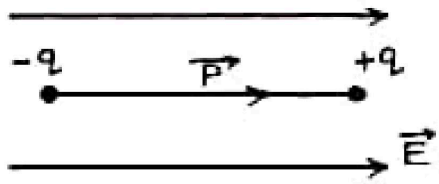
or $$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

The direction of $\vec{\tau}$ is given by right handed screw rule and is perpendicular to $\vec{P}$ and $\vec{E}$.

* When $\vec{P}$ is along $\vec{E}$, $\theta = 0^\circ$

$\tau = PE\sin 0^\circ = 0$

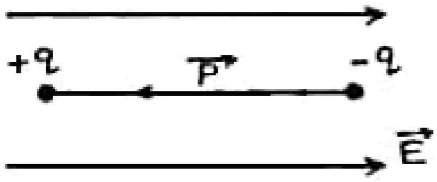
The dipole is in stable equilibrium.



* When $\vec{P}$ is along $\vec{E}$, $\theta = 180^\circ$

$\tau = PE\sin 180^\circ = 0$

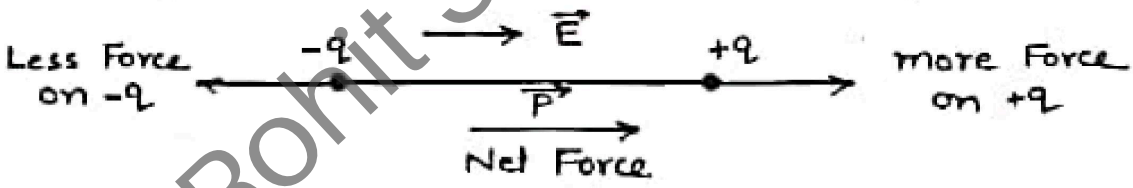
The dipole is in unstable equilibrium.



Dipole in a non-uniform Electric Field -

In either case, when $\vec{P}$ is parallel to $\vec{E}$ or antiparallel to $\vec{E}$, the net torque is zero, but there is a net force on the dipole if $\vec{E}$ is not uniform.

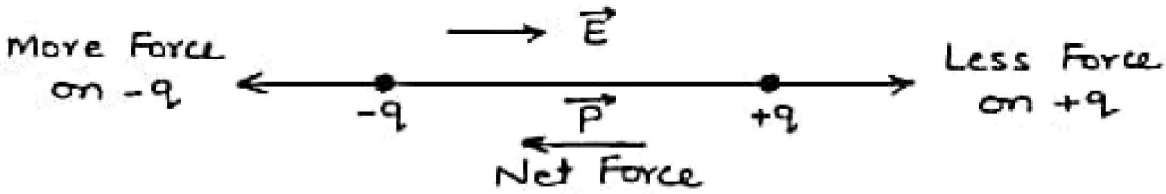
(a) When Field is increasing along $\vec{P}$



Net Force on dipole is along $\vec{E}$.

Net Torque $= PE\sin\theta = 0$ (as $\theta = 0^\circ$)

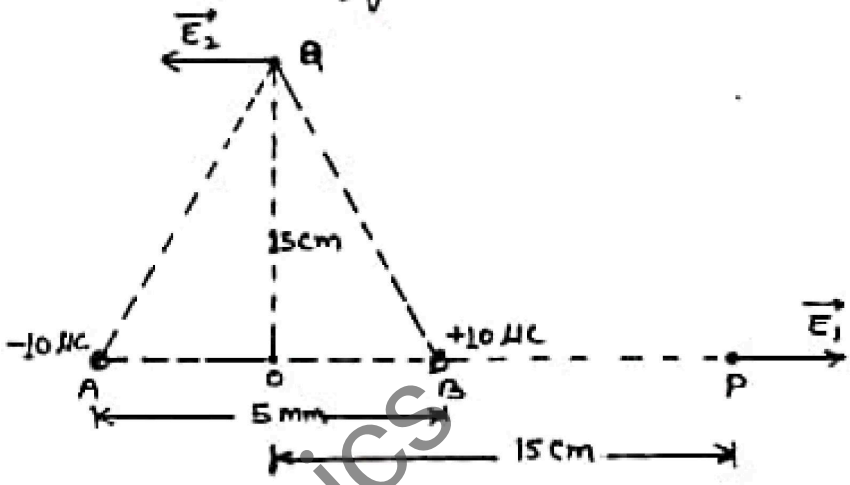
(b) When Field is decreasing along $\vec{P}$



Net Force on dipole is apposite to $\vec{E}$.

Net Torque $= PE\sin\theta = 0$ as $\theta = 0^\circ$

Q. Two charges $\pm 10\ \mu C$ are placed 5 mm apart. Determine the electric field at (a) a point P on the axis of dipole 15 cm away from its centre on the side of the positive charge. (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole as shown in figure.



Sol:

Here $|\vec{P}| = q \times 2a$

$|\vec{P}| = 10 \times 10^{-6} \times 5 \times 10^{-3}$

$= 5 \times 10^{-8}$ C.m.

Also $2a << r$.

(a) As point P lies on axial line of dipole so -

$$E_1 = \frac{2KP}{r^3} = \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

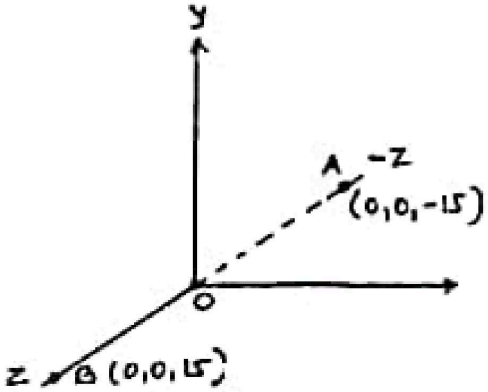
so $E_1 = 2.67 \times 10^5$ N/C (along the direction of $\vec{P}$)

(b) As point Q lies on equatorial line of dipole so -

$$E_2 = \frac{KP}{r^3} = \frac{9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

$E_2 = 1.33 \times 10^5$ N/C (opposite to the direction of $\vec{P}$)

Q. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0,0,-15 cm) and B: (0,0,15 cm) respectively. What are the total charge and electric dipole moment of the system?



Sol:

Total charge of the system

$q = q_A + q_B$

$q = 2.5 \times 10^{-7} C - 2.5 \times 10^{-7} C$

$q = 0$

Electric dipole moment of the system is given by-

$$|\vec{P}| = q \times 2a = 2.5 \times 10^{-7} \times 0.30$$

so $|\vec{P}| = 7.5 \times 10^{-8}$ C.m (along Positive z-axis).

Q. In a certain region of space, electric field is along the z-direction. The magnitude of electric field is not Constant and increases uniformly along positive z-axis, at the rate of 10^5 N/c Per metre. Calculate the Force and torque experienced by a system having a total dipole moment 10^{-7} C.m. in the negative z direction?

Sol. Dipole moment of the system $|\vec{P}| = q \times dL$

$$|\vec{P}| = q \times dL = -10^{-7} \text{ C.m.}$$

Rate of increase of electric field per unit length,

$$\frac{dE}{dL} = 10^5 \text{ N/c per meter}$$

Force experienced by the system is given by

$$F = qE$$

or $$F = q \times dL \times \frac{dE}{dL}$$

$$F = -10^{-7} \times 10^{-5} = -10^{-2} \text{ N (along -z axis)}$$

Here the angle between electric field and dipole moment is 180°.

So Torque $\tau = PE\sin\theta$

$$\tau = PE\sin 180^\circ = 0$$

Charge Distribution -

There are two types of distribution of charge on a body-

(1) Discontinuous Charge Distribution -

In this distribution the charges are distributed randomly (non uniformly) in a body at different places.

(2) Continuous charge distribution -

In this distribution the charges are equally distributed in a body or object.

There are three types of continuous charge distribution -

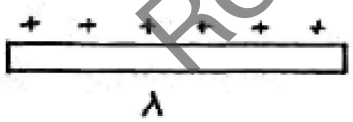
(a) Linear Charge Distribution -

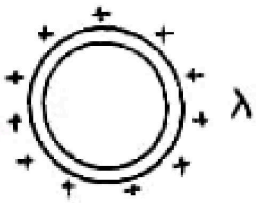
It is represented by linear charge density (λ).

$$\boxed{\text{Linear charge density } (\lambda) = \frac{\text{Charge}}{\text{Length}} = \frac{\Delta Q}{\Delta l}}$$

* SI Unit → C/m.

Examples - Charged straight wire, Charged circular ring etc.





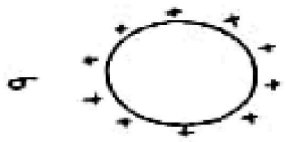
(b) Surface charge Distribution

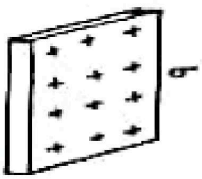
It is represented by surface charge density σ.

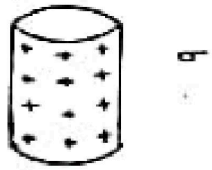
$$\boxed{\text{Surface charge density } (\sigma) = \frac{\text{Charge}}{\text{Area}} = \frac{\Delta Q}{\Delta S}}$$

SI Unit → C/m^2.

Example - Charged Plane sheet, Charge Conducting sphere, cylinder etc.





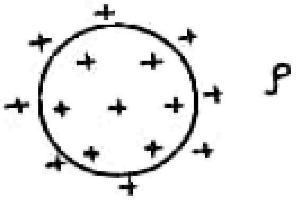


(c) Volume Charge Distribution

$$\boxed{\text{Volume Charge density } (\rho) = \frac{\text{Charge}}{\text{Volume}} = \frac{\Delta Q}{\Delta V}}$$

SI Unit → C/m^3.

Examples – charged insulating sphere etc.

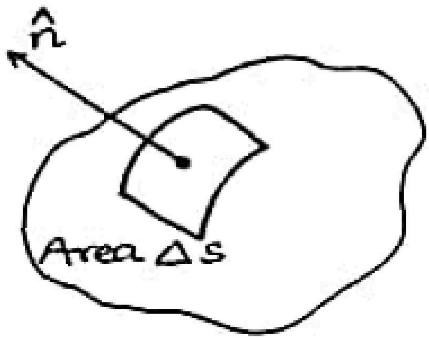


Area Vector -

By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

$$\boxed{\Delta \vec{S} = (\Delta S)\hat{n}}$$

Here ΔS is magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.



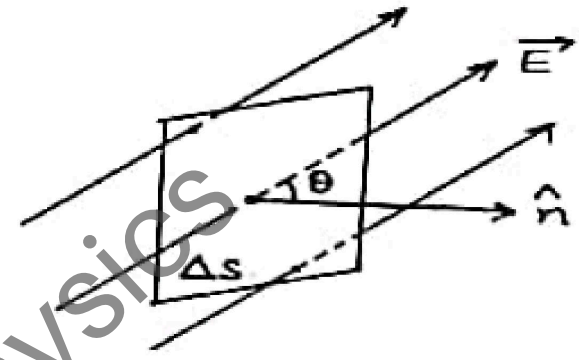
Electric Flux (ϕ) -

Electric flux over an area in an electric field represents the total number of electric field lines crossing this area normally.

The electric flux $\Delta\phi$ through an area element $\Delta\vec{s}$ in an electric field $\vec{E}$ is defined as -

$$\boxed{\Delta\phi = \vec{E}\cdot\Delta\vec{s} = E(\Delta s)\cos\theta}$$

where θ is the angle between $\vec{E}$ and $\Delta\vec{s}$



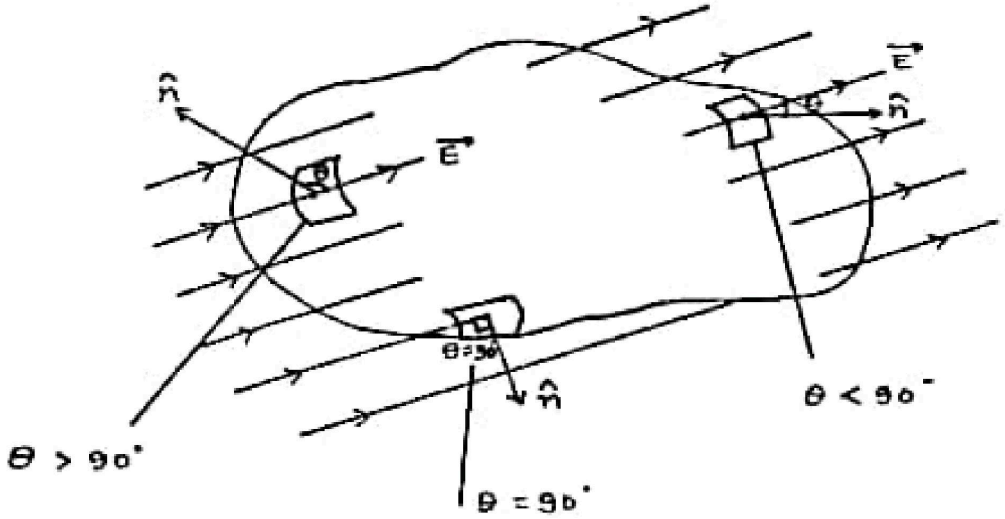
For a non uniform surface total electric flux is -

$$\boxed{\phi = \oint \vec{E}\cdot\vec{ds} = \oint E\,ds\cos\theta}$$

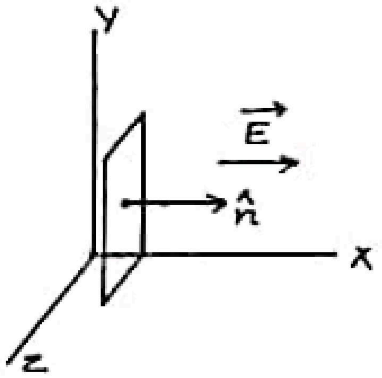
The circle on the integral sign indicates that the surface of integration is a closed surface.

* Electric flux is a scalar quantity.

* For a closed surface the flux comes outside from the surface is taken positive and the flux going inside to the surface is taken negative.



Q. Consider a uniform electric field $\vec{E} = 3\times10^3\hat{i}$ N/C (a) What is the flux of this field through a square of 10 cm. on a side whose plane is parallel to the YZ plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?



Sol: Here Electric Field $\vec{E} = 3\times10^3\hat{i}$ N/C

Side of the square = 0.1 m.

Area of the square $(A) = (0.1)^2$

So $A = 0.01\ m^2$

(a)

When the plane of the square is parallel to the Y-Z plane then the angle between the area vector and electric field is $\theta = 0°$.

So Electric Flux $\phi = EA\cos\theta$

$\phi = 3\times10^3 \times 0.01 \times \cos 0° = 30\ Nm^2/C$

(b) When plane makes an angle of 60° with the x-axis then $\theta = 60°$.

So Electric flux $\phi = EA\cos\theta$

$= 3\times10^3 \times 0.01 \times \cos 60°$

$\phi = 15\ Nm^2/C$

Q. The electric field in a certain region of space is $(5\hat{i} + 4\hat{j} - 4\hat{k})\times10^5$ N/C. Calculate electric flux due to this field over an area of $(2\hat{i} - \hat{j})\times10^{-2}\ m^2$.

Sol. We Know that Electric Flux $\phi = \vec{E}\cdot\vec{ds}$

$$\phi = (5\hat{i} + 4\hat{j} - 4\hat{k})\times10^5 \cdot (2\hat{i} - \hat{j})\times10^{-2}$$

$$\phi = (10 - 4)\times10^3$$

$$\phi = 6\times10^3\ Nm^2/C$$

$$\left[\begin{array}{l} \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0 \end{array}\right.$$

Gauss's Law -

The total electric flux over the closed surface in vacuum is $1/\epsilon_0$ times the total charge Q contained inside surface.

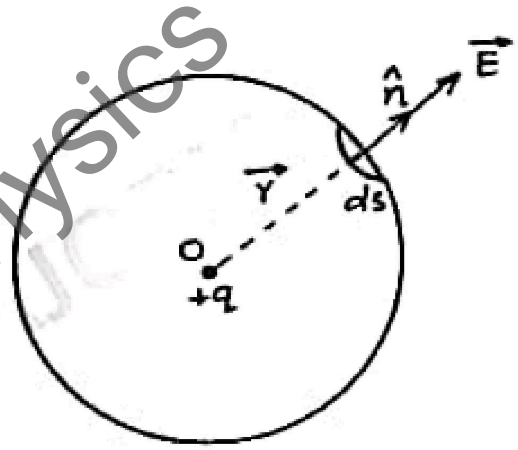
$$\boxed{\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}}$$

Proof of Gauss's Law -

Suppose an isolated positive point charge q is situated at the centre o of a sphere of radius r.

The electric field intensity at any point P on the surface of the sphere is

$$\boxed{\vec{E} = \frac{Kq}{r^2}\hat{r}}$$



where $\hat{r}$ is unit vector directed from o to P.

So the flux over the area element is -

$$d\phi = \vec{E} \cdot \vec{ds} = \frac{Kq}{r^2} \times ds \cos 0^\circ$$

or $$d\phi = \frac{Kq}{r^2} ds$$

So Total electric flux over the entire surface of sphere

$$\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{Kq}{r^2} \oint_s ds$$

$$\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0} \qquad \left(\oint_s ds = 4\pi r^2 \right)$$

Hence $$\boxed{\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}}$$

It proves the Gauss's theorem.

Q. The electric field components in figure are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$ in which $\alpha = 800\ N/C\,m^{1/2}$. Consider the cube as shown in fig and calculate -

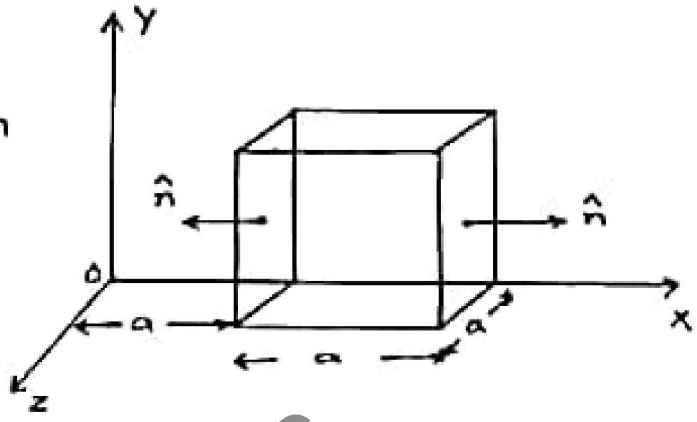
(a) Total flux through the cube

(b) Charge within the cube. Assume that $a = 0.1\,m$.

Sol. Here $E_x = \alpha x^{1/2}$

$E_y = E_z = 0$,

$\alpha = 800\ N/C\text{-}m^{1/2}$, $a = 0.1\,m$



(a) The electric field has only x component, so

$\phi = \vec{E}\cdot\vec{ds} = 0$ for each of four faces of cube perpendicular to Y-Z plane.

At the left face $x = a$ so $E_L = \alpha a^{1/2}$

so $\phi_L = \vec{E_L}\cdot\vec{\Delta S} = \alpha a^{1/2} \times a^2 \cos 180^\circ = -\alpha a^{5/2}$

At the Right Face $x = a + a = 2a$ so $E_R = \alpha (2a)^{1/2}$

so $\phi_R = \vec{E_R}\cdot\vec{\Delta S} = \alpha (2a)^{1/2} \times a^2 \cos 0^\circ$

$\phi_R = \alpha a^{5/2}\sqrt{2}$

So Net flux through the cube $\phi = \phi_R + \phi_L$

$\phi = \alpha a^{5/2}\sqrt{2} - \alpha a^{5/2} = \alpha a^{5/2}(\sqrt{2} - 1)$

$\phi = 800\,(0.1)^{5/2}(\sqrt{2} - 1) = 1.05\ Nm^2/C.$

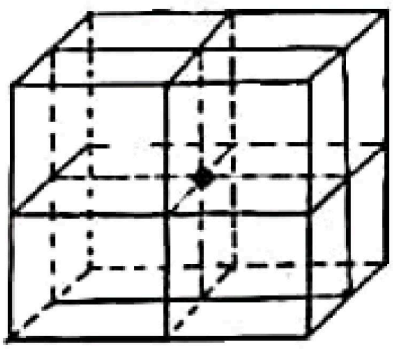
(b) By Gauss's theorem

$q = \epsilon_0 \phi$

so $q = 8.85 \times 10^{-12} \times 1.05 = 9.27 \times 10^{-12}\ C.$

Q. A charge of $2 \times 10^{-9}\,C$ is placed on a corner of a cube of side $1\,m$. Find the electric flux passing through this cube.

Sol.

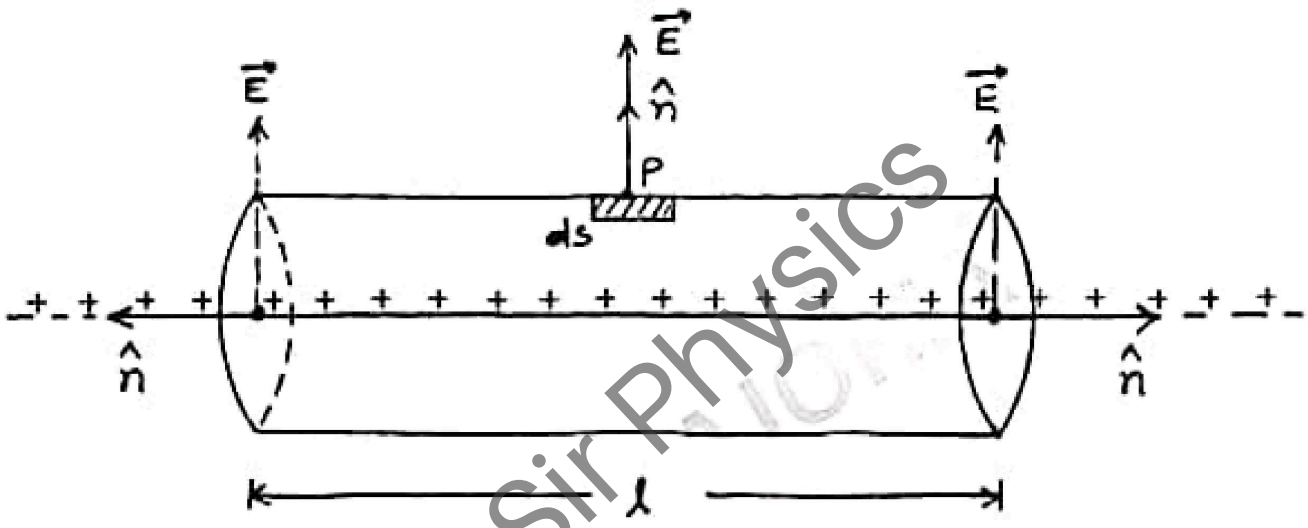


Application of Gauss's Theorem

(1) Field due to an infinitely Long straight Uniformly charged wire -

Consider an infinitely long thin wire with uniform linear charge density λ.

The electric field at every point in the plane cutting the wire normally is radial and its magnitude depends only on the radial distance r.



Now consider a circular closed cylinder of radius r and length l, with the infinitely long line of charge as its axis.

So the total electric flux over the surface -

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 90^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = E\int_s ds \quad \text{or} \quad \phi = E \times 2\pi r l$$

Charge inside the cylinder $(q) = \lambda l$

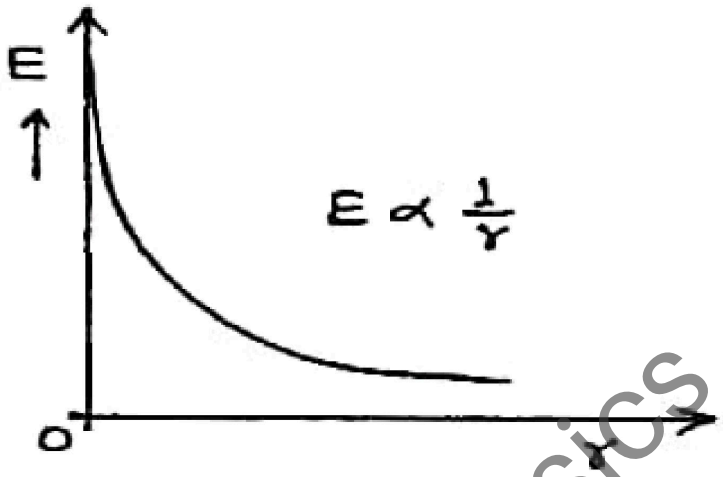
Now according to Gauss's theorem $\phi = \frac{q}{\epsilon_0}$

so $$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

or $$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

or $$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{n}}$$ Clearly $E \propto \frac{1}{r}$

* If $\lambda > 0$, the direction of electric field at every point is radially outwards.
* If $\lambda < 0$, the direction of electric field at every point is radially inwards.



Q. A plastic rod of length 2.2 m and radius 3.6 mm carries a negative charge of 3.8×10^{-7} C which is spread uniformly over its surface. What is the electric field near the mid point of the rod, on its surface?

Sol: Given that – $l = 2.2$ m.

$r = 3.6$ mm $= 3.6 \times 10^{-3}$ m. , $q = -3.8 \times 10^{-7}$ C.

Linear charge density $\lambda = \frac{q}{l}$

$$\lambda = \frac{-3.8 \times 10^{-7}}{2.2} = -1.73 \times 10^{-7} \text{ C/m}$$

As $$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r}$$

So $$E = \frac{2 \times 9 \times 10^{9} \times (-1.73 \times 10^{-7})}{3.6 \times 10^{-3}}$$

or $$E = -8.6 \times 10^{5} \text{ N/C}$$

(2) Electric Field Intensity due to a Uniformly Charged Spherical shell -

(a) Electric Field Outside the shell -

Consider a thin spherical shell of radius R and centre o with charge +q distributed over the surface of the shell uniformly

To calculate electric field intensity at any point P, imagine a sphere (Gaussian surface) with centre o and radius r.

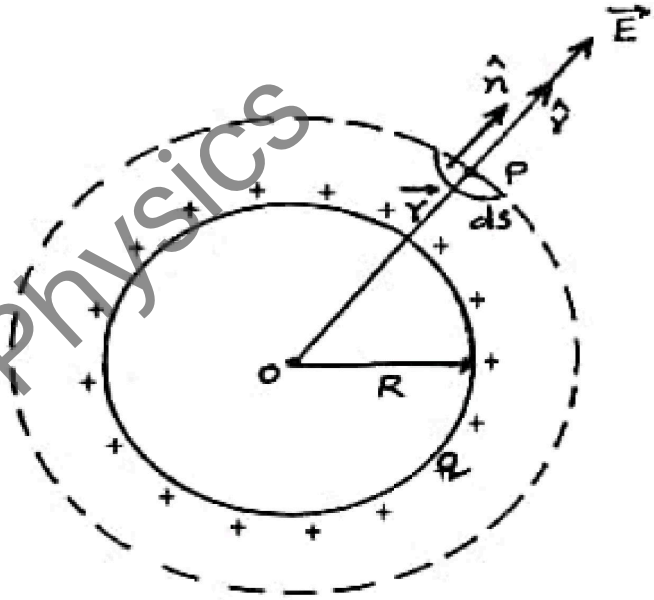
According to Gauss's Law

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

or $$\phi = \oint_s E\,ds\cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E\oint_s ds = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$



* If $q > 0$ then $\vec{E}$ is radially outwards and
* If $q < 0$ then $\vec{E}$ is radially inwards.

(b) At a point on the surface of the shell -

At the surface of the spharical shell

$r = R$ so $$\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$$

If σ is surface charge density on the shell then -

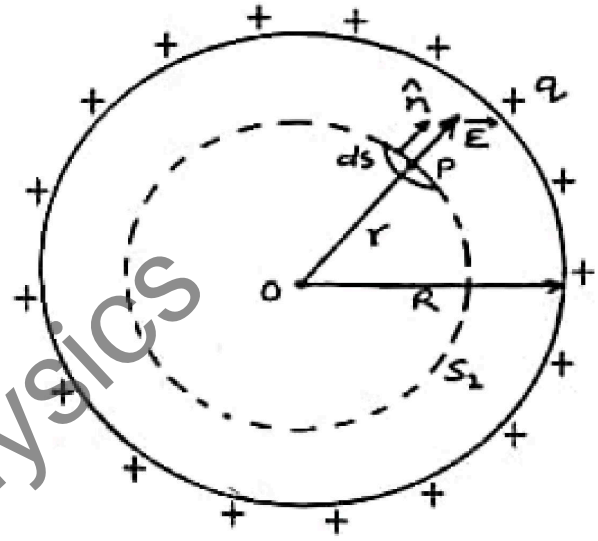
$$\boxed{E = \frac{\sigma}{\epsilon_0} = \text{Constant}} \qquad \left(\sigma = \frac{q}{4\pi R^2}\right)$$

(c) Electric Field inside the shell -

To find field at point P inside the shell consider a Gaussian surface of a sphere S_2 passing through P and with centre at O. Here $r < R$.

As the charge inside the spherical shell is zero the Gaussian surface encloses no charge.

By Gauss's theorem-

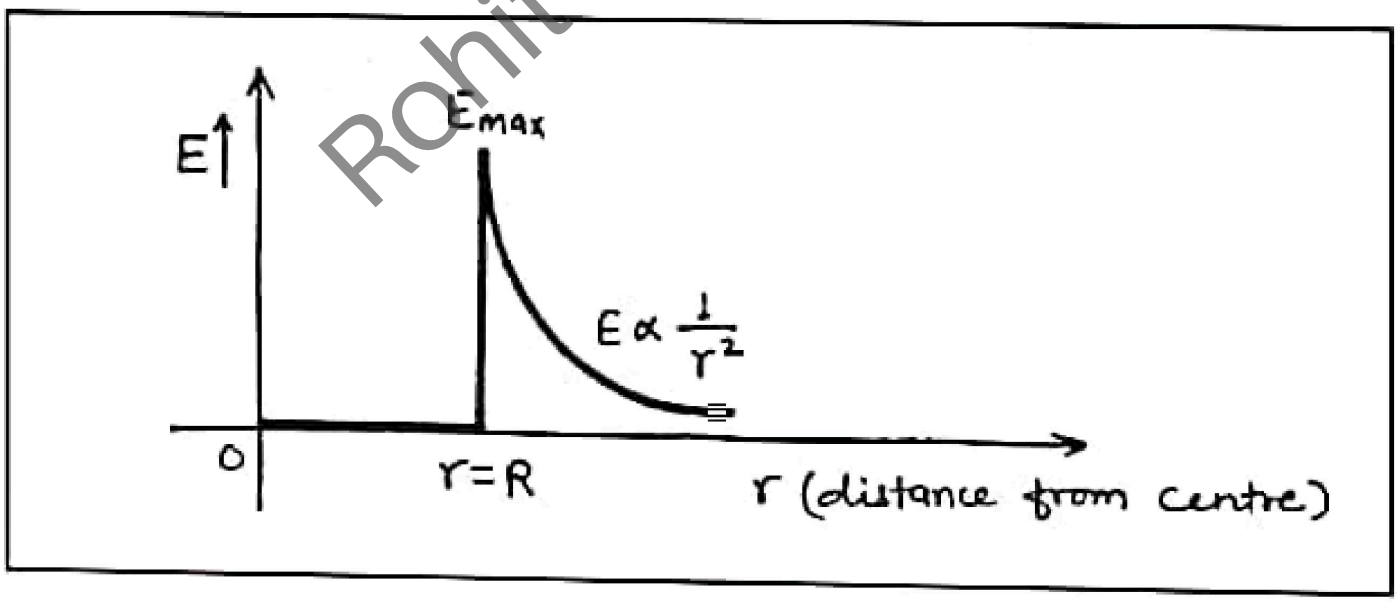


$$\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\phi = E \times 4\pi r^2 = 0$$

so $\boxed{E = 0}$ for $r<R$

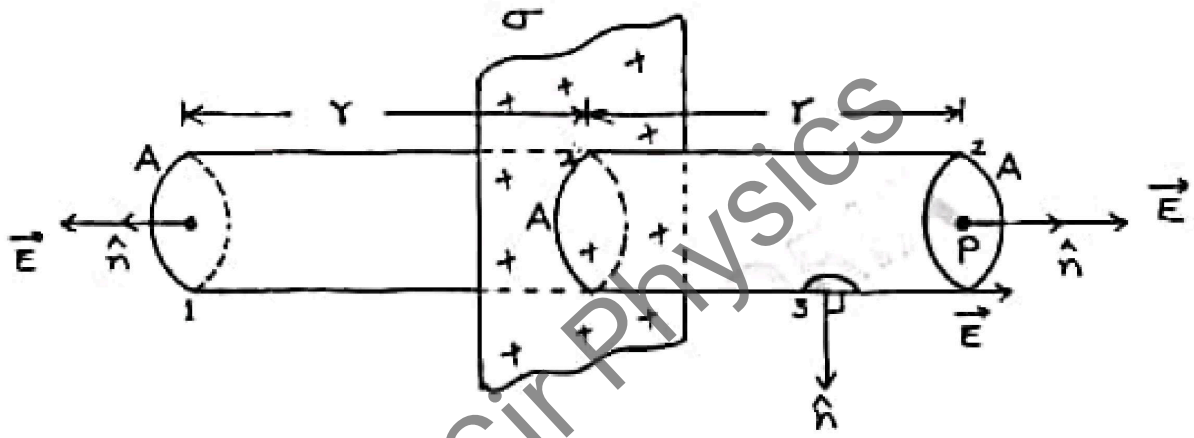
Hence the electric field inside a uniformly charged spherical shell is zero



(3) Electric Field intensity due to a thin infinite plane sheet of charge -

Consider a thin infinite plane sheet of charge with surface charge density σ.
We have to find the electric field at point P at a perpendicular distance r from the sheet.

Now imagine a cylinderical Gaussian surface of cross-sectional area A around P and length 2r, passing through the sheet.



Total flux over the entire surface of cylinder is -

$$\phi = \int_1 \vec{E}\cdot\vec{ds} + \int_2 \vec{E}\cdot\vec{ds} + \int_3 \vec{E}\cdot\vec{ds}$$

$$\phi = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 0^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = 2EA$$

Total Charge enclosed by the cylinder $= \sigma A$

According to Gauss's Law - $\phi = \frac{q}{\epsilon_0}$

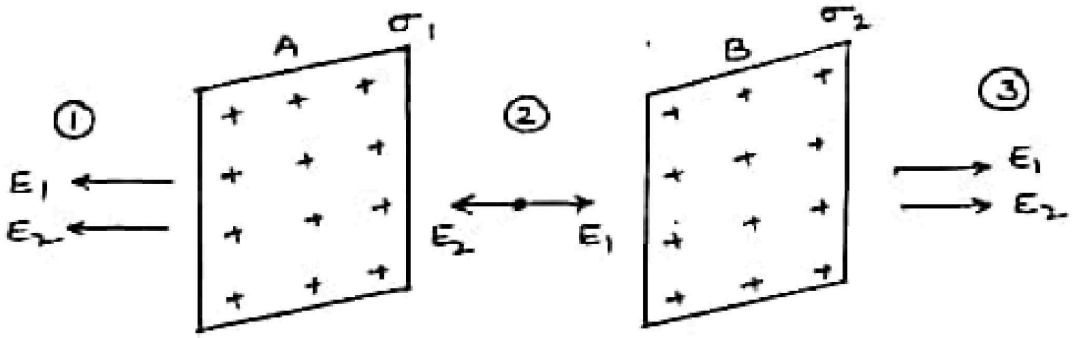
$$2EA = \frac{\sigma A}{\epsilon_0} \qquad E = \frac{\sigma}{2\epsilon_0}$$

so $\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$ ★ E is independent of r.

★ If $\sigma > 0$ then electric field is normally outward and
★ If $\sigma < 0$ then electric field is normally inward to the sheet.

(4) Electric Field intensity due to two thin infinite parallel sheets of charge -

Case 1: - Here $\sigma_1 > \sigma_2 > 0$



In region ① & ③

$$E = E_1 + E_2$$

$$E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0}$$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)}$$

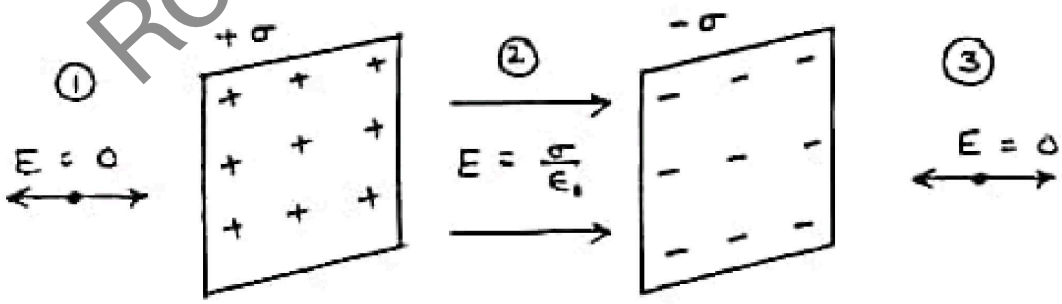
In region ②

$$E = E_1 - E_2$$

$$E = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0}$$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)}$$

Case 2: - Here $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$



In region ① & ③

$$E = E_1 - E_2$$

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E = 0}$$

In region ②

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Q. A large plane sheet of charge having surface charge density 5×10^{-6} C/m² lie in xy plane. Find the electric flux through a circular area of radius 0.1 m if the normal to the circular area makes an angle of 60° with the z-axis.

Sol. Here $\sigma = 5 \times 10^{-6}$ C/m², $r = 0.1$ m, $\theta = 60°$

So $\phi = \vec{E} \cdot \vec{ds} = E ds \cos\theta$

$$\phi = \left(\frac{\sigma}{2\epsilon_0}\right) \times \pi r^2 \cos\theta$$

$$\phi = \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times 3.14 \times (0.1)^2 \cos 60°$$

$$\phi = 4.44 \times 10^3 \text{ Nm}^2/\text{C}$$ — Ans.

Very Important Questions

1 Mark Questions -

Q.1. Which is bigger, a coulomb or charge on an electron? How many electronic charges form one coulomb of charge?
(Pb. Board 2011)

Q.2. What is the cause of charging?
(Jharkhand Board 2011)

Q.3. What is ment by Quantization of charge?
(CBSE 2005) (Bihar Board 2011)

Q.4. What is the value of charge on an electron? Is a charge less than this value possible?
(Manipur CHSE 2011)

Q.5. Why no two electric lines of force can intersect each other?
(Hr. Board 2007)

Q.6. Force of attraction between two point charges placed at a distance d is F. What distance apart should they be kept in the same medium so that force between them is F/3?
(CBSE 2011)

Q.7. In Coulomb's Law, on what factors the value of electrostatic force constant K depends?
(Pb. Board 2011)

Q.8. Define electric dipole moment. Write its SI Unit.
(Jharkhand Board 2011, Raj Board 2011, CBSE 2011)

Q.9. In which orientation, a dipole placed in a uniform field is in (a) stable (b) Unstable equilibrium?
(CBSE 2011)

Q.10. Name the physical quantity whose SI Unit is N/C.
(Pb. Board 2011)

2 Marks Questions -

Q.1. Explain what is ment by dielectric Constant? (CBSE 2005)

Q.2. Define electric flux. Write its SI Unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change? (Raj Board 2008, CBSE 2009, 12)

Q.3. A copper sphere of mass 2g. Contains nearly 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a Charge of $+2\mu C$?
Ans = 2.16×10^{-11} (CBSE 2009)

Q.4. An area $5 \times 10^{-3} \hat{j}\ m^2$ is placed in an uniform electric field of $(200\hat{i} + 300\hat{j})$ N/C. Calculate the electric flux passing through this area. (Raj Board 2002)
Ans = 1.5 Nm^2/C

Q.5. The magnitude of entered and exerted electric flux from a close surface are 4×10^{12} Nm^2/C and 8×10^{12} Nm^2/C respectively. Calculate the charge enclosed by this surface.
Ans = 35.6 C (Raj Board 2003)

Q.6. An electric dipole of dipole moment 20×10^{-6} cm is placed inside a closed surface. Calculate the net electric flux passing through this surface.
Ans. = Zero (CBSE 2005, 06, 10)

Q.7. What do you mean by electric field lines? Give three main properties of electric field lines. (Raj. Board 2006, CBSE 2005)

Q.8. Plot a graph showing the variation of Coulomb force (F) versus $(1/r^2)$, where r is the distance between two Charges of each pair of charges ($1\mu C$, $2\mu C$) and ($2\mu C$, $-3\mu C$). (CBSE 2011)

5 Marks Questions -

Q. 1. Using Gauss's Law, derive an expression for the electric field intensity at any point near a uniformly charged thin wire of charge/length = λ C/m.

(CBSE 2008, 5, 7, 9)

Q. 2. Derive an expression for dipole field intensity at any point on (a) axial line of dipole and (b) on equatorial line of dipole.

(CBSE 2004, 5, 11, 12)

Q. 3. Derive an expression for electric field due to a uniformly charged infinite plane sheet using Gauss's Law.

(CBSE 2009, 12)

Q. 4. State Gauss's Law in electrostatics and derive an expression for electric field intensity at a point due to -

(a) a uniformly charged thin spherical shell (CBSE 3, 5, 7, 9, 11)

(b) two parallel sheets of charge with charge densities $+\sigma$ and $-\sigma$ (CBSE 2009, 6)

Q. 5. Derive an expression for torque acting on an electric dipole in a uniform electric field.

(Raj Board 2008, 11, CBSE 2005)