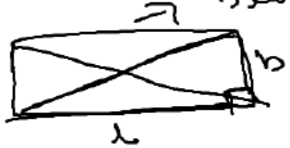
Mensuration → Area & volume

2-D figures

Diag. → $\sqrt{l^2+b^2}$

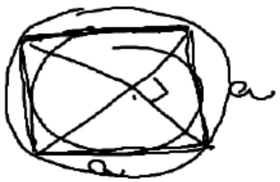
1. Rectangle —



Area: $l \times b$

Perimeter: $2(l+b)$

2. Square —



Area: ~~Area~~ $a \times a$ = a^2

Perimeter: $2(a+a)$ = $\underline{4a}$

3. Triangle (Scalene) —

$= \frac{1}{2} \times b \times h$

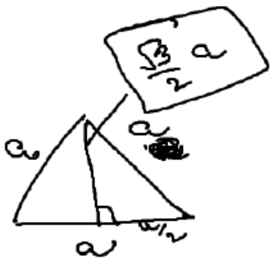
Heron's formula

$s = \frac{a+b+c}{2}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Per.: $a+b+c$

4. Equilateral triangle —



$A = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a$

$$\boxed{A = \frac{\sqrt{3}}{4} a^2}$$

Per.: $3a$

5. Isosceles triangle –

$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$

$= \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{\frac{4a^2 - b^2}{4}} = \frac{\sqrt{4a^2 - b^2}}{2}$

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times b \times \frac{\sqrt{4a^2 - b^2}}{2}$

$= \frac{b}{4} \sqrt{4a^2 - b^2}$

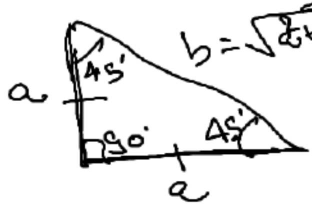
Per. $2a + b$

6. Right angled triangle –

– $A = \frac{1}{2} \times b \times p$

$P = p + b + h$

7. Isosceles Right angled triangle –

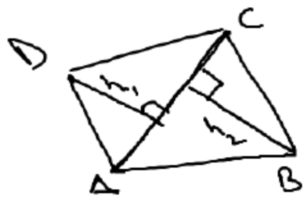


$b = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a$

$A = \frac{1}{2} \times a \times a$

$P = 2a + b$

8. Quadrilateral –



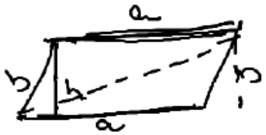
$P = AB + BC + CD + DA$

$A =$ Ar. ΔACD + Ar ΔACB

$= \frac{1}{2} \times AC \times h_1 + \frac{1}{2} \times AC \times h_2$

$= \frac{1}{2} \times AC(h_1 + h_2)$

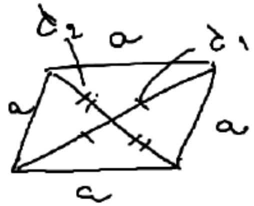
9. Parallelogram



$A = \underset{\text{base}}{a} \times \underset{\text{height}}{h}$

$P = 2(a+b)$

10. Rhombus



$A = \frac{1}{2} \times d_1 \times d_2$

$P = 4a$

(when diagonals are perpendicular)

$\frac{1}{2} \times \sqrt{2}a \times \sqrt{2}a = a^2$

regular polygon $A = \frac{na^2}{4} \cot\left(\frac{180}{n}\right)$

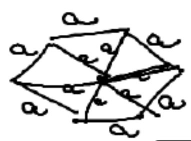
$= \frac{6a^2}{4} \times \sqrt{3} = \frac{3\sqrt{3}}{2}a^2$

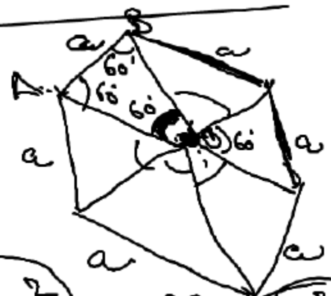
11.

$A = \frac{1}{2}(a+b) \times h$

$P = a+b+c+d$

12. Regular Hexagon



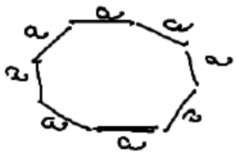


$A = 6 \times \frac{\sqrt{3}}{4}a^2$

$\frac{3\sqrt{3}}{2}a^2$

$P = 6a$

13. Regular octagon -



$$A = \frac{n a^2}{4} \cot\left(\frac{180}{n}\right) \qquad P = 8a$$

$$= \frac{8a^2}{4} \cot\left(\frac{180}{8}\right)$$

$$= 2a^2 \cot(22.5^\circ)$$

$$= 2a^2 (1+\sqrt{2})$$

$$1 + \cos 2\theta = 2\cos^2\theta$$

$$1 - \cos 2\theta = 2\sin^2\theta$$

$$\sin(\underbrace{22.5^\circ}_{\theta}) = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\theta = 22.5^\circ$$

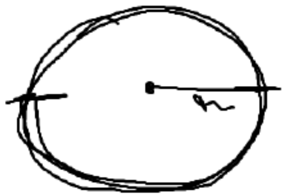
$$= \sqrt{1-\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$= \sqrt{\frac{2\sqrt{2}-\sqrt{2}+1}{2\sqrt{2}}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}}{\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}}$$

$$= \sqrt{\frac{(\sqrt{2}+1)^2}{1}} = \sqrt{2}+1$$

14. Circle — 

$A = \pi r^2$, $C/P = 2\pi r$

15. Semicircle — 

$A = \frac{\pi r^2}{2}$, $C/P = \pi r + 2r$

16. Quadrant — 

$A = \frac{\pi r^2}{4}$, $C/P = \frac{\pi r}{2} + 2r$

17. Ring or Circular Path = 

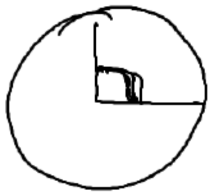
$A = \pi R^2 - \pi r^2$
$= \pi (R^2 - r^2)$
$= \pi (R-r)(R+r)$

$P_o = 2\pi R$
$P_i = 2\pi r$

18. Sector of a circle — 

$\frac{\pi R^2 \times 90}{360} = \frac{\pi R^2}{4}$

$\frac{360}{90} = 4$



$360° \rightarrow \pi R^2$

$1° \rightarrow \frac{\pi R^2}{360}$

$\theta° \rightarrow \boxed{\frac{\pi R^2 \theta}{360}}$

$A = \frac{\pi R^2 \theta}{360}$

$P = \frac{2\pi R\theta}{360} + 2R$

$360° \rightarrow 2\pi R$

$1° \rightarrow \frac{2\pi R}{360}$

$\theta° \rightarrow \frac{2\pi R\theta}{360°}$

14. Segment of a circle – Region b/w chord & Arc

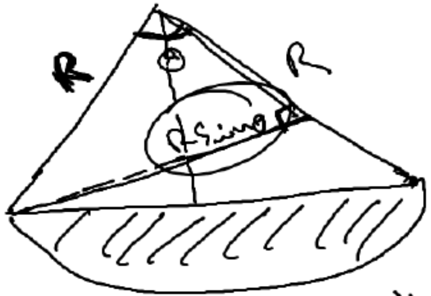
$\Rightarrow$ Section − Triangle

$$\frac{\pi R^2 \theta}{360} - \frac{1}{2} R^2 \sin\theta$$

$R\sin\theta$

$R\cos\theta$

Section → The region b/w Arc & 2 radii of Circle



$\frac{1}{2} \times R \times R\sin\theta$

$\boxed{\frac{1}{2} R^2 \sin\theta}$

$h = b\sin\theta$

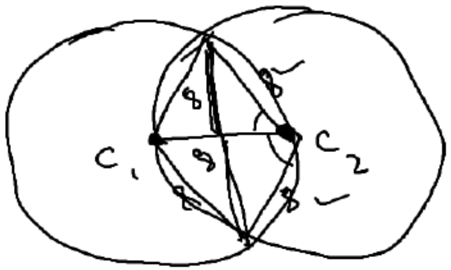
$A = ab\sin\theta$

Equilateral Δ

Inradius $= \frac{\text{Side}}{2\sqrt{3}}$ → a

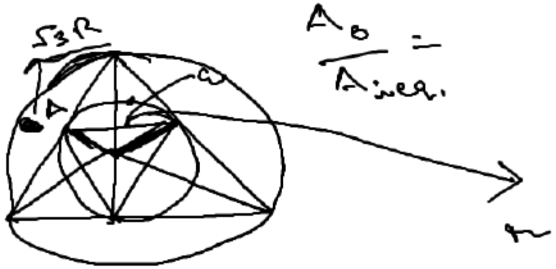
Circumradius $= \frac{\text{Side}}{\sqrt{3}}$

Q.11



$2 \times$ Area of segment

Q.14



$\frac{A_o}{A_{i\,eq.}} =$

$r = \frac{Side}{\sqrt{3}}$

R

$\boxed{R = \frac{A}{\sqrt{3}}} \Rightarrow A = \sqrt{3}R$

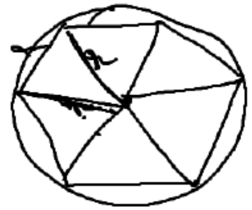
$\frac{R}{2} = \frac{Side}{\sqrt{3}}$

$r = \frac{Side}{2\sqrt{3}} = \frac{\cancel{\sqrt{3}}R}{2\cancel{\sqrt{3}}}$

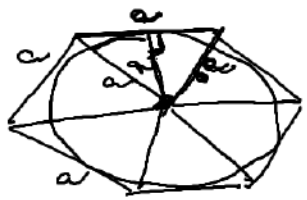
$\boxed{r = \frac{R}{2}}$

$\boxed{\underset{(a)}{Side} = \frac{\sqrt{3}R}{2}}$

Q.15



$A_i = 6 \times \frac{\sqrt{3}}{4} r^2$



$\frac{\sqrt{3}}{2}a = r$

$\boxed{a = \frac{2r}{\sqrt{3}}}$

$A_o = 6 \times \frac{\sqrt{3}}{4} a^2$

$= 6 \times \frac{\sqrt{3}}{4} \times \left(\frac{2r}{\sqrt{3}}\right)^2$

Mensuration (3-D)

$V = \text{Area} \times h$

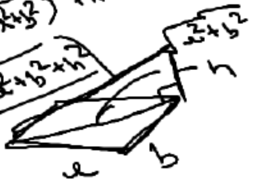
Volume	L.S.A / C.S.A	T.S.A
$l \times b \times h$	$2(l+b)h$	$2(lb+bh+hl)$

1. Cuboid —

$d = \sqrt{l^2+b^2+h^2}$

$2bh + 2lh = 2(l+b)h$

Area of 4 walls



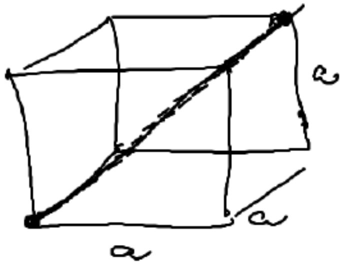
$d = \sqrt{(\sqrt{l^2+b^2})^2+h^2}$

$d = \sqrt{l^2+b^2+h^2}$

$lb + lb$

$2lb$

2. Cube —



$d = \sqrt{a^2+a^2+a^2}$

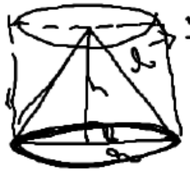
$= \sqrt{3a^2} = \sqrt{3}a$

vol.	C.S.A	T.S.A
$l \times b \times h$	$4a^2$	$6a^2$
$a \times a \times a$		
a^3		

3. Right Circular cylinder —

vol.	C.S.A	T.S.A
$v = \pi r^2 h$	$2\pi r \times h$	$2\pi rh + 2\pi r^2$
$A \times h$		

4. Right Circular Cone



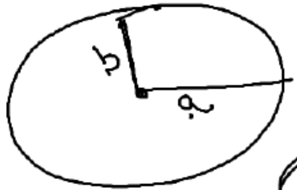
$l = \sqrt{h^2+r^2}$

$V = \frac{1}{3}\pi r^2 h$

C.S.A: $\pi r l$

T.S.A: $\pi r l + \pi r^2$

5.



$\pi \times a \times b$

5. Right triangular Prism

V = Area of base $\times h$

$\pi r^2 \times h$

L.S.A / C.S.A

Peri. of base $\times$ height

$2\pi r \times h$

T.S.A

T = C.S.A + 2(Area of base)

Cuboid cube cylinder

$2\pi r h + 2\times\pi r^2$

T.S.A = C.S.A + Base Area

6. Right Pyramid

vol.

$V = \frac{1}{3} \times$ Area of base $\times$ height

$\boxed{V = \frac{1}{3} \times \pi r^2 \times h}$

L.S.A / C.S.A

$\frac{1}{2} \times$ Perim. of base $\times$ slant height

C.S.A $= \frac{1}{2} \times 2\pi r \times l = \pi r l$

7. Sphere

$V = \frac{4}{3}\pi r^3$

S.A $= 4\pi r^2$

C.S.A = T.S.A

8. Hemisphere

$V = \frac{\frac{4}{3}\pi r^3}{2}$

$\boxed{V = \frac{2}{3}\pi r^3}$

C.S.A $= 2\pi r^2$

T.S.A $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

$\pi R^2 - \pi r^2$

9. Spherical shell

$V = \frac{4}{3}\pi (R^3 - r^3)$

$4\pi(R^2 + r^2)$

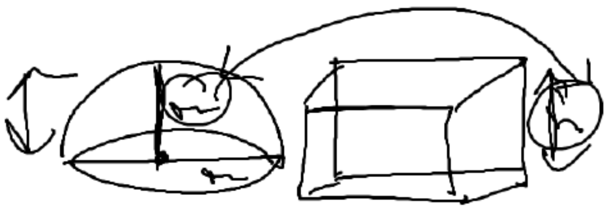
10. Frustum of a cone

$V = \frac{\pi}{3} h(R^2 + r^2 + Rr)$

C.S.A $= \pi(r+R)l$

T.S.A $=$ C.S.A $+ \pi r^2 + \pi R^2$

$4l = 2\pi r$



$r = h$

$\frac{V_c - V_H}{V_c} \times 100$

$2\cancel{4}l = \cancel{2}\pi r$

$r = \frac{2l}{\pi}$

Cub. → $v = l^2 \times h = l^2 \times \frac{2l}{\pi}$

$= \frac{2l^3}{\pi}$

Hem. → $v = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times \frac{8l^3}{\pi^{3\,2}}$

$\frac{V_c}{V_H} = \frac{3\pi}{8} = \frac{\cancel{66}\,33}{7 \times \cancel{8}\,4} = \frac{33}{28}$

$= \frac{\cancel{16}\,8}{3}\frac{l^3}{\pi^2}$

$\cancel{\pi}(5+x)^2 \cancel{5} = \cancel{\pi}(5)^2(5+3x)$

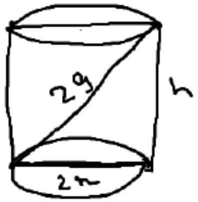
$\cancel{25} + x^2 + 10x = \cancel{25} + 15x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0,\ x = 5$

$\frac{5}{\cancel{33}\,3} \times \cancel{100} = 15\%$



$$\boxed{h^2 + \underbrace{4r^2}_{(2r)^2} = 841}$$

$$\cancel{2}\pi rh = \cancel{2640}\ 1320$$

$$\cancel{\frac{22}{7}} \times rh = \cancel{1320}\ 60$$

$$\boxed{rh = 420}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(2r+h)^2 = 4r^2 + h^2 + 4rh$$

$$= 841 + 1680$$

$$(2r+h)^2 = 2521$$

$$\boxed{2r+h = \sqrt{2521}}$$

50.2

$$(2r-h)^2 = 4r^2 + h^2 - 4rh$$

$$= 841 - 1680$$

$$=$$

$$l = \sqrt{\frac{a}{\sqrt{2}}}$$

$$h = 3 \times \sqrt{2}\, a$$

$$\boxed{h = 3\sqrt{2}\, a}$$

$P =$

$$l = \sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + (3\sqrt{2}a)^2}$$

$$= a\sqrt{\frac{1}{2} + 18}$$

$$= a\sqrt{\frac{37}{2}}$$

$$h_s = \sqrt{(3\sqrt{2}a)^2 + \frac{a^2}{4}}$$

$$= a\sqrt{18 + \frac{1}{4}}$$

$$= \frac{a\sqrt{73}}{2}$$

$$C.S.A = \frac{1}{2} \times \cancel{4}a \times a\sqrt{\frac{37}{2}}$$

Δ

$$= \frac{2a^2\sqrt{\frac{37}{2}}}{\cancel{4}\ 2}$$

$$= \frac{a^2\sqrt{37}}{2\sqrt{2}}$$