

9. Each of the four entries of a 2×2 matrix is filled by independently choosing either 1 or -1 uniformly at random. What is the probability that the matrix is singular?

- A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

10. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are f_1 and f_2 respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn uniformly from $[0, 1]$. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from $[0, 1]$. Total demand of vaccine is 1 unit and if firm $i \in \{1, 2\}$ supplies $q_i \in [0, 1]$ units and has a per unit cost of incurs a manufacturing cost of $c_i q_i$ (along with f_i). Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

- A. $f_1 + f_2 + \frac{1}{2}$ B. $f_1 + f_2 + \frac{1}{3}$ C. $f_1 + f_2 + \frac{2}{3}$ D. $f_1 + f_2 + \frac{3}{4}$

11. Suppose that there are 30 MCQ type questions where each question has four options: A, B, C, D. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, & 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5, option B is correct is 0.3, option C is correct is 0.2, & option D is correct is 0. Two students Gupi & Bagha have no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5 but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

- A. Both Gupi & Bagha have expected scores more than 30
B. Gupi's expected score is greater than or equal to 30 & Bagha's expected score is strictly less than 30
C. Gupi's expected score is less than or equal to 30 & Bagha's expected score is strictly more than 30
D. None of the other options are correct

12. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D ?

- A. 10 B. 1 C. 0 D. None of them

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 playing cards, with four suits of 13 cards each, which has been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

- A. $\frac{C(4,1)C(13,5)}{C(52,5)}$ B. $\frac{C(4,2)C(13,4)}{C(52,5)}$ C. $\frac{P(4,1)C(13,5)}{P(52,5)}$ D. $\frac{C(4,1)C(12,5)}{C(52,5)}$