## **Excerpts From** CWT (Advanced): Probability & Probability Distributions

| 1. The probability of getting head in a toss of a biased coin is $\frac{2}{3}$ . Let the coin be tossed three |
|---|
| times independently. Then the probability of getting head in the first two tosses and tail in                 |
| the final toss is   |

A.  $\frac{4}{27}$  B.  $\frac{1}{8}$  C.  $\frac{2}{27}$  D.  $\frac{23}{27}$ 

2. Let M and N be events defined on the sample space S. If  $P(M) = \frac{1}{3} \& P(N^c) = \frac{1}{4}$  then which one of the following is necessarily CORRECT?

(A) M and N are disjoint

(B) M and N are not disjoint

(C) M and N are independent

(D) M and N are not independent

3. From a set comprising of 10 students, four girls  $G_i = 1, 2, 3, 4$  and six boys  $B_i = 1, 2, 3, 4, 5, 6$ , a team of FIVE students are to be formed. The probability that a randomly selected team comprises of 2 girls and 3 boys, with at least one of them to be  $\boldsymbol{B}_1$  or  $\boldsymbol{B}_2$  , is equal to

A.  $\frac{3}{7}$  B.  $\frac{6}{7}$  C.  $\frac{8}{21}$  D.  $\frac{5}{21}$ 

4. If X denotes the sum of the numbers appearing on a throw of two fair six-faced dice then the probability  $P(7 < X < 10) = \underline{\qquad}$  (round off to 2 decimal places).

5. The number of ways in which the word PANDEMIC can be arranged such that the vowels appear together is

(A)  $6 \times (3!)(5!)$  (B)  $5 \times (3!)(5!)$  (C)  $4 \times (3!)(5!)$  (D)  $1 \times (3!)(5!)$ 

6. A family has two children. What is the probability that both are boys given that at least one is a boy?

A.  $\frac{1}{2}$  B.  $\frac{2}{3}$  C.  $\frac{1}{3}$  D.  $\frac{1}{4}$ 

7. Consider two boxes, one containing one black ball and one white ball, the other containing two white balls and one black ball. A box is selected at random, and a ball is selected at random from the

A.  $\frac{5}{12}$  B.  $\frac{2}{5}$  C.  $\frac{1}{6}$  D.  $\frac{5}{11}$ 

selected box. What is the probability that the ball is black?

8. A number is chosen randomly from the first billion natural numbers. The probability that the product of the number with its two immediate successors is divisible by 24 is closest to

A.  $\frac{1}{2}$  B.  $\frac{3}{4}$  C.  $\frac{5}{8}$  D.  $\frac{2}{3}$ 

| 9. Each of the four entries of a $2 \times 2$ matrix is filled by independently choosing either 1 or $-1$ uniformly at random. What is the probability that the matrix is singular?                           |
|---|
| A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$  |
| 10. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are $f_1$ and $f_2$ respectively. Once R&D is |

10. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are  $f_1$  and  $f_2$  respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn uniformly from [0,1]. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from [0,1]. Total demand of vaccine is 1 unit and if firm  $i \in \{1,2\}$  supplies  $qi \in [0,1]$  units and has a per unit cost of incurs a manufacturing cost of  $c_iq_i$  (along with  $f_i$ ). Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

$$\mathsf{A.}\ f_1 + f_2 + \tfrac{1}{2} \qquad \ \ \mathsf{B.}\ \ f_1 + f_2 + \tfrac{1}{3} \qquad \ \ \mathsf{C.}\ \ f_1 + f_2 + \tfrac{2}{3} \qquad \qquad \mathsf{D.}\ \ f_1 + f_2 + \tfrac{3}{4}$$

11. Suppose that there are 30 MCQ type questions where each question has four options: A, B,C,D. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, & 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5, option B is correct is 0.3, option C is correct is 0.2, & option D is correct is 0. Two students Gupi & Bagha have no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5 but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

- A. Both Gupi & Bagha have expected scores more than 30
- B. Gupi's expected score is greater than or equal to 30 & Bagha's expected score is strictly less than 30
- C. Gupi's expected score is less than or equal to 30 & Bagha's expected score is strictly more than 30
- D. None of the other options are correct
- 12. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D?

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 playing cards, with four suits of 13 cards each, which has been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

A. 
$$\frac{C'(4,1) C(13,5)}{C(52,5)}$$
 B.  $\frac{C'(4,2) C(13,4)}{C(52,5)}$  C.  $\frac{P(4,1) C(13,5)}{P(52,5)}$  D.  $\frac{C'(4,1) C(12,5)}{C(52,5)}$